Διάλεξη 7 (συνέχεια)

Θα δείξουμε ότι

$$\int_{\mathbb{R}^n_+} \left| \frac{\partial w_m}{\partial x_n} - \frac{\partial u}{\partial x_n} \right|^p dx \xrightarrow[m \to +\infty]{} 0$$

$$\left| \frac{\partial w_m}{\partial x_n} - \frac{\partial u}{\partial x_n} \right| \leq \left|\frac{\partial u}{\partial x_n}\right| |\zeta_m(x)| + m u \zeta'(x)$$

$\Rightarrow$

$$\left| \frac{\partial w_m}{\partial x_n} - \frac{\partial u}{\partial x_n} \right|^p \leq C \left|\frac{\partial u}{\partial x_n}\right|^p |\zeta_m(x)|^p + C m^p u^p |\zeta'(x)|^p$$

$$\int_{\mathbb{R}^+_n} \left| \frac{\partial u}{\partial x_n} \right|^p |\zeta_m(x)|^p dx \xrightarrow[m \to +\infty]{} 0.$$

$$m^p \int_0^{2/m} \left(\int_{\mathbb{R}^{n-1}} |u|^p dx' \right) dt \overset{(6.56)}{\leq} m^p \left(\int_0^{2/m} t^{p-1} dt \right) \left(\int_0^{2/m} \int_{\mathbb{R}^{n-1}} |\nabla u|^p dx' dt \right)$$

[Προκύπτει από ολοκλήρωση της (6.56) ως προς t

$$\int_0^{2/m} \left(t^{p-1} \int_0^t \int_{\mathbb{R}^{n-1}} |\nabla u|^p dx' d\tau \right) dt$$]

$$\leq C \int_0^{2/m} \int_{\mathbb{R}^{n-1}} |\nabla u|^p dx' d\tau \xrightarrow[m \to +\infty]{} 0$$

$\square$

Αποδείξαμε ότι $\exists \{w_m\} \subset W^{1,p}(\mathbb{R}^n_+)$

με $\quad spt\, w_m \subset \{x_n \geq \frac{1}{m}\}$,

$$w_m \longrightarrow u \quad \text{στο} \quad W^{1,p}(\mathbb{R}^n_+).$$

Ομαλοποιούμε τώρα την w_m με $\eta_{1/5m} * w_m =: \tilde{w}_m$

που είναι $C_c^\infty(\{x_n > 0\})$,

$$\tilde{w}_m \longrightarrow u \quad \text{στο} \quad W^{1,p}(\mathbb{R}^n_+)$$

$\Longrightarrow$

$$\underset{\substack{\parallel \\ 0}}{T\tilde{w}_m} \xrightarrow{L^p(\mathbb{R}^{n-1})} Tu.$$

$\Longrightarrow \quad Tu = 0.$

$\square$