1.4 Validity, Truth, Soundness, Strength, Cogency

PREVIEW • *Suppose you are undecided about your major, and a friend tells you that you should major in criminal justice because everyone in your sorority or fraternity is majoring in that field. You know that this is a really bad argument, but what is it, exactly, that makes it bad? In this section you will learn what causes inductive and deductive arguments to be good or bad and the language used to classify them as such.*

This section introduces the central ideas and terminology needed to evaluate arguments—to distinguish good arguments from bad arguments. Regardless of the type of argument, whether deductive or inductive, the evaluation of any argument involves answering two

distinct questions: (1) Do the premises support the conclusion? (2) Are all the premises true? The answer to the first question is the more important one, because if the premises fail to support the conclusion (that is, if the reasoning is bad), the argument is worthless. The material that follows first considers deductive arguments and then inductive.

Deductive Arguments

The previous section defined a deductive argument as one incorporating the claim that it is impossible for the conclusion to be false given that the premises are true. If this claim is true, the argument is said to be valid. Thus, a **valid deductive argument** is an argument in which it is impossible for the conclusion to be false given that the premises are true. In these arguments the conclusion follows with strict necessity from the premises. Conversely, an **invalid deductive argument** is a deductive argument in which it *is* possible for the conclusion to be false given that the premises are true. In these arguments the conclusion does not follow with strict necessity from the premises, even though it is claimed to.

An immediate consequence of these definitions is that there is no middle ground between valid and invalid. There are no arguments that are "almost" valid and "almost" invalid. If the conclusion follows with strict necessity from the premises, the argument is valid; if not, it is invalid.

To test an argument for validity we begin by assuming that all the premises are true, and then we determine if it is possible, in light of that assumption, for the conclusion to be false. Here is an example:

> All television networks are media companies.
> NBC is a television network.
> Therefore, NBC is a media company.

In this argument both premises are actually true, so it is easy to *assume* that they are true. Next we determine, in light of this assumption, if it is possible for the conclusion to be false. Clearly this is not possible. If NBC is included in the group of television networks (second premise) and if the group of television networks is included in the group of media companies (first premise), it necessarily follows that NBC is included in the group of media companies (conclusion). In other words, assuming the premises to be true and the conclusion false entails a strict *contradiction.* Thus, the argument is valid.

Here is another example:

> All automakers are computer manufacturers.
> United Airlines is an automaker.
> Therefore, United Airlines is a computer manufacturer.

In this argument, both premises are actually false, but it is easy to assume that they are true. Every automaker could have a corporate division that manufactures computers. Also, in addition to flying airplanes, United Airlines could make cars. Next, in light of these assumptions, we determine if it is possible for the conclusion to be false. Again, we see that this is not possible, by the same reasoning as the previous example.

Assuming the premises to be true and the conclusion false entails a contradiction. Thus, the argument is valid.

Another example:

All banks are financial institutions.
Wells Fargo is a financial institution.
Therefore, Wells Fargo is a bank.

As in the first example, both premises of this argument are true, so it is easy to assume they are true. Next we determine, in light of this assumption, if it is possible for the conclusion to be false. In this case it *is* possible. If banks were included in one part of the group of financial institutions and Wells Fargo were included in another part, then Wells Fargo would *not* be a bank. In other words, assuming the premises to be true and the conclusion false does not involve any contradiction, and so the argument is invalid.

In addition to illustrating the basic idea of validity, these examples suggest an important point about validity and truth. In general, validity is not something that is uniformly determined by the actual truth or falsity of the premises and conclusion. Both the NBC example and the Wells Fargo example have actually true premises and an actually true conclusion, yet one is valid and the other invalid. The United Airlines example has actually false premises and an actually false conclusion, yet the argument is valid. Rather, validity is something that is determined by the *relationship* between premises and conclusion. The question is not whether the premises and conclusion are true or false, but whether the premises *support* the conclusion. In the examples of valid arguments the premises do support the conclusion, and in the invalid case they do not.

Nevertheless, there is *one* arrangement of truth and falsity in the premises and conclusion that does determine the issue of validity. Any deductive argument having actually true premises and an actually false conclusion is invalid. The reasoning behind this fact is fairly obvious. If the premises are actually true and the conclusion is actually false, then it certainly is *possible* for the premises to be true and the conclusion false. Thus, by the definition of invalidity, the argument is invalid.

The idea that any deductive argument having actually true premises and a false conclusion is invalid may be the most important point in all of deductive logic. The entire system of deductive logic would be quite useless if it accepted as valid any inferential process by which a person could start with truth in the premises and arrive at falsity in the conclusion.

Table 1.1 presents examples of categorical syllogisms (deductive arguments) that illustrate the various combinations of truth and falsity in the premises and conclusion. In the examples having false premises, both premises are false, but it is easy to construct other examples having only one false premise. When examining this table, note that the only combination of truth and falsity that does not allow for *both* valid and invalid arguments is true premises and false conclusion. As we have just seen, any argument having this combination is necessarily invalid.

TABLE 1.1 Deductive Arguments

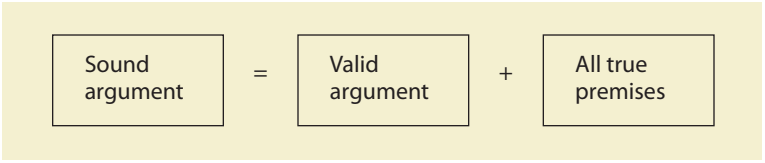
	Valid	Invalid
True premises	All flowers are plants. All daisies are flowers.	All flowers are plants. All daisies are plants.
True conclusion	Therefore, all daisies are plants. **[sound]**	Therefore, all daisies are flowers. **[unsound]**
True premises	None exist	All roses are flowers. All daisies are flowers.
False conclusion		Therefore, all daisies are roses. **[unsound]**
False premises	All flowers are dogs. All poodles are flowers.	All dogs are flowers. All poodles are flowers.
True conclusion	Therefore, all poodles are dogs. **[unsound]**	Therefore, all poodles are dogs. **[unsound]**
False premises	All flowers are dogs. All tigers are flowers.	All roses are cats. All daisies are cats.
False conclusion	Therefore, all tigers are dogs. **[unsound]**	Therefore, all daisies are roses. **[unsound]**

The relationship between the validity of a deductive argument and the truth or falsity of its premises and conclusion, as illustrated in Table 1.1, is summarized as follows:

Premises	Conclusion	Validity
T	T	?
T	F	Invalid
F	T	?
F	F	?

This short summary table reinforces the point that merely knowing the truth or falsity of the premises and conclusion tells us nothing about validity except in the one case of true premises and false conclusion. Any deductive argument having true premises and a false conclusion is necessarily invalid.

A **sound argument** is a deductive argument that is *valid* and has *all true premises.* Both conditions must be met for an argument to be sound; if either is missing the argument is unsound. Thus, an **unsound argument** is a deductive argument that is invalid, has one or more false premises, or both. Because a valid argument is one such that it is impossible for the premises to be true and the conclusion false, and because a sound argument does in fact have true premises, it follows that every sound argument, by definition, will have a true conclusion as well. A sound argument, therefore, is what is meant by a good, or successful, deductive argument in the fullest sense of the term.



In connection with this definition of soundness, a single proviso is required: For an argument to be unsound, the false premise or premises must actually be needed to support the conclusion. An argument having a conclusion that is validly supported by true premises but having a superfluous false premise would still be sound. By similar reasoning, no addition of a false premise to an originally sound argument can make the argument unsound. Such a premise would be superfluous and should not be considered part of the argument. Analogous remarks, incidentally, extend to induction.

Since (at least from the standpoint of logic) every premise is either true or false, and every deductive argument is either valid or invalid, it follows that every deductive argument is either sound or unsound. However, given that many, if not most, premises have truth values that are unknown or impossible to determine, it is not always possible to determine the soundness of a deductive argument. But that does not mean that soundness is unimportant in logic. It is crucially important that soundness be recognized as a criterion of evaluation that is distinct from validity and that the evaluator be ever vigilant never to confuse soundness with validity.

Inductive Arguments

Section 1.3 defined an inductive argument as one incorporating the claim that it is improbable that the conclusion be false given that the premises are true. If this claim is true, the argument is said to be strong. Thus, a **strong inductive argument** is an inductive argument in which it is improbable that the conclusion be false given that the premises are true. In such arguments, the conclusion does in fact follow probably from the premises. Conversely, a **weak inductive argument** is an argument in which the conclusion does not follow probably from the premises, even though it is claimed to.

All inductive arguments depend on what philosophers call the uniformity of nature. According to this principle, the future tends to replicate the past, and regularities that prevail in one spatial region tend to prevail in other regions. For example, in the past, sugar has always tasted sweet. According to the uniformity of nature, sugar will continue to taste sweet in the future. Also, just as sugar tastes sweet in Los Angeles, so does it in New York, London, and everywhere else. The uniformity of nature is the ultimate basis for our judgments about what we naturally expect to occur. Good inductive arguments are those that accord with the uniformity of nature. They have conclusions that we naturally expect to turn out true. If the conclusion of such an argument should turn out to be false, in violation of our expectations, this occurrence would cause us to react with surprise.

The procedure for testing the strength of inductive arguments runs parallel to the procedure for deduction. First we assume the premises are true, and then we determine whether, based on that assumption, the conclusion is probably true. This determination is accomplished by linking up the premises with regularities that exist in our experiential background. For example, if the argument is a causal inference, we link the information in the premises with known causal patterns. If the argument is an argument from signs, we connect the information in the premises with what we know about signs: some kinds of signs are trustworthy, others are not. If the argument is a generalization, we connect the information in the premises with what we know about a sample being representative of a population. All of these regularities are instances of the uniformity of nature. Here is an example of a prediction:

> All dinosaur bones discovered to this day have been at least 50 million years old. Therefore, probably the next dinosaur bone to be found will be at least 50 million years old.

In this argument the premise is actually true. Given that all dinosaur bones discovered to date have been over 50 million years old (and that thousands of such bones have been discovered), the uniformity of nature dictates that the next one to be discovered will also be over 50 million years old. This is what we would naturally expect, and anything to the contrary would be highly surprising. Thus, the conclusion is probably true, and so the argument is strong.

Here is another example:

> All meteorites found to this day have contained salt. Therefore, probably the next meteorite to be found will contain salt.

The premise of this argument is clearly false; but if we assume it to be true, then we would naturally expect that the next meteorite to be found would contain salt. Thus, the argument is strong.

The next example is an argument from analogy:

> Dom Pérignon champagne, which is made in France, sells for over $100 per bottle. Marquis de la Tour is also a French champagne. Therefore probably it, too, sells for over $100 per bottle.

In this argument the premises are actually true, but our background experience tells us that the mere fact that two wines come from the same country does not imply that they sell for the same price. Thus, the argument is weak. The conclusion, incidentally, happens to be false.

Another example:

> During the past fifty years, inflation has consistently reduced the value of the American dollar. Therefore, industrial productivity will probably increase in the years ahead.

In this argument, the premise is actually true and the conclusion is probably true in the actual world, but the probability of the conclusion is in no way based on the assumption that the premise is true. Because there is no direct connection between inflation and increased industrial productivity, the premise is irrelevant to the conclusion and it

provides no probabilistic support for it. The conclusion is probably true independently of the premise. As a result, the argument is weak.

This last example illustrates an important distinction between strong inductive arguments and valid deductive arguments. As we will see in later chapters, if the conclusion of a deductive argument is necessarily true independently of the premises, the argument is still considered valid. But if the conclusion of an inductive argument is probably true independently of the premises, the argument is weak.

These four examples show that in general the strength or weakness of an inductive argument results not from the actual truth or falsity of the premises and conclusion, but from the probabilistic support the premises give to the conclusion. The dinosaur argument has a true premise and a probably true conclusion, and the meteorite argument has a false premise and a probably false conclusion; yet both are strong because the premise of each provides probabilistic support for the conclusion. The industrial productivity argument has a true premise and a probably true conclusion, but the argument is weak because the premise provides no probabilistic support for the conclusion. As in the evaluation of deductive arguments, the only arrangement of truth and falsity that establishes anything is true premises and probably false conclusion (as in the Dom Pérignon argument). Any inductive argument having true premises and a probably false conclusion is weak.

TABLE 1.2 Inductive Arguments

	Strong	Weak
True premise **Probably true conclusion**	Every previous U.S. president was older than 40. Therefore, probably the next U.S. president will be older than 40. **[cogent]**	A few U.S. presidents were lawyers. Therefore, probably the next U.S. president will be older than 40. **[uncogent]**
True premise **Probably false conclusion**	None exist	A few U.S. presidents were unmarried. Therefore, probably the next U.S. president will be unmarried. **[uncogent]**
False premise **Probably true conclusion**	Every previous U.S. president was a TV debater. Therefore, probably the next U.S. president will be a TV debater. **[uncogent]**	A few U.S. presidents were dentists. Therefore, probably the next U.S. president will be a TV debater. **[uncogent]**
False premise **Probably false conclusion**	Every previous U.S. president died in office. Therefore, probably the next U.S. president will die in office. **[uncogent]**	A few U.S. presidents were dentists. Therefore, probably the next U.S. president will be a dentist. **[uncogent]**

Before proceeding further, however, we must qualify and explain this last statement. When we speak of the premises being true, we mean "true" in a complete sense. The premises must not exclude or overlook some crucial piece of evidence that undermines the stated premises and requires a different conclusion. This proviso is otherwise called the *total evidence requirement*. If the total evidence requirement is not met, an argument might have literally true premises and a probably false conclusion and still be strong. Also, when we speak of the conclusion being probably false, we mean probably false in the actual world in light of all the known evidence.

Table 1.2 presents several predictions (inductive arguments) that illustrate the various combinations of truth and falsity in the premises and conclusion. Note that the only arrangement of truth and falsity that is missing for strong arguments is true premises and probably false conclusion.

The relationship between the strength of an inductive argument and the truth or falsity of its premises and conclusion, as illustrated in Table 1.2, is summarized as follows:

Premises	Conclusion	Strength
T	probably T	?
T	probably F	Weak
F	probably T	?
F	probably F	?

Like the summary table for deduction, this brief table reinforces the point that merely knowing the truth conditions of the premises and conclusion tells us nothing about the strength of an argument except in the one case of true premises and probably false conclusion. Any inductive argument having true premises (in the sense just explained) and a probably false conclusion is weak.

Unlike the validity and invalidity of deductive arguments, the strength and weakness of inductive arguments allow for degrees. To be considered strong, an inductive argument must have a conclusion that is more probable than improbable. In other words, given that the premises are true, the likelihood that the conclusion is true must be more than 50 percent, and as the probability increases, the argument becomes stronger. For this purpose, consider the following pair of arguments:

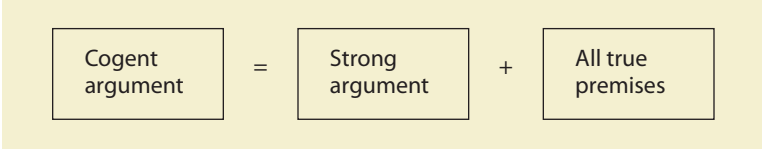
This barrel contains 100 apples.
Three apples selected at random were found to be ripe.
Therefore, probably all 100 apples are ripe.

This barrel contains 100 apples.
Eighty apples selected at random were found to be ripe.
Therefore, probably all 100 apples are ripe.

The first argument is weak and the second is strong. However, the first is not absolutely weak nor the second absolutely strong. Both arguments would be strengthened

or weakened by the random selection of a larger or smaller sample. For example, if the size of the sample in the second argument were reduced to seventy apples, the argument would be weakened. The incorporation of additional premises into an inductive argument will also generally tend to strengthen or weaken it. For example, if the premise "One unripe apple that had been found earlier was removed" were added to either argument, the argument would be weakened.

A **cogent argument** is an inductive argument that is *strong* and has *all true premises*. Also, the premises must be true in the sense of meeting the *total evidence requirement.* If any one of these conditions is missing, the argument is *uncogent*. Thus, an **uncogent argument** is an inductive argument that is weak, has one or more false premises, fails to meet the total evidence requirement, or any combination of these. A cogent argument is the inductive analogue of a sound deductive argument and is what is meant by a good, or successful, inductive argument without qualification. Because the conclusion of a cogent argument is genuinely supported by true premises, it follows that the conclusion of every cogent argument is probably true in the actual world in light of all the known evidence.



As an illustration of the need for the total evidence requirement, consider the following argument:

> Swimming in the Caribbean is usually lots of fun. Today the water is warm, the surf is gentle, and on this beach there are no dangerous currents. Therefore, it would be fun to go swimming here now.

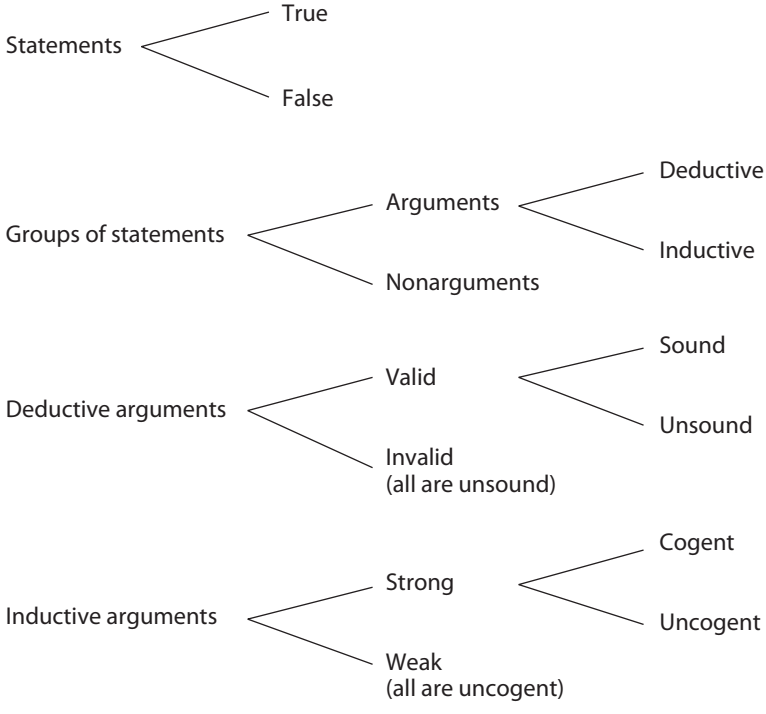
If the premises reflect all the important factors, then the argument is cogent. But if they ignore the fact that several large dorsal fins are cutting through the water (suggesting sharks), then obviously the argument is not cogent. Thus, for cogency the premises must not only be true but also not overlook some important fact that requires a different conclusion.

Finally, just as it is not always possible to determine the soundness of a deductive argument, it is not always possible to determine the cogency of an inductive argument. And this follows for two reasons. Many inductive arguments, especially those about complex real-life subjects, are not susceptible to being evaluated as clearly strong or clearly weak. And many premises have truth values that are unknown or impossible to determine. Yet, it remains important that cogency be recognized as a criterion for evaluating inductive arguments and that it not be confused with strength and weakness.

Summary

For both deductive and inductive arguments, two separate questions need to be answered: (1) Do the premises support the conclusion? (2) Are all the premises true? To answer the first question we begin by *assuming* the premises to be true. Then, for deductive arguments we determine whether, in light of this *assumption*, it necessarily follows that the conclusion is true. If it does, the argument is valid; if not, it is invalid. For inductive arguments we determine whether it probably follows that the conclusion is true. If it does, the argument is strong; if not, it is weak. For inductive arguments we keep in mind the requirements that the premises actually support the conclusion and that they not ignore important evidence. Finally, if the argument is either valid or strong, we turn to the second question and determine whether the premises are actually true. If all the premises are true, the argument is sound (in the case of deduction) or cogent (in the case of induction). All invalid deductive arguments are unsound, and all weak inductive arguments are uncogent.

The various alternatives open to statements and arguments may be diagrammed as follows. Note that in logic one never speaks of an argument as being "true" or "false," and one never speaks of a statement as being "valid," "invalid," "strong," or "weak."



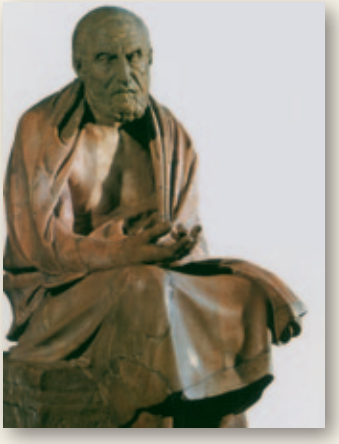
eminentlogicians

Chrysippus 280–206 B.C.

Chrysippus was born in Soli, a city located on the southeast coast of Asia Minor. Early in life he moved to Athens, where he studied under the Stoic philosopher Cleanthes, who in turn was a student of Zeno of Citium, the founder of Stoicism. Upon Cleanthes' death in 232 B.C., Chrysippus took over as leader of the school, and he produced over 700 treatises that systematized Stoic teaching. All of these works have been lost, but fragments survive in the writings of Cicero, Seneca, and others. Because of his extraordinary contribution, Chrysippus is considered to be the second founder of Stoicism.

Stoicism derives its name from the Greek word *stoa,* which means porch; Stoic philosophers used to gather on a porch in the Agora (public square) in Athens to discuss their views. The Stoics prized the virtue of self-sufficiency, and they emphasized the importance of not allowing oneself to be carried away by emotions or passions such as fear or love. The Stoics considered emotions to be false judgments about the goodness or badness of something. The proper therapy for those victimized by emotions is to persuade them that these judgments are indeed false because they constitute obstacles to true happiness.

Chrysippus is often considered to be the originator of propositional logic. Unlike Aristotelian logic, where the fundamental components are terms, in propositional logic the fundamental components are whole propositions or statements. Aristotle had overlooked this kind of logic, but his close friend and successor Theophrastus worked out some of the logic of the pure hypothetical syllogism (If *A* then *B; If B, then C; therefore, If A, then C*). Also, Philo of Megara introduced the truth functional interpretation of the material conditional (If *A,* then *B*). Beginning at this point, Chrysippus advanced propositional logic to a high level of development.



Chrysippus divided propositions into simple and compound, and he introduced a set of connectives that were used to produce compound propositions from one or more simple propositions. The compound propositions included negation, conjunction, exclusive disjunction, and implication, and Chrysippus showed how the truth value of a compound statement is a function of the truth values of its simple components. Chrysippus also introduced a set of rules of inference including what is today called *modus ponens, modus tollens,* disjunctive syllogism, and a rule similar to De Morgan's rule. Finally, he introduced the theory of natural deduction by which the conclusion of an argument can be derived from its premises through a series of discrete steps.

The broader philosophy of Chrysippus is characterized by monism and determinism. While most of us think that the universe is made up of millions of discrete entities, Chrysippus argued that in fact only one substance exists, and what appear to be individual substances are really parts of this one primary substance. Furthermore, everything that occurs is strictly governed by fate. Yet, in the face of this rigid causal determinism Chrysippus held that humans are responsible for their actions, and he tried in many ways to prove that the two viewpoints are in fact compatible with each other.

I. The following arguments are deductive. Determine whether each is valid or invalid, and note the relationship between your answer and the truth or falsity of the premises and conclusion. Finally, determine whether the argument is sound or unsound.

★**1.** Since *Moby Dick* was written by Shakespeare, and *Moby Dick* is a science-fiction novel, it follows that Shakespeare wrote a science-fiction novel.

2. Since London is north of Paris and south of Edinburgh, it follows that Paris is south of Edinburgh.

3. If George Washington was beheaded, then George Washington died. George Washington died. Therefore, George Washington was beheaded.

★**4.** The longest river in South America is the Amazon, and the Amazon flows through Brazil. Therefore, the longest river in South America flows through Brazil.

5. Since the Spanish-American War occurred before the U.S. Civil War, and the U.S. Civil War occurred after the Korean War, it follows that the Spanish-American War occurred before the Korean War.

6. The Empire State Building is taller than the Statue of Liberty, and the Statue of Liberty is taller than the Eiffel Tower. Therefore, the Empire State Building is taller than the Eiffel Tower.

★**7.** All leopards with lungs are carnivores. Therefore, all leopards are carnivores.

8. Chicago is a city in Michigan and Michigan is part of the United States. Therefore, Chicago is a city in the United States.

9. If President Barack Obama was born in Massachusetts, then he is a native of New England. Barack Obama is not a native of New England. Therefore, Barack Obama was not born in Massachusetts.

★**10.** Every province in Canada has exactly one city as its capital. Therefore, since there are thirty provinces in Canada, there are thirty provincial capitals.

11. Since the Department of Defense Building outside Washington, D.C., has the shape of a hexagon, it follows that it has seven sides.

12. Since Winston Churchill was English, and Winston Churchill was a famous statesman, we may conclude that at least one Englishman was a famous statesman.

★**13.** Since some fruits are green, and some fruits are apples, it follows that some fruits are green apples.

14. All physicians are individuals who have earned degrees in political science, and some lawyers are physicians. Therefore, some lawyers are persons who have earned degrees in political science.

15. The United States Congress has more members than there are days in the year. Therefore, at least two members of Congress have the same birthday.

II. The following arguments are inductive. Determine whether each is strong or weak, and note the relationship between your answer and the truth or falsity of the premise(s) and conclusion. Then determine whether each argument is cogent or uncogent.

★1. The grave marker at Arlington National Cemetery says that John F. Kennedy is buried there. It must be the case that Kennedy really is buried in that cemetery.

2. The ebb and flow of the tides has been occurring every day for millions of years. But nothing lasts forever. Therefore, probably the motion of the tides will die out within a few years.

3. The vast majority of Rose Bowl games (in Pasadena, California) have been played in freezing-cold weather. Therefore, probably the next Rose Bowl game will be played in freezing-cold weather.

★4. Franklin Delano Roosevelt said that we have nothing to fear but fear itself. Therefore, women have no reason to fear serial rapists.

5. Most popular film stars are millionaires. Viola Davis is a popular film star. Therefore, probably Viola Davis is a millionaire.

6. Constructing the great pyramid at Giza required lifting massive stone blocks to great heights. Probably the ancient Egyptians had some antigravity device to accomplish this feat.

★7. People have been listening to rock and roll music for over a hundred years. Probably people will still be listening to it a year from now.

8. Paleontologists have unearthed the fossilized bones of huge reptiles, which we have named dinosaurs. Tests indicate that these bones are more than 50 million years old. Therefore, probably dinosaurs really did roam the earth 50 million years ago.

9. The Declaration of Independence says that all men are endowed by their creator with certain unalienable rights. Therefore it probably follows that a creator exists.

★10. Coca-Cola is an extremely popular soft drink. Therefore, probably someone, somewhere, is drinking a Coke right this minute.

11. Every map of the United States shows that Alabama is situated on the Pacific coast. Therefore, Alabama must be a western state.

12. When Neil Armstrong landed on the moon, he left behind a gold-plated Schwinn bicycle, which he used to ride around on the moon's surface. Probably that bicycle is still up there on the moon.

★13. The African American athlete Adrian Peterson is able to withstand tremendous impacts on the football field. However, Serena Williams, like Adrian Peterson, is a great African American athlete. Therefore, Serena Williams should be able to withstand tremendous impacts on the football field.

14. Unlike monkeys, today's humans have feet that are not suited for grasping objects. Therefore, a thousand years from now, probably humans will still have feet that are not suited for grasping objects.

15. A random sample of twenty-five famous country and western singers, which included Garth Brooks and Dolly Parton, revealed that every single one of them studied music in Tasmania. Therefore, probably the majority of famous country and western singers studied music in Tasmania.

III. Determine whether the following arguments are inductive or deductive. If an argument is inductive, determine whether it is strong or weak. If it is deductive, determine whether it is valid or invalid.

★1. Since Tom is the brother of Agatha, and Agatha is the mother of Raquel, it follows that Tom is the uncle of Raquel.

2. When a cook cannot recall the ingredients in a recipe, it is appropriate that she refresh her memory by consulting the recipe book. Similarly, when a student cannot recall the answers during a final exam, it is appropriate that she refresh her memory by consulting the textbook.

3. The Broadway Theater marquee says that *The Phantom of the Opera* is playing nightly. Therefore, it must be that case that *Phantom* is playing there tonight.

★4. Since Christmas is always on a Thursday, it follows that the day after Christmas is always a Friday.

5. Suppose figure *A* is a triangle having two equal angles. It follows that figure *A* has two equal sides.

6. By accident Karen baked her brownies two hours longer than she should have. Therefore, they have probably been ruined.

★7. After taking LSD, Alice said she saw a flying saucer land in the shopping center parking lot. Since Alice has a reputation for always telling the truth, we must conclude that a flying saucer really did land there.

8. Since Phyllis is the cousin of Denise, and Denise is the cousin of Harriet, it follows necessarily that Harriet is the cousin of Phyllis.

9. The picnic scheduled in the park for tomorrow will most likely be cancelled. It's been snowing for six days straight.

★10. Circle A has exactly twice the diameter of circle B. From this we may conclude that circle A has exactly twice the area of circle B.

11. Robert has lost consistently at blackjack every day for the past several days. Therefore, it is very likely that he will win today.

12. Since John loves Nancy and Nancy loves Peter, it follows necessarily that John loves Peter.

★13. This cash register drawer contains over 100 coins. Three coins selected at random were found to have dates earlier than 1960. Therefore, probably all of the coins in the drawer have dates earlier than 1960.

14. The Japanese attack on Pearl Harbor happened in either 1941 or 1951. But it didn't happen in 1941. Therefore, it happened in 1951.

15. Harry will never be able to solve that difficult problem in advanced calculus in the limited time allowed. He has never studied anything beyond algebra, and in that he earned only a C−.

★16. Since $x + y = 10$, and $x = 7$, it follows that $y = 4$.

17. If acupuncture is hocus pocus, then acupuncture cannot relieve chronic pain. But acupuncture can relieve chronic pain. Therefore, acupuncture is not hocus pocus.

18. If inflation heats up, then interest rates will rise. If interest rates rise, then bond prices will decline. Therefore, if inflation heats up, then bond prices will decline.

★19. Statistics reveal that 86 percent of those who receive flu shots do not get the flu. Jack received a flu shot one month ago. Therefore, he should be immune, even though the flu is going around now.

20. Since Michael is a Pisces, it necessarily follows that he was born in March.

IV. Define the following terms:

valid deductive argument	strong inductive argument
invalid deductive argument	weak inductive argument
sound argument	cogent argument
unsound argument	uncogent argument

V. Answer "true" or "false" to the following statements:

1. Some arguments, while not completely valid, are almost valid.
2. Inductive arguments allow for varying degrees of strength and weakness.
3. Invalid deductive arguments are basically the same as inductive arguments.
4. If a deductive argument has true premises and a false conclusion, it is necessarily invalid.
5. A valid argument may have a false premise and a false conclusion.
6. A valid argument may have a false premise and a true conclusion.
7. A sound argument may be invalid.
8. A sound argument may have a false conclusion.
9. A strong argument may have false premises and a probably false conclusion.
10. A strong argument may have true premises and a probably false conclusion.
11. A cogent argument may have a probably false conclusion.
12. A cogent argument must be inductively strong.
13. If an argument has true premises and a true conclusion, we know that it is a perfectly good argument.
14. A statement may legitimately be spoken of as "valid" or "invalid."
15. An argument may legitimately be spoken of as "true" or "false."

1.5 Argument Forms: Proving Invalidity

PREVIEW • *While at a party you overhear someone talking about you. Disdainfully, the person observes that you don't wear designer clothing, and then comments that losers don't wear designer clothing either. The implication is that you are a loser. Hearing this infuriates you. You know the argument is clearly invalid, but how do you prove it? In this section you will learn about a simple, intuitive method to prove invalid deductive arguments invalid.*

This section explores the idea that the validity of a deductive argument is determined by its form. This idea was suggested by the arguments in Table 1.1 in the previous section. All the arguments in the Valid column have the same valid form, and all the arguments in the Invalid column have the same invalid form. The form of an argument illustrates the argument's internal structure or pattern of reasoning. If the pattern of reasoning is good, the argument will be valid; if not, it will be invalid.

In reference to Table 1.1, all the valid arguments have this form:

All *A* are *B*.
All *C* are *A*.
All *C* are *B*.

If *A, B,* and *C* are thought of as referring to groups of things, it is easy to see that this form is valid. Assume, by the second premise, that the *C*s (whatever they might be) are included in the *A*s, and, by the first premise, that the *A*s (whatever they might be) are included in the *B*s. Then it necessarily follows that the *C*s are included in the *B*s, which is what the conclusion asserts.

We can use this example to define what we mean by an argument form. An **argument form**, for the present purpose, is an arrangement of letters (in this case *A, B,* and *C*) and words (in this case "all" and "are") such that the uniform substitution of words or phrases in the place of the letters results in an argument. For this form, the words or phrases being substituted must refer to groups of things. Thus, if we substitute "sporting events," "engaging pastimes," and "baseball games" in the place of *A, B,* and *C,* respectively, in the argument form (left), we obtain the following argument (right):

All *A* are *B*.	All sporting events are engaging pastimes.
All *C* are *A*.	All baseball games are sporting events.
All *C* are *B*.	All baseball games are engaging pastimes.

This argument is called a **substitution instance** of the argument form. Any substitution instance of a valid argument form is a valid argument.

Before proceeding to invalid arguments, we must briefly consider valid arguments in which the form is not apparent. Many of the arguments in the previous set of exercises were like this. How can we reconcile the existence of such arguments with the claim that validity is determined by form? The answer is that these arguments are incomplete, so the form is not explicit. But once such arguments are completed and

correctly phrased (which we address later in this book), the form becomes apparent. For example, consider the following valid argument:

Geese are migratory waterfowl, so they fly south for the winter.

This argument is missing a premise:

Migratory waterfowl fly south for the winter.

The argument can now be rephrased to make its form apparent:

All migratory waterfowl are birds that fly south for the winter.
All geese are migratory waterfowl.
Therefore, all geese are birds that fly south for the winter.

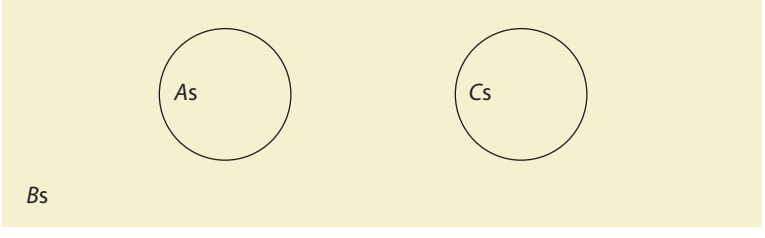
The form of the argument is

All *A* are *B*.
All *C* are *A*.
All *C* are *B*.

This form is identical to the form we just considered and is valid.
Let us now consider an invalid argument form:

All *A* are *B*.
All *C* are *B*.
All *A* are *C*.

In this argument form, if we assume that the *A*s are in the *B*s and that the *C*s are in the *B*s, it does not *necessarily* follow that the *A*s are in the *C*s. It would not follow if the *A*s were in one part of the *B*s and the *C*s were in another part, as the following diagram illustrates:



This diagram suggests that we can prove the form invalid if we can find a substitution instance having actually true premises and an actually false conclusion. In such a substitution instance the *A*s and the *C*s would be separated from each other, but they would both be included in the *B*s. If we substitute "cats" for *A*, "animals" for *B*, and "dogs" for *C*, we have such a substitution instance:

All *A* are *B*.	All cats are animals.	True
All *C* are *B*.	All dogs are animals.	True
All *A* are *C*.	Therefore, all cats are dogs.	False

This substitution instance proves the form invalid, because it provides a concrete example of a case where the *A*s are in the *B*s, the *C*s are in the *B*s, but the *A*s are *not* in the *C*s.

Now, since the form is invalid, can we say that any argument that has this form is invalid? Unfortunately, the situation with invalid forms is not quite as simple as it is with valid forms. Every substitution instance of a valid form is a valid argument, but it is not the case that every substitution instance of an invalid form is an invalid argument. The reason is that some substitution instances of invalid forms are also substitution instances of valid forms.* However, we can say that any substitution instance of an invalid form is an invalid argument *provided* that it is not a substitution instance of any valid form. Thus, we will say that an argument actually *has* an invalid form if it is a substitution instance of that form and it is not a substitution instance of any valid form.

The fact that some substitution instances of invalid forms are also substitution instances of valid forms means simply that we must exercise caution in identifying the form of an argument. However, cases of ordinary language arguments that can be interpreted as substitution instances of both valid and invalid forms are so rare that this book chooses to ignore them. With this in mind, consider the following argument:

All romantic novels are literary pieces.
All works of fiction are literary pieces.
Therefore, all romantic novels are works of fiction.

This argument clearly has the invalid form just discussed. This invalid form captures the reasoning process of the argument, which is obviously defective. Therefore, the argument is invalid, and it is invalid precisely because it has an invalid form.

Counterexample Method

A substitution instance having true premises and a false conclusion (like the cats-and-dogs example just constructed) is called a counterexample, and the method we have just used to prove the romantic-novels argument invalid is called the **counterexample method**. It consists of isolating the form of an argument and then constructing a substitution instance having true premises and a false conclusion. This proves the form invalid, which in turn proves the argument invalid. The counterexample method can be used to prove the invalidity of any invalid argument, but it cannot prove the validity of any valid argument. Thus, before the method is applied to an argument, the argument must be known or suspected to be invalid in the first place. Let us apply the counterexample method to the following invalid categorical syllogism:

Since some employees are not social climbers and all vice presidents are employees, we may conclude that some vice presidents are not social climbers.

**For example, the following valid argument is a substitution instance of the invalid form we have been discussing:*

All bachelors are persons.
All unmarried men are persons.
Therefore, all bachelors are unmarried men.

However, because "bachelors" is equivalent in meaning to "unmarried men," the argument is also a substitution instance of this valid form:

All *A* are *B*.
All *A* are *B*.
All *A* are *A*.

This argument is invalid because the employees who are not social climbers might not be vice presidents. Accordingly, we can *prove* the argument invalid by constructing a substitution instance having true premises and a false conclusion. We begin by isolating the form of the argument:

Some *E* are not *S*.
All *V* are *E*.
Some *V* are not *S*.

Next, we select three terms to substitute in place of the letters that will make the premises true and the conclusion false. The following selection will work:

E = animals
S = mammals
V = dogs

The resulting substitution instance is this:

Some animals are not mammals.
All dogs are animals.
Therefore, some dogs are not mammals.

The substitution instance has true premises and a false conclusion and is therefore, by definition, invalid. Because the substitution instance is invalid, the form is invalid, and therefore the original argument is invalid.

In applying the counterexample method to categorical syllogisms, it is useful to keep in mind the following set of terms: "cats," "dogs," "mammals," "fish," and "animals." Most invalid syllogisms can be proven invalid by strategically selecting three of these terms and using them to construct a counterexample. Because everyone agrees about these terms, everyone will agree about the truth or falsity of the premises and conclusion of the counterexample. Also, in constructing the counterexample, it often helps to begin with the conclusion. First, select two terms that yield a false conclusion, and then select a third term that yields true premises. Another point to keep in mind is that the word "some" in logic always means "at least one." For example, the statement "Some dogs are animals" means "At least one dog is an animal"—which is true. Also note that this statement does not imply that some dogs are not animals.

Not all deductive arguments, of course, are categorical syllogisms. Consider, for example, the following hypothetical syllogism:

If the government imposes import restrictions, the price of automobiles will rise. Therefore, since the government will not impose import restrictions, it follows that the price of automobiles will not rise.

This argument is invalid because the price of automobiles might rise even though import restrictions are not imposed. It has the following form:

If *G*, then *P*.
Not *G*.
Not *P*.

This form differs from the previous one in that its letters stand for complete statements. *G*, for example, stands for "The government imposes import restrictions." If we make the substitution

G = Abraham Lincoln committed suicide.
P = Abraham Lincoln is dead.

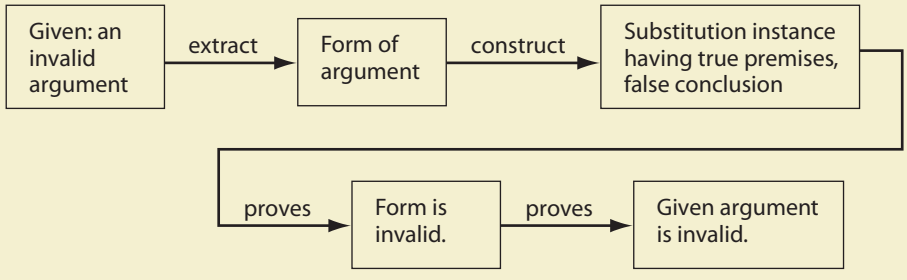
we obtain the following substitution instance:

If Abraham Lincoln committed suicide, then Abraham Lincoln is dead.
Abraham Lincoln did not commit suicide.
Therefore, Abraham Lincoln is not dead.

Since the premises are true and the conclusion false, the substitution instance is clearly invalid. Therefore, the form is invalid, and this proves the original argument invalid.

When applying the counterexample method to an argument having a conditional statement as a premise (such as the one just discussed), it is recommended that the statement substituted in place of the conditional statement express some kind of necessary connection. In the Lincoln example, the first premise asserts the necessary connection between suicide and death. There can be no doubt about the truth of such a statement. Furthermore, if it should turn out that the conclusion is a conditional statement, note that one sure way of producing a false conditional statement is by joining a true antecedent with a false consequent. For example, the conditional statement "If Lassie is a dog, then Lassie is a cat" is clearly false.

Counterexample method



Being able to identify the form of an argument with ease requires a familiarity with the basic deductive argument forms. The first task consists in distinguishing the premises from the conclusion. Always write the premises first and the conclusion last. The second task involves distinguishing what we may call "form words" from "content words." To reduce an argument to its form, leave the form words as they are, and replace the content words with letters. For categorical syllogisms, the words "all," "no," "some," "are," and "not" are form words, and for hypothetical syllogisms the words "if," "then," and "not" are form words. Additional form words for other types of arguments are "either," "or," "both," and "and." For various kinds of hybrid arguments, a more intuitive approach may be needed. Here is an example:

All movie stars are actors who are famous, because all movie stars who are famous are actors.

If we replace "movie stars," "actors," and "famous" with the letters *M, A,* and *F,* this argument has the following form:

All *M* who are *F* are *A*.
All *M* are *A* who are *F*.

Here is one possible substitution instance for this form:

All humans who are fathers are men.
Therefore, all humans are men who are fathers.

Because the premise is true and the conclusion false, the form is invalid and so is the original argument.

Using the counterexample method to prove arguments invalid requires a little ingenuity because there is no rule that will automatically produce the required term or statement to be substituted into the form. Any term or statement will work, of course, provided that it yields a substitution instance that has premises that are indisputably true and a conclusion that is indisputably false. Ideally, the truth value of these statements should be known to the average individual; otherwise, the substitution instance cannot be depended on to prove anything. If, for example, *P* in the earlier hypothetical syllogism had been replaced by the statement "George Wilson is dead," the substitution instance would be useless, because nobody knows whether this statement is true or false.

The counterexample method is useful only for proving invalidity, because the only arrangement of truth and falsity that proves anything is true premises and false conclusion. If a substitution instance is produced having true premises and a true conclusion, it does *not* prove that the argument is valid. Furthermore, the method is useful only for deductive arguments because the strength and weakness of inductive arguments is only partially dependent on the form of the argument. Accordingly, no method that relates exclusively to the form of an inductive argument can be used to prove the argument weak.

EXERCISE 1.5

I. Use the counterexample method to prove the following categorical syllogisms invalid. In doing so, follow the suggestions given in the text.

★**1.** All galaxies are structures that contain black holes in the center, so all galaxies are quasars, since all quasars are structures that contain black holes in the center.

2. Some evolutionists are not people who believe in the Bible, for no creationists are evolutionists, and some people who believe in the Bible are not creationists.

3. No patents are measures that discourage research and development, and all patents are regulations that protect intellectual property. Thus, no measures

that discourage research and development are regulations that protect intellectual property.

★4. Some farm workers are not people who are paid decent wages, because no undocumented individuals are people who are paid decent wages, and some undocumented individuals are not farm workers.

5. Some politicians are people who will stop at nothing to win an election, and no people who will stop at nothing to win an election are true statesmen. Hence, no politicians are true statesmen.

6. All meticulously constructed timepieces are true works of art, for all Swiss watches are true works of art and all Swiss watches are meticulously constructed timepieces.

★7. No patrons of fast-food restaurants are health-food addicts. Consequently, no patrons of fast-food restaurants are connoisseurs of fine desserts, since no connoisseurs of fine desserts are health-food addicts.

8. Some toxic dumps are sites that emit hazardous wastes, and some sites that emit hazardous wastes are undesirable places to live near. Thus, some toxic dumps are undesirable places to live near.

9. All persons who assist others in suicide are people guilty of murder. Accordingly, some individuals motivated by compassion are not persons guilty of murder, inasmuch as some people who assist others in suicide are individuals motivated by compassion.

★10. Some school boards are not groups that oppose values clarification, because some school boards are not organizations with vision, and some groups that oppose values clarification are not organizations with vision.

11. All super PACs are unlimited spenders. For this reason, some big-time power brokers are not super PACs, in as much as some unlimited spenders are big-time power brokers.

12. No movie producers are uncompetitive business executives, and some Hollywood moguls are movie producers. It follows that no Hollywood moguls are uncompetitive business executives.

★13. Some improvers of humankind are not exploiters of personal information. As a result, some corporate social networks are not improvers of humankind, seeing that all corporate social networks are exploiters of personal information.

14. Some drone attacks are assaults on human life, given that some stealth operations are assaults on human life and all drone attacks are stealth operations.

15. Some near-death experiences are supernatural phenomena, and no near-death experiences are easily forgotten happenings. Consequently some easily forgotten happenings are not supernatural phenomena.

II. Use the counterexample method to prove each of the following arguments invalid.

★1. If animal species are fixed and immutable, then evolution is a myth. Therefore, evolution is not a myth, since animal species are not fixed and immutable.

I

2. If carbon dioxide is present in the atmosphere, then plants have a source of carbon. Hence, since plants have a source of carbon, carbon dioxide is present in the atmosphere.

3. If human rights are recognized, then civilization flourishes. If equality prevails, then civilization flourishes. Thus, if human rights are recognized, then equality prevails.

★4. If energy taxes are increased, then either the deficit will be reduced or conservation will be taken seriously. If the deficit is reduced, then inflation will be checked. Therefore, if energy taxes are increased, then inflation will be checked.

5. All homeless people who are panhandlers are destitute individuals. Therefore, all homeless people are destitute individuals.

6. Some wrestlers are colorful hulks, since some wrestlers are colorful and some wrestlers are hulks.

★7. All community colleges with low tuition are either schools with large enrollments or institutions supported by taxes. Therefore, all community colleges are institutions supported by taxes.

8. All merchandisers that are retailers are businesses that are inventory rotators. Therefore, all merchandisers are inventory rotators.

9. All diabetes victims are either insulin takers or glucose eliminators. Accordingly, some diabetes victims are glucose eliminators, since some diabetes victims are insulin takers.

★10. All FHA loans are living-standard enhancers for the following reasons. All reverse mortgages that are FHA loans are either living-standard enhancers or home-equity depleters, and all reverse mortgages are home-equity depleters.