ΓΕ77
COMPUTATIONAL LINGUISTICS

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FORMAL LANGUAGES AND LOGIC

FORMAL LANGUAGES

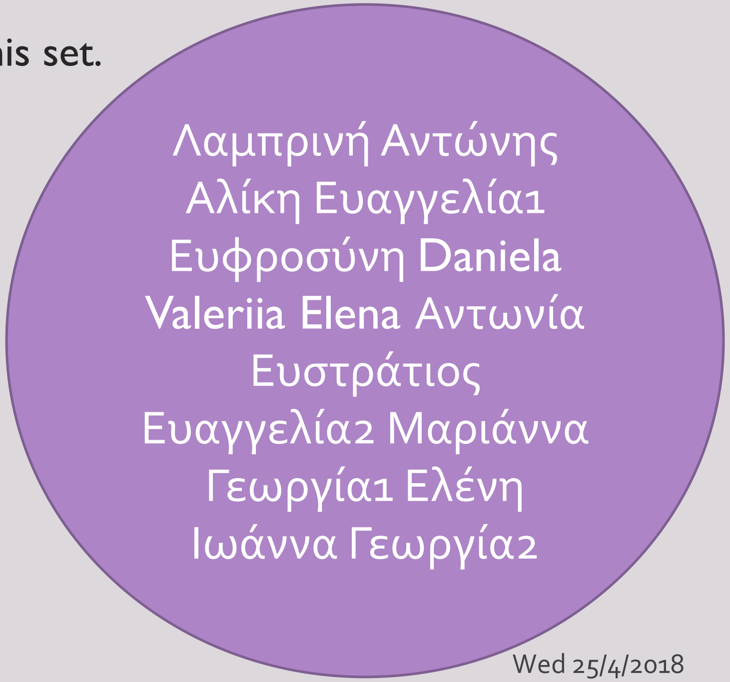
- **Formal language** is created by humans for knowledge description and representation.
- **Natural languages** are used by humans to communicate, exchange ideas and knowledge, express feelings and emotions, etc.
- There are several formal languages that try to describe and represent the natural languages. They do not try to explain how a human acquires knowledge. They try to represent the mechanics and the device of how linguistic utterances and meaning are generated.

SETS: DEFINITION

- Sets are collections of discrete objects, either of our reality or of our imagination or our abstract knowledge.
  - A set **contains** objects or **is consisted** by objects that are called **elements** or **members** of the set and they **belong** to a set.
  - A set of all the students that participate in the module ΓΛ545.
  - A set of all the furniture of my house.
  - A set of ideas from the European Enlightenment
  - A set of heroes from the trilogy The Lord of the Rings.

SETS: SYMBOLS

- ΓΕ77 = {Ψυχογυιού Γεωργία, Χαρμαντζή Ιωάννα, Τσίλη Ελένη, Τρίτου Μαριάννα,…, Alafuzova Elena}
  - … symbolize a series of elements that are known in this set.
  - The order of the elements is insignificant.
- ΓΕ77 = { x|x a ΓΕ77 undergraduate student}
- The Venn diagram



ELEMENT SYMBOLS

- Αντωνία Κατσιγιάννη ∈ ΓΕ77
- Αθανάσιος Καρασίμος ∉ ΓΕ77

- Every element is a distinct object/element of a set. You can not have a set that contains two or more identical objects/elements.
- However you can have ΓΕ77 = {Ελένη, Ιωάννα, Γεωργία1, Ευστράτιος, Elena Ευφροσύνη, ... Γεωργία2} if Γεωργία1 and Γεωργία2 are two different and distinct objects (persons).
- We can have an **empty set**, like A = { }.

SUBSETS

- If you have two sets and the elements of a set are also elements of the other set, then the first set is a **subset** of the second set.
  - ΓΕ77α= {Αντώνης, Ευστράτιος, Νίκος} is a subset of the ΓΕ77 set.

ΓΕ77α ⊆ ΓΕ77

If ΓΕ77 contains at least one more element than the ΓΕ77α1 set, then this set (ΓΕ77α) is a **proper subset**.

ΓΕ77α ⊂ ΓΕ77

⊂ = part of

⊆ = part of or equal to

SUBSETS

- **Task 1: Which is the correct statement?**
  - ΓΕ77 ⊆ ΓΕ77
  - ΓΕ77 ⊂ ΓΕ77
- **Task 2:**
  - TC1 = {Mission Impossible 1, Jack Reacher, Mission Impossible II, Top Gun}
  - TC2 = {Mission Impossible II, Top Gun, Mission Impossible I, Jack Reacher}
    - TC1 = TC2
    - TC1 ≠ TC2
    - TC1 ⊂ TC2
    - TC1 ⊆ TC2

SET RELATIONS

- **Union**

When we have two sets (A and B) and we combined them, we create a new set (C) which contains all the elements from sets A and B.

$\mathbf{A \cup B = C}$

$\mathbf{A \cup \emptyset = A}$

- **Intersection**

When we have two sets (A and B) and we want to keep the common elements, we create an new set (C) which contains only the common elements from the two sets.

$\mathbf{A \cap B = C}$

$\mathbf{A \cap \emptyset = \emptyset}$

Two sets are **disjoint sets if** $A \cap B = \emptyset$

SET RELATIONS

- **Task 3: Union of three sets**
  - ΓΛ1 = {Γεωργία, Δήμητρα, Valerria, Ιώαννα}
  - ΓΛ2 = {Μαριάνα, Daniela, Elena}
  - ΓΛ3 = {Χρήστος, Ευστράτιος}
- **Task 4: Union of two sets?**
  - ΓΛ1 = {Ευαγγελία, Μαριάννα, Valeriia}
  - ΓΛ2 = {Αλίκη, Ειρήνη}
  - ΓΛ = {Ειρήνη, Valeriia, Μαριάννα, Ευαγγελία, Χρήστος, Αλίκη}
- **Task 5: Intersection of three sets**
  - ΓΛ1 = {Χρήστος, Μαριάννα, Γεωργία, Ελένη, Elena, Ευστράτιος, Ευαγγελία}
  - ΓΛ2 = { x | x male student}
  - ΓΛ3 = { x | x older than 19 years}
- **Task 6: Intersection of two sets?**
  - ΓΛ1 = {Ευαγγελία, Γεωργία1, Γεωργία2, Μαριάννα}
  - ΓΛ2 = {Γεωργία1, Daniela, Χρήστος, Ευαγγελία}
  - ΓΛ = {Ευαγγελία, Γεωργία2}

SET RELATIONS: CARTESIAN PRODUCT

- **Cartesian Product**
  - is a mathematical operation that returns a set from multiple sets. That is, for sets A and B, the Cartesian product $A \times B$ is the set of all **ordered pairs** (a, b) where $a \in A$ and $b \in B$. Products can be specified using set-builder notation.
- **Ordered Pair**
  - an ordered pair <a, b> is a pair of elements. The order in which the objects appear in the pair is significant: the ordered pair <a, b> is different from the ordered pair <b, a> unless a = b (In contrast, the unordered pair {a, b} equals the unordered pair {b, a}).

SET RELATIONS: CARTESIAN PRODUCT

- If you have a set A = {A, B, C} and a set B = {α, β}, then:
- A x B = {<A, α>, <A, β>, <B, α>, <B, β>, <C, α>, <C, β>}
- This Cartesian Product can describe a parent-child relation, where the new set C (A x B): {x∈A parent of the y∈B}
- The **reverse relation** describes the opposite relation between two sets and its symbol is C^{-1}.
- >> C^{-1}: {x∈B child of y∈A}

SET RELATIONS: ADVANCED EDITION

- Relation over a set X is **reflexive** if every element of X is related to itself. Formally, this may be written ∀x ∈ X : {x, x} OR x R x.
  - An example of a reflexive relation is the relation "is equal to" on the set of real numbers, since every real number is equal to itself.
- Relation over a set X is **symmetric** if it holds for all a and b in X that a is related to b if and only if b is related to a.
  - In mathematical notation, this is: x ∈A and y∈A: <x, y>∈R = <y, x>∈R (i.e. the siblings relation).
- Relation over a set X is **transitive** if whenever an element a is related to an element b and b is related to an element c then a is also related to c.
  - Transitivity is a key property of both partial order relations and equivalence relations (i.e. older – younger relation). <x,y>∈R and <y,z>∈R then <x,z>∈R

LOGIC

A short Introduction to Logic and Calculus

LOGIC: PROPOSITIONAL CALCULUS

- **Propositional calculus** (also called propositional logic) is the branch of logic concerned with the study of propositions (whether they are true or false) that are formed by other propositions with the use of logical connectives, and how their value depends on the truth value of their components.
- Logical connectives are found in natural languages. In English for example, some examples are "and" (**conjunction**), "or" (**disjunction**), "not" (**negation**) and "if" (but only when used to denote **material conditional**).

LOGIC: PROPOSITIONAL CALCULUS

- Premise 1: If she is a professor, the she has a PhD diploma
- Premise 2: She is a professor
- Conclusion: She has a PhD diploma
- Both **premises** and the **conclusion** are **propositions**. The premises are taken for granted and then with the application of modus ponens (an inference rule) the conclusion follows.

TRUTH TABLES

- Every statement of propositional calculus should have one of the two possible values: 0 = **False** or 1 = **True**.
- A **truth table** has one column for each input variable, and one final column showing all of the possible results of the logical operation that the table represents.
- Each row of the truth table contains one possible configuration of the input variables, and the result of the operation for those values. See the examples below for further clarification.

p	q	p	¬ p
1	0	0	1
		1	0

- **Double negation** is actually a zero negation! **¬ ¬ p -> p**
  - **Subtask:** Does double negation have the same operation in natural language? i.e. I will not do nothing -> I will do everything.

TRUTH TABLES: CONJUNCTION

- **Logical conjunction** is an operation on two logical values, typically the values of two propositions, that produces a value of true if both of its operands are true.
- The truth table for p AND q (also written as p ∧ q) is as follows:

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

TRUTH TABLES: DISJUNCTION

- **Logical disjunction** is an operation on two logical values, typically the values of two propositions, that produces a value of true if at least one of its operands is true.
- The truth table for p OR q (also written as p ∨ q) is as follows:
- p q p ∨ q
- T T T
- T F T
- F T T
- F F F

TRUTH TABLES: IMPLICATIONS

- Logical implication and the material conditional are both associated with an operation on two logical values, typically the values of two propositions, which produces a value of false if the first operand is true and the second operand is false, and a value of true otherwise.
- The truth table associated with the logical implication p implies q (symbolized as p ⇒ q) is as follows:

p	q	p ⇒ q
T	T	T
T	F	F
F	T	T
F	F	T

TRUTH TABLES: EQUALITY

- Logical equality is an operation on two logical values, typically the values of two propositions, that produces a value of true if both operands are false or both operands are true.
- The truth table for p XNOR q (also written as p ↔ q, Epq, p = q, or p ≡ q) is as follows:
- p q p ↔ q
- T T T
- T F F
- F T F
- F F T

DISTRIBUTIVE LAW

- The **Distributive Law** says that multiplying a number by a group of numbers added together is the same as doing each multiplication separately.
  - $(p \lor (q \land r)) \leftrightarrow (p \lor q) \land (p \lor r)$
  - $(p \land (q \lor r))\ ))\ \leftrightarrow\ (p \land q) \lor (p \land r)$ >> Task 5: Present the truth table.

p	q	r	$(q \land r)$	$(p \lor (q \land r))$	$(p \lor q)$	$(p \lor r)$	$(p \lor q) \land (p \lor r)$
1	1	1	1	1	1	1	1
1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1
0	0	1	0	0	0	1	0
1	1	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0

FIRST ORDER PREDICATE CALCULUS

FIRST ORDER LOGIC

- **First-order logic**(aka first-order predicate calculus, FOL) uses quantified variables over non-logical objects and allows the use of sentences that contain variables, so that rather than propositions.
  - such as Socrates is a man one can have expressions in the form "there exists X such that X is Socrates and X is a man" and there **exists** is a quantifier while **X** is a **variable**.
- This distinguishes it from propositional logic, which does not use quantifiers or relations.

PREDICATES AND TERMS

- In FOL we have the ability to compile a sentence using **predicates** and **terms** that are distinct in fixed and variable terms. If we ‘transform’ the statements with the following correspondence:
- (1) John walks ---- walk (John)
- (2) Mark is fat. ---- be_fat (Mark)
- (3) Mary teaches Philosophy. ---- teach (Mary, Philosophy)

Predicates: walk, be_fat, teach

Terms (fixes): John, Mark, Mary, Philosophy

Terms (variable): x, y, z

PREDICATES AND TERMS: TASK

- **Task 6:** *Write the predicates and terms for:*
  - *Jim is tall.*
  - *Jessica eats a burger.*
  - *Paul gives Peter a bottle.*
  - *Ο Βασίλης χτίζει ένα σπίτι.*
  - *Ο Κώστας χαιρετάει την Ελένη.*
  - *Η Νάντια πλέκει ένα κασκόλ.*

QUANTIFIERS

Universal quantifier

- The expression: ∀ x P(x), denotes the universal quantification of the atomic formula P(x). Translated into the English language, the expression is understood as: "for each x, P(x) holds“. The universal quantifier means all the objects x in the universe. If this is followed by P(x) then the meaning is that P(x) is true for every object x in the universe.
  - i.e. “All cars have wheels“ > ∀ x has wheels, where x=cars.

Universal Quantifier and Connective AND

- If all the elements in the universe of discourse can be listed then the universal quantification ∀x P(x) is equivalent to the conjunction: P(x1) ∧ P(x2) ∧ P(x3) ... P(xn) .
- For example, in the above example of x P(x), if we knew that there were only 4 cars in our universe of discourse (c1, c2, c3 and c4) then we could also translate the statement as: P(c1) ∧ P(c2) ∧ P(c3) ∧ P(c4).

QUANTIFIERS

Existential Quantifier

- The expression: $\exists x P(x)$, denotes the existential quantification of P(x): "there is at least one x such that P(x)…". $\exists x$ means at least one object x in the universe.
  - i.e. "Someone loves you" could be transformed into the propositional form, $\exists x\ P(x)$, where: P(x) is the predicate meaning: x loves you.

Existential Quantifier and Connective OR

- If all the elements in the universe of discourse can be listed, then the existential quantification $\exists x P(x)$ is equivalent to the disjunction: $P(x1) \vee P(x2) \vee P(x3)$ ... $P(xn)$.
- For example, in the above example of x P(x), if we knew that there were only 5 living creatures in our universe of discourse (say: me, he, she, rex and fluff), then we could also write the statement as: $P(me) \vee P(he) \vee P(she) \vee P(rex) \vee P(fluff)$

QUANTIFIERS

- Statements with quantifiers:
  - Everyone speaks > (∀x) *speak* (x)
  - At least someone is smart > (∃y) *be_smart* (y)

Scope of the quantifier

(∀x) *τρέχω* (x) Λ (∀y) *τραγουδώ* (y)	scope of (∀x) and (∀y)
(∀x) [*τρέχω* (x) Λ (∃y) *τραγουδώ* (x,y)]	scope of (∀x)
(∀x) [*τρέχω* (x) Λ *τραγουδώ* (x,y)]	scope of (∀x)
(∀x) [*τρέχω* (x) Λ (∃y) *τραγουδώ* (x,y)]	scope of (∃y)

QUANTIFIERS: TASKS

- **Task 7:** *Write five sentences that can be described with quantifiers.*
- **Task 8:** *Convert these sentences into FOL statements.*
- **Task 9:** *Convert the Greek sentences into FOL statements.*
  - Κάθε κατεργάρης στον πάγκο του.
  - Όλοι αντάμα και ο ψωριάρης χώρια.
  - Μερικοί το προτιμούν καυτό
  - Μου τό 'πε ένα πουλάκι.
  - Κάποιο λάκκο έχει η φάβα

NEGATION OF QUANTIFIERS

- For the relation between negation and quantifiers, the Law of Negation on Quantifiers is:
- $\neg(\forall x)\ P(x) \leftrightarrow (\exists\ x)\ \neg P\ (x)$
- The law of Negation on Quantifiers, to its right part $\exists(x)\ \neg P(x)$, states that there is at least one entity in our model for which statement P(x) has a truth value 0 (false). Therefore, P(x) does not apply to all entities of the model, which says the left part of the equivalence $\neg(\forall x)\ P(x)$.

QUANTIFIER DISTRIBUTION

- The relations of the quantifiers and the basic binders (conjunction and disjunction).
- We see the equivalence relation existing for statements containing the universal quantifier and conjunctions and statements that contain the existential quantifier and disjunction.
  - $(\forall x)\ (P(x) \land R\ (x)) \leftrightarrow (\forall x\ (P(x)) \land (\forall x\ (R\ (x))$
  - $(\exists x)\ (P(x) \lor R\ (x)) \leftrightarrow (\exists x\ (P(x)) \lor (\exists x\ (R\ (x))$

QUANTIFIER DISTRIBUTION

Task 10: *Which of the following statements are true?*

- Δεν είναι όλοι έξυπνοι ↔ Υπάρχει τουλάχιστον ένας που δεν είναι έξυπνος
- Δεν είναι όλα παγωμένα ↔ Υπάρχει κάτι που είναι καυτό
- Όλοι τρέχουν και όλοι σκοντάφτουν ↔ Όλοι τρέχουν και σκοντάφτουν
- Κάτι τρέχει ή κάτι σκοντάφτει ↔ Κάτι τρέχει ή σκοντάφτει

READING

- Τάντος, Α., Μαρκωντονάτου, Σ., Συμεωνίδου-Αναστασιάδη, Α. & Τ. Κυριακοπούλου (2015). Υ*πολογιστική Γλωσσολογία.* Πανεπιστημιακές εκδόσεις, σσ.27-37.