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Testability and Meaning—*Continued*

BY

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IV. THE CONSTRUCTION OF A LANGUAGE-SYSTEM

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17. The Problem of a Criterion of Meaning

IT IS not the aim of the present essay to defend the principle of empiricism against apriorism or anti-empiricist metaphysics. Taking empirism[1] for granted, we wish to discuss, the question what is meaningful. The word 'meaning' will here be taken in its empiricist sense; an expression of language has meaning in this sense if we know how to use it in speaking about empirical facts, either actual or possible ones. Now our problem is what expressions are meaningful in this sense. We may restrict this question to sentences because expressions other than sentences are meaningful if and only if they can occur in a meaningful sentence.

Empiricists generally agree, at least in general terms, in the view that the question whether a given sentence is meaningful is closely connected with the questions of the possibility of verification, confirmation or testing of that sentence. Sometimes the two questions have been regarded as identical. I believe that this identification can be accepted only as a rough first approximation. Our real problem now is to determine the precise relation between the two questions, or generally, to state the criterion of meaning in terms of verification, confirmation or testing.

I need not emphasize that here we are concerned only with the problem of meaning as it occurs in methodology, epistemology or applied logic,[2] and not with the psychological question of meaning. We shall not consider here the questions whether any images and, if so, what images are connected with a given sentence. That these questions belong to psychology and do not touch the methodological question of meaning, has often been emphasized.[3]

[1] The words 'empiricism' and 'empiricist' are here understood in their widest sense, and not in the narrower sense of traditional positivism or sensationalism or any other doctrine restricting empirical knowledge to a certain kind of experience.

[2] Our problem of meaning belongs to the field which *Tarski* [1] calls *Semantic*; this is the theory of the relations between the expressions of a language and things, properties, facts etc. described in the language.

[3] Comp. e.g. Schlick [4] p. 355.

It seems to me that the question about the criterion of meaning has to be construed and formulated in a way different from that in which it is usually done. In the first place we have to notice that this problem concerns the structure of language. (In my opinion this is true for all philosophical questions, but that is beyond our present discussion.) Hence a clear formulation of the question involves reference to a certain language; the usual formulations do not contain such a reference and hence are incomplete and cannot be answered. Such a reference once made, we must above all distinguish between two main kinds of questions about meaningfulness; to the first kind belong the questions referring to a historically given language-system, to the second kind those referring to a language-system which is yet to be constructed. These two kinds of questions have an entirely different character. A question of the first kind is a theoretical one; it asks, what is the actual state of affairs; and the answer is either true or false. The second question is a practical one; it asks, how shall we procede; and the answer is not an assertion but a proposal or decision. We shall consider the two kinds one after the other.

A *question of the first kind* refers to a given language-system L and concerns an expression E of L (i.e. a finite series of symbols of L). The question is, whether E is meaningful or not. This question can be divided into two parts: a) "Is E a sentence of L"?, and b) "If so, does E fulfill the empiricist criterion of meaning"? Question (a) is a formal question of logical syntax (comp. Chapter II); question (b) belongs to the field of methodology (comp. Chapter III). It would be advisable to avoid the terms 'meaningful' and 'meaningless' in this and in similar discussions – because these expressions involve so many rather vague philosophical associations – and to replace them by an expression of the form "a . . . sentence of L"; expressions of this form will then refer to a specified language and will contain at the place '. . .' an adjective which indicates the methodological character of the sentence, e.g. whether or not the sentence (and its negation) is verifiable or completely or incompletely confirmable or completely or incompletely testable and the like, according to what is intended by 'meaningful'.

18. The Construction of a Language-System L

A *question of the second kind* concerns a language-system L which is being proposed for construction. In this case the rules of L are not given, and the problem is how to choose them. We may construct L in whatever way we wish. There is no question of right or wrong, but only a practical question of convenience or inconvenience of a system form, i.e. of its suitability for certain purposes. In this case a theoretical discussion is possible only concerning the consequences which such and such a choice of rules would have; and obviously this discussion belongs to the first kind. The special question whether or not a given choice of rules will produce an empiricist language, will then be contained in this set of questions.

In order to make the problem more specific and thereby more simple, let us suppose that we wish to construct L as a physical language, though not as a language for all science. The problems connected with specifically biological or psychological terms, though interesting in themselves, would complicate our present discussion unnecessarily. But the main points of the philosophical discussions of meaning and testability already occur in this specialized case.

In order to formulate the rules of an intended language L, it is necessary to use a language L′ which is already available. L′ must be given at least practically and need not be stated explicitly as a language-system, i.e. by formulated rules. We may take as L′ the English language. In constructing L, L′ serves for two different purposes. First, L′ is the syntax-language[4] in which the rules of the object-language L are to be formulated. Secondly, L′ may be used as a basis for comparison for L, i.e. as a first object-language with which we compare the second object-language L, as to richness of expressions, structure and the like. Thus we may consider the question, to which sentences of the English language (L′) do we wish to construct corresponding sentences in L, and to which not. For example, in constructing the language of Principia Mathematica, Whitehead and Russell wished to have

[4] Comp. Carnap [4] §1; [5], p. 39.

available translations for the English sentences of the form "There is something which has the property φ"; they therefore constructed their language-system so as to contain the sentence-form "$(\exists x) \cdot \varphi x$". A difficulty occurs because the English language is not a language-system in the strict sense (i.e. a system of fixed rules) so that the concept of translation cannot be used here in its exact syntactical sense. Nevertheless this concept is sufficiently clear for our present practical purpose. The comparison of L with L′ belongs to the rather vague, preliminary considerations which lead to decisions about the system L. Subsequently the result of these decisions can be exactly formulated as rules of the system L.

It is obvious that we are not compelled to construct L so as to contain sentences corresponding to all sentences of L′. If e.g. we wish to construct a language of economics, then its sentences correspond only to a small part of the sentences of the English language L′. But even if L were to be a language adequate for all science there would be many – and I among them – who would not wish to have in L a sentence corresponding to every sentence which usually is considered as a correct English sentence and is used by learned people. We should not wish e.g. to have corresponding sentences to many or perhaps most of the sentences occurring in the books of metaphysicians. Or, to give a non-metaphysical example, the members of our Circle did not wish in former times to include into our scientific language a sentence corresponding to the English sentence

S_1: "This stone is now thinking about Vienna."

But at present I should prefer to construct the scientific language in such a way that it contains a sentence S_2 corresponding to S_1. (Of course I should then take S_2 as false, and hence $\sim S_2$ as true.) I do not say that our former view was wrong. Our mistake was simply that we did not recognize the question as one of decision concerning the form of the language; we therefore expressed our view in the form of an assertion – as is customary among philosophers – rather than in the form of a proposal. We used to say: "S_1 is not false but meaningless"; but the careless use of the word 'meaningless' has its dangers and is the second point in which we would like at present to modify the previous formulation.

We return to the question how we are to proceed in constructing a physical language L, using as L′ the English physical language.

The following list shows the items which have to be decided in constructing a language L.

I. Formative rules (= definition of 'sentence in L').
- A. Atomic sentences.
  - 1. The form of atomic sentences.
  - 2. The atomic predicates.
    - a. Primitive predicates.
    - b. Indirectly introduced atomic predicates.
- B. Formative operations of the first kind: Connections; Molecular sentences.
- C. Formative operations of the second kind: Operators.
  - 1. Generalized sentences. (This is the *critical point.*)
  - 2. Generalized predicates.

II. Transformative rules (= definition of 'consequence in L').
- A. L-rules. (The rules of logical deduction.)
- B. P-rules. (The physical laws stated as valid.)

In the following sections we shall consider in succession items of the kind I, i.e. the formative rules. We will choose these rules for the language L from the point of view of empiricism; and we shall try, in constructing this empiricist language L, to become clear about what is required for a sentence to have meaning.

19. Atomic Sentences: Primitive Predicates

The suitable method for stating formative rules does not consist in describing every single form of sentence which we wish to admit in L. That is impossible because the number of these forms is infinite. The best method consists in fixing

1. The forms of some sentences of a simple structure; we may call them (elementary or) *atomic sentences* (I A);

2. Certain *operations* for the formation of compound sentences (I B, C).

I A 1. Atomic sentences. As already mentioned, we will consider only predicates of that type which is most important for physical language, namely those predicates whose arguments are individual constants i.e. designations of space-time-points. (It may be remarked that it would be possible and even convenient to admit also full sentences of physical functors as atomic sentences of L, e.g. 'te(a) = r', corresponding to the sentence of L': "The temperature at the space-time-point *a* is r". For the sake of simplicity we will restrict the following considerations to predicate-sentences. The results can easily be applied to functor-sentences also.) An atomic sentence is a full sentence of an atomic predicate (Definition 15a, §9). An atomic predicate is ether primitive or introduced by an atomic chain (Definition 14b, §9). Therefore we have to answer the following questions in order to determine the form of the atomic sentences of L:

I A 2. a) Which predicates shall we admit as primitive predicates of L?

b) Which forms of atomic introductive chains shall we admit?

I A 2a: *Primitive predicates.* Our decision concerning question (a) is obviously very important for the construction of L. It might be thought that the richness of language L depends chiefly upon how rich is the selection we make of primitive predicates. If this were the case the philosophical discussion of what sentences were to be included in L – which is usually formulated as: what sentences are meaningful? – would reduce to this question of the selection of primitive predicates. But in fact this is not the case. As we shall see, the main controversy among philosophers concerns the formation of sentences by operators (I C 1). About the selection of primitive predicates agreement can easily be attained, even among representatives of the most divergent views regarding what is meaningful and what is meaningless. This is easily understood if we remember our previous considerations about sufficient bases. If a suitable predicate is selected as the primitive predicate of L, all other physical predicates can be introduced by reduction chains.

To illustrate how the selection of primitive predicates could be carried out, let us suppose that the person N_1 who is constructing the language L trusts his sense of sight more than his other senses. That may lead him to take the colour-predicates (attributed to things or space-time-points, not to acts of perception, compare the example given on p. 466, vol. 3) as primitive predicates of L. Since all other physical predicates are reducible to them, N_1 will not take any other primitive predicates. It is just at this point in selecting primitive predicates, that N_1 has to fact the question of observability. If N_1 possesses a normal colour sense each of the selected predicates, e.g. 'red', is observable by him in the sense explained before (§ 11). Further, if N_1 wishes to share the language L with other people – as is the case in practice – N_1 must inquire whether the predicates selected by him are also observable by them; he must investigate whether they are able to use these predicates in sufficient agreement with him, – whether it be subsequent to training by him or not. We may suppose that N_1 will come to a positive result on the basis of his experience with English-speaking people. Exact agreement, it is true, is not obtainable; but that is not demanded. Suppose however that N_1 meets a completely colour-blind man N_2. N_1 will find that he cannot get N_2 to use the colour predicates in sufficient agreement with him, in other words, that these predicates are not observable by N_2. If nevertheless N_1 wishes to have N_2 in his language-community, N_1 must change his selection of primitive predicates. Perhaps he will take the brightness-predicates which are also observable by him. But there might be a completely blind man N_3, for whom not one of the primitive predicates selected by N_1 is observable. Is N_3 now unable to take part in the total physical language of N_1? No, he is not. N_1 and N_3 might both take e.g. the predicate 'solid' as primitive predicate for their common language L. This predicate is observable both for N_3 and N_1, and it is a sufficient confirmation basis for the physical language L, as we have seen above. Or, if N_1 prefers to keep visual predicates as primitive predicates for L, he may suggest to N_3 that he take 'solid' as primitive predicate of N_3's language L_3 and then introduce the other predicates by reduction in such a way that they agree with the predicates of N_1's language L. Then L and L_3 will be completely congruent even as to the stock of predicates, though the selections of primitive predicates are different. How far N_1 will go in accepting people with restricted sensual faculties into his language-community, is a matter of practical decision. For our further considerations we shall suppose that only observable predicates are selected as primitive predicates of L. Obviously this restriction is not a necessary one. But, as empiricists, we want every predicate of our scientific language to be confirmable, and we must therefore select observable predicates as primitive ones. For the following considerations we suppose that the primitive predicates of L are observable without fixing a particular selection.

Decision 1. Every primitive descriptive predicate of L is observable.

20. *The Choice of a Psychological or a Physical Basis*

In selecting the primitive predicates for the physical language L we must pay attention to the question whether they are observable, i.e. whether they can be directly tested by perceptions. Nevertheless we need not demand the existence of sentences in L – either atomic or other kinds – corresponding to perception-sentences of L′ (e.g. "I am now seeing a round, red patch"). L may be a physical language constructed according to the demands of empiricism, and may nevertheless contain no perception-sentences at all.

If we choose a basis for the whole scientific language and if we decide as empiricists, to choose observable predicates, two (or three) different possibilities still remain open for specifying more completely the basis, apart from the question of taking a narrower or wider selection. For, if we take the concept 'observable' in the wide sense explained before (§ 11) we find two quite different kinds of observable predicates, namely physical and psychological ones.

1. Observable *physical predicates of the thing-language*, attributed to perceived things of any kind or to space-time-points. All examples of primitive predicates of L mentioned before belong to this kind. Examples of full sentences of such predicates: "This thing is brown," "This spot is quadrangular," "This space-time-point is warm," "At this space-time-point is a solid substance."

2. Observable *psychological predicates*. Examples: "having a feeling of anger," "having an imagination of a red triangle," "being in the state of thinking about Vienna," "remembering the city hall of Vienna." The perception predicates also belong to this kind, e.g. "having a perception (sensation) of red," ". . . of sour"; these perception predicates have to be distinguished from the corresponding thing-predicates belonging to the first kind (see vol. 3, p. 466). These predicates are observable in our sense in so far as a person N who is in such a state can, under normal conditions, be aware of this state and can therefore directly confirm a sentence attributing such a predicate to himself. Such an attribution is based upon that kind of observation which psychologists call introspection or self-observation, and which philosophers sometimes have called perception by the inner sense. These designations are connected with and derived from certain doctrines to which I do not subscribe and which will not be assumed in the following; but the fact referred to by these designations seems to me to be beyond discussion. Concerning these observable psychological predicates we have to distinguish two interpretations or modes of use, according to which they are used either in a phenomenological or in a physicalistic language.

2a. Observable psychological predicates *in a phenomenological language*. Such a predicate is attributed to a so-called state of consciousness with a temporal reference (but without spatial determination, in contradistinction to 2b). Examples of full sentences of such predicates (the formulation varies according to the philosophy of the author): "My consciousness is now in a state of anger" (or: "I am now . . .," or simply: "Now anger"); and analogously with "such and such an imagination," ". . . remembrance," ". . . thinking," ". . . perception," etc. These predicates are here interpreted as belonging to a phenomenological language, i.e. a language about conscious phenomena as non-spatial events. However, such a language is a purely subjective one, suitable for soliloquy only, while the intersubjective thing-language is suitable for use among different subjects. For the construction of a subjective language predicates of this kind may be taken as primitive predicates. Several such subjective languages constructed by several subjects may then be combined for the construction of an intersubjective language. But the predicates of this kind cannot be taken directly as observable primitive predicates of an intersubjective language.

2b. Observable psychological predicates *in a physicalistic language*. Such a predicate is attributed to a person as a thing with spatio-temporal determination. (I believe that this is the use of psychological predicates in our language of everyday life, and that they are used or interpreted in the phenomenological way only by philosophers.) Examples of full sentences: "Charles was angry yesterday at noon," "I (i.e. this person, known as John Brown) have now a perception of red," etc. Here the psychological predicates belong to an intersubjective language. And they are intersubjectively confirmable. N_2 may succeed in confirming such a sentence as "N_1 is now thinking of Vienna" (S), as is constantly done in everyday life as well as in psychological investigations in the laboratory. However, the sentence S is confirmable by N_2 only incompletely, although it is completely confirmable by N_1. [It seems to me that there is general agreement about the fact that N_1 can confirm more directly than N_2 a sentence concerning N_1's feelings, thoughts, etc. There is disagreement only concerning the question whether this difference is a fundamental one or only a difference in degree. The majority of philosophers, including some members of our Circle in former times, hold that the difference is fundamental inasmuch as there is a certain field of events, called the consciousness of a person, which is absolutely inaccessible to any other person. But we now believe, on the basis of physicalism, that the difference, although very great and very important for practical life, is only a matter of degree and that there are predicates for which the directness of confirmation by other persons has intermediate degrees (e.g. 'sour' and 'quadrangular' or 'cold' when attributed to a piece of sugar in my mouth). But this difference in opinion need not be discussed for our present purposes.] We may formulate the fact mentioned by saying that the psychological predicates in a physicalistic language are intersubjectively confirmable but only *subjectively observable*. [As to testing, the difference is still greater. The sentence S is certainly not completely testable by N_2; and it seems doubtful whether it is at all testable by N_2, although it is certainly confirmable by N_2.] This feature of the predicates of kind 2b is a serious disadvantage and constitutes a reason against their choice as primitive predicates of an inter-subjective language. Nevertheless we would have to take them as primitive predicates in a language of the whole of science if they were not reducible to predicates of the kind 1, because in such a language we require them in any case. But, if physicalism is correct they are in fact reducible and hence dispensable as primitive predicates of the whole language of science. And certainly for the physical language L under construction we need not take them as primitive.

According to these considerations, it seems to be preferable to choose the primitive predicates from the predicates of kind 1, i.e. of the observable thing-predicates. These are the only inter-subjectively observable predicates. In this case, therefore, the same choice can be accepted by the different members of the language community. We formulate our decision concerning L, as a supplement to Decision 1:

Decision 2. Every primitive predicate of L is a thing predicate.

The choice of primitive predicates is meant here as the choice of a basis for possible confirmation. Thus, in order to find out whether the choice of primitive predicates of the kind 1 or 2a or 2b corresponds to the view of a certain philosopher, we have to examine what he takes as the basis for empirical knowledge, for confirmation or testing. *Mach*, by taking the sensation elements ('Empfindungselemente') as basis, can be interpreted as a representative of the standpoint 2a; and similarly other positivists, sensationalists and idealists. The views held in the first period of the Vienna Circle were very much influenced by positivists and above all by Mach, and hence also show an inclination to the view 2a. I myself took elementary experiences ('Elementar-erlebnisse') as basis, (in [1]). Later on, when our Circle made the step to physicalism, we abandoned the phenomenological language recognizing its subjective limitation.[5] *Neurath*[6] requires for the basic

[5] Comp. Carnap [2], §6.
[6] Neurath [5] and [6] p. 361.

sentences ('Protokollsätze'), i.e. those to which all confirmation and testing finally goes back, the occurrence of certain psychological terms of the kind 2b—or: of biological terms, as we may say with Neurath in order to stress the physicalistic interpretation—namely designations of actions of perception (as physicalistic terms). He does not admit in these basic sentences such a simple expression as e.g. "a black round table" which is observable in our sense but requires instead "a black round table perceived (or: seen) by Otto." This view can perhaps be interpreted as the choice of predicates of the kind 2b as primitive ones. We have seen above the disadvantages of such a choice of the basis. *Popper*[7] rejects for his basic sentences reference to mental events, whether it be in the introspective, phenomenological form, or in physicalistic form. He characterizes his basic sentences with respect to their form as singular existential sentences and with respect to their content as describing observable events; he demands that a basic sentence must be intersubjectively testable by observation. Thus his view is in accordance with our choice of predicates of the kind 1 as primitive ones. He was, it seems to me, the first to hold this view. (The only inconvenient point in his choice of basic sentences seems to me to be the fact that the negations of his basic sentences are not basic sentences in his sense.)

I wish to emphasize the fact that I am in agreement with Neurath not only in the general outline of empiricism and physicalism but also in regard to the question what is to be required for empirical confirmation. Thus I do not deny—as neither Popper nor any other empiricist does, I believe—that a certain connection between the basic sentences and our perceptions is required. But, it seems to me, it is sufficient that the biological designations of perceptive activity occur in the sormulation of the methodological requirement concerning the basic fentences—as e.g. in our formulation "The primitive descriptive predicates have to be observable," where the term "observable" is a biological term referring to perceptions—and that they need not occur in the basic sentences themselves. Also a language restricted to physics as e.g. our language L without containing any biological or perception terms may be an empiricist language provided its primitive descriptive predicates are observable; it may even fulfill the requirement of empiricism in its strictest form inasmuch as all predicates are completely testable. And this language is in its nature quite different from such a language as e.g. that of theoretical physics. The latter language—although as a part

[7] Popper [1] p. 58 f.

of the whole language of science, it is an empiricist language because containing only confirmable terms—does not contain observable predicates of the thing-language and hence does not include a confirmation basis. On the other hand, a physical language like L contains within itself its basis for confirmation and testing.

21. Introduced Atomic Predicates

Beside the question just discussed concerning the choice of a psychological or a physical basis no problems of a fundamental, philosophical nature arise in selecting primitive predicates. In practice, an agreement about the selection can easily be obtained, because every predicate whose observability could be doubted—as e.g. electric field or the like – can easily be dispensed with. As mentioned before, the whole situation described here is not logically necessary, but a contingent character of the system of predicates in their relation to reducibility and consequently to the laws of science. This character of the system of science explains the historical fact that nearly all controversies among contemporary philosophers – at least among those who reject trans-empirical speculative metaphysics – about the limitation of language do not concern the selection of primitive predicates but the selection of formative operations to be admitted. These operations will be considered later on.

As we have seen, the question of observability has to be decided only for the predicates to be chosen as primitive predicates. Our description of the process of their selection has shown that it is an empirical question, not a logical one. All other questions of confirmability of a given predicate concern indirect confirmation, which depends upon the logical, i.e. syntactical relations between the predicate in question and observable predicates. Thus these further questions of confirmability concern the structure of the language, namely the form of definitions and reduction sentences. However, the question of testability of a given predicate involves, in addition, another empirical question, namely whether certain confirmable predicates are realizable.

IA2b. Indirectly introduced atomic predicates. In addition to the primitive predicates of the physical language L other predicates have to be introduced by introductive chains. We have to decide – first for atomic predicates, and later on also for predicates of other kinds – whether to admit in introductive chains definitions only, or also reduction sentences of the general form. In our previous considerations we have seen that the introduction by reduction is practically indispensable. Therefore we decide to admit it. There are two possibilities: we may or may not restrict the introductive chains in L to test chains. We will leave this point undecided and formulate the two possible forms of our decision:

Decision 3. Introductive chains containing reduction pairs are admitted in L,
either a) *only* in the form of *test chains*,
or b) without restriction to test chains.

Theorem 15. If the primitive predicates of a language are observable – as e.g. in our language L according to Decision 1 —all atomic predicates are completely confirmable; moreover, they are completely testable if only test chains are admitted – as e.g. in L in the case of Decision 3a. – This follows from Theorem 8 (§ 12).

22. *Molecular Sentences*

After considering the question of the atomic sentences of L (I A in the list of p. 6), we have to consider the second part of the formative rules, namely the rules determining what operations for the formation of compound sentences are to be admitted. We have to distinguish two main kinds of such operations:

1) the formation of molecular sentences with the aid of connections (I B);
2) the formation of generalized sentences with the aid of operators (I C).

IB: Connections. There are two kinds of sentential connections. The so-called extensional connections or truth-functions are characterized by the fact that the truth-value of any compound sentence constructed with their help depends only upon the truth-values of the component sentences. The connections of the usual sentential calculus mentioned before are extensional (see § 5): negation, disjunction, conjunction, implication, equivalence. The non-extensional connections are called intensional[8]; to them belong e.g. Lewis' strict implication[9] and the so-called modal functions.[10] In the case of an intensional connection the truth-value of a compound sentence depends upon the truth-values as well as the forms of the component sentences. (Here it is presupposed that sufficient L-rules are stated for the connective symbol in question; if that is not the case the symbol is, strictly speaking, not a logical, but a descriptive one[11] and hence would have to be introduced on the basis of the primitive descriptive predicates.)

That the extensional connections are admissible and even necessary (at least a sufficient selection of one or two of them by which the others can be defined if desired) is not in doubt. But whether or not they are sufficient, i.e. whether or not intensional connections are also desirable or perhaps necessary for the expressiveness of the language, is still discussed by logicians. I believe that we can dispense with them without making the language poorer.[12] However, the question is not important for our present problem concerning meaningfulness, because those who prefer not to introduce the connections of this kind, do not deny that they are meaningful.

For the sake of simplicity we will not use intensional connections in language L.

Decision 4. The sentential connections in L are extensional. This decision seems to be justified by the fact that so far no concept needed for a language of science is known which could not be expressed in a language having extensional connections only; e.g. the concept of probability can also be expressed extensionally. Of course this decision is here made only for the language L as an

[8] For the lack of better terms I keep Russell's terms 'extensional' and 'intensional'; it is to be noticed that here they have only the above given meaning, not the meaning they have in traditional philosophy.

[9] C. I. Lewis and C. H. Langford [1].

[10] Comp. Carnap [4] §69.

[11] Comp. Carnap [4] §50 and 62.

[12] Comp. Carnap [4] §70.

object of our present considerations and does not at all intend to dispose of the whole problem. – The restriction to extensional predicates was presupposed in our former definitions of 'molecular form', 'molecular predicate', 'molecular sentence'; hence these definitions can now be applied to L.

Theorem 16. If the primitive predicates of a language are observable – as they are e.g. in L according to Decision 1 – the following is true. a. All molecular predicates are completely confirmable and all molecular sentences are bilaterally completely confirmable. b. If only test chains are admitted – as e.g. in L in the case of Decision 3a – all molecular predicates are completely testable and all molecular sentences are bilaterally completely testable. This follows from Theorems 8 and 9 (§ 12).

A universal or existential sentence which is restricted to a finite field (as e.g. the sentences constructed with restricted operators in the languages I and II dealt with in Carnap [4]) can be transformed into a conjunction or a disjunction respectively and therefore has the same character as a molecular sentence. It is also completely confirmable, if the predicates occurring are completely confirmable. If such sentences occurred in L it would be convenient to include them among the molecular sentences. But we will suppose that L does not contain sentences of this kind.

23. Molecular Languages

The fact that the molecular sentences are completely confirmable and, in the case of Decision 3a, also completely testable, is an important advantage of these sentences over the essentially generalized sentences. Let us call a language limited to molecular sentences exclusively, a *molecular language.* Such a language fulfills the requirements of confirmability and testability in its most radical form. Hence we understand the fact that certain epistemologists, especially positivists, propose or demand a molecular language as the language of science. We shall regard as examples the views of Russell, Wittgenstein, Schlick and Ramsey.

In a molecular language unrestricted universality cannot be expressed. Therefore, if such a language is chosen, we have to face the problem of how to deal with the physical laws. There seem to be in the main two possible ways. A law may be expressed in the form of a molecular sentence, namely a restricted universal sentence or a conjunction, concerning those instances of the law which have been observed so far. On the other hand a law may be taken, not as a sentence, but as a rule of inference according to which one molecular sentence (e.g. a prediction about a future event) can be inferred from other ones (e.g. sentences about observed events). Each of these ways has actually been followed, as we shall see.

Russell asserts the following thesis in discussing the "question of the verifiability of physics"[13]: "Empirical knowledge is confined to what we actually observe."[14] This view is perhaps influenced by *Mach's* positivism.[15] If we wish to interpret this thesis we have to make it clearer by translating it from the material idiom into a formal (or a semi-formal) one (comp. § 4): "The assertions of empirical science are confined to those sentences which are deducible from stated observation-sentences" (i.e. from sentences about actual observations). As this thesis is true for a molecular language of a certain kind, but not for a language containing physical laws in the form of unrestricted universal sentences, we may interpret Russell's view as presupposing a molecular language.

Wittgenstein, perhaps influenced by Mach and Russell, requires that every sentence must be completely verifiable.[16] Thus we might expect him to acknowledge as legitimate only a molecular language. And indeed he asserts that "propositions are truth-functions of elementary propositions,"[17] "all propositions are results of truth-operations on the elementary propositions";[18] here truth-functions are conceived as not including general operators.[19] In consequence of this, Wittgenstein does not acknowledge physical laws as sentences in the proper sense, but takes them as rules for forming (or rather, stating) sentences, thus choosing the

[13] Russell [2] p. 110.
[14] l.c., p. 112.
[15] Comp. l.c., p. 123.
[16] Comp. *Waismann* [1], p. 229.
[17] Wittgenstein [1], prop. 5, p. 103.
[18] l.c., prop. 5.3, p. 119.
[19] l.c., prop. 5.521, p. 135.

second of the two ways mentioned above. This view of Wittgenstein is reported by *Schlick* who is himself in agreement with it.[20]

Ramsey propounds a quite similar view, perhaps influenced by Wittgenstein. A universal sentence like "All men are mortal" – he calls it a variable hypothetical – is not a conjunction, because "it cannot be written out as one";[21] "if then it is not a conjunction, it is not a proposition at all";[22] "variable hypotheticals are not judgments, but rules for judging 'If I meet a ϕ, I shall regard it as a ψ' ";[23] a variable hypothetical "is not strictly a proposition at all, but a formula from which we derive propositions."[24]

Previously, influenced also by Mach and Russell, I too accepted a molecular language.[25] According to the positivistic principle of testability in its most radical form, I restricted the atomic sentences to sentences about actual experiences. The laws of physics as well as all predictions were interpreted as records of present and (remembered) past experiences, namely those experiences from which the law or the prediction is usually said to be inferred by induction. Thus I followed the first of the two ways mentioned above; the physical laws also were interpreted as molecular sentences. At present I no longer hold this view. But I do not think – as Lewis and Schlick do – that it was false. I think it is

[20] *Schlick* [1] p. 150: "A definitive verification" of a natural law "is, strictly speaking, impossible"; it follows from this that a law, "logically considered, does not have the character of an assertion, for a genuine assertion must admit of being definitively verified." It follows from the fact "that one can never actually speak of an absolute verification of a natural law" that "a natural law essentially does not possess the logical character of an 'assertion,' but rather presents an 'instruction for the formation of assertions' (I am indebted to Ludwig Wittgenstein for these ideas and terms)" (l. c. p. 151). "Instructions of this kind occur grammatically in the guise of ordinary sentences." By this explanation, "the problem of induction becomes pointless," i.e. "the question of the logical justification of universal sentences about reality." "We recognize with Hume that there is no logical justification for them; there can be none because they are not genuine sentences. Natural laws are not 'general implications' (to use the language of the logician); because they cannot be verified for *all* cases; rather, they are prescriptions, rules of procedure for the investigator to discover true sentences" (l. c. p. 156).

[21] Ramsey [1], p. 237.

[22] l.c., p. 238.

[23] l.c., p. 241.

[24] l.c., 251.

[25] Carnap [1].

true concerning a molecular language (of a special kind). But I was wrong in thinking that the language I dealt with was *the* language, i.e. the only legitimate language, – as Wittgenstein, Schlick and Lewis likewise seem to think concerning the language-forms accepted by them. Consequently I made the mistake of formulating my epistemological view in the form of an assertion – as most philosophers do – instead of in the form of a suggestion concerning the form of language. At present I think that the whole question is a matter of choice, of convention; and further, that a molecular language can be chosen as the language of science, but that a non-molecular, generalized one is much more suitable and, in addition, closer to the actual practice of science. This will soon be explained.

It may be mentioned that in the discussion about the logical foundations of mathematics, some finitists or intuitionists, e.g. *Weyl*, *Brouwer* and *Kaufmann*, sometimes express opinions which are related to those just quoted and which may be understood as arguing in favor of a molecular language. Thus for instance *Kaufmann*[26] rejects unrestricted universal sentences (except the *a priori* ones), because they are not verifiable. In *Weyl's*[27] opinion a pure existential judgment (as he calls it) is not a proper judgment, but a 'judgment-abstract', similar to a description of a hidden treasure without indication of its place; and a universal judgment is not a proper judgment, but a rule for judgments ('Urteilsanweisung'). We will not analyse here the views of these authors in detail, because they are chiefly concerned with mathematics rather than empirical science.

24. The Critical Problem: Universal and Existential Sentences

So far we have considered the first kind of operations by which compound sentences may be constructed out of atomic sentences, namely the construction of molecular sentences by the help of connections. Now we have to deal with the second kind of operations (I c in the list of p. 6), namely the construction of generalized sentences with the aid of universal and existential operators.

[26] Kaufmann [1], p. 10.

[27] Weyl [1], p. 19.

We shall suppose, that no sentences occur in language L with finitely restricted operators or with free variables. As mentioned before, the former ones have the same character as molecular sentences; the latter ones have the same character as sentences with universal operators. For the sake of simplicity we will consider in the following only operators of the lowest type, i.e. those with individual variables, not with predicate- or functor-variables. The operators of the lowest type are the most important ones in physics and generally in science; and all fundamental problems of meaning, confirmation and testing discussed in present philosophy already arise in connection with these operators. – Accordingly the term 'operator (in L)' is to be understood in the following as 'operator (not finitely restricted) with an individual variable'.

The purpose of the following considerations is to enable us to decide whether or not we will admit the application of operators in L and, if so, to what extent. In the following, 'M_1', 'M_2', etc. are taken as molecular predicates. Any molecular sentence can be transformed into (i.e. is equipollent to) a full sentence of a molecular predicate defined in a suitable way.

If we at all admit operators in L we may allow beside generalized sentences of the simplest form, such as '$(x)M(x)$' and '$(\exists x)M(x)$', also those with a more complicated form, as e.g. '$(\exists x)(y)M_1(x, y)$' or '$(x)(\exists y)(z)M_2(x, y, z)$'. The last example corresponds to the English sentence (of L'): "For every point *x* there exists a point *y* such that for every point *z* $M_2(x, y, z)$".

In Theorem 3, § 6, we stated a certain relation between '$(x)P_1(x)$' (S_1) and the full sentences of 'P_1'. Now the same relation subsists between '$(x)(y)P_2(x, y)$' (S_2) and the full sentences of 'P_2' because '$P_2(a, b)$' is a consequence of '$(y)P_2(a, y)$'; this last is a consequence of S_2, so that '$P_2(a, b)$' is itself a consequence of S_2, although S_2 is not a consequence of any finite class of full sentences of 'P_2'. Furthermore, the relation which we stated in Theorem 4 (§ 6) between '$(\exists x)P_1(x)$' (S_3) and the full sentences of 'P_1' also subsists between '$(\exists x)(\exists y)P_2(x, y)$' ($S_4$) and the full sentences of 'P_2'; for S_4 is a consequence of '$(\exists y)P_2(a, y)$', which is a consequence of '$P_2(a, b)$', so that S_4 is a consequence of '$P_2(a, b)$', although $\sim S_4$ is not a consequence of any finite class of negations of full sentences of 'P_2'. Thus we see that for the question of confirmation a series of several operators of the same kind – that is to say all of them universal or all of them existential – has the same character as one operator of that kind.

First we will deal with only such generalized sentences of L as contain molecular predicates only. A sentence of this kind is constructed out of molecular predicates with the help of connections and operators. As is well-known such a sentence can be transformed into the so-called normal form[28] consisting of an operand which does not contain operators and is preceded by a series of operators without negation symbols. With the help of a molecular predicate defined in a suitable way we may transform the operand into '$M(x, ..)$'. We next divide the series of operators of such a sentence S into sub-series each containing one or several operators of the same kind, that is to say, all of them universal or all of them existential; we call these sub-series the operator sets of S. Finally, we classify the sentences of the form described in the following way. The class of those sentences which have n operator-sets is called U_n, if the first operator is a universal one, and E_n, if the first operator is an existential one. The class U_o is the same as E_o; it is the class of the molecular sentences. Instead of "a sentence of the form U_n" we shall write shortly "a U_n"; and analogously "an E_n". A U_1 has one or more universal operators only, an E_1 one or more existential operators. To U_2 belong the sentences of the form '$(x)(\exists\, y)M(x, y)$', but likewise '$(x_1)(x_2)(\exists\, y_1)(\exists\, y_2)(\exists\, y_3)M(x_1,x_2,y_1,y_2,y_3)$' etc., and generally every sentence consisting of a set of universal operators succeeded by a set of existential operators and by a molecular operand. To U_3 belongs every sentence constructed in the following way: first a set of universal operators, then a set of existential operators, then a set of universal operators, and finally a molecular operand.

Theorem 17. If S is a U_{n+1}, the confirmation of S is incompletely reducible to that of certain E_n, and the confirmation of $\sim$ S is completely reducible to that of each among certain U_n.

Proof. For n = o, this follows easily from Theorem 3 (§ 6).

[28] Comp. Hilbert [1] p. 63; Carnap [4b] §34b, RR 9.

For n $>$ 0, let S be '$(x_1)(\exists x_2)(x_3) \ldots x_{n+1})M(x_1, \ldots x_{n+1})$'. We define 'P' by '$P(x_1) \equiv (\exists x_2)(x_3) \ldots x_{n+1})M(x_1, \ldots x_{+1})$'. Then S can be transformed into '$(x_1)P(x_1)$'. Therefore, according to Theorem 3 (§ 6), the confirmation of S is incompletely reducible to that of the full sentences of 'P'; and the confirmation of $\sim$ S is completely reducible to anyone of their negations. Now a full sentence of 'P', say 'P(a)', can be transformed into '$(\exists x_2)(x_3) \ldots x_{n+1})M(a, x_2, \ldots x_{n+1})$' and is therefore an E_n. '$\sim P(a)$' can be transformed into '$(x_2)(\exists x_3) \ldots x_{n+1})$ $[\sim M(a, x_2, \ldots x_{n+1})]$' and is therefore a U_n.

Theorem 18. If S is an E_{n+1}, the confirmation of S is completely reducible to that of each among certain U_n, and the confirmation of $\sim$ S is incompletely reducible to that of certain E_n.

Proof. For n = 0, this follows easily from Theorem 4 (§ 6). For n $>$ 0, let S be '$(\exists x_1)(x_2)(\exists x_3) \ldots x_{n+1})M(x_1, \ldots x_{n+1})$'. We define 'P' by '$P(x_1) \equiv (x_2)(\exists x_3) \ldots x_{n+1})M(x_1, \ldots x_{n+1})$'. Then S can be transformed into '$(\exists x_1)P(x_1)$'. Therefore, according to Theorem 4 (§ 6), the confirmation of S is completely reducible to that of any full sentence of 'P'; and the confirmation of $\sim$ S is incompletely reducible to that of the negations of the full sentences of 'P'. A full sentence 'P(a)' can be transformed into '$(x_2)(\exists x_3) \ldots x_{n+1})M(a, x_2, \ldots x_{n+1})$' and is therefore a U_n. '$\sim P(a)$' can be transformed into '$(\exists x_2)(x_3) \ldots x_{n+1})$ $[\sim M(a, x_2, \ldots x_{n+1})]$' and is therefore an E_n.

Theorem 19. If the primitive predicates of a language are observable – as they are e.g. in L – and if S is a U_1, i.e. of the form '$(x)M(x)$', the following is true. a. S is incompletely confirmable and $\sim$ S completely confirmable. b. If only test chains are admitted – as e.g. in L in the case of Decision 3a – S is incompletely testable and $\sim$ S completely testable. – This follows from Theorem 3 (§ 6) and Theorem 16 (§ 22).

Theorem 20. If the primitive predicates of a language are observable – as they are e.g. in L – and if S is an E_1, i.e. of the form '$(\exists x)M(x)$', the following is true. a. S is completely confirmable and $\sim$ S incompletely confirmable. b. If only test chains are admitted – as e.g. in L in the case of Decision 3a – S is completely testable and $\sim$ S incompletely testable. – This follows from Theorem 4 (§ 6) and Theorem 16 (§ 22).

The Theorems 19 and 20 correspond to the customary but not quite correct formulation: "a universal sentence is not verifiable but falsifiable; an existential sentence is verifiable but not falsifiable."

Theorem 21. If the primitive predicates of a language are observable – as they are e.g. in L – and if S is a U_n or an E_n with $n > 1$, thus containing at least one universal operator and simultaneously at least one existential operator, the following is true. a. Both S and $\sim$ S are incompletely confirmable, and hence S is bilaterally confirmable. b. If only test chains are admitted both S and $\sim$ S are incompletely testable, and hence S is bilaterally testable. – This follows from Theorems 17 and 18.

Thus we have seen that all generalized sentences of L of the forms described before are confirmable, and, in the case of Decision 3a, testable. The E_1 and the negations of U_1 are completely confirmable (or completely testable, respectively); all the other generalized sentences – provided they are essentially generalized— are only incompletely confirmable (or incompletely testable, respectively). No essentially generalized sentence is bilaterally completely confirmable or bilaterally completely testable.

25. The Scale of Languages

This being the case, how shall we decide about admitting of generalized sentences in the language L? This is the most critical question. In regard to it there are fundamental differences among philosophers, which are very sharply discussed. There is an infinite number of possible answers, i.e. of possible choices concerning the limitation of language. Among the possible language-forms we may choose the chief ones and order them in a series with regard to the highest degree of complexity admitted in them. But how may we determine this degree? It is natural to assume, if $m > n$, that a U_m is more complicated than a U_n, and an E_m as more so than an E_n. But how are we to decide the order of U_n with respect to E_n? We may do so by establishing the convention to take U_n as simpler than E_n. This convention is practically justified by the fact that some philosophers admit

U_1 but not E_1, or U_2 but not E_2; the attempt to give theoretical reasons for this convention has been made by Popper, as we shall see. Thus we obtain a progression of languages L_o, L_1, etc., starting with the molecular language L_o and going on to languages of greater and greater extentions. Every language in the following table contains the sentences of the previous languages and, in addition, the sentences of the class given in the second column. After this endless series we may put the language L_∞ which is to contain all the sentences of the languages of the series L_o, L_1 . . . L_n, . . . (with finite n) but no others.

Language	Sentences of maximal complexity admitted in L_n	
	Class	Example
L_0	U_0, E_0 (both molecular)	$M_1(a)$
L_1	U_1	$(x)M_1(x)$
L_2	E_1	$(\exists x)M_1(x)$
L_3	U_2	$(x)(\exists y)M_2(x, y)$
L_4	E_2	$(\exists x)(y)M_2(x, y)$
L_5	U_3	$(x)(\exists y)(z)M_3(x, y, z)$
L_6	E_3	$(\exists x)(y)(\exists z)M_3(x, y, z)$
.	.	.
.	.	.
.	.	.
L_∞	no maximal complexity; sentences of any such class with any number of operator sets are admitted.	

Note on L_o, molecular language. We have considered above some examples of philosophers who propose or require L_o, that is, who demand the limitation to molecular sentences. From our last considerations it is clear that to accept the requirement of complete confirmability or that of complete testability means to exclude generalized sentences and hence to state L_o. The step of dropping that requirement and choosing one of the wider languages instead of L_o is a decisive one. One of the chief reasons in favour of this decision is the fact, that both methods of interpreting physical laws in the case of L_o which we mentioned above

(§ 23) are not very convenient for practical use and, above all, are not in close conformity with the actual method adopted by physicists. For in the first place, in actual practice laws are not dealt with as reports; and secondly, they are connected with one another or with singular sentences in a form of a disjunction or conjunction or implication or equivalence, etc.; in other words: they are manipulated like sentences, not like rules. (These reason are not proofs for an assertion, but motives for a decision.)

I believe that *Morris*[29] is right in saying that by the step described, i.e. the adoption of a generalized language which is able to express physical laws in a satisfactory way, we ("logical positivists") come to a closer agreement with pragmatism. Morris[30] considers the two movements as complementary in their views, and as convergent in the directions of their present development.

Note on L_1. We may take *Popper's*[31] *principle of falsifiability* as an example of the choice of this language. Popper is however very cautious in the formulation of his limiting principle ("Abgrenzungskriterium"); he does not call the sentences E_1 meaningless, but only non-empirical and metaphysical. (Perhaps he wishes to exclude existential sentences and other metaphysical sentences not from the language altogether, but only from the language of empirical science.) At first sight, universal and existential sentences seem to be coördinate with each other. In pure logic there is indeed a complete symmetry between them (principle of duality), but in epistemology, i.e. in applied logic considered from the point of view of confirmation and testing, there is difference[32] which has often been noticed. – Also some intuitionists object more to existential than to universal sentences, and sometimes only to the former ones. Therefore they may perhaps be taken as supporters of L_1. – I[33] have stated a language

[29] Morris [1] p. 6.

[30] l.c., p. 1.

[31] Popper [1], p. 12, 33.

[32] Popper ([1] Ch. II and IV) especially has emphasized the fact that for scientific testing falsifiability is more important than verifiability, and therefore (in our terminology:) sentences whose negations are completely confirmable are preferable to those whose negations are only incompletely confirmable though they are themselves completely confirmable, and hence U_1 preferable to E_1.

[33] Carnap [4], Language I.

which contains U_1 (with free variables, not with operators) but not E_1 and therefore may also be taken as an example of L_1; but this language has not been proposed as the language of science.

Note on L_3. While Popper in theory states the principle of falsifiability and in consequence takes the language-form L_1, in practice he seems to me to take the more liberal form L_3. He shows that probability-sentences are sentences of the form U_2 which he calls existential hypotheses ("Es-gibt-Hypothesen"[34]). He admits that probability-sentences are essential for physics, and therefore he includes them into the language of physics, which thus seem to have the form L_3. The way in which he tries to show that the admission of existential hypotheses is compatible with his requirement of falsifiability, is less important for our present consideration. He admits that they are neither falsifiable nor verifiable[35] – in our terminology: neither their negations nor they themselves are completely confirmable – but he tries to show that according to certain methodological rules they are manipulated like falsifiable sentences and actually are sometimes falsified.[36]

Note on L_∞. I am at present inclined to accept this most liberal form of language, including sentences with any number of operator-sets. If one sees, e.g. from Popper's explanations, how convenient and even essential the sentences U_2 are for physics, and if in consequence one decides to admit this form, then it seems rather arbitrary to limit the number of operator-sets to two or any fixed higher number and not to admit more complicated forms. It is true that the greater the number of operator-sets in a sentence S is, the greater is the distance of S from the empirical basis, i.e. from the atomic sentences, and hence the more indirect and incomplete is the possibility of confirming or testing S and $\sim$ S. But there is no number of operator-sets for which the connection with the empirical basis would completely vanish. If operators once are admitted and thereby the requirement of complete confirmability or complete testability is dropped, there

[34] Popper [1], p. 135.

[35] l.c., p. 134.

[36] l.c., p. 140, 144.

seems to me to be no natural limit at any finite number of operator-sets.

After anyone of the languages $L_o, L_1 \ldots L_\infty$ is chosen we may decide between Decision 3a and 3b (§ 21). In the case of Decision 3a all introductive chains are test chains and hence all predicates and all sentences of the language are *testable*. A language L_n restricted in this way, may be designated by 'L_n^t'. Thus we have a second series of languages: $L_0^t, L_1^t, L_2^t, \ldots L^t_\infty$.

I C 2: Generalized predicates. If we have a language in which operators are admitted then we may also admit them in definitions, i.e. state generalized definitions and general introductive chains containing such definitions.

We have considered so far only such generalized sentences as have a molecular operand. We did this for the sake of simplicity, because the definition of the single languages of the series L_o, L_1, etc. can be stated more easily in this case. But if we come to language L_∞ in which the use of operators is not limited then for this language we may also admit the occurrence of any number of generalized predicates in the operand.

26. Incompletely Confirmable Hypotheses in Physics

Now let us consider under what circumstances a physicist might find it necessary or desirable to state an hypothesis in a generalized form. Let us begin with one operator. The full sentences of a molecular predicate 'M_1' (i.e. '$M_1(a)$', etc.) are bilaterally completely confirmable. Suppose some of them are confirmed by observations, but not the negation of any of them so far. This fact may suggest to the physicist the sentence '$(x)M_1(x)$' of U_1 as a physical law to be adopted, i.e. a hypothesis whose negation is completely confirmable and which leads to completely confirmable predictions as consequences of it (e.g. '$M_1(b)$' etc.). If more and more such predictions are confirmed by subsequent observations, but not the negation of any of them, we may say that the hypothesis, though never confirmed completely, is confirmed in a higher and higher degree.

Considerations of this kind are very common; they are often used in order to explain that the admission of not completely confirmable ("unverifiable") universal hypotheses does not infringe the principle of empiricism. Such considerations are, I think, agreed to by all philosophers except those who demand complete confirmability ("verifiability") and thereby the limitation to a molecular language.

Now it seems to me that a completely analogous consideration applies to sentences with any number of operator sets, i.e. to sentences of U_n or E_n for any n. The following diagram may serve as an example. A *broken* arrow running from a sentence S to a class C of sentences indicates that the confirmation of S is *incompletely* reducible to that of C. S is in this case a universal sentence and C the class of its instances; each sentence of C is therefore a consequence of S, but S is not a consequence of any finite sub-class of C. A *solid* arrow running from S_1 to S_2 indicates that the confirmation of S_1 is *completely* reducible to that of S_2. In this case, S_1 is an existential sentence and a consequence of S_2. The relation of reducibility of confirmation as indicated in the diagram is in accordance with Theorems 17 and 18 (§ 24), but, for these cases, can easily be seen by glancing at the sentences. At the left side are indicated the classes to which the sentences belong.

Let us start at the bottom of the diagram. The sentences of C_1 are molecular, and hence bilaterally completely testable. Let us suppose that a physicist confirms by his observations a good many of the sentences of C_1 without finding a confirmation for the negation of any sentence of C_1. According to the customary procedure described above these experiences will suggest to him the adoption of S_1 as a well-confirmed hypothesis, which, by further confirmation of more and more sentences of C_1, may acquire an even higher degree of confirmation. Let us suppose that likewise the sentences of C_2 are confirmed by observations, further those of C_3, etc. Then the physicist will state S_2, S_3 etc. as well-confirmed hypotheses. If now sentences of the form E_2 are admitted in L, then the first sentence of C is a sentence of L, is also a consequence of S_1 and is therefore confirmed to the same degree as S_1. In order to make feasible the formulation of this well-confirmed hypothesis the physicist will be inclined to admit

U_5: $(x)P'(x)$

(S':) $(v)(\exists w)(x)(\exists y)(z)M'(v, w, x, y, z)$

$\downarrow$

C'

E_4: $P'(d_1)$ $\quad$ $P'(d_2)$ $\quad$ $P'(d_3)$ $\cdots$

$(\exists w)(x)(\exists y)(z)M'(d_1, w, x, y, z)$

$\downarrow$

$(x)(\exists y)(z)M'(d_1, e_1, x, y, z)$

U_3: $(x)P(x)$

(S:) $(x)(\exists y)(z)M(x, y, z)$

$\downarrow$

C

E_2:

$P(a_1)$	$P(a_2)$	$P(a_3)$	
$(\exists y)(z)M(a_1, y, z)$	$(\exists y)(z)M(a_2, y, z)$	$(\exists y)(z)M(a_3, y, z)$	$\cdots$

$\downarrow$ $\quad$ $\downarrow$ $\quad$ $\downarrow$

U_1:

(S_1:) $(z)M(a_1, b_1, z)$	(S_2:) $(z)M(a_2, b_2, z)$	(S_3:) $(z)M(a_3, b_3, z)$	$\cdots$
$\downarrow$	$\downarrow$	$\downarrow$	
C_1	C_2	C_3	

$U_0(E_0)$: (molecular)

$M(a_1, b_1, c_1)$	$M(a_2, b_2, c_2)$	$M(a_3, b_3, c_3)$	
$M(a_1, b_1, c_1')$	$M(a_2, b_2, c_2')$	$M(a_3, b_3, c_3')$	
$M(a_1, b_1, c_1'')$	$M(a_2, b_2, c_2'')$	$M(a_3, b_3, c_3'')$	$\cdots$

the sentences of E_2 in L. If he does so he can go one step further. He will adopt the second sentence of C as a consequence of the stated hypothesis S_2, the third one as a consequence of S_3, etc. If now the sentences of a sufficient number of classes of the series C_1, C_2, etc. are confirmed by observations, the corresponding number of sentences of the series S_1, S_2, etc. and likewise of sentences of C will be stated as well-confirmed hypotheses. If we define 'P' by '$P(x) \equiv (\exists y)(z)M(x, y, z)$', we may abbreviate the sentences of C by '$P(a_1)$', '$P(a_2)$', etc. The fact that these sentences are well-confirmed hypotheses will suggest to the physicist the sentence '$(x)P(x)$', that is S, as a hypothesis to be adopted provided he admits at all sentences of the form U_3 in L. The statement of S as confirmed by C is quite analogous to that of S_1 as confirmed by C_1. If somebody asserted that S – belonging to U_3 – is meaningless while the sentences of C – belonging to E_2 – are meaningful, he would thereby assert that it is meaningless to assume hypothetically that a certain condition which we have already assumed to subsist at several points a_1, a_2, a_3, etc. subsists at every point. Thus no reason is to be seen for prohibiting sentences of U_3, if sentences of E_3 are admitted.

This same procedure can be continued to higher and higher levels. Suppose that in the definition of 'M' two individual constants occur, say 'd_1' and 'e_1'; then we may write S in the form '$(x)(\exists y)(z)M'(d_1, e_1, x, y, z)$'. According to our previous supposition this is a hypothesis which is incompletely confirmed to a certain degree by our observations, namely by the sentences of C_1, C_2, etc. Then the first sentence of C′, being a consequence of S, is confirmed to at least the same degree. If we define 'P″' by '$P'(v) \equiv (\exists w)(x)(\exists! y)(z)M'(v, w, x, y, z)$' we may abbreviate the first sentence of C′ by '$P'(d_1)$'. Now let us suppose that analogous sentences for d_2, d_3, etc. are likewise found to be confirmed by our observations. Then by these sentences of C′ (belonging to E_4) S′ (belonging to U_5) is incompletely confirmed.

On the basis of these considerations it seems natural and convenient to make the following decisions.

Decision 5. Let S be a universal sentence (e.g. '$(x)Q(x)$') – which is being considered either for admission to or exclusion from L – and C be the class of the corresponding full sentences ('$Q(a_1)$', '$Q(a_2)$', etc.). Then obviously the sentences of C are consequences of S, and the confirmation of S is incompletely reducible to that of C.

a. If the sentences of C are admitted in L we will admit the sentences of the form S, i.e. a class U_n for a certain n ($n > 0$).

b. If the sentences of C are stated as hypotheses with a sufficiently high degree of confirmation, we will admit S to be stated as a hypothesis with a certain degree of confirmation, if no other reasons are against this, e.g. the negation of one of the sentences of C being confirmed to a sufficiently high degree.

Decision 6. Let S be an existential sentence (e.g. '$(\exists x)Q(x)$') – which is being considered either for admission to or exclusion from L – and C be the class of the corresponding full sentences ('$Q(a_1)$', '$Q(a_2)$', etc.) Then obviously S is a consequence of every sentence of C, and hence the confirmation of S is completely reducible to that of C.

a. If the sentences of C are admitted in L we will admit the sentences of the form S, i.e. a class E_n for a certain n ($n > 0$).

b. If at least one sentence of C, say S′, is stated as a hypothesis with a sufficiently high degree of confirmation, we will admit S to be stated as a hypothesis with a certain degree of confirmation at least equal to that of S′.

The acceptance of Decisions 5 and 6 leads in the first place, as shown by the example explained before, to the admission of U_1, E_2, U_3, E_4, U_5, etc. in L; and it also leads to the admission of E_1, U_2, E_3, U_4, etc. Hence the result is the choice of a language L_∞ or, if Decision 3a is made, language L_∞^t.

As an objection to our proposal of language L_∞ the remark will perhaps be made that the statement of hypotheses of a high complexity, say U_{10} or E_{10}, will never be necessary or desirable in science, and that therefore we need not choose L_∞. Our reply is, that the proposal of L_∞ by no means requires the statement of hypotheses of such a kind; it simply proposes not to prohibit their statement *a priori* by the formative rules of the language. It seems convenient to give the scientist an open field for possible formulations of hypotheses. Which of these admitted possibilities will actually be applied, must be learned from the further evolution of science, – it cannot be foreseen from general methodological considerations.

27. The Principle of Empiricism

It seems to me that it is preferable to formulate the principle of empiricism not in the form of an assertion – "all knowledge is empirical" or "all synthetic sentences that we can know are based on (or connected with) experiences" or the like – but rather in the form of a proposal or requirement. As empiricists, we require the language of science to be restricted in a certain way; we require that descriptive predicates and hence synthetic sentences are not to be admitted unless they have some connection with possible observations, a connection which has to be characterized in a suitable way. By such a formulation, it seems to me, greater clarity will be gained both for carrying on discussion between empiricists and anti-empiricists as well as for the reflections of empiricists.

We have seen that there are many different possibilities in framing an empiricist language. According to our previous considerations there are in the main four different requirements each of which may be taken as a possible formulation of empiricism; we will omit here the many intermediate positions which have been seen to consist in drawing a rather arbitrary boundary line.

RCT. *Requirement of Complete Testability:* "Every synthetic sentence must be completely testable". I.e. if any synthetic sentence S is given, we must know a method of testing for every descriptive predicate occurring in S so that we may determine for suitable points whether or not the predicate can be attributed to them; moreover, S must have such a form that at least certain sentences of this form can possibly be confirmed in the same degree as particular sentences about observable properties of things. This is the strongest of the four requirements. If we adopt it, we shall get a *testable molecular language* like L_0^t, i.e. a language restricted to molecular sentences and to test chains as the only introductive chains, in other words, to those reduction sentences whose first predicate is realizable.

RCC. Requirement of Complete Confirmability: "Every synthetic sentence must be completely confirmable." I.e. if any synthetic sentence S is given, there must be for every descriptive predicate occurring in S the possibility of our finding out for suitable points whether or not they have the property designated by the predicate in question; moreover, S must have a form such as is required in RCT, and hence be molecular. Thus the only difference between RCC and RCT concerns predicates. By RCC predicates are admitted which are introduced by the help of reduction sentences which are not test sentences. By the admission of the predicates of this kind the language is enlarged to a *confirmable molecular language* like L_0. The advantages of the admission of such predicates have been explained in §14. It seems however that there are not very many predicates of this kind in the language of science and hence that the practical difference between RCT and RCC is not very great. But the difference in the methodological character of L_0^t and L_0 may seem important to those who wish to state RCT.

RT. Requirement of Testability: "Every synthetic sentence must be testable." RT is more liberal than RCT, but in another direction than RCC. RCC and RT are incomparable inasmuch as each of them contains predicates not admitted in the other one. RT admits incompletely testable sentences—these are chiefly universal sentences to be confirmed incompletely by their instances—and thus leads to a *testable generalized language*, like L_∞^t. Here the new sentences in comparison with L_0^t are very many; among them are the laws of science in the form of unrestricted universal sentences. Therefore the difference of RCT and RT, i.e. of L_0^t and L_∞^t, is of great practical importance. The advantages of this comprehensive enlargement have been explained in §§ 25 and 26.

RC. Requirement of Confirmability: "Every synthetic sentence must be confirmable". Here both restrictions are dispensed with. Predicates which are confirmable but not testable are admitted; and generalized sentences are admitted. This simultaneous enlargement in both directions leads to a *confirmable generalized language* like L_∞. L_∞ contains not only L_0^t but also L_0 and L_∞^t as proper sub-languages. RC is the most liberal of the four requirements. But it suffices to exclude all sentences of a non-empirical nature, e.g. those of transcendental metaphysics inasmuch as they are not confirmable, not even incompletely. Therefore it seems to me that RC suffices as a formulation of the principle of empiricism; in other words, if a scientist chooses any language fulfilling this requirement no objection can be raised against this choice from the point of view of empiricism. On the other hand, that does not mean that a scientist is not allowed to choose a more restricted language and to state one of the more restricting requirements for himself—though not for all scientists. There are no theoretical objections against these requirements, that is to say, objections condemning them as false or incorrect or meaningless or the like; but it seems to me that there are practical objections against them as being inconvenient for the purpose of science.

The following table shows the four requirements and their chief consequences.

Requirement	restriction to *molecular* sentences	restriction to *test* chains	language
RCT: complete testability	+	+	L_0^t
RCC: complete confirmability	+	—	L_0
RT: testability	—	+	L_∞^t
RC: confirmability	—	—	L_∞

28. Confirmability of Predictions

Let us consider the nature of a *prediction*, a sentence about a future event, from the point of view of empiricism, i.e. with respect to confirmation and testing. Modifying our previous symbolism, we will take 'c' as the name of a certain physical system, 'x' as a corresponding variable, 't' as the time-variable, 't_0' as a value of 't' designating a moment at which we have made observations about c, and 'd' as a constant designating a certain time interval, e.g. one day or one million years. Now let us consider the following sentences

(S) $$(t)[P_1(c, t) \supset P_2(c, t + d)]$$

in words: "For every instant t, if the system c has the state P_1 at the time t, then it has the state P_2 at the time $t + d$";

(S_1) $$P_1(c, t_0)$$

"The system c has the state P_1 at the time t_0 (of our observation)";

(S_2) $$P_2(c, t_0 + d)$$

"The system c will have the state P_2 at the time $t_0 + d$". Now let us make the following suppositions. There is a set C of laws about physical systems of that kind to which c belongs such that S can be derived from C; the predicates occurring in the laws of C, and among them 'P_1' and 'P_2', are completely testable; the laws of C have been tested very frequently and each tested instance had a positive result; S_1 is confirmed to a high degree by observations. From these suppositions it follows, that S_1 and S_2, having molecular form and containing only predicates which are completely testable, are themselves completely testable; that the laws of C are incompletely testable, but (incompletely) confirmed to a rather high degree; that S, being a consequence of C, is also confirmed to a rather high degree; that S_2, being a consequence of S and S_1, is also confirmed to a rather high degree. If we wait until the time $t_0 + d$ it may happen that we shall confirm S_2 by direct observations to a very high degree. But, as we have seen, a prediction like S_2 may have even at the present time a rather high degree of confirmation dependent upon the degree of confirmation of the laws used for the derivation of the prediction. The nature of a prediction like S_2 is, with respect to confirmation and testing, the same as that of a sentence S_3 about a past event not observed by ourselves, and the same as that of a sentence S_4 about a present event not directly observed by us, e.g. a process now going on in the interior of a machine, or a political event in China. S_3 and S_4 are, like S_2, derived from sentences based on our direct observations with the help of laws which are incompletely confirmed to some degree or other by previous observations.[37]

To give an example, let c be the planetary system, C the set of the differential equations of celestial mechanics from which S may be derived by integration, S_1 describing the present constellation of c—the positions and the velocities of the bodies—and d the interval of one million years. Let '$P_3(t)$' mean: "There are no living beings in the world at the time t," and consider the following sentence.

$$(S_5) \qquad P_3(t_0 + d) \supset P_2(t_0 + d)$$

meaning that, if in a million years there will be no living beings in the world then at that time the constellation of the planetary system will be P_2 (i.e. that which is to be calculated from the present constellation with the help of the laws confirmed by past observations). S_5 may be taken as a convenient formulation of the following sentence discussed by *Lewis*[38] and *Schlick*:[39] "If all minds (or: living beings) should disappear from the universe, the stars would still go on in their courses". Both Lewis and Schlick assert that this sentence is not verifiable. This is true if 'verifiable' is interpreted as 'completely confirmable'. But the sentence is confirmable and even testable, though incompletely. We have no well-confirmed predictions about the existence or non-existence of organisms at the time $t_0 + d$; but the laws C of celestial mechanics are quite independent of this question. Therefore, irrespective of its first part, S_5 is confirmed to the same degree as its second part, i.e. as S_2, and hence, as C. Thus we see that an indirect and incomplete testing and confirmation of S_2—and thereby of S_5—is neither logically nor physically nor even practically impossible, but has been actually carried out by

[37] *Reichenbach* ([3], p. 153) asks what position the Vienna Circle has taken concerning the methodological nature of predictions and other sentences about events not observed, after it gave up its earlier view influenced by Wittgenstein (comp. §23). The view explained above is that which my friends—especially Neurath and Frank—and I have held since about 1931 (compare Frank [1], Neurath [3], Carnap [2a], p. 443, 464 f.; [2b], p. 55 f., 99 f.).

[38] Lewis [2], p. 143.

[39] Schlick [4], p. 367.

astronomers. Therefore I agree with the following conclusion of Schlick concerning the sentence mentioned above (though not with his reasoning): "We are as sure of it as of the best founded physical laws that science has discovered." The sentence in question is meaningful from the point of view of empiricism, i.e. it has to be admitted in an empiricist language, provided generalized sentences are admitted at all and complete confirmability is not required. The same is true for any sentence about past, present or future events, which refers to events other than those we have actually observed, provided it is sufficiently connected with such events by confirmable laws.—

The object of this essay is not to offer definitive solutions of problems treated. It aims rather to stimulate further investigation by supplying more exact definitions and formulations, and thereby to make it possible for others to state their different views more clearly for the purposes of fruitful discussion. Only in this way may we hope to develop convergent views and so approach the objective of *scientific empiricism* as a movement comprehending all related groups,—the development of an increasingly scientific philosophy.

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