Answers to Exercises 23

A Suppose 'm' denotes Socrates, 'n' denotes Plato, 'o' denotes Aristotle, 'Fx' means *x is a philosopher*, 'Gx' means *x is wise*, 'Mxy' means *x taught y*. Take the domain of discourse to consists of people. And then translate the following into **QL**:

1 Socrates taught Plato and Plato taught Aristotle
(Mmn ∧ Mno)

2 Aristotle taught neither Socrates nor Plato
¬(Mom ∨ Mon) *or* (¬Mom ∧ ¬Mon)

3 Plato taught someone
∃x Mnx

4 Some philosophers are wise
∃x(Fx ∧ Gx)

5 Some wise people aren't philosophers
∃x(Gx ∧ ¬Fx)

6 No one taught Socrates
¬∃x Mxm *or* ∀x¬ Mxm

7 If Socrates taught Plato, then someone taught Plato
(Mmn ⊃ ∃x Mxn)

8 Whoever Socrates taught is wise
∀x(Mmx ⊃ Gx)

9 Any philosopher who was taught by Plato taught Aristotle
i.e. Everyone *x* is such that, if *x* is a philosopher and *x* was taught by Plato, then *x* taught Aristotle, so:
∀x((Fx ∧ Mnx) ⊃ Mxo)

10 No wise philosopher was taught by Aristotle
¬∃x((Fx ∧ Gx) ∧ Mox) *or* ∀x((Fx ∧ Gx) ⊃ ¬Mox)

B Which of the following pairs of wffs are equivalent (i.e. imply each other), and why? When they aren't equivalent, give interpretations to illustrate the non-equivalence.

1. ∃x∀y∃z Ryxz; ∃z∀y∃x Ryzx

Swapping 'x' and 'z' in the first (which of course doesn't change what it means) turns it into the second, so these are equivalent.

2. ∃x∀y∃z Ryxz; ∃z∀x∃y Rxyz

Swapping 'x' and 'z' in the first (which of course doesn't change what it means) turns it into

∃z∀y∃x Ryzx

Now swap 'x' and 'y' to get

∃z∀x∃y Rxzy

The initial block of quantifiers is the same as in the original second wff. But the following expressions are now 'Rxzy' and 'Rxyz' are plainly not equivalent – so the originals are not equivalent.

3. (∀xFx ⊃ Fn); (∀zFz ⊃ Fn)

Plainly equivalent.

4. $(\forall xFx \supset \forall xFx)$; $(\forall zFz \supset \forall yFy)$

Also plainly equivalent, since each of '$\forall xFx$', '$\forall yFy$' and '$\forall zFz$' are equivalent to each other.

5. $\exists x\exists yLxy$; $\exists y\exists xLxy$

These are equivalent – see §24.3 for more explanation.

6. $\forall x\forall yLxy$; $\forall y\forall xLxy$

These too are equivalent – see §24.3 for more explanation.

7. $\forall x(Fx \wedge Gx)$; $(\forall xFx \wedge \forall xGx)$

These are equivalent. If everything is *F* and *G*, then everything is *F* and everything is *G*; and equally, if everything is *F* and everything is *G*, then everything is *F* and *G*.

8. $\forall x(Fx \vee Gx)$; $(\forall xFx \vee \forall xGx)$

Not equivalent. Compare 'everyone is male or female' with 'everyone is male or everyone is female'.

9. $\exists x(Fx \wedge Gx)$; $(\exists xFx \wedge \exists xGx)$

Not equivalent. Compare 'someone is male and female' with 'someone is male and someone is female'.

10. $\exists x(Fx \vee Gx)$; $(\exists xFx \vee \exists xGx)$

Equivalent. If something is *F* or *G*, then either something is *F* or something is *G*; and vice versa. And if something is *F* or something is *G*,then something is *F*-or-*G* or something is *G*-or-*F*; so something is *F*-or-*G*.

C We can render 'Plato and Aristotle are philosophers' by e.g. '$(Fm \wedge Fn)$'. Why can't we render 'Plato and Aristotle are classmates' by something like '$(Gm \wedge Gn)$'? Consider other cases of predicates *F* where we can't render something of the form 'Plato and Aristotle are *F*' by something of the type '$(Fm \wedge Fn)$'. What can be learnt from such cases about the expressive limitations of **QL**?

$(Gm \wedge Gn)$ entails Gm. But *Plato and Aristotle are classmates* does not entail *Plato is a classmate* (which hardly makes sense). Likewise *Socrates and Plato and Aristotle surrounded the escaped goat* doesn't entail *Socrates surrounded the escaped goat* (it takes more than one to do that!).

Let's say that a predicate *F* is *distributive* if it sustains the inference *if m and n is F, then m is F.* Then not all English predicates are distributive: but **QL** can only represent distributive predicates – so that's an expressive limitation.