Answers to Exercises 25

A *Show the following simple arguments are valid by translating into* **QL** *and using trees.*

1. *Everyone is rational; hence Socrates is rational.*

Translation: $\forall x\, Fx \therefore Fn$ *Here and throughout we'll assume the domain is* people. *And of course the particular choice of predicate and constant letters is arbitrary.*

(1)	$\forall x\, Fx$		*premiss*
(2)	$\neg Fn$		*negated conclusion*
(3)	Fn		*from (1) by* ($\forall$) *rule*
	✻		

The tree immediate closes and the argument is valid.

2. *No-one loves Angharad; hence Caradoc doesn't love Angharad.*

Translation (a): $\forall x \neg Lxn \therefore \neg Lmn$
Translation (b): $\neg \exists x Lxn \therefore \neg Lmn$

(1a)	$\forall x \neg Lxn$		*premiss*
(2a)	$\neg\neg Lmn$		*negated conclusion*
(3a)	$\neg Lmn$		*from (1a) by* ($\forall$) *rule*
	✻		

(We don't need to unpack (2a) further – remember, any contradiction closes a tree!)

(1b)	$\neg \exists x Lxn$	√	*premiss*
(2b)	$\neg\neg Lmn$		*negated conclusion*
(3b)	$\forall x \neg Lxn$		*from (1b) by* ($\neg\exists$) *rule*
(4b)	$\neg Lmn$		*from (3b) by* ($\forall$) *rule*
	✻		

(Check you understand why there is a line checked off in the second proof and not the first!)

3. *No philosopher speaks Welsh; Jones is a philosopher; hence Jones does not speak Welsh.*

Translation (a): $\forall x(Fx \supset \neg Gx), Fn \therefore \neg Gn$
Translation (b): $\neg \exists x(Fx \wedge Gx), Fn \therefore \neg Gn$

(1a)	$\forall x(Fx \supset \neg Gx)$			*premiss*
(2a)	Fn			*premiss*
(3a)	$\neg\neg Gn$			*negated conclusion*
(4a)	$(Fn \supset \neg Gn)$		√	*from (1a) by* ($\forall$) *rule*
(5a)	$\neg Fn$	$\neg Gn$		*from (4a) by* $\supset$ *rule*
	✻	✻		

(1b)	$\neg \exists x(Fx \wedge Gx)$		√	*premiss*
(2b)	Fn			*premiss*
(3b)	$\neg\neg Gn$			*negated conclusion*
(4b)	$\forall x \neg(Fx \wedge Gx)$			*from (1b) by* ($\neg\exists$) *rule*
(5b)	$\neg(Fn \wedge Gn)$		√	*from (4b) by* ($\forall$) *rule*
(6b)	$\neg Fn$	$\neg Gn$		*from (5b) by rule for negated conjunctions*
	✻	✻		

4. *Jones doesn't speak Welsh; hence not everyone speaks Welsh.*

Translation: $\neg Fn \therefore \neg \forall x\, Fx$

(1)	$\neg Fn$		*premiss*
(2)	$\neg\neg \forall x\, Fx$	√	*negated conclusion*
(3)	$\forall x\, Fx$		*from (2) by rule for double negations*
(4)	Fn		*from (3) by* ($\forall$) *rule*
	✻		

Note that we can't apply the (∀) rule directly to (2) to derive '$\neg\neg$Fn'. The (∀) rule applies to wffs that *start* with a universal quantifier.

Note too the relationship between our examples (1) and (4). Quite generally, if the argument (i) *A, so C* is valid, so must be the argument (ii) *not-C, so not-A*. Putting that in terms of trees, if the tree headed

A	*premiss of (i)*
$\neg C$	*negated conclusion of (i)*

eventually closes, so must the tree headed

$\neg C$	*premiss of (ii)*
$\neg\neg A$	*negated conclusion of (ii)*
A	*removing the double negation.*

5. *Socrates is rational; hence someone is rational.*

Translation: Fn ∴ ∃x Fx

(1)	Fn		*premiss*
(2)	¬∃x Fx	√	*negated conclusion*
(3)	∀x¬ Fx		*from (2) by (¬∃) rule*
(4)	¬Fn		*from (3) by (∀) rule*
	✲		

6. *Some philosophers speak Welsh; all Welsh speakers sing well; hence some philosophers sing well.*

Translation: ∃x(Fx ∧ Gx), ∀x(Gx ⊃ Hx) ∴ ∃x(Fx ∧ Hx)

(1)	∃x(Fx ∧ Gx)	√	*premiss*
(2)	∀x(Gx ⊃ Hx)		*premiss*
(3)	¬∃x(Fx ∧ Hx)	√	*negated conclusion*
(4)	∀x¬(Fx ∧ Hx)		*from (3) by (¬∃) rule*
(5)	(Fa ∧ Ga)		*from (1) by (∃) rule*
(6)	(Ga ⊃ Ha)		*from (2) by (∀) rule*
(7)	¬(Fa ∧ Ha)		*from (4) by (∀) rule*

That's all pretty much automatic. We write down the premiss and negated conclusion: look to see if there are any candidates for applying the negated-quantifier rules. Then at step (5) the only thing we can do is instantiate the first premiss with some new name. Obviously the next move is to instantiate the universal quantifiers.

Now we check off (5), (6), (7) in turn, using the familiar rules for connectives …

(8)		Fa		*from (5)*
(9)		Ga		*from (5)*
(10)	¬Ga		Ha	*from (6)*
	✲			
(11)		¬Fa	¬Ha	*from (7)*
		✲	✲	

7. *All electrons are leptons; all leptons have half-integral spin; hence all electrons have half-integral spin.*

Translation: ∀x(Fx ⊃ Gx), ∀x(Gx ⊃ Hx) ∴ ∀x(Fx ⊃ Hx)

(1)	$\forall x(Fx \supset Gx)$		*premiss*
(2)	$\forall x(Gx \supset Hx)$		*premiss*
(3)	$\neg\forall x(Fx \supset Hx)$	√	*negated conclusion*
(4)	$\exists x\neg(Fx \supset Hx)$	√	*from (3) by* ($\neg\forall$) *rule*
(5)	$\neg(Fa \supset Ha)$		*from (4) by* ($\exists$) *rule*
(6)	$(Fa \supset Ga)$		*from (1) by* ($\forall$) *rule*
(7)	$(Ga \supset Ha)$		*from (2) by* ($\forall$) *rule*

As in the last example, getting here is pretty much automatic. Then we just apply the familiar connective rules to complete the tree:

(8)		Fa		*from (5)*
(9)		$\neg Ha$		*from (5)*
(10)	Fa ✻		Ga	*from (6)*
(11)		$\neg Ga$ ✻	Ha ✻	*from (7)*

8. *All logicians are philosophers; all philosophers are rational people; no rational person is a flat-earther; hence no logician is a flat-earther.*

Translation (a): $\forall x(Fx \supset Gx)$, $\forall x(Gx \supset Hx)$, $\forall x(Hx \supset \neg Jx) \therefore \forall x(Fx \supset \neg Jx)$
Translation (b): $\forall x(Fx \supset Gx)$, $\forall x(Gx \supset Hx)$, $\neg\exists x(Hx \wedge Jx) \therefore \neg\exists x(Fx \wedge Jx)$

(1a)	$\forall x(Fx \supset Gx)$		*premiss*
(2a)	$\forall x(Gx \supset Hx)$		*premiss*
(3a)	$\forall x(Hx \supset \neg Jx)$		*premiss*
(4a)	$\neg\forall x(Fx \supset \neg Jx)$	√	*negated conclusion*
(5a)	$\exists x\neg(Fx \supset \neg Jx)$	√	*from (4a) by* ($\neg\forall$) *rule*
(6a)	$\neg(Fa \supset \neg Ja)$		*from (5a) by* ($\exists$) *rule*
(7a)	$(Fa \supset Ga)$		*from (1) by* ($\forall$) *rule*
(8a)	$(Ga \supset Ha)$		*from (2) by* ($\forall$) *rule*
(9a)	$(Ha \supset \neg Ja)$		*from (3) by* ($\forall$) *rule*

And then the rest is just the application of standard connective rules. Similarly we have

(1b)	$\forall x(Fx \supset Gx)$		*premiss*
(2b)	$\forall x(Gx \supset Hx)$		*premiss*
(3b)	$\neg\exists x(Hx \wedge Jx)$	√	*premiss*
(4b)	$\neg\neg\exists x(Fx \wedge Jx)$	√	*negated conclusion*
(5b)	$\forall x\neg(Hx \wedge Jx)$		*from (3b) by* ($\neg\exists$) *rule*
(6b)	$\exists x(Fx \wedge Jx)$	√	*from (4b) by double negation rule*
(7b)	$(Fa \wedge Ja)$		*from (5b) by* ($\exists$) *rule*
(8b)	$(Fa \supset Ga)$		*from (1) by* ($\forall$) *rule*
(9b)	$(Ga \supset Ha)$		*from (2) by* ($\forall$) *rule*
(10b)	$(Ha \supset \neg Ja)$		*from (3) by* ($\forall$) *rule*

And then we apply connective rules again.

9. *If Jones is a bad philosopher, then some Welsh speaker is irrational; but every Welsh speaker is rational; hence Jones is not a bad philosopher.*

Translation: $(Fn \supset \exists x(Gx \wedge \neg Hx))$, $\forall x(Gx \supset Hx) \therefore \neg Fn$

Strictly speaking, we don't need to discern any internal complexity in 'Jones is a bad philosopher'. At the end of §26.1, we add propositional letters to QL for use in such cases. But that minro point apart, the key thing to note is that the first premiss is a conditional whose antecedent is 'Jones is a bad philosopher' and consequent is 'some Welsh speaker is irrational'. Hence the translation.

Once the translation is in place, the tree is straightforward

(1)	$(Fn \supset \exists x(Gx \wedge \neg Hx))$ √			*premiss*
(2)	$\forall x(Gx \supset Hx)$			*premiss*
(3)	$\neg\neg Fn$			*negated conclusion*
(4)	$\neg Fn$	$\exists x(Gx \wedge \neg Hx)$ √		*from (1) by ⊃ rule*
(5)	✻	$(Ga \wedge \neg Ha)$ √		*from (4) by* (∃) *rule*
(6)		$(Ga \supset Ha)$ √		*from (2) by* (∀) *rule*
(7)		Ga		*from (5)*
(8)		$\neg Ha$		*from (5)*
(9)		$\neg Ga$	Ha	*from (6)*
		✻	✻	

B *Consider the following rule*

($\neg\exists'$) If $\neg\exists v C(\ldots v\ldots v\ldots)$ appears on an open path, then we can add $\neg C(\ldots c\ldots c\ldots)$ to that path, where c is any constant which already appears on the path.

Show informally that this rule would do as well as our rule ($\neg\exists$). *What would be the analogous rule for dealing with negated universal quantifiers without turning them first into existential quantifiers?*

Our current rule ($\neg\exists$) allows us to extend a tree like this:

$$\neg\exists v C(\ldots v\ldots v\ldots)$$
$$\forall x\neg C(\ldots v\ldots v\ldots)$$

Now, in our system, what can we do with a universally quantified wff like that, once we've got it? Just two things (i) instantiate it, or (ii) use it, if we already have the wff $\neg\forall x\neg C(\ldots v\ldots v\ldots)$ on the tree, to close off a branch as having hit a contradiction. Now, in case (i) our ($\forall$) rule allows us to continue the tree by adding

$$\neg C(\ldots c\ldots c\ldots)$$

where c is any constant which already appears on the path. So an application of ($\neg\exists$) plus instantiation is equivalent to an application of ($\neg\exists'$) which just wraps these two steps into one. In case (ii), if $\neg\exists v C(\ldots v\ldots v\ldots)$ and $\neg\forall x\neg C(\ldots v\ldots v\ldots)$ are on a branch which closes by using our old ($\neg\exists$), we can use the ($\neg\forall$) rule to infer $\exists x\neg\neg C(\ldots v\ldots v\ldots)$ and then instantiate with a new name to get $\neg\neg C(\ldots a\ldots a\ldots)$. Now applying ($\neg\exists'$) we get $\neg C(\ldots a\ldots a\ldots)$ and our new rule allows the branch to close again. So either way, in case (i) and (ii) our new rule gives us the effect of the old one.

C *Suppose we had set up predicate logic with a single quantifier formed using the symbol 'N', so that $NvCv$ holds when nothing is C. Show that the resulting language would be expressively equivalent to our now familiar two-quantifier language **QL**. What would be an appropriate set of rules for tree-building in a language with this single quantifier?*

Expressive equivalence is trivial, since because $NvCv$ is equivalent to $\forall v\neg Cv$ and $\neg\exists vCv$;so $\forall vCv$ is equivalent to $Nv\neg Cv$, and $\exists vCv$ is equivalent to $\neg NvCv$.

Appropriate quantifier rules (to add to the usual connective rules) would be: from $NvCv$ infer $\neg Cc$ where c is a name already on the branch; and from $\neg NvCv$ infer Cc where c is a new name.