An Introduction to

Formal Logic

Answers to Exercises 29

A *Use QL trees to evaluate the entailment claims (1) to (10) in Exercises 28A.*

1. ∀x(Fx ⊃ Gx) ⊨ ∀x(Gx ⊃ Fx)

(1) ∀x(Fx ⊃ Gx)
(2) ¬∀x(Gx ⊃ Fx)
(3) ∃x¬(Gx ⊃ Fx)
(4) ¬(Ga ⊃ Fa)
(5) Ga
(6) ¬Fa
(7) (Fa ⊃ Ga)

(8) ¬Fa   Ga
(9) ✻

The tree doesn't close and there are no more rules to apply. We can read off the open branch a valuation which makes the premisses true and conclusion false, and (recall) *the trick is to pick a valuation which makes the 'primitives' on the branch, i.e. the atoms and negated atoms, all true,* and which puts into the domain just enough objects to give references to every constant on the branch. We want a valuation with just the object named by 'a' in the domain and which makes '¬Fa' and 'Ga' both true. So, put just the number 0 in the domain as its sole member, and let F' have as extension the empty set, and 'G' has the extension {0}. Then, as desired, '∀x(Fx ⊃ Gx)' is true and '∀x(Gx ⊃ Fx)' is false.

2. ∀x(Fx ⊃ Gx) ⊨ ∀x(¬Gx ⊃ ¬Fx)

(1) ∀x(Fx ⊃ Gx)
(2) ¬∀x(¬Gx ⊃ ¬Fx)
(3) ∃x¬(¬Gx ⊃ ¬Fx)
(4) ¬(¬Ga ⊃¬ Fa)
(5) ¬Ga
(6) ¬¬Fa
(7) (Fa ⊃ Ga)

(8) ¬Fa   Ga
✻   ✻

The inference is valid and the q-validity claim is true.

3. ∀x∃yLxy ⊨ ∀y∃xLyx

(1)	∀x∃yLxy		
(2)	¬∀y∃xLyx	√	
(3)	∃y¬∃xLyx	√	From 2
(4)	¬∃xLax	√	From 3
(5)	∀x¬Lax	√	From 4
(6)	∃yLay		From 1
(7)	Lab		From 2
(8)	¬Lab		From 5
	✻		

The inference is valid and the q-validity claim is true.

4. $\forall x((Fx \land Gx) \supset Hx) \vDash \forall x(Fx \supset (Gx \supset Hx))$

(1)			$\forall x((Fx \land Gx) \supset Hx)$			
(2)			$\neg\forall x(Fx \supset (Gx \supset Hx))$		√	
(3)			$\exists x\neg(Fx \supset (Gx \supset Hx))$		√	From 2
(4)			$\neg(Fa \supset (Ga \supset Ha))$		√	From 3
(5)			Fa			From 4
(6)			$\neg(Ga \supset Ha)$		√	From 4
(7)			Ga			From 6
(8)			$\neg Ha$			From 6
(9)			$((Fa \land Ga) \supset Ha)$		√	From 1
(10)		$\neg(Fa \land Ga)$ √		Ha		From 9
				✻		
(11)	$\neg Fa$		$\neg Ga$			
	✻		✻			

The inference is valid and the q-validity claim is again true.

5. $(\forall xFx \lor \forall xGx) \vDash \forall x(Fx \lor Gx)$

(1)		$(\forall xFx \lor \forall xGx)$		√	
(2)		$\neg\forall x(Fx \lor Gx)$		√	
(3)		$\exists x\neg(Fx \lor Gx)$		√	From 2
(4)		$\neg(Fa \lor Ga)$		√	From 3
(5)		$\neg Fa$			From 4
(6)		$\neg Ga$			From 4
(7)	$\forall xFx$		$\forall xGx$		From 1
(8)	Fa		Ga		From 7
	✻		✻		

The inference is valid and the q-validity claim is again true.

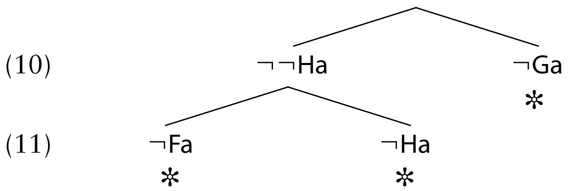
6. $\forall x(Fx \supset Gx), \forall x(\neg Gx \supset Hx) \vDash \forall x(Fx \supset \neg Hx)$

(1)		$\forall x(Fx \supset Gx)$				
(2)		$\forall x(\neg Gx \supset Hx)$				
(3)		$\neg\forall x(Fx \supset \neg Hx)$			√	
(4)		$\exists x\neg(Fx \supset \neg Hx)$			√	
(5)		$\neg(Fa \supset \neg Ha)$			√	
(6)		Fa				
(7)		$\neg\neg Ha$			√	
(8)		Ha				
(9)		$(Fa \supset Ga)$			√	From 1
(10)		$(\neg Ga \supset Ha)$			√	From 2
(11)	$\neg Fa$		Ga			
	✻					
(12)		$\neg\neg Ga$ √		Ha		
(13)		Ga				

The tree doesn't close and there are no more rules to apply. We can read off each open branch a valuation which makes the premisses true and conclusion false – in fact the same valuation, as each branch contains the same primitives, 'Fa', 'Ga' and 'Ha'. We want a valuation with just the object named by a in the domain and which makes each of those primitives true. So, put just the number 0 in the domain as its sole member, and let 'F', 'G' and 'H' have the extension {0}. Then, as desired, '$\forall x(Fx \supset Gx)$' and '$\forall x(\neg Gx \supset Hx)$' are true and '$\forall x(Fx \supset \neg Hx)$' is false.

7. $\exists x(Fx \wedge Gx), \forall x(\neg Hx \supset \neg Gx) \vDash \exists x(Fx \wedge Hx)$

(1)	$\exists x(Fx \wedge Gx)$		
(2)	$\forall x(\neg Hx \supset \neg Gx)$		
(3)	$\neg\exists x(Fx \wedge Hx)$	√	
(4)	$\forall x\neg(Fx \wedge Hx)$	√	
(5)	$(Fa \wedge Ga)$	√	From 1
(6)	Fa		
(7)	Ga		
(8)	$(\neg Ha \supset \neg Ga)$	√	From 2
(9)	$\neg(Fa \wedge Ha)$	√	From 4



The inference is valid and the q-validity claim is again true.

8. $\forall x\exists y(Fy \supset Gx) \vDash \forall y\exists x(Gx \supset Fy)$

(1)	$\forall x\exists y(Fy \supset Gx)$		
(2)	$\neg\forall y\exists x(Gx \supset Fy)$	√	
(3)	$\exists y\neg\exists x(Gx \supset Fy)$	√	
(4)	$\neg\exists x(Gx \supset Fa)$	√	
(5)	$\forall x\neg(Gx \supset Fa)$		From 1

The only name in play is a so let's now instantiate both universal quantifiers with this name

(6)	$\exists y(Fy \supset Ga)$	√
(7)	$\neg(Ga \supset Fa)$	
(8)	Ga	
(9)	$\neg Fa$	

Now, the tree hasn't finished, and indeed the tree will never close if we carry on applying every rule we can. For then we'd instantiate (6) to get

(10) $(Fb \supset Ga)$ √

And instantiating both our universal quantifers with the new name b we'd get

(11)	$\exists y(Fy \supset Gb)$	√
(12)	$\neg(Gb \supset Fb)$	

And now we've got another existential quantifier to instantiate, which introduces another name, and off we go down an infinite tree.

But go back and look at (8) and (9). In fact a minimal valuation that makes these two primitives true makes (6) true. So consider the valuation with just the number 0 in the domain as its sole member, and let 'F' have as extension the empty set, and 'G' has the extension {0}. Then, this makes the premiss of the argument true and conclusion false.

9. $\forall x\forall y(Lxy \supset Lyx) \vDash \forall xLxx$

(1)		$\forall x\forall y(Lxy \supset Lyx)$			
(2)		$\neg\forall xLxx$		√	
(3)		$\exists x\neg Lxx$		√	From 2
(4)		$\neg Laa$		√	From 3
(5)		$\forall y(Lay \supset Lya)$			From 1
(6)		$(Laa \supset Laa)$ √			From 1
(10)	$\neg Laa$		Laa		From 9
			✱		

There are no more moves to make. So consider the valuation with just the number 0 in the domain as its sole member, and let ‘L’ have as extension the empty set. Then that makes the premiss true and conclusion false.

10. $\forall x(\exists yLxy \supset \forall zLzx) \vDash \forall x\forall y(Lxy \supset Lyx)$

(1)		$\forall x(\exists yLxy \supset \forall zLzx)$			
(2)		$\neg\forall x\forall y(Lxy \supset Lyx)$		√	
(3)		$\exists x\neg\forall y(Lxy \supset Lyx)$		√	From 2
(4)		$\exists y\neg(Lay \supset Lya)$		√	From 3
(5)		$\neg(Lab \supset Lba)$		√	From 4
(6)		Lab			
(7)		$\neg Lba$			
(6)		$(\exists yLay \supset \forall zLza)$			From 1
(10)	$\neg\exists yLay$		$\forall zLza$		From 9
(11)	$\forall y\neg Lay$		Lba		From 10
(12)	$\neg Lab$		✱		
	✱				

The inference is valid and the q-validity claim is again true.

B *Using trees, show the following arguments are valid:*

1. *Some philosophers admire Jacques. No one who admires Jacques is a good logician. So some philosophers are not good logicians.*

$\exists x(Fx \wedge Gx), \forall x(Gx \supset \neg Hx) \therefore \exists x(Fx \wedge \neg Hx)$

(1)		$\exists x(Fx \wedge Gx)$		√	
(2)		$\forall x(Gx \supset \neg Hx)$			
(3)		$\neg\exists x(Fx \wedge \neg Hx)$		√	Negated conclusion
(4)		$\forall x\neg(Fx \wedge \neg Hx)$			From 3
(5)		$(Fa \wedge Ga)$		√	Instantiating 1
(6)		$(Ga \supset \neg Ha)$		√	From 2
(7)		$\neg(Fa \wedge \neg Ha)$		√	From 4
(8)		Fa			
(9)		Ga		√	
(11)	$\neg Ga$	$\neg Ha$			
	✱				
(12)		$\neg Fa$	$\neg\neg Ha$		
		✱	✱		

2. *Some philosophy students admire all logicians; no philosophy student admires any rotten lecturer; hence, no logician is a rotten lecturer.*

∃x(Fx ∧ ∀y(Gy ⊃ Rxy)), ¬∃x(Fx ∧ ∃y(Hy ∧ Rxy)) ∴ ¬∃x(Gx ∧ Hx)

Other translations of the 'no' propositions are possible. For example, we could have translated the second premiss as '∀x(Fx ⊃ ¬∃y(Hy ∧ Rxy))' or '∀x(Fx ⊃ ∀y(Hy ⊃ ¬Rxy))'. The conclusion can be translated '∀x(Gx ⊃ ¬Hx)'. The tree will go similarly with each combination of translations:

(1) ∃x(Fx ∧ ∀y(Gy ⊃ Rxy))
(2) ¬∃x(Fx ∧ ∃y(Hy ∧ Rxy)) √
(3) ¬¬∃x(Gx ∧ Hx) √ Negated conclusion
(4) ∃x(Gx ∧ Hx) From 3
(5) ∀x¬(Fx ∧ ∃y(Hy ∧ Rxy)) From 2

We now have two existentials to instantiate: we should start with (1) — as the other involves predicates buried inside the wffs (1) and (5).

(6) (Fa ∧ ∀y(Gy ⊃ Ray)) √ From 1

And now we immediately use the new name to instantiate the universal quantifier to get

(7) ¬(Fa ∧ ∃y(Hy ∧ Ray)) √ From 5
(8) Fa }Unpacking 6
(9) ∀y(Gy ⊃ Ray)

(10) ¬Fa | ¬∃y(Hy ∧ Ray) √ Unpacking 7
(11) ✻ | ∀y¬(Hy ∧ Ray) Pushing in the negation sign

At this point, we have three universals and an unchecked existential in play: so we now instantiate the existential and unpack the result …

(12) (Gb ∧ Hb) √
(13) Gb
(14) Hb

We now instantiate the two universals we haven't so far used and the rest is plain sailing:

(15) (Gb ⊃ Rab) √
(16) ¬(Hb ∧ Rab) √

(17) ¬Gb | Rab
✻

(18) ¬Hb | ¬Rab
✻ | ✻

3. *There's a town to which all roads lead. So all roads lead to a town.*

∃x(Fx ∧ ∀y(Gy ⊃ Ryx)) ∴ ∀x(Gx ⊃ ∃y(Fy ∧ Rxy))

where 'Rab' expresses *a leads to b.*

(1) ∃x(Fx ∧ ∀y(Gy ⊃ Ryx))
(2) ¬∀x(Gx ⊃ ∃y(Fy ∧ Rxy)) √ Negated conclusion
(3) ∃x¬(Gx ⊃ ∃y(Fy ∧ Rxy))

We'll instantiate the first wff and unpack the result to get …

(4) (Fa ∧ ∀y(Gy ⊃ Rya)) √
(5) Fa
(6) ∀y(Gy ⊃ Rya)

Now we'll instantiate the other existential wff and unpack the result to get …

(7) ¬(Gb ⊃ ∃y(Fy ∧ Rby)

(8) Gb
(9) $\neg\exists y(Fy \land Rby)$
(10) $\forall y\neg(Fy \land Rby)$

Finally, we instantiate the two universal quantifiers (6) and (10) so as to give two occurrences of Rba …

(11) $(Gb \supset Rba)$
(12) $\neg(Fa \land Rba)$

And now the rest is again plain sailing …

(13) $\neg Gb$ ✻ | Rab
(14) under Rab: $\neg Fa$ ✻ | $\neg Rab$ ✻

4. *Some good philosophers admire Frank; all wise people admire any good philosopher; Frank is wise; hence there is someone who both admires and is admired by Frank.*

$$\exists x(Fx \land Rxn), \forall x(Gx \supset \forall y(Fy \supset Rxy)), Gn \therefore \exists x(Rxn \land Rnx)$$

'F' means *good philosopher*, 'n' denotes Frank, etc.,

(1) $\exists x(Fx \land Rxn)$
(2) $\forall x(Gx \supset \forall y(Fy \supset Rxy))$
(3) Gn
(4) $\neg\exists x(Rxn \land Rnx)$ √ Negated conclusion
(5) $\forall x\neg(Rxn \land Rnx)$

The obvious first move is to instantiate (2) to get 'Gn' as the antecedent to combine with (3) …

(6) $(Gn \supset \forall y(Fy \supset Rny))$
(7) $\neg Gn$ ✻ | $\forall y(Fy \supset Rny)$

We now have the initial existential wff at (1) plus two universals at (5) and (7) which we haven't yet made use of. So we now proceed in the obvious way:

(8) $(Fa \land Ran)$
(9) $\neg(Ran \land Rna)$
(10) $(Fa \supset Rna)$

And now everything quickly closes:

(11) Fa
(12) Ran
(13) $\neg Ran$ ✻ | $\neg Rna$
(14) under $\neg Rna$: $\neg Fa$ ✻ | Rna ✻

5. *Any true philosopher admires some logician. Some students admire only existentialists. No existentialists are logicians. Therefore not all students are true philosophers.*

$\forall x(Fx \supset \exists y(Gy \wedge Rxy)), \exists x(Hx \wedge \forall y(Rxy \supset Ey)), \forall x(Ex \supset \neg Gx) \therefore \neg\forall x(Hx \supset Fx)$

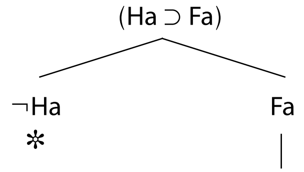
(1) $\forall x(Fx \supset \exists y(Gy \wedge Rxy))$
(2) $\exists x(Hx \wedge \forall y(Rxy \supset Ey))$
(3) $\forall x(Ex \supset \neg Gx)$
(4) $\neg\neg\forall x(Hx \supset Fx)$ √ Negated conclusion
(5) $\forall x(Hx \supset Fx)$

The first move must be to instantiate the existential quantifier (2) to give

(6) $(Ha \wedge \forall y(Ray \supset Ey))$
(7) Ha
(8) $\forall y(Ray \supset Ey)$

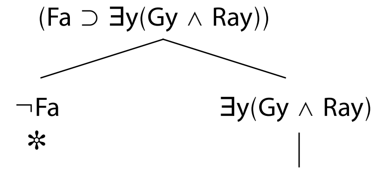
The obvious next move is to instantiate (5) in order to use the antecedent 'Ha':

(9)
(10)



And we now instantiate (1) in order to use the antecedent 'Fa':

(11)
(12)

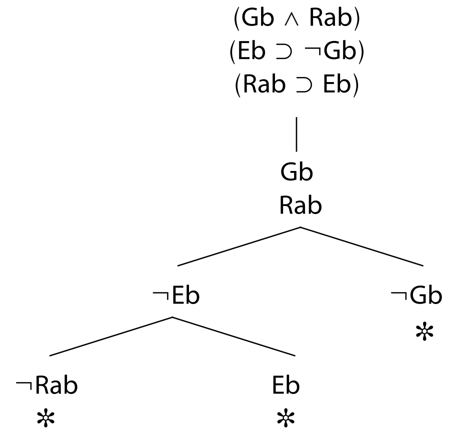


We now have an uninstantiated existential wff at (12), and two as-yet-unused universally quantified wffs at (3) and (8). So let's proceed in the obvious way to get

(13)
(14)
(15)

The last steps are trivial!

(16)
(17)
(18)
(19)



6. *Everyone loves a lover; hence if someone is a lover, everyone loves everyone!*

For the translation, see E, p. 278: *someone is a lover* is equivalent to there is s*omeone who is such that there is someone that they love*, so

$$\forall x\forall y(\exists z\, Lyz \supset Lxy) \therefore (\exists x\exists y\, Lxy \supset \forall x\forall y\, Lxy)$$

(1)	$\forall x\forall y(\exists z\, Lyz \supset Lxy)$		
(2)	$\neg(\exists x\exists y\, Lxy \supset \forall x\forall y\, Lxy)$	√	Negated conclusion
(3)	$\exists x\exists y\, Lxy$		
(4)	$\neg\forall x\forall y\, Lxy$	√	
(5)	$\exists x\neg\forall y\, Lxy$		

We've now got a lot of existentials to instantiate!

(6)	$\exists y\, Lay$	√	From 3, now checked off
(7)	Lab		From 6
(8)	$\neg\forall y\, Lcy$	√	From 5, now checked off
(9)	$\exists y\neg Lcy$	√	From 8
(10)	$\neg Lcd$		From 9

Everything other than primitive wffs is now checked off, except (1), so we now need to use that. Let's first instantiate to get 'Lcd' as the consequent, to conflict with (10) …

(11)	$\forall y(\exists z\, Lyz \supset Lcy)$	
(12)	$(\exists z\, Ldz \supset Lcd)$	
(13)	$\neg\exists z\, Ldz$	Lcd
(14)	$\forall z\neg Ldz$	✻

But now what? Well, we want eventually to make use of (7), so we'll aim to eventually get an occurrence of '$\neg Lab$' to contradict (7). But how are we going to get that? Presumably by using (1) again. But the consequent of instantiations of (1) don't involve negations: so our needed wff will come – if at all – via the antecedent of that instantiation. Which means that the 'y' variable will need to be instantiated with 'a'. But now, if we also instantiate the 'x' variable in (1) with 'd' we'll get an occurrence of 'Lda' as the consequent, which will contradict (14). So, let's try that line …

(15)	$\forall y(\exists z\, Lyz \supset Ldy)$		
(16)	$(\exists z\, Laz \supset Lda)$		
(17)	$\neg\exists z\, Laz$	Lda	
(18)	$\forall z\neg Laz$	$\neg Lda$	From 17 | From 14
(19)	$\neg Lab$	✻	
	✻		

And we are done!

Note that the argument is intuitively valid. Assume everyone loves a lover. Then, supposing someone is a lover, everyone loves him (because everyone loves a lover)! So everyone is a lover. So everyone loves everyone (again because everyone loves a lover)! This double invocation of the premiss in the informal argument is matched by the double invocation in our formal tree-argument.

7. *If anyone speaks to anyone, then someone introduces them; no one introduces anyone to anyone unless they know them both; everyone speaks to Frank; therefore everyone is introduced to Frank by someone who knows him. [Use ‘Rxyz’ to render ‘x introduces y to z’.]*

   For the translation, use ‘Sxy’ for *x speaks to y*, and ‘Kxy’ for *x knows y*. Translation requires a bit of thought. (a) The first premiss is plainly intended to involve a universal generalization (that any pair of people, *x*, *y*, if *x* talks to *y*, then they’ve been introduced). (b) Being introduced is (strictly speaking) a matter of someone (i) introducing the first to the second and (ii) the second to the first, though it doesn’t in fact matter for the validity of this argument if you forget about (ii).

   $\forall x\forall y(Sxy \supset \exists z\{Rzxy \wedge Rzyx\}), \forall x\forall y\forall z(Rxyz \supset (Kxy \wedge Kxz)), \forall xSxn$
   $\therefore\ \forall x\exists y(Ryxn \wedge Kyn)$

(1)	$\forall x\forall y(Sxy \supset \exists z\{Rzxy \wedge Rzyx\})$		
(2)	$\forall x\forall y\forall z(Rxyz \supset (Kxy \wedge Kxz))$		
(3)	$\forall xSxn$		
(4)	$\neg\forall x\exists y(Ryxn \wedge Kxn)$	√	Negated conclusion
(5)	$\exists x\neg\exists y(Ryxn \wedge Kxn)$		

The first move has to be to instantiate the existential quantifier (5) to give

(6)	$\neg\exists y(Ryan \wedge Kyn)$	√
(7)	$\forall y\neg(Ryan \wedge Kyn)$	

We now have two names in play, ‘n’ and ‘a’: that’s not enough to make use of the triply quantified (2), so forget that for the moment. But if we instantiate (3) with ‘a’ to get ‘San’ , and (1) with both names we’ll get an occurrence of ‘San’ as the antecedent of a conditional, thus …

(8)	San	
(9)	$\forall y(Say \supset \exists z\{Rzay \wedge Rzya\})$	
(9)	$(San \supset \exists z\{Rzan \wedge Rzna\})$	
(10)	$\neg San$	$\exists z\{Rzan \wedge Rzna\}$
	✻	

Obviously, we now instantiate our new existential wff to get:

(11)	$\{Rban \wedge Rbna\}$
(11)	$Rban$
(12)	$Rbna$

We’ve now got two universals that we haven’t yet made use of, at (2) and (7). Take the simpler one first and instantiate with ‘b’ (of course! — to give us an occurrence of ‘Rban’ in the scope of a negation, to contradict (11)):

(13)	$\neg(Rban \wedge Kbn)$	
(14)	$\neg Rban$	$\neg Kbn$
	✻	

We now at last use (2): to get something contradicting ‘$\neg Kbn$’, we must instantiate ‘x’ by ‘b’:

(13)	$\forall y\forall z(Rbyz \supset (Kby \wedge Kbz))$

Now it should be obvious how to continue …

(14)	$\forall z(Rbaz \supset (Kba \wedge Kbz))$	
(15)	$(Rban \supset (Kba \wedge Kbn))$	
(16)	$\neg Rban$	$(Kba \wedge Kbn)$
(17)	✻	Kba
(18)		Kbn
		✻

8. *Any elephant weighs more than any horse. Some horse weighs more than any donkey. If a first thing weighs more than a second thing, and the second thing weighs more than a third, then the first weighs more than the third. Hence any elephant weighs more than any donkey.*

$\forall x\forall y((Fx \wedge Gy) \supset Rxy)$, $\exists x(Gx \wedge \forall y(Hy \supset Rxy))$, $\forall x\forall y\forall z((Rxy \wedge Ryz) \supset Rxz)$
$\therefore \forall x\forall y((Fx \wedge Hy) \supset Rxy)$

(1)	$\forall x\forall y((Fx \wedge Gy) \supset Rxy)$		
(2)	$\exists x(Gx \wedge \forall y(Hy \supset Rxy))$		
(3)	$\forall x\forall y\forall z((Rxy \wedge Ryz) \supset Rxz)$		
(4)	$\neg\forall x\forall y((Fx \wedge Hy) \supset Rxy)$	√	Negated conclusion
(5)	$\exists x\neg\forall y((Fx \wedge Hy) \supset Rxy)$		

The first move has to be to instantiate our two existential quantifiers to give

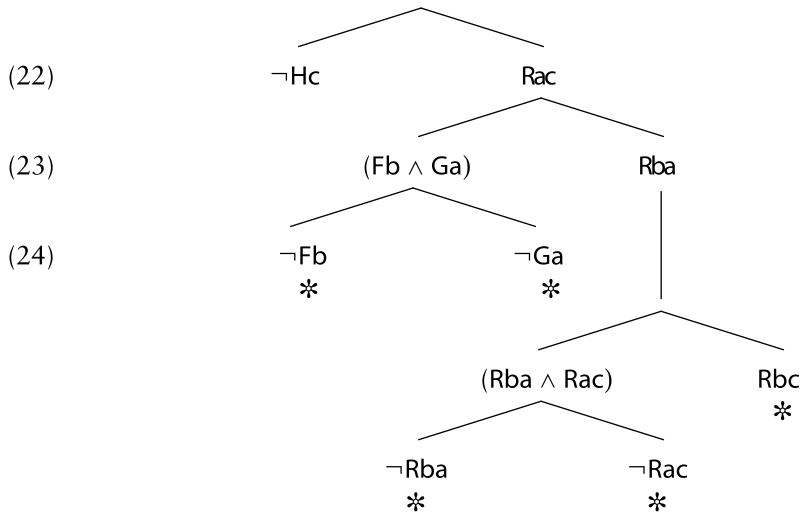
(6)	$(Ga \wedge \forall y(Hy \supset Ray))$	
(7)	$\neg\forall y((Fb \wedge Hy) \supset Rby)$	√
(8)	$\exists x\neg((Fb \wedge Hy) \supset Rby)$	√
(9)	$\neg((Fb \wedge Hc) \supset Rbc)$	

where we've just instantiated the new existential at (8) too. Let's just now unpack (6) and (9) to give

(10)	Ga	
(11)	$\forall y(Hy \supset Ray)$	
(12)	$(Fb \wedge Hc)$	√
(13)	$\neg Rbc$	
(14)	Fb	
(15)	Hc	

We now have three names in play, and three universals at (1), (3) and (11) to instantiate. Obviously we should chose instantiations which neatly tie in with the primitives at (10), (13), (14), (15), thus …

(16)	$\forall y((Fb \wedge Gy) \supset Rby)$
(17)	$((Fb \wedge Ga) \supset Rba)$
(18)	$(Hc \supset Rac)$
(19)	$\forall y\forall z((Rby \wedge Ryz) \supset Rbz)$
(20)	$\forall z((Rba \wedge Raz) \supset Rbz)$
(21)	$((Rba \wedge Rac) \supset Rbc)$



C Redo the first three examples of §29.2 as *signed* trees (as in §28.2).

A $\exists xFx, \forall x\forall y(Fy \supset \neg Lxy) \therefore \exists x\forall y\neg Lyx$

We suppose that there is a valuation q such that

(1) $\exists xFx \Rightarrow_q \mathrm{T}$

(2) $\forall x\forall y(Fy \supset \neg Lxy) \Rightarrow_q \mathrm{T}$

(3) $\neg\exists x\forall y\neg Lyx \Rightarrow_q \mathrm{T}$

So from (3) we get

(4) $\forall x\neg\forall y\neg Lyx \Rightarrow_q \mathrm{T}$

(1) tells us that there then there must be an extension q^+ of q to cover the new name 'a', such that

(5) $Fa \Rightarrow_{q^+} \mathrm{T}$

So from (4) — since the extended valuation doesn't change what's in the domain or anything's properties, but just dubs something with a new name —we know

(6) $\neg\forall y\neg Lya \Rightarrow_{q^+} \mathrm{T}$

(7) $\exists y\neg\neg Lya \Rightarrow_{q^+} \mathrm{T}$

(1) tells us that there then there must be a further extension q^{++} of q to cover the new name 'b', such that

(8) $\neg\neg Lba \Rightarrow_{q^{++}} \mathrm{T}$

Whence (why??) …

(9) $\forall y(Fy \supset \neg Lby) \Rightarrow_{q^{++}} \mathrm{T}$

(10) $(Fa \supset \neg Lba) \Rightarrow_{q^{++}} \mathrm{T}$

(11) $\neg Fa \Rightarrow_{q^{++}} \mathrm{T}$ $\qquad$ $\neg Lba \Rightarrow_{q^{++}} \mathrm{T}$

✻ ✻

B $\forall x\exists y(Fy \wedge Lxy), \forall x\forall y(Lxy \supset Mxy) \therefore \forall x\exists y(Fx \wedge Mxy)$

We suppose that there is a valuation q such that

(1) $\forall x\exists y(Fy \wedge Lxy) \Rightarrow_q \mathrm{T}$

(2) $\forall x\forall y(Lxy \supset Mxy) \Rightarrow_q \mathrm{T}$

(3) $\neg\forall x\exists y(Fx \wedge Mxy) \Rightarrow_q \mathrm{T}$

So from (3) we get

(4) $\exists x\neg\exists y(Fx \wedge Mxy) \Rightarrow_q \mathrm{T}$

(4) tells us that there then there must be an extension q^+ of q to cover the new name 'a', such that

(5) $\neg\exists y(Fa \wedge May) \Rightarrow_{q^+} \mathrm{T}$

whence …

(6) $\forall y\neg(Fa \wedge May) \Rightarrow_{q^+} \mathrm{T}$

(7) $\exists y(Fa \wedge Lay)) \Rightarrow_{q^+} \mathrm{T}$

(8) $\forall y(Lay \supset May)) \Rightarrow_{q^+} \mathrm{T}$

(7) tells us that there then there must be a further extension q^{++} of q to cover the new name 'b', such that

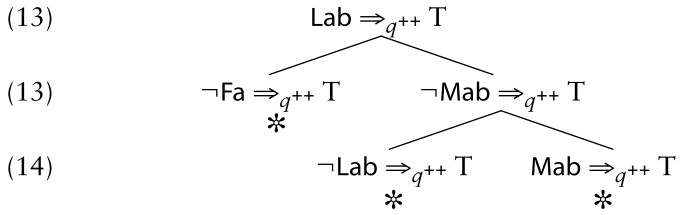
(9) $(Fa \wedge Lab) \Rightarrow_{q^{++}} \mathrm{T}$

whence …

(10) $\neg(Fa \wedge Mab) \Rightarrow_{q^{++}} \mathrm{T}$

(11) $(Lab \supset Mab) \Rightarrow_{q^{++}} \mathrm{T}$

(12) $Fa \Rightarrow_{q^{++}} \mathrm{T}$



Similarly for **C**.