16.1 a) $m=3$

$(x_0, y_0) = (-1, 2)$

$y - y_0 = m(x - x_0)$

$y - 2 = 3(x+1)$

$y - 2 = 3x + 3$

$y = 3x + 5$

b) $(5, 6) \quad (-2, -1)$

$m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-1-6}{-2-5} = \dfrac{-7}{-7} \Rightarrow$

$m = 1$

$y - 6 = 1(x - 5) \Rightarrow y = x + 1$

c) $y = 10x + 5$

d) $\varepsilon \parallel \varepsilon'$ $\quad \varepsilon' : y = 2x - 3 \Rightarrow m = 2$

$$y - 1 = 2(x - 1)$$
$$y = 2x - 1$$

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2) Έστω $P(x_0, y_0)$ το σημείο τομής των $\varepsilon, \varepsilon'$, τότε P ικανοποιεί τις εξισώσεις και των δύο ευθειών

$$\left.\begin{aligned} 3x_0 + 7y_0 &= -4 \\ 5x_0 - 11y_0 &= 16 \end{aligned}\right| \Leftrightarrow \begin{aligned} x_0 &= -\frac{7}{3}y_0 - \frac{4}{3} \\ 5x_0 - 11y_0 &= 16 \end{aligned} \Leftrightarrow$$

$$\Leftrightarrow \begin{aligned} x_0 &= -\frac{7}{3}y_0 - \frac{4}{3} \\ -\frac{35}{3}y_0 - \frac{20}{3} - 11y_0 &= 16 \end{aligned} \Leftrightarrow$$

$$\Leftrightarrow \begin{aligned} x_0 &= -\frac{7}{3}y_0 - 4/3 \\ -\frac{68}{3}y_0 &= \frac{48 + 20}{3} \end{aligned} \Leftrightarrow \begin{aligned} x_0 &= -\frac{7}{3}y_0 - \frac{4}{3} \\ -68y_0 &= 68 \end{aligned}$$

$$\Leftrightarrow \begin{aligned} x_0 &= -\frac{7}{3}y_0 - 4/3 \\ y_0 &= -1 \end{aligned} \;\Bigg|\; \begin{aligned} x_0 &= 1 \\ y_0 &= -1 \end{aligned}$$

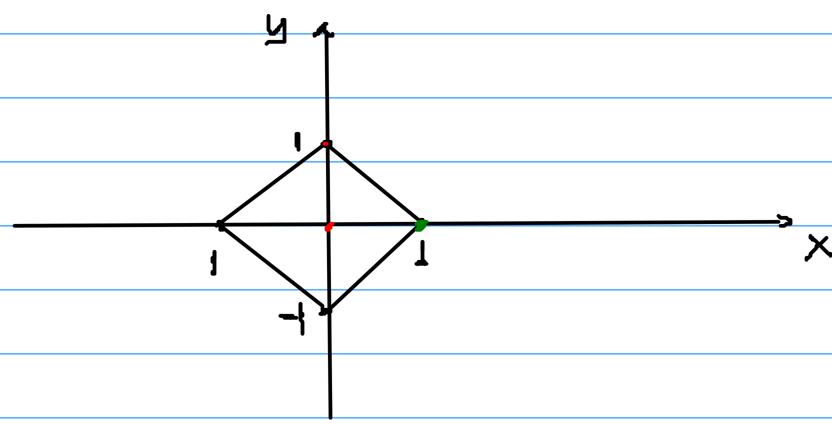
$P(1, -1)$.

3) $|x| + |y| = 1$

Για να διώξουμε τις απόλυτες τιμές λαμβάνουμε περιπτώσεις:

$\underline{y = mx + b}$ $\qquad b = 1$, $m = -1$

$x \geq 0$	$y \geq 0$:	$x + y = 1 \Rightarrow y = 1 - x$
$x \geq 0$	$y < 0$:	$x - y = 1 \Rightarrow y = x - 1$
$x < 0$	$y \geq 0$:	$-x + y = 1 \Rightarrow y = 1 + x$
$x < 0$	$y < 0$:	$-x - y = 1 \Rightarrow y = 1 - x$



16.4. $m = \dfrac{0-2}{-1-0} = 2$ (η κλίση της ΓΔ)

$y = mx + b$ $\qquad m = 2$, $b = 0$ $\qquad \boxed{y = 2x}$

Κάθε ευθεία που διέρχεται από την αρχή των αξόνων, (0,0), είναι της μορφής

$$y = mx$$

18.1

Ευθύ Αν $\left.\begin{array}{l} x^2+y^2+Ax+By+C=0 \\ (x-x_0)^2+(y-y_0)^2=R^2 \end{array}\right\} \Rightarrow A^2+B^2-4C>0$

$$(x-x_0)^2+(y-y_0)^2=R^2 \Leftrightarrow$$

$$\Leftrightarrow x^2+x_0^2-2xx_0+y^2+y_0^2-2yy_0=R^2$$

$$\Leftrightarrow x^2+y^2-2x_0x-2y_0y+(x_0^2+y_0^2-R^2)=0$$

$$A=-2x_0 \qquad C=x_0^2+y_0^2-R^2$$
$$B=-2y_0$$

$$A^2+B^2-4C=\cancel{4x_0^2}+\cancel{4y_0^2}-\cancel{4y_0^2}-\cancel{4x_0^2}+4R^2$$
$$=4R^2>0$$

Αντίστροφο

Αν $A^2+B^2-4C>0 \Rightarrow x^2+y^2+Ax+By+C=0$ παριστάνει κύκλο.

$$x^2+y^2+Ax+By+C=0 \Leftrightarrow$$

$$\Rightarrow x^2-2\left(-\frac{A}{2}\right)x+\left(-\frac{A}{2}\right)^2-\left(\frac{A}{2}\right)^2+$$
$$+y^2-2\left(-\frac{B}{2}\right)y+\left(-\frac{B}{2}\right)^2-\left(\frac{B}{2}\right)^2+C=0$$

$$\Rightarrow \left(x-\left(-\frac{A}{2}\right)\right)^2+\left(y-\left(-\frac{B}{2}\right)\right)^2+C-\left(\frac{A}{2}\right)^2-\left(\frac{B}{2}\right)^2=0$$

$$\left(x-\left(-\frac{A}{2}\right)\right)^2+\left(y-\left(-\frac{B}{2}\right)\right)^2=\left(\frac{A}{2}\right)^2+\left(\frac{B}{2}\right)^2-C$$

$$\left(x-\left(-\frac{A}{2}\right)\right)^2+\left(y-\left(-\frac{B}{2}\right)\right)^2=\frac{A^2+B^2-4C}{4}>0$$

$$R=\frac{\sqrt{A^2+B^2-4C}}{2}$$

$$\left(x-\left(-\frac{A}{2}\right)^2\right)+\left(y-\left(-\frac{B}{2}\right)\right)^2=R^2$$

Εξίσωση κύκλου ακτίνας $R=\frac{\sqrt{A^2+B^2-4C}}{2}$

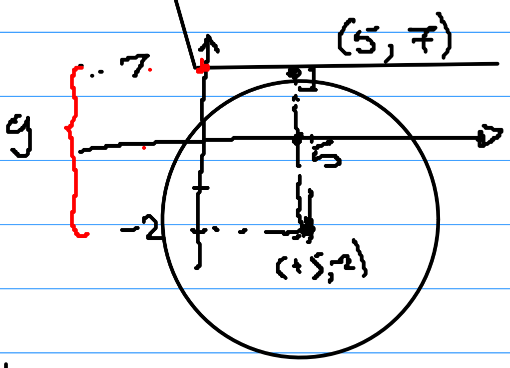
και κέντρου $\rho\left(-\frac{A}{2},-B/2\right)$.

— - —

18.2 α) $(x-x_0)^2+(y-y_0)^2=R^2$

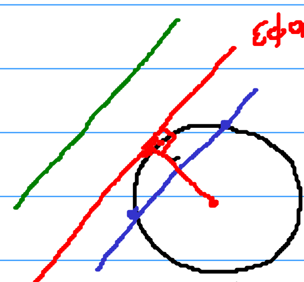
$$(x+3)^2+(y-4)^2=5^2 \quad ✓$$

β) $(x-5)^2+(y+2)^2=9^2$



Εφαπτομένη $\perp$ στην ακτίνα

εφαπτομένη;



Η εφαπτομένη του κύκλου είναι κάθετη στην ακτίνα του

Εφόσον η εφαπτομένη είναι $\parallel$ στον άξονα xx'

Τότε η κάθετη ευθεία στην εφαπτομένη που περνάει από το κέντρο του κύκλου

Θα είναι παράλληλη στην ευθεία yy'.
Άρα, το σημείο επαφής θα έχει την ίδια τεταγμένη με το κέντρο του κύκλου.

3β) $4x^2+4y^2-4x+20y+36=0 \Rightarrow$

$$x^2+y^2-x+5y+9=0$$

Από θεώρημα ①

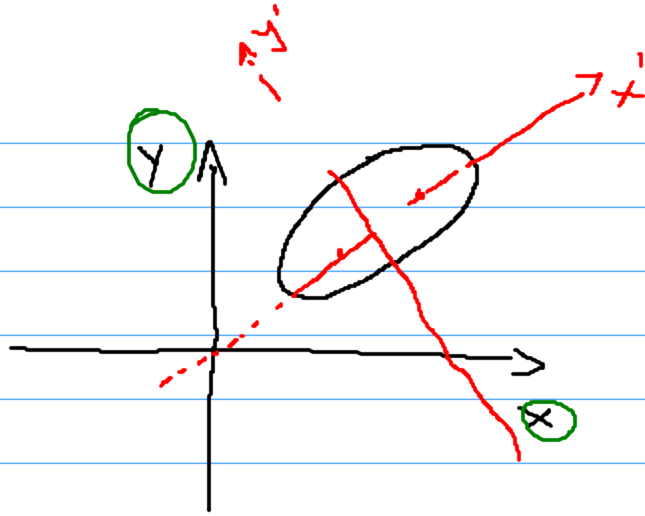
$A=-1$, $B=5$, $C=9$

$A^2+B^2-4C>0$

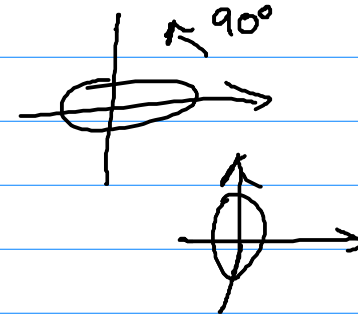
$(-1)^2+(5)^2-4\cdot 9$

$26-36<0$

δεν είναι εξ. κύκλου.



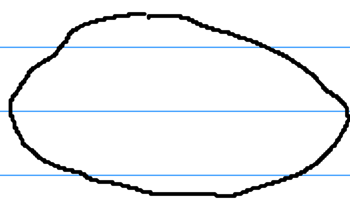
$$\cancel{\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1}$$

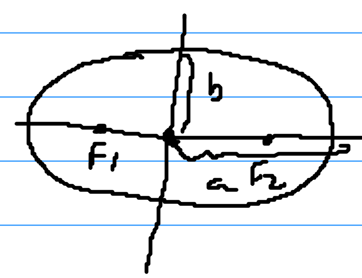


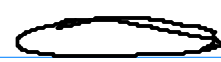
$$0 \le e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \left(\frac{b}{a}\right)^2} < 1$$

$$e^2 = 1 - \left(\frac{b}{a}\right)^2 \Rightarrow (b/a)^2 = 1 - e^2 \Rightarrow b/a = \pm\sqrt{1 - e^2}$$

$$\Rightarrow b/a = \sqrt{1 - e^2}$$



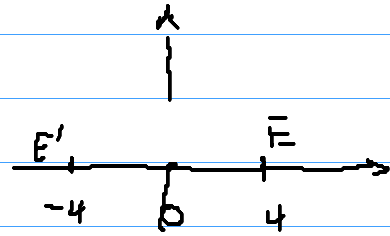




23.1 a)

$E'(-4,0)$

$E\ (4,0)$



$$\left.\begin{array}{l} 2a = 10 \\ (EE') = 2c = 8 \end{array}\right\} \Rightarrow \begin{array}{l} a = 5 \\ c = 4 \end{array}$$

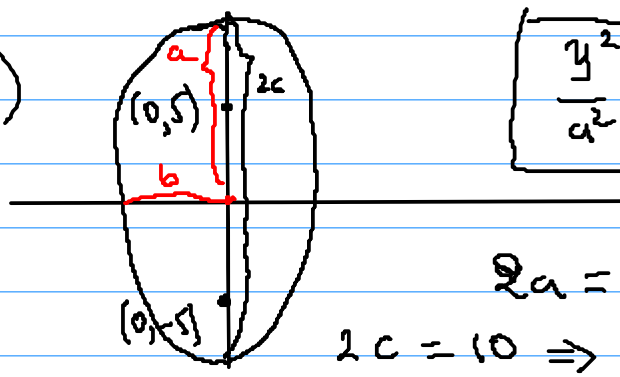
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$$b = \sqrt{a^2 - c^2}$$

$$= \sqrt{25 - 16} = \sqrt{9} \Rightarrow$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

23.1b)



$$\boxed{\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1}$$

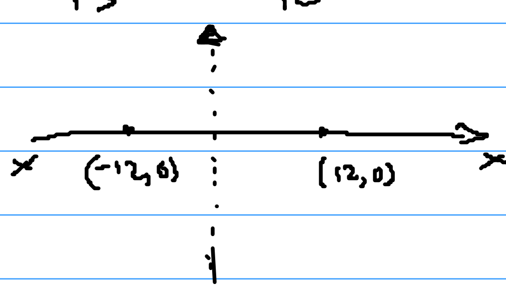
$$2a = 26 \Rightarrow a = 13$$

$$2c = 10 \Rightarrow c = 5$$

$$b = \sqrt{a^2 - c^2} \Rightarrow b = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$$

$$\frac{y^2}{13^2} + \frac{x^2}{12^2} = 1$$

23.1c)



$e = 12/13$

$c = 12$

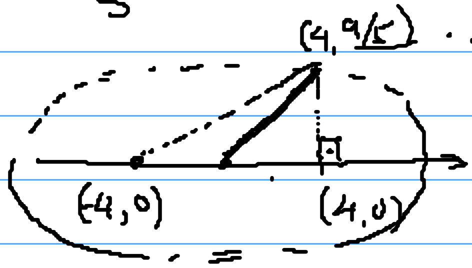
$$e = \frac{c}{a} \Rightarrow a = \frac{c}{e} = \frac{12}{12/13}$$

$a = 13$ $\qquad b = \sqrt{a^2 - c^2} = \sqrt{13^2 - 12^2}$

$$= \sqrt{(13-12)(13+12)}$$
$$= \sqrt{25} = 5$$

$$\frac{x^2}{13^2} + \frac{y^2}{5^2} = 1$$

d)



$c = 4$

$b^2 = a^2 - c^2 > 0$

$b^2 = a^2 - 16 > 0$

$$\frac{x^2}{\alpha^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{\alpha^2 - c^2} = 1$$

Το σημείο $(4, 9/5)$ ικανοποιεί την εξίσωσή μου

δηλ. $\frac{16}{a^2} + \frac{81/25}{\alpha^2 - 16} = 1$

$\alpha^2 > 16$

$$\frac{16}{a^2} + \frac{81}{25(a^2-16)} = 1$$

$$25a^2(a^2-16)\left[\frac{16}{a^2} + \frac{81}{25(a^2-16)}\right] = 25a^2(a^2-16)$$

$$16 \cdot 25(a^2-16) + 81a^2 = 25a^2(a^2-16)$$

$$\ldots \quad 25a^4 - 881a^2 + 6400 = 0$$

Θέτουμε $k = a^2$

$$25k^2 - 881k + 6400 = 0$$

$$\Delta = 881^2 - 4 \cdot 25 \cdot 6400 = 369^2$$

$$k = \frac{881 \pm 369}{2 \cdot 25} = \begin{cases} 25 \\ 10{,}24 \end{cases}$$

$$a^2 = \begin{cases} 25 \\ \text{ή} \\ 10{,}24 < 16 \end{cases}$$

απορρίπτεται

$$b^2 = a^2 - c^2 = 25 - 16 = 9$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$