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THE CHARYBDIS OF REALISM:
EPISTEMOLOGICAL IMPLICATIONS OF BELL'S
INEQUALITY*

In scepticism and realism, empiricist epistemology has its Scylla and Charybdis. The main role of scepticism today is in *reductio*: if a position is shown to lead to scepticism, it is thereby refuted. But fleeing from that danger, we are hard put to steer clear of the metaphysical rocks and shoals of realism. I shall leave the first danger aside for now.¹ Concerning epistemic realism I shall argue that, given one plausible way to make it precise, it is refuted by *Bell's Inequality Argument*. Realists will presumably wish to formulate their views on epistemology so as to avoid this refutation, and I shall end with some helpful suggestions.

1. EPISTEMIC REALISM

The medieval nominalist-realist debate was to a large extent about what we would today call causal properties and dispositions. These figured in the explanation of regularities in nature; for they determined how a given sort of thing could be or behave, how it would develop if left alone, and how react if acted upon. When the nominalist critique led to scepticism about the reality of these properties, the realist response was not simply to declare that the observed regularities would be unintelligible without them, though that was part of it. They also argued, unnervingly: if there is nothing to explain the regularity, no reason for it, there is also no reason for it to continue, and hence we can have no reason to expect its persistence. The nominalist position in philosophy of nature would, in other words, lead to scepticism, to the impossibility of reasonable expectations about the future.

This argument can easily be found also in later philosophy, connected with either metaphysical or scientific realism. Thus Peirce's critique of Mill, and his arguments for "Thirdness" (law or physical necessity) are prime examples; not surprisingly perhaps given his avowed debt to Scotus.² Similar arguments in contemporary philosophy of science I have discussed elsewhere.³ A recent book that

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straddles the two concerns, by Harré and Madden, provides a further striking example.⁴

Let me attempt a preliminary statement of the doctrine I shall call epistemic realism. Consider the question: *How is reasonable expectation about future events possible?* (“Future” may be replaced by “unobserved” for generality.) The recurrent idea that there is some rational form of simple extrapolation from the past, something like rules of induction, may be especially appealing to empiricists because it holds out hope for a presuppositionless, non-metaphysical answer. But it is an idea that goes into bankruptcy with every new philosophical generation.⁵ The answer that I shall call “realist” is: *Reasonable expectation of future events is possible only on the basis of some understanding of (or, reasonable certainty about) causal mechanisms that produce those events.*⁶

Support for this answer springs easily to mind. If there were no causal mechanism that makes litmus paper turn red in acid, then the past regularity (it always has) is a mere accident or coincidence, and there is no reason to think it will in the future. What is more, if there is granted to be a cause, everything hinges on what it is like. This is brought out graphically by Bertrand Russell’s refutation of simple induction. If there were a simple inductive rule, it would have to go from premises of the form “proportion m/n of past X ’s were A ’s” to some conclusion about future X ’s. Being a rule, it must lead from structurally similar premises to structurally similar conclusions. But now consider two persons applying this rule. The first looks back over past days and, arriving at the premise that the sun has always risen, concludes that it will continue to rise. The second is Russell’s example of a man falling down the Empire State building. As he passes an open window on the twenty-seventh floor, he is heard to say “Well, so far so good.”

The difference between the two cases lies clearly in the underlying mechanism that produces the two sequences. Their initial segments are structurally alike – but that means nothing at all. To think that it does is exactly like making one of two assumptions: either that all structurally similar observed sequences are produced by similar causal mechanisms, or else that the producing mechanism is irrelevant to the continuation. Since both those assumptions are manifestly unreasonable, to think or reason as if one had made them is unreasonable too.

This is the case for epistemic realism. To discuss it properly, we

must make it more precise, and this means mainly that we must make the notion of causal mechanism more precise.

2. CAUSALITY

Since I shall attack the position, I need to explicate the notion of causal mechanism in a way that is, while non-trivial, as weak as possible. This was also the problem that Hans Reichenbach faced when he wished to reconcile the ideas of causality that had played such a central role in the development of relativity, with the indeterminism of quantum mechanics. He thus developed the concept of *common cause* which has more recently been explored by Salmon and Suppes.⁷

Suppose there is a correlation between two (sorts of) events, such as lung cancer and heavy smoking. That is a correlation in the simultaneous presence of two factors: having lung cancer *now* and being a heavy smoker *now*. An explanation that has at least the form to satisfy us traces both back to a common cause (in this case, a history of smoking which both produced the smoking habit and irritated the lungs). Characteristic of such a common cause is that, relative to it, the two events are independent. Thus present smoking is a good indication of lung cancer in the population as a whole; but it carries no information of that sort for people whose past smoking history is already known. The second characteristic is that the cause lies in the past (if the two events are spatially separated, in the common part of their absolute past cones). Obviously A and B are independent relative to $(A \& B)$; so if the first characteristic alone were taken into account, the notion would be trivial.

The common cause picture is one that can certainly fit an indeterministic world: correlations, rather than events as such, require an explanation, and this is given by tracing (stochastic) processes back to their intersections.

There is a positive correlation between A and B exactly if $P(A|B) > P(A)$, or equivalently if $P(A \& B) > P(A)P(B)$, a symmetric relationship. Similarly we call quantities X and Y uncorrelated only if $P(A = a|Y = b) = P(X = a)$ for all values a and b of these quantities (assuming throughout that the events in question have positive probability). Negative correlation of A and B is obviously equivalent to positive correlation of \bar{A} and B , hence if all positive correlations are explained, so are all negative ones. A common cause

C for the correlation of A and B must have, by Reichenbach's definition, the property that A and B are independent (not correlated) conditional on C , and similarly conditional on \bar{C} .

Generalized to the case in which the cause itself is a variable factor Z – and not just a yes-no event – this becomes

$$P(A \& B|Z = x) = P(A|Z = x)P(B|Z = x)$$

for all values x of that quantity.

In realistic examples, the events A and B are often outcomes of experiments. That the experiment is going to be done at all, is of course an independent point; what we are meant to explain causally is that the outcome is thus and so if the experiment is done. Hence the statement of correlation takes the form: $P(A \& B|A^* \& B^*) = P(A|A^* \& B^*)P(B|A^* \& B^*)$. If there is a space-like separation between the two experiments, we suppose that either could be stopped at will before termination, and therefore that $P(A|A^* \& B^*) = P(A|A^*)$. This supposition may be false, for it is conceivable that there is a pre-established harmony, and the experimenters are caused to perform the B^* -experiment in just those cases in which experiment A^* is performed and has outcome A . A little common-sense should help us here when we are discussing a specific, realizable experimental arrangement, though we must keep the pre-established harmony possibility in mind if we contemplate general conclusions.

I am now going to describe a conceivable phenomenon (the one described by Bell) in which there is a correlation for which there can exist no common cause. The argument presupposes no physics at all. But we can remark that quantum mechanics allows for phenomena of this sort, and predicts the correlation when they occur. Since quantum mechanics is a well-supported theory, it is reasonable to have expectations in accord with its predictions. Therefore it is possible to have reasonable expectations of future events not based on any understanding of, or certainty about, causal mechanisms that produce those events. My analysis of Bell's Argument will give this conclusion a large degree of philosophical autonomy and generality.⁸

3. SURFACE DESCRIPTION OF A PHENOMENON

There are two generals, Alfredo and Armand, who wish to strike a common enemy simultaneously, unexpectedly, and very far apart. To

guarantee spy-proof surprise, they ask a physicist to construct a device that will give them a simultaneous signal, whose exact time of occurrence is not predictable. This is a science-fiction story – their physics is like ours but their technology much advanced (it happened in a galaxy long ago and far, far away . . .). The physicist gives each a receiver with three settings, and constructs a source which produces pairs of particles travelling toward those receivers. In each receiver is a barrier; if a received particle passes the barrier, a bell rings. The probability of this depends on the setting chosen. But when the two generals choose the same setting, one member of the pair of particles passes if and only if the other does not. Alfredo and Armand agree to choose a common setting at predetermined time t , and then Alfredo will strike the first time his bell does not ring while Armand will strike as soon as his bell does ring.

The story makes clear that no theory is presupposed in the description of what happens. Before looking at possible theories that might explain this curious correlation (which in itself is perfectly possible so far even from a classical point of view) I shall make this description precise, and general.

3-1. *The Experimental Situation*

Two experiments will be made, one on each of a pair of particles produced by a common source, referred to as the left (L) and the right (R). Each experiment can be of three sorts, or be said to have one of three *settings*. The proposition that the *first* kind of experiment is done on the *left* particle will be symbolized L1; and so forth.

Each experiment has two possible outcomes, *zero* or *one*. The proposition that the *second* kind of experiment is done on the *right* particle and has outcome *zero* will be symbolized R10; and so forth. Note that L1 is equivalent to the disjunction of L11 and L10. To allow general descriptions, I shall use indices i, j, k to range over $\{1, 2, 3\}$ and a, b over $\{0, 1\}$. In addition let $\bar{x} = 1 - x$, so that \bar{a} is the opposite outcome of a .

A situation in which the two experiments are going to be done on a single particle-pair, can be described in terms of a small field of propositions, generated by the logical partition:

$$PR_{surface} = \{Lia \ \& \ Rjb : i, j = 1, 2, 3 \text{ and } a, b = 0, 1\}$$

which has thirty-six distinct members.

3-2. *Surface Probabilities*

Probabilities for these propositions come from two sources. First, we may have some information about how the two settings will be chosen (possibly, to ensure randomness, by tossing dice). This gives us probabilities for the propositions in a coarser partition:

$$PRchoice = \{Li \ \& \ Rj : i, j = 1, 2, 3\}$$

Secondly we may have a hypothesis or theory which gives information about how likely the outcomes are for different sorts of experiments. Because there may be correlation, the information optimally takes the form of a function

$$P(Lia \ \& \ Rjb | Li \ \& \ Rj) = p$$

giving the probability of the (a, b) outcome for the (Li, Rj) experimental set-up. Let us call this function P a *surface state*. Note that it is not a probability function on our field; but it can be extended to one by combining it with a probability assignment to $PRchoice$ (which may be called a *choice weighting*). Such a probability function on the whole field may be called a *total state*. We can also derive marginal probabilities for the individual experiments, such as $P(Lia | Li) = \Sigma\{P(Lia \ \& \ Rjb | Li \ \& \ Rj) : j = 1, 2, 3 \text{ and } b = 0, 1\}$. Note that hypotheses concerning the surface state are directly testable: we simply choose the settings, and start the source working, and do the relevant frequency counts – see how often the bells ring, to follow our story.

3-3. *Perfect Correlation*

The special case I wish to examine satisfies two postulates for the surface states.

- I. *Perfect Correlation* $P(Lia \ \& \ Ria | Li \ \& \ Ri) = 0$
- II. *Surface Locality* $P(Lia | Li \ \& \ Rj) = P(Lia | Li)$
 $P(Rjb | Li \ \& \ Rj) = P(Rjb | Rj)$

It should be emphasized again that these probability assertions are directly testable by observed frequencies.

The Perfect Correlation Principle can be stated conveniently as: *parallel experiments have opposite outcomes*. My formulation, symmetric in L and R , is a simple condition on the surface states. If both

principles hold, we obviously have

$$P(Lia|Li) = P(Lia \& Ri\bar{a}|Li \& Ri)$$

but the reader is asked to resist counterfactual (and dubious) inferences such as that if the L1 experiment has outcome *one* then if the R1 experiment had been done, it would have had outcome *zero*.¹⁰

4. COMMON CAUSES AS HIDDEN VARIABLES

When principles I and II hold, there is a clear correlation between outcomes in the two experiments. What would a causal theory of this phenomenon be like? It would either postulate or exhibit a factor, associated with the particle source, that acts as common cause for the two separate outcomes, in the examined probabilistic sense. I shall refer to this as “the hidden factor”; not because I assume that we cannot have experimental or observational access to it, but because it does not appear in the surface description (i.e. in the statement of the problem).

Symbolizing the proposition that this hidden factor has value *q* as *Aq*, the space of possibilities now has the still finer partition

$$PR_{total} = \{Lia \& Rjb \& Aq : i, j = 1, 2, 3; \\ a, b = 0, 1; \text{ and } q \in I\}$$

where *I* is the set of possible values of that factor. A total state must be a probability function defined on the (sigma-) field generated by this partition. (Let us not worry about how to restate this in case *I* is uncountable; as will shortly turn out, that precaution is not needed.)

- III. *Causality* $P(Lia \& Rjb|Li \& Rj \& Aq) = \\ = P(Lia|Li \& Rj \& Aq) \\ \times P(Rjb|Li \& Rj \& Aq)$
- IV. *Hidden Locality* $P(Lia|Li \& Rj \& Aq) = P(Lia|Li \& Aq) \\ P(Rjb|Li \& Rj \& Aq) = P(Rjb|Rj \& Aq)$
- V. *Hidden Autonomy* $P(Aq|Li \& Rj) = P(Aq)$

I shall break up the ensuing argument into three sub-arguments, in which these postulates are separately exploited. But I shall say a few words here to defend the idea that a proper causal theory must satisfy all three.¹¹

Causality is just the probabilistic part of the Common Cause

principle stated before. The other two, *Hidden Locality* and *Hidden Autonomy* are meant to spin out implications of the idea that it is the common cause alone, and not special arrangements or relationships between the two separate experimental set-ups, that accounts for the correlation. If we had only III to reckon with, it could be satisfied simply by setting

$$Aq = (Li, a_i \& Rj, b_j)$$

where the *actual* settings chosen are (Li, Rj) and the *actual* outcomes are (a_i, b_j) . But the common cause is meant to be located at the particle source, in the absolute past of the two events, which have space – like separation. Now the choices of the experimental settings, and of the particular type of source used, can all be made beforehand, or else in any temporal order, and by means of any chance mechanisms or experimenters' whims you care to specify.

To put it conversely, if the probability of a given outcome at L is dependent not merely on the putative common cause, but also on what happens at R, or if the character of that putative common cause itself depends on which experimental arrangement is chosen (even if after the source has been constructed) then I say that the two outcome events have *not* been traced back to a common cause which explains their correlation. Of course I am not saying that nature must be such as to obey these postulates – quite the opposite. These postulates describe causal models, in the “common cause” sense of “causes”, and the question before us is whether all correlation phenomena can be embedded in such models.

4-1. *Causality Alone: A Deduction of Partial Determinism*

Principles I and III alone already imply that when parallel settings are chosen, the process is deterministic, the common cause determines the outcomes of the experiments with certainty. For abbreviating “ $Li \& Rj \& Aq$ ” to “ $Bijq$ ” we derive from those two principles:

$$\begin{aligned} 0 &= P(Lia \& Ria | Li \& Ri) \\ &= P(Lia \& Ria | Li \& Ri \& Aq) \\ &= P(Lia | Biiq)P(Ria | Biiq) \end{aligned}$$

But since the product is *zero*, one of the two multiplicands must be *zero*. The other will be *one*. For example, if $P(Li1 | Biiq) = 0$ then

$P(Li0|Biiq) = 1$. But setting $a = 0$ in the above deduction we conclude that if $P(Li0|Biiq) \neq 0$ then $P(Ri0|Biiq) = 0$, and hence $P(Ri1|Biiq) = 1$. So we see that, conditional on $Biiq$, all experimental outcomes have probability zero or one.

I doubt very much that Reichenbach can have perceived this consequence of his principle, because he had explicitly designed it so as not to require determinism for causal explanation. Had Einstein read Reichenbach and perceived this consequence in time, he could have added a little codicil to the Einstein-Podolski-Rosen paradox: according to the Common Cause principle, conditional certainties of the sort found in that paradox can exist only if they are the result of a hidden deterministic mechanism, so quantum mechanics is incomplete. See how much we have got – and we have hardly begun!

4-2. *Hidden Locality: A Deduction of Complete Determinism*

We have just deduced that conditional on the antecedent $(Li \& Ri \& Aq)$, all probabilities for outcomes are zero or one. But *Hidden Locality* says that this antecedent contains irrelevant information as far as the outcome at either side, separately, is concerned:

$$P(Lia|Li \& Aq) = P(Lia|Li \& Ri \& Aq) = \text{zero or one}$$

This follows from I, III, and IV together. It says that, given the value of the hidden variable that acts as common cause, the outcome of any performable experiment on either side is determined with certainty.

4-3. *Hidden Autonomy: The Testable Consequences*

It is a tenet of modern philosophy, owed perhaps to Kant, that the mere assertion of causality, or even determinism, has no empirical consequences. Any phenomena at all can be embedded in a causal story; only specific causal hypotheses have testable consequences. Nothing we have seen so far refutes that tenet, for all the consequences drawn have been about the hidden variable and not about the surface phenomena themselves. But we come now to the peculiar twist that Bell discerned.

We can begin with Wigner's observation that, given the preceding, there are only eight relevant classes of values for the hidden vari-

able.¹² (And accordingly, no generality in the causal theory will be lost if we say that the variable has only eight possible values.) For these values can be classified by their answers to the questions:

- (a) Suppose L_i . Is it the case that L_i ?
- (b) Suppose R_j . Is it the case that R_j ?

Given A_q , each of these questions receives a definite *yes* or *no* answer (with probability *one*). And indeed, the answers to the second type of question are determined by those to the first:

$$\begin{aligned} P(R_j1|R_j) &= P(R_j1|L_j \& R_j) \\ &= P(L_j0 \& R_j1|L_j \& R_j) \\ &= P(L_j0|L_j \& R_j) \\ &= P(L_j0|L_j) \\ &= 1 - P(L_j1|L_j) \end{aligned}$$

which deduction uses principles I, III, IV.

Since there are three questions of form (a), each with two possible answers, these answers divide the hidden variable values into $2^3 = 8$ types. Let us say that q is of type (a_1, a_2, a_3) when this value q predicts outcomes a_1, a_2, a_3 for arrangements L1, L2, L3 respectively. And let us abbreviate the assertion that the actual value is of this type to $Ca_1a_2a_3$. Precisely:

$$\begin{aligned} Ca_1a_2a_3 = \vee \{Aq : &P(L1a_1|L1 \& Aq) = \\ &P(L2a_2|L2 \& Aq) = \\ &= P(L3a_3|L3 \& Aq) = 1\} \end{aligned}$$

This is an ordinary finite disjunction of form $(Aq_1 \vee \dots \vee Aq_m)$ if the set of values of the hidden variables is finite (and we know now that we can assume that without loss of generality). Thus we have not introduced new propositions; we are still working within the field generated by PR_{total} .

Suppose now that we have chosen settings L1 and R2. What is the probability that we shall get outcomes L11 and R21? Well, let us put it a different way. Supposing A_q , what must the value q be like if we are to get outcomes L11 and R21? It must clearly be of type $(1, 0, b)$ for some value b or other. In other words, this outcome will happen only if $(C101 \vee C100)$ is the case. But that proposition has a probability of its own – and that is our answer.

The argument I have just given tacitly presupposes Principle IV of

Hidden Autonomy for it assumes that the choice of L1 and R2 as settings does not affect the probabilities for value q . Let us state the argument precisely. To begin

$$\begin{aligned} p(1; 2) &= P(L11 \& R21 | L1 \& R2) \\ &= \Sigma \{P(Aq)P(L11 \& R21 | L1 \& R2 \& Aq) : q \in I\} \end{aligned}$$

The notation $p(1; 2)$ is our abbreviation for future reference. We notice now that in the summation, the conditional probability equals zero except in cases where q is of type (1, 0, 1) or of type (1, 0, 0). Hence we have:

$$\begin{aligned} p(1; 2) &= \Sigma \{P(Aq) : q \text{ is of type } (1, 0, 1) \text{ or } (1, 0, 0)\} \\ &= P(C101) \vee P(C100) \\ &= P(C101) + P(C100) \end{aligned}$$

In just the same way we deduce

$$\begin{aligned} p(2; 3) &= P(C110) + P(C010) \\ p(1; 3) &= P(C110) + P(C100) \end{aligned}$$

Adding up the first two equations we get the sum of four probabilities, two of which appear again in the equation for $p(1; 3)$. Hence

$$p(1; 2) + p(2; 3) \geq p(1; 3)$$

And this is Bell's famous Inequality.

It hardly needs pointing out that the numbers $p(i; j)$ are surface probabilities by their definition (in which the hidden variable does not occur). So this Inequality is testable directly by means of observable frequencies. So our quite metaphysical looking principles have led us to an empirical prediction! What is more, the Einstein-Podolski-Rosen paradox has variants which satisfy our surface description and for which Quantum Mechanics predicts the violation of this Inequality. And finally, experimentation so far has produced overwhelming support for the quantum-mechanical predictions. The conclusion is surely inevitable: there are well-attested phenomena which cannot be embedded in any common-cause model.

5. EPISTEMOLOGICAL CONCLUSION

I have made an effort to present the deduction of Bell's Inequality shorn of all superfluous mathematical technicalities and woolly inter-

pretative commentary. (A reader as yet unfamiliar with the literature will be astounded to see the incredible metaphysical extravaganzas to which this subject has led.) In addition I began the deduction with theoretical postulates (III-V) that follow directly from the idea that correlation phenomena must have common-cause explanations.

Returning now to epistemology, let us again ask when it is possible to have reasonable expectations about future events. Assuming (as we surely all agree) that it is reasonable to base one's expectations on well-supported scientific theories, we are reasonable to expect the persistence, whenever the relevant conditions obtain, of the correlations predicted by such theories. And this point is quite independent of whether we are provided with a causal explanation – or even with the possibility thereof.

In response to the situation highlighted by Bell's Inequality, I suggest a picture of theories, of the enterprise of theorizing, and of how we justify our expectations that has nothing to do with causation or determination (however partial) of the future by the past. When I gave a surface description I was doing (in a modest way) what Suppes calls constructing a model of the data. When I followed this with postulates about a hidden factor acting as common cause, I was constructing a family of theoretical models in which such phenomenal structures were to be embeddable. And when the Inequality was deduced it became clear that only a proper subclass of the data models could be embedded in any of the theoretical models. Well, empirical adequacy of a theory consists in it having a model that *all* the (models of) *actual* phenomena will fit into. In some cases, the methodological tactic of developing a causal theory will achieve this aim of empirical adequacy, in other cases it will not, and that is just the way the world is. The causal terminology is descriptive, in any case, not of the (models of the) phenomena, but of the proffered theoretical models.¹³ So pervasive has been the success of causal models in the past, especially in a rather schematic way at a folk-scientific level, that a mythical picture of causal processes got a grip on our imagination. But to say that is itself as metaphysical as any other causal talk; is eloquent testimony, perhaps, that the grip is firm.

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NOTES

* This research was supported by National Science Foundation grant SES 80-005827, hereby gratefully acknowledged. The paper began as a commentary on Hilary Putnam's contribution to this volume; although it took independent shape, the reader will discern my debt in the way I see and state the problems addressed. This paper is also deeply indebted to Richmond H. Thomason, 'Prescience and Statistical Laws' (ms. circulated November 1980) which argues cogently for theses about knowledge similar to the ones I defend for reasonable expectation, in relation to causal dependence.

¹ There seem to me two main types of solutions to the problem of scepticism; they are the solutions of idealism and of voluntarism. I would classify Putnam's response to the question whether we could be brains in a vat as belonging to the idealist type. The voluntarist tradition, associated with Augustine, may be briefly, if perhaps cryptically, conveyed by the slogan that scepticism is the ass' side of Buridan's Ass' problem.

² See especially pages 157–158 and 166 of Charles S. Peirce, *Essays in the Philosophy of Science* (ed. V. Thomas; New York: Liberal Arts Press, 1957) and John F. Bolen, *Charles Peirce and Scholastic Realism* (Seattle: University of Washington Press, 1963).

³ In Chapter Two of *The Scientific Image* (Oxford University Press, 1980).

⁴ R. Harré and E.H. Madden, *Causal Powers* (Oxford: Blackwell, 1975); see especially pp. 70–71.

⁵ As Putnam points out, and I agree, the logical problems brought to light by Goodman's analysis cannot be circumvented, a fact sometimes obscured by their arid logical form.

⁶ There is no implication here that the understanding need go very deep; *perhaps* it is quite possible to have reasonable certainty that there is some causal mechanism or other, producing sequences of some vaguely described type or structure, that is producing the observed sequence, without any certainty at all about more intimate features of the mechanism.

⁷ For fuller presentation and references, see note 3 above.

⁸ I circulated this analysis in ditto form under the name of 'Baby Bell' in January 1981. The literature on the subject is voluminous; see especially J.S. Bell, *Physics* 1 (1965), 195–200, and *Rev. Modern Physics* 38 (1966), 447–452; J.F. Clauser and A. Shimony, *Rep. Prog. Phys.* 41 (1978), 1881–1927.

⁹ It will be clear, because II holds (note also that II follows from IV) that there is no question here at all of the two generals being able to signal each other faster than light. It is true that by their arrangement, they reap some benefits which, as their enemy may surmise, they would reap from having signals faster than light if there were such. Imagined conflicts between the described situation and relativity theory lie solely in controversial assumptions about what a relativistic indeterministic theory would have to be like.

¹⁰ Certain elegant simplifications of Bell's argument (P.H. Eberhard, *Il Nuovo Cimento* 38B (1977), 75–79; N. Herbert and J. Karush, *Foundations of Physics* 8 (1978), 313–317, rest on assumed principles about counter-factual conditionals that have been controversial or definitely rejected in the general theory of such conditionals developed by Stalnaker, Lewis, and others. This is discussed in forthcoming papers by Brian Skyrms and Geoffrey Hellman.

¹¹ Since II follows from the postulate that IV holds (for all values q), II will not play an overt role in the argument.

¹² E.P. Wigner, 'On Hidden Variables and Quantum Mechanical Probabilities', *American Journal of Physics* 38 (1970), 1005–1009.

¹³ A burning question at this point is clearly, what account can be given of testing and confirmation, the process that leads to reasonable theory choice or acceptance? I have given a preliminary statement of an account of theory choice as a case of decision making *sub specie* conflicting criteria in 'Glymour on Evidence and Explanation' forthcoming in a volume edited by John Earman.