Incompleteness, Nonlocality, and Realism

A PROLEGOMENON TO THE PHILOSOPHY OF QUANTUM MECHANICS

MICHAEL REDHEAD

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4

Nonlocality and the Bell Inequality

4.1. The Bell Inequality

In the preceding chapter we presented the Einstein dilemma, that the minimal instrumentalist interpretation F of QM either implied nonlocality or that F was incomplete. Einstein, as we have seen, chose the incompleteness horn of this dilemma and concluded that, at any rate in certain states, observables for which these states were not eigenstates nevertheless possessed sharp values. This suggested the programme of 'completing' QM in the style advocated in what we referred to as view A in Chapter 2. View A, it will be recalled, says that all observables, in all states, have sharp values. But then, in a famous paper published in 1964, Bell showed that view A, in conjunction with a locality principle appropriate to view A, which we shall term LOC_3, implied a certain inequality between measurable correlation coefficients in a slight extension of the Bohm spin example for the EPR argument. And this inequality, now usually referred to as the Bell inequality, turns out to be in disagreement, over a certain range of conditions, with the predictions of F itself.

This raises two questions. First there is a logical point, that filling out the interpretation F to the complete interpretation of view A is not consistently possible unless the locality principle LOC_3 used in deriving the Bell inequality is violated. In other words, following the incompleteness horn of the Einstein dilemma has not allowed us to escape nonlocality, but has itself landed us in a violation of LOC_3. The second question is whether the predictions of F for the particular set-up envisaged by Bell actually agree with experiment. It might be that the Bell inequality is not violated by experiment, so that view A and LOC_3 could be maintained, but F itself is what is wrong.

We shall now proceed to develop this circle of ideas in more detail. First, let us state the locality principle, LOC_3, appropriate to view A.

LOC_3:
A sharp value for an observable cannot be changed into another sharp value by altering the setting of a remote piece of apparatus.

Let us now see how view A supplemented by LOC_3 leads to the Bell inequality. In the original proof given by Bell, the hidden-variable version of view A, described in Section 2.1 above, was employed. But, as noted there, this involves additional assumptions over and above the existence of sharp values for all observables in all states. In particular, the hidden-variable approach commits us to the existence of joint probabilities for incompatible observables. This 'hidden' assumption might then be incriminated as responsible for the Bell inequality, leaving the locality assumption unchallenged. It is important, therefore, that a proof of the Bell inequality can be given which does not make use of the hidden-variable machinery, and which makes no assumption of joint probability distributions for incompatible observables. This we proceed to do.

Consider again Bohm's version of the EPR argument. Two spin-$\frac{1}{2}$ particles emerge from a source S in a singlet spin state, and move in opposite directions towards two spin-meters which can measure the spin-projection of either particle along any specified direction. We shall consider two directions or 'settings' for each spin-meter, viz. $\mathbf{a}$ and $\mathbf{a}'$ for A and $\mathbf{b}$ and $\mathbf{b}'$ for B. For the n^{th} pair of particles emitted from the source, denote by a_n the spin-component of the A-particle (i.e. the particle travelling towards spin-meter A) projected in the direction $\mathbf{a}$ in units of $\hbar/2$ when the A-meter is set parallel to $\mathbf{a}$. Similarly for a'_n, b_n, and b'_n in an obvious notation. QM of course dictates that, in so far as measurements by the spin-meters merely reveal these values, they are restricted always to be ± 1.

The situation is sketched in Fig. 9, where the two settings of each spin-meter are indicated by the possible positions labelled $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, and

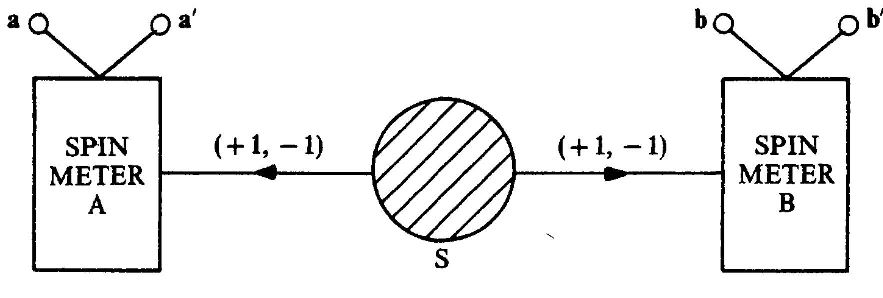

Fig. 9. Schematic illustration of the Bell experiment. S is the source emitting two spin-$\frac{1}{2}$ particles with possible spin-components in any direction of value ± 1 in units of $\hbar/2$. A and B are two spin-meters which can be adjusted to measure spin-components parallel to $\mathbf{a}$ or $\mathbf{a}'$ for the A-meter and $\mathbf{b}$ or $\mathbf{b}'$ for the B-meter.

$\mathbf{b}'$ of a 'knob' or joystick attached to the corresponding meter. Now form the expression

$$\gamma_n = a_n b_n + a_n b'_n + a'_n b_n - a'_n b'_n \tag{1}$$

γ_n clearly has integral values which can at most lie between -4 and $+4$ inclusive. The trick here is that the value of the fourth term (with the minus sign) is the product of the first three. Thus

$$a_n b_n \cdot a_n b'_n \cdot a'_n b_n = a_n^2 \cdot b_n^2 \cdot a'_n b'_n = a'_n b'_n$$

A little thought will show that this fact restricts the value of γ_n to ± 2.

This is confirmed by the following simple argument. Write

$$\gamma_n = a_n (b_n + b'_n) + a'_n (b_n - b'_n) \tag{2}$$

Now b_n and b'_n must have either the same sign or opposite sign. In either case, only one term in (2) is non-vanishing, and its value is clearly ± 2.

Now consider N events and form

$$\left|\frac{1}{N}\sum_{n=1}^{N} \gamma_n\right| = \left|\frac{1}{N}\sum_{n=1}^{N} a_n b_n + \frac{1}{N}\sum_{n=1}^{N} a_n b'_n + \frac{1}{N}\sum_{n=1}^{N} a'_n b_n - \frac{1}{N}\sum_{n=1}^{N} a'_n b'_n\right|$$

But, since $\gamma_n = \pm 2$ for all n, this expression must be less than or equal to 2.

Define correlation coefficients

$$\left.\begin{aligned}
c(\mathbf{a}, \mathbf{b}) &= \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} a_n b_n \\
c(\mathbf{a}, \mathbf{b}') &= \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} a_n b'_n \\
c(\mathbf{a}', \mathbf{b}) &= \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} a'_n b_n \\
c(\mathbf{a}', \mathbf{b}') &= \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} a'_n b'_n
\end{aligned}\right\} \tag{3}$$

Then in the limit as $N \to \infty$ we can conclude

$$|c(\mathbf{a}, \mathbf{b}) + c(\mathbf{a}, \mathbf{b}') + c(\mathbf{a}', \mathbf{b}) - c(\mathbf{a}', \mathbf{b}')| \leqslant 2 \tag{4}$$

The inequality (4) is one form of the so-called Bell inequality.

If we remember that the mean values $\overline{a_n}, \overline{b_n}, \overline{a'_n}$ and $\overline{b'_n}$ are all zero for $|\Psi_{\text{singlet}}\rangle$, and the variances $\overline{(a_n - \bar{a}_n)^2}$, etc. are all unity, then the definition of the correlation coefficients given in (3) agrees with the usual definition given in statistics, that the correlation coefficient between two statistical variables a_n, b_n is given in general by

$$\frac{\overline{(a_n - \bar{a}_n)(b_n - \bar{b}_n)}}{(\overline{(a_n - \bar{a}_n)^2} \cdot \overline{(b_n - \bar{b}_n)^2})^{\frac{1}{2}}}$$

where we use the bar to denote average or expectation values.

It is now easy to show that, for suitable choice of the direction **a**, **a**′, **b**, and **b**′, the QM predictions for the correlation coefficients violate the Bell inequality. We have in fact already done the necessary calculations in Chapter 1. Thus from Eq. (1.105) we have immediately

$$c(\mathbf{a}, \mathbf{b}) = -\cos\theta_{ab} \tag{5}$$

where θ_{ab} is the angle between the directions **a** and **b**. Similarly

$$\begin{aligned} c(\mathbf{a}, \mathbf{b}') &= -\cos\theta_{ab'} \\ c(\mathbf{a}', \mathbf{b}) &= -\cos\theta_{a'b} \\ c(\mathbf{a}', \mathbf{b}') &= -\cos\theta_{a'b'} \end{aligned} \tag{6}$$

Choose the directions **a**, **a**′, **b**, **b**′ to be coplanar and take **a** parallel to **b** and $\theta_{ab'} = \theta_{a'b} = \phi$, say, so $\theta_{a'b'} = 2\phi$ as illustrated in Fig. 10. For this special choice of directions, the Bell inequality will be satisfied by the QM predictions provided

$$F(\phi) \underset{\mathrm{Df}}{=} |1 + 2\cos\phi - \cos 2\phi| \leqslant 2 \tag{7}$$

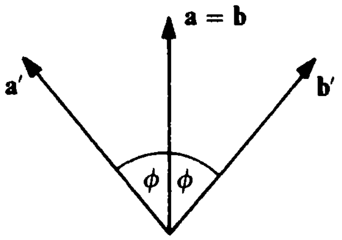

Fig. 10. Special choice of directions for illustrating the violation of the Bell inequality.

In Fig. 11 we show $F(\phi)$ plotted as a function of ϕ in the range $0° \leqslant \phi \leqslant 180°$. The Bell inequality is violated for all values of ϕ between 0° and 90°. It is easily checked that the maximum value for $F(\phi)$ is $2\frac{1}{2}$ and is attained for $\phi = 60°$.

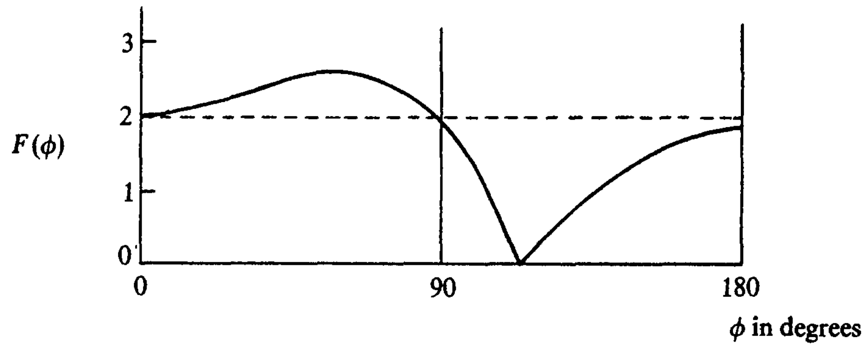

Fig. 11. Graph of $F(\phi)$ given in Eq. (7) against the angle ϕ in the range 0 to 180°. The Bell limit is given by $F = 2$. The inequality is violated for all values of ϕ between 0° and 90°.

It is instructive to consider an example of correlation in classical physics for which the Bell inequality is of course satisfied. Consider two wheels which spin with angular momentum **J** and $-$**J** about their common axle, so that the total angular momentum of the system is zero, just as in the QM spin example we have been discussing. Now let the two wheels fly apart, and measure the sign of the component of each wheel's angular momentum along arbitrary directions **a** for the first wheel and **b** for the second. Now consider an ensemble of such wheel and axle systems with the axles distributed isotropically in space, and let a_n and b_n be the signs of the angular momentum components for the n^{th} axle. So, just as in the QM case, a_n and b_n are always ± 1.

If we draw great circles on the unit sphere whose planes are perpendicular to **a** and **b**, the surface of the sphere is divided into four lunes of aperture θ_{ab}, $\pi - \theta_{ab}$, θ_{ab}, and $\pi - \theta_{ab}$, as illustrated in Fig. 12.

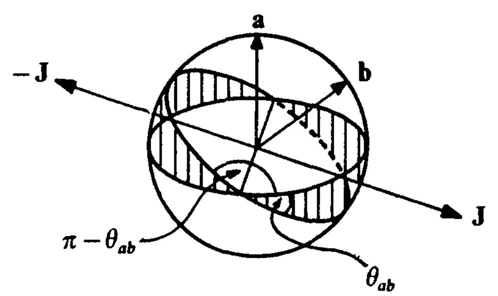

Fig. 12. Classical example of the Bell inequality for two wheels which fly apart with angular momenta **J** and $-$**J**.

If the axle cuts the sphere in the region of the two shaded lunes of aperture θ_{ab}, as shown in the figure, then clearly $a_n b_n$ will be $+1$, while if it cuts the sphere in the region of the two unshaded lunes of aperture $\pi-\theta_{ab}$, $a_n b_n$ will be -1. With an isotropic distribution of axle directions we have then the simple result

$$c(\mathbf{a},\mathbf{b}) = \overline{a_n b_n} = \frac{2\theta_{ab}(+1)+2(\pi-\theta_{ab})(-1)}{2\pi}$$

$$= -1+\frac{2\theta_{ab}}{\pi} \tag{8}$$

With the choice of the four directions **a**, **a**′, **b**, and **b**′ shown in Fig. 10, it is easily checked that the LHS of the Bell inequality comes out equal to 2; so in this particular example the Bell inequality is saturated but not, of course, violated.

The reason why the Bell inequality is violated in QM is due to the angular dependence of the correlation coefficients specified in Eqs. (5) and (6). Comparing (5) and (8), notice how for small θ_{ab} the QM prediction 'hangs on' to perfect anti-correlation more tightly than in the classical example. Thus from (5)

$$c(\mathbf{a},\mathbf{b}) = -\cos\theta_{ab} \simeq -1+\tfrac{1}{2}\theta_{ab}^2 \ldots$$

So de-correlation is proportional to θ_{ab}^2 rather than θ_{ab}, as specified in (8).

The essential ingredient that has gone into the proof of Bell's inequality is the assumption of LOC_3. For example, we have assumed that the value of a_n is the same whether we are measuring b_n or b_n', that the change in setting of the knob on the spin-meter B from **b** to **b**′ does not affect the value of a_n, which is 'discovered' by the spin-meter A with knob set in the direction **a**. This means that both occurrences of a_n in the expression (1) for γ_n have the same value, similarly for a_n', b_n, and b_n'. This is crucial to the proof that $\gamma_n = \pm 2$. Notice that the definition of a_n does allow for a dependence of the spin-projection of the A-particle parallel to **a**, on the setting of the spin-meter A.

There are four separate correlation experiments involved in testing the Bell inequality in the form in which we have presented it. These involve combining setting **a** with **b**, **a** with **b**′, **a**′ with **b**, and **a**′ with **b**′ respectively. We are regarding the four experiments as mutually exclusive, in the sense that each knob can have only one setting for any given experiment; so we are taking here the strong line that incompatible observables cannot be *measured* simultaneously. Nevertheless, we are assuming that a_n, b_n, a'_n, b'_n all have definite *values* which can be measured simultaneously in pairs: a_n with b_n, a_n with b'_n, a'_n with b_n, and a'_n with b'_n.

We illustrate what is going on by considering the table of values for a_n, a'_n, b_n and b'_n (see Fig. 13), in which the four correlation experiments are distinguished as I, II, III, and IV. Each of the four values occurs twice in each row of the table, since it figures in two correlation experiments. The fact that each of the two occurrences has the same numerical value (indicated by the 'ties' in Fig. 13) we term the Matching Condition, which essentially incorporates the assumption of LOC_3. Each pair of columns for a given correlation experiment enables one to compute, in the limit as $n \to \infty$, a correlation coefficient with respect to possessed values using all values of n.

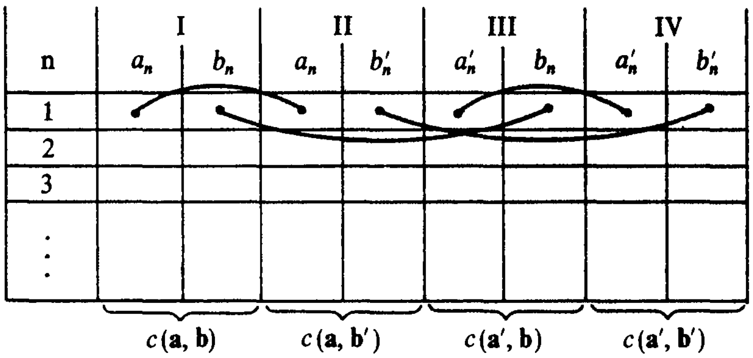

Fig. 13. Schematic table of values for the four correlation experiments I, II, III, and IV in the Bell experiment. The Matching Condition is illustrated by the 'ties' connecting values of the same spin-component in different experiments.

A very important point to notice here is that the possessed values are in general counterfactually possessed. Thus, if the spin-meter A is set parallel to **a** and the spin-meter B parallel to **b**, then the entries for a_n and b_n in the first two columns are actual possessed values. But what about the value for a_n in the third column? This is the value a_n would possess if spin-meter B were set parallel to $\mathbf{b}'$ instead of **b**. We shall argue later that these counterfactuals cause no difficulty under the assumption of determinism, i.e. that the values for a_n, a'_n, b_n, and b'_n are deterministically related to the total 'hidden' state of the two particles emerging from the source. But, equally, we shall argue that, if we give up determinism, then the Matching Condition is not licensed by appeal to LOC_3. So we have uncovered one 'hidden' assumption in the proof of Bell's inequality, viz. determinism.

But there are other assumptions we need to be explicit about. In order to carry out the correlation experiments we must perform a place selection on the sequence of (in general counterfactually) possessed values which tells us which values of n are to be the subject of which measurement procedure (I, II, III, or IV). We can now isolate two assumptions we have tacitly made:

1. Limiting frequencies computed under these place selections have the same value as those computed with all values of n.
2. The correlation coefficients evaluated with respect to measured values are the same as those evaluated with respect to these selected possessed values.

The first assumption, which we shall term the randomness assumption, is simply that each sequence of a_n's, b_n's, etc. is a random sequence in the Church–von Mises sense, if we suppose that the selection of measurement procedures is governed by some effectively computable rule, and furthermore remains random when we conditionalize on any specified value of properties possessed by the particle entering the opposite wing of the apparatus. This is needed to ensure that sequences such as $\{ a_n b_n \}$ are random in addition to $\{ a_n \}$ and $\{ b_n \}$. The second assumption is justified by a

Principle of Faithful Measurement (FM):

The result of measurement is numerically equal to the value possessed by an observable immediately prior to measurement.

Note that it is conceivable that FM is true, so that every time we make a measurement we reveal what is there, and yet the frequency distribution of measured values might not be equal to the unselected frequency distribution of the possessed values, because measurement selection might skew the underlying distribution. But this would suggest a remarkable conspiracy on the part of nature, that is clearly inconsistent with the experimenter's freedom to choose which possessed values to subject to measurement and, in particular, to specify a rule for sèlecting measurement procedures.

We can now collect up our various assumptions and write schematically

$$\text{View } A \wedge LOC_3 \wedge \text{Determinism} \wedge \text{Randomness} \wedge FM \rightarrow \text{Bell Inequality} \quad (9)$$

Looking at this rag-bag of assumptions made in the proof of the Bell inequality, there may be thought to be a tension or even a downright inconsistency between the assertions of Determinism and Randomness. Determinism said that the values of the spin-projections just before measurement were related deterministically to the state of the two particles produced in the source, while Randomness said that the sequence of successive values of a_n, for example, was a random sequence. But even if states of the source were produced deterministically from previous states of the system, this does not mean that the outcomes of measurement cannot be a random sequence—just that the determination is not ultimately specifiable by a rule which is effectively computable. Randomness in the Church—von Mises sense is compatible with an *ontological* determinism—a given state at one time issues in a *unique* state at a later time—but not with what we may term *pragmatic determinism*, that the prediction of future states can be effectively computed.

We have stressed so far the assumptions that *are* made in the proof of the Bell inequality. But, equally importantly, we stress the assumptions that have *not* been made:

1. We do not assume that the a_n's and a'_n's, for example, have a well-defined joint probability distribution, the correlation functions actually used in deriving the Bell inequality always referring to compatible (commuting) observables. In particular we do not assume that $\frac{1}{N}\sum_{n=1}^{N} a_n a'_n$ has any well-defined limit as $N \to \infty$.
2. We do not assume that a_n and a'_n can be measured simultaneously, contrary to the view expressed by Brody and de la Peña-Auerbach (1979).

4.2. Counterfactuals and Indeterminism

In this section we shall discuss the question of whether we can block the proof of the Bell inequality by giving up determinism.

We shall begin by sketching an argument, due to Stapp and Eberhard, to the effect that a proof can be given of the Bell inequality that is formulated entirely in terms of actual or possible *measurement results*, i.e. the responses of macroscopic measuring apparatus, and which is neutral with regard to possible interpretations of QM such as the views we have labelled *A*, *B*, and *C*. The locality assumption used in this type of argument we shall term LOC_4, which is formulated as follows:

LOC_4:

A macroscopic object cannot have its classical state changed by altering the setting of a remote piece of apparatus.

So the purported theorem is in essence

$$LOC_4 \rightarrow \text{Bell Inequality} \tag{10}$$

If this result could be established, it would provide a very powerful and comprehensive proof of nonlocality in QM. Schematically

$$F \rightarrow \sim(\text{Bell}) \rightarrow \sim(LOC_4) \tag{11}$$

and this result would apply just as well to view *B*, for example, as to view *A*. We have already seen (3.10 above) that view *B* implies a violation of LOC_1. But (10) would demonstrate that view *B* also involves a violation of LOC_4. In other words, we would now not need to go through the EPR argument plus the Bell inequality argument to demonstrate nonlocality in QM. Furthermore, the escape from the Einstein dilemma provided by view *C* would also be blocked, since not only LOC_2 would be (acceptably) violated, but also (unacceptably) LOC_4.

Let us then attempt, following the ideas of Stapp and Eberhard, to prove(10). We begin by simply taking over the mathematics of the proof of the Bell inequality given in the preceding section, but with appropriately altered definitions of the symbols a_n, a'_n, b_n, and b'_n. We interpret these quantities now, not as possessed values for the microsystems, but as the responses of the macroscopic spin-meters when set to measure these quantities. Thus a_n = response of *A*-meter when set to measure $\boldsymbol{\sigma}(A) \cdot \mathbf{a}$ for the n^{th} pair of particles emitted by the source. Similarly for a'_n, b_n and b'_n. But since the four correlation experiments I, II, III and IV are mutually exclusive, we must proceed counterfactually.

a_n = response *A*-meter *would* show *if* Experiment I or Experiment II were performed.

Now, the essential crux of the derivation of the Bell inequality was the Matching Condition. Applied to a_n, for example, this says that the result recorded by the *A*-meter would be the same whether Experiment I or Experiment II were performed. But Experiments I and II differ only in the setting of a remote piece of apparatus, spin-meter *B*. Thus the Matching Condition would follow, and hence the Bell inequality could be proved if the following principle could be sustained.

Principle of Local Counterfactual Definiteness (PLCD):

The result of an experiment which *could* be performed on a microscopic system has a definite value which does not depend on the setting of a remote piece of apparatus.

Clearly then, modulo the randomness assumption involved in the proof,

$$\text{PLCD} \rightarrow \text{Bell Inequality} \tag{12}$$

Suppose now we could show that

$$\text{LOC}_4 \rightarrow \text{PLCD} \tag{13}$$

Then from (13) and (12) we would at once obtain the result we are trying to prove, viz. (10).

Arguing contrapositively, (13) says that violation of PLCD implies a violation of LOC_4. This is the crucial claim that we want to examine. Let us consider two simple thought experiments.

In the first, a clock is situated at one end of a table. At time t_2, say 9 o'clock, the clock strikes. I stand at the other hand of the table and at time t_1, say 1 sec. before 9 o'clock, I raise my hand. I now ask the question: 'If I had not raised my hand at time t_1 would the clock have struck at time t_2?' Assuming no mysterious connection between my hand and the mechanism of the clock, the intuitively correct answer to this counterfactual query would seem to be 'Yes', in accordance with PLCD. And, moreover, if the clock had not struck at t_2, when I did not raise my hand at t_1, then I think we could have concluded that the macroscopic behaviour of the clock must depend on the remote setting of my hand, i.e. a violation of LOC_4.

Let us compare this conclusion with what happens in a second thought experiment. In this, the clock is replaced by an atom of radium which decays (emits an α-particle) at time t_2. Again at time t_1, just before t_2, I raise my hand. The question I now ask is: 'If I had not raised my hand at time t_1 would the atom still have decayed at time t_2?' It is not so obvious in this example what the answer to the question should be. Suppose the decay of the radium atom is a truly indeterministic process; then, if I imagine running the course of events through again, with my hand not raised this time, the outcome at time t_2 might just as well be that the atom did not decay. Or, to take a slightly different example, suppose I placed a bet on number 17 turning up for a truly indeterministic roulette wheel, and in fact number 16 turns up. Should I correctly say: 'If only I had bet on number 16 I would have won my bet'? This is a conundrum about which philosophers have different views, but it is certainly problematic in a way that is not the case for the deterministic clock example.

One popular way of analysing the truth conditions for counterfactuals is in terms of possible worlds. Let us apply this type of analysis to the counterfactual $\phi \;\Box\!\!\rightarrow \psi$, where ϕ denotes the condition that I do not raise my hand at time t_1, and ψ the state of affairs that the atom decays at time t_2, and $\Box\!\!\rightarrow$ is a convenient symbol for denoting a counterfactual conditional.

Let the world in which I raise my hand at t_1 and the atom decays at t_2 be denoted by W_i. Let possible worlds W_j for variable j be ordered in respect of 'nearness' to W_i. More specifically, we collect all worlds into classes each of which is composed of worlds 'equidistant' from W_i, and then order these classes in respect of 'distance' from W_i. If W_l is nearer to W_i than W_k, I write $W_l \underset{i}{<} W_k$. Then I analyse $\phi \;\Box\!\!\rightarrow \psi$ as

$$\exists W_k [\exists W_l ((W_l \underset{i}{<} W_k) \wedge W_l(\phi)) \wedge \forall W_j ((W_j \underset{i}{<} W_k) \rightarrow (W_j(\phi) \rightarrow W_j(\psi)))]$$

thus reducing the counterfactual conditional $\Box\!\!\rightarrow$ in terms of the material conditional $\rightarrow$. $W_p(\phi)$ signifies that ϕ is true in W_p. We are assuming then $\sim W_i(\phi)$. In words: there is a world sufficiently close to W_i such that there exist some worlds closer to W_i in which ϕ holds and such that for any such worlds ψ holds.

This somewhat involved analysis can be illustrated as follows. Represent the classes of equidistant worlds as spheres centred on W_i. Then $\phi \;\Box\!\!\rightarrow \psi$ would be true in the situation shown in Fig. 14, where the conditions ϕ and ψ are identified with the classes of worlds in which they hold. ϕ is shown shaded and ψ is the region inside the dotted line. W_k is some world on the sphere S_k, and W_j a world on the sphere S_j.

Let us take the example given by David Lewis in his book *Counterfactuals*. If a kangaroo did not have a tail it would fall over. The material conditional is not required to hold for worlds sufficiently far from W_i, i.e. worlds in which the kangaroo has crutches!

In our example time enters in an essential way—we are dealing with a tensed counterfactual. It is important here that the specification of

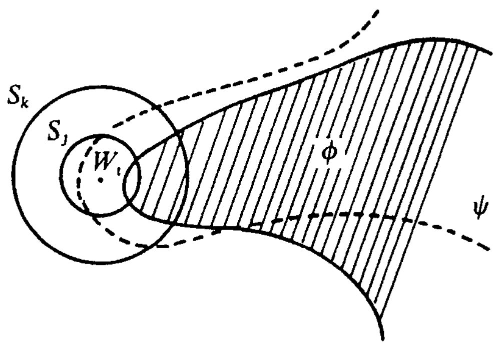

Fig. 14. Truth conditions for the counterfactual conditional ϕ □→ ψ. W_i denotes the actual world. S_j and S_k are 'spheres' of possible worlds 'equidistant' from W_i.

'sufficiently close' must refer to states of the world up to but not including t_2—i.e. we must not include the fact of ψ obtaining at t_2 as part of the specification of a sufficiently close world. This proviso is required to deal with the following sort of counter-example: 'If I were to pull the switch at t_1 the hydrogen bomb would explode at t_2.' If I include in the specification of 'sufficiently close world' what happens after the explosion, this statement comes out false, while the statement: 'If I were to pull the switch, the switch would jam' comes out true, since a jammed switch is closer to the actual world than one devastated by the explosion of the hydrogen bomb! This is very counter-intuitive, and can be dealt with as suggested by restricting the specification of 'sufficiently close' to states of the world up to but not including t_2.

Informally, then, we consider a world which differs from our actual world just in the fact that ϕ obtains in the alternative world but not in the actual world; everything else, including the laws of nature, are the same, and we let the world run on to the instant before t_2 and ask: 'Must now ψ occur at t_2?' But if the occurrence of ψ is essentially probabilistic, then there is no necessity for ψ to occur at t_2 in the alternative world. This is not paradoxical. It is just what we mean by saying that the occurrence of ψ is indeterministic (essentially probabilistic). We just cannot refine the description of the world prior to t_2 so as to force the occurrence of ψ at t_2. If such a refinement were possible the occurrence of ψ would be deterministic, not indeterministic.

To return to our original example, the fact that keeping my hand down at t_1 allows the atom not to decay at t_2 has nothing to do with a violation of locality, in particular LOC_4, assuming the decay of the atom is used to trigger some macroscopic recording device. It just involves the recognition of what is meant by the claim that the decay of a radium atom is indeterministic. So what is wrong with PLCD is the meaning attached to 'definite'. The outcome of an essentially indeterministic situation is definite in the sense that on a particular occasion, in a particular world, whatever does happen is the determinate outcome; but it is not definite in the sense that it is both determinate, i.e. necessarily either true or false, and determined, i.e. is rendered either necessarily true or necessarily false, by any possible specification of that world prior to the occurrence of that particular outcome.

The conclusion of this discussion is that, if we assume determinism, PLCD is a valid principle which can indeed be licensed by appeal to LOC_4. Under this assumption, violation of the Bell inequality shows that LOC_4 is violated. Indeed, if one can change the possessed value of some attribute at a distance, and this possessed value is linked deterministically to a macroscopic pointer reading as measurement outcome, then the state of the pointer can be altered 'at a distance'. In other words, LOC_4 holds if and only if LOC_3 holds; so violation of LOC_3 commits us to violation of LOC_4.

But in an indeterministic framework such as that envisaged in view *B*, for example, it is at least highly questionable whether PLCD can be invoked as a valid principle licensed by an appeal to LOC_4; and if PLCD cannot be used, then the generality claimed by Stapp and Eberhard for their method of proof must be denied.

A final comment on the Stapp–Eberhard proof. There is an alternative way that Eberhard refers to in his (1977) for expressing his results which does not employ counterfactuals. The idea is to record in a table, such as that illustrated in Fig. 13 above, not the results of four correlation experiments which could have been performed, although not simultaneously, but the results of four correlation experiments which are all actually performed. In other words, each pair of columns records a sequence of measurements made with the appropriate pair of knob settings. When four such correlation sequences have been obtained, they are written down side by side to form the complete eight-column table, but each row now refers to four different particle pairs emitted by the source.

So clearly, in general, the Matching Condition will not hold for any particular row. But, by chance, it may. So now perform a place selection on the columns which consists in selecting those rows for which the Matching Condition does hold. If we calculate new correlation coefficients using only these place-selected rows of the table, then Eberhard points out that the results cannot in general agree with the correlation coefficients calculated with the whole table. This is perfectly true, since the former, by construction, satisfy the Bell inequality, whereas the latter, in general, violate it. But all this has nothing to do with LOC_4. It just demonstrates the familiar fact that place selections in a random sequence, made in the light of the actual outcomes, can change the limiting frequencies for these outcomes.

There are, however, claims in the literature that sticking with view A but giving up determinism does allow the derivation of the Bell inequality by a *different* line of argument from that used by Stapp and Eberhard. In Section 4.4 we shall consider the case of so-called stochastic hidden-variable theories in relation to questions of nonlocality. But first we shall interrupt the main line of discussion to consider briefly some mathematical manipulations that enable us to give useful alternative formulations of the basic Bell inequality (4).

4.3. Alternative Forms of the Bell Inequality

We start from the inequality (4)

$$|c(\mathbf{a}, \mathbf{b}) + c(\mathbf{a}, \mathbf{b}') + c(\mathbf{a}', \mathbf{b}) - c(\mathbf{a}', \mathbf{b}')| \leqslant 2.$$

By interchanging $\mathbf{b}$ and $\mathbf{b}'$ we have also

$$|c(\mathbf{a}, \mathbf{b}) + c(\mathbf{a}, \mathbf{b}') - (c(\mathbf{a}', \mathbf{b}) - c(\mathbf{a}', \mathbf{b}'))| \leqslant 2.$$

Define

$$X = c(\mathbf{a}, \mathbf{b}) + c(\mathbf{a}, \mathbf{b}') \tag{14}$$

$$Y = c(\mathbf{a}', \mathbf{b}) - c(\mathbf{a}', \mathbf{b}') \tag{15}$$

Then we have shown

$$|X + Y| \leqslant 2 \tag{16}$$

$$|X - Y| \leqslant 2 \tag{17}$$

Suppose X has the same sign as Y. Then using (16)

$$|X| + |Y| = |X + Y| \leqslant 2$$

while if X has the opposite sign to Y, using (17)

$$|X| + |Y| = |X - Y| \leqslant 2$$

So in all cases

$$|X| + |Y| \leqslant 2 \tag{18}$$

or
$$|Y| \leqslant 2 - |X| \tag{19}$$
Now consider an experiment for which
$$c(\mathbf{b}, \mathbf{b}) = -1 \tag{20}$$
and choose the direction $\mathbf{a}$ parallel to $\mathbf{b}$ (the notation $c(\mathbf{b}, \mathbf{b})$ just means $c(\mathbf{a}, \mathbf{b})$ when $\mathbf{a}$ is chosen parallel to $\mathbf{b}$). Then (19) becomes
$$\begin{aligned}|c(\mathbf{a}', \mathbf{b}) - c(\mathbf{a}', \mathbf{b}')| &\leqslant 2 - |c(\mathbf{b}, \mathbf{b}) + c(\mathbf{b}, \mathbf{b}')| \\ &= 2 - |-1 + c(\mathbf{b}, \mathbf{b}')| \\ &= 2 - |1 - c(\mathbf{b}, \mathbf{b}')| \\ &= 2 - (1 - c(\mathbf{b}, \mathbf{b}')) \\ &= 1 + c(\mathbf{b}, \mathbf{b}')\end{aligned}$$
So
$$|c(\mathbf{a}', \mathbf{b}) - c(\mathbf{a}', \mathbf{b}')| \leqslant 1 + c(\mathbf{b}, \mathbf{b}') \tag{21}$$
for arbitrary directions $\mathbf{b}$, $\mathbf{a}'$ and $\mathbf{b}'$. This is the original form in which Bell presented his inequality. But notice that it is only applicable under the assumption (20).

Although (20) *is* true for the state $|\Psi_{\text{singlet}}\rangle$, the inequality (4) is of more general applicability, since it holds in the more general case where the strict anti-correlation expressed in (20) does not apply.

Another useful way of expressing the basic inequality (4) is in terms of probabilities rather than correlation functions. Denote by Prob $(\varepsilon_a, \varepsilon_b)_{a,b}$, for example, the joint probability that measurement of $\boldsymbol{\sigma}(A)\cdot\mathbf{a}$ yields the value ε_a and measurement of $\boldsymbol{\sigma}(B)\cdot\mathbf{b}$ yields the value ε_b, where ε_a and ε_b have the value ± 1 and $\boldsymbol{\sigma}(A)$ and $\boldsymbol{\sigma}(B)$ are as usual the Pauli spin-vectors for the A-particle and the B-particle respectively (i.e. the particles moving towards the A-meter and the B-meter). Similarly for Prob $(\varepsilon_a, \varepsilon_{b'})_{a,b'}$, Prob $(\varepsilon_{a'}, \varepsilon_b)_{a',b}$ and Prob $(\varepsilon_{a'}, \varepsilon_{b'})_{a',b'}$. Also let Prob $(\varepsilon_a)_a$ be the probability of measuring the value ε_a for $\boldsymbol{\sigma}(A)\cdot\mathbf{a}$, irrespective of what is measured on the spin-meter B. Similarly for Prob $(\varepsilon_b)_b$. Then we have
$$\text{Prob}\,(+1)_a = \text{Prob}\,(+1,+1)_{a,b} + \text{Prob}\,(+1,-1)_{a,b} \tag{22}$$
$$\text{Prob}\,(+1)_b = \text{Prob}\,(+1,+1)_{a,b} + \text{Prob}\,(-1,+1)_{a,b} \tag{23}$$
and
$$\begin{aligned}\text{Prob}\,(+1,+1)_{a,b} + \text{Prob}\,(-1,-1)_{a,b} + \text{Prob}\,(+1,-1)_{a,b}& \\ + \text{Prob}\,(-1,+1)_{a,b} = 1&\end{aligned} \tag{24}$$
From Eqs. (22) to (24) we easily derive
$$\text{Prob}\,(+1,-1)_{a,b} = \text{Prob}\,(+1)_a - \text{Prob}\,(+1,+1)_{a,b} \tag{25}$$
$$\text{Prob}\,(-1,+1)_{a,b} = \text{Prob}\,(+1)_b - \text{Prob}\,(+1,+1)_{a,b} \tag{26}$$

and

$$\mathrm{Prob}(-1,-1)_{a,b} = 1 - \mathrm{Prob}(+1,+1)_{a,b} - \mathrm{Prob}(+1,-1)_{a,b}$$
$$- \mathrm{Prob}(-1,+1)_{a,b} = 1 + \mathrm{Prob}(+1,+1)_{a,b} - \mathrm{Prob}(+1)_a - \mathrm{Prob}(+1)_b \quad (27)$$

From Eqs. (25) to (27) we obtain

$$c(\mathbf{a},\mathbf{b}) = \mathrm{Prob}(+1,+1)_{a,b} + \mathrm{Prob}(-1,-1)_{a,b}$$
$$- \mathrm{Prob}(+1,-1)_{a,b} - \mathrm{Prob}(-1,+1)_{a,b} = 4\,\mathrm{Prob}(+1,+1)_{a,b} - 2\,\mathrm{Prob}(+1)_a - 2\,\mathrm{Prob}(+1)_b + 1 \quad (28)$$

Writing down similar expressions for $c(\mathbf{a},\mathbf{b}')$, $c(\mathbf{a}',\mathbf{b})$ and $c(\mathbf{a}',\mathbf{b}')$, we easily derive the following inequality from (4)

$$-1 \leqslant \mathrm{Prob}(+1,+1)_{a,b} + \mathrm{Prob}(+1,+1)_{a,b'} + \mathrm{Prob}(+1,+1)_{a',b}$$
$$- \mathrm{Prob}(+1,+1)_{a',b'} - \mathrm{Prob}(+1)_a - \mathrm{Prob}(+1)_b \leqslant 0 \quad (29)$$

More generally, if we choose $\varepsilon_a = \varepsilon_{a'} = \varepsilon_A$, say, and $\varepsilon_b = \varepsilon_{b'} = \varepsilon_B$, say, then we can write

$$-1 \leqslant \mathrm{Prob}(\varepsilon_A,\varepsilon_B)_{a,b} + \mathrm{Prob}(\varepsilon_A,\varepsilon_B)_{a,b'}$$
$$+ \mathrm{Prob}(\varepsilon_A,\varepsilon_B)_{a',b} - \mathrm{Prob}(\varepsilon_A,\varepsilon_B)_{a',b'}$$
$$- \mathrm{Prob}(\varepsilon_A)_a - \mathrm{Prob}(\varepsilon_B)_b \leqslant 0 \quad (30)$$

where $\varepsilon_A = \pm 1$, $\varepsilon_B = \pm 1$.

The set of four inequalities comprised in (30) are actually equivalent to (4), in the sense that we can derive (4) from (30) by multiplying the two inequalities for which $\varepsilon_A \neq \varepsilon_B$ by -1 and adding to the two inequalities for which $\varepsilon_A = \varepsilon_B$.

4.4. Stochastic Hidden-Variable Theories

We now revert to the question of whether a proof can be given of the Bell inequality if we combine view *A* with indeterminism. The framework of so-called stochastic hidden-variable theories will provide a basis for this discussion. (We use the term 'theory' rather than 'interpretation' to allow for the possibility of not reproducing all the empirical predictions of QM.) The idea of such theories is that the 'complete' hidden-variable description of the source does not determine the values of local observables possessed by the two particles in the Bell type of experiment, but only the probabilities for possible values to occur. We can think picturesquely that the values of the spin-components in any given direction are developing in time stochastically, the state of the source controlling only the probabilities that particular values will be revealed when subsequent measurements are performed. We will still suppose that faithful measurement is true, i.e. that measurement at time t reveals the value possessed at time t. As we have said before, if measurement results were themselves linked stochastically to possessed values, it would be difficult to know in what sense one could talk of measurement at all. More formally, we shall assume the existence of a joint probability density

$$\text{Prob}\,(\varepsilon_a, \varepsilon_b, \lambda)_{a,b}^{\eta_A,\eta_B}$$

for the values ε_a and ε_b to be possessed by the observables a and b, which are shorthand for $\boldsymbol{\sigma}(A)\cdot\mathbf{a}$ and $\boldsymbol{\sigma}(B)\cdot\mathbf{b}$ respectively, and the value λ for the hidden-variable specifying the state of the source. The superscripts η_A and η_B indicate the settings of the two spin-meters A and B respectively.

If $\eta_A = \mathbf{a}$ and $\eta_B = \mathbf{b}$, then this will be the probability of finding the results ε_a and ε_b on measuring a and b, together with the value λ for the hidden variable. This joint probability in terms of measurement results will be denoted simply by

$$\text{Prob}\,(\varepsilon_a, \varepsilon_b, \lambda)_{a,b}$$

But even if $\eta_A \neq \mathbf{a}$, $\eta_B \neq \mathbf{b}$, the joint probability $\text{Prob}\,(\varepsilon_a, \varepsilon_b, \lambda)_{a,b}^{\eta_A,\eta_B}$ is supposed to exist, although its values will not translate immediately in terms of the probabilities for measurement results.

We now write

$$\begin{aligned}\text{Prob}\,(\varepsilon_a, \varepsilon_b, \lambda)_{a,b}^{\eta_A,\eta_B} &= \text{Prob}\,(\varepsilon_a/\varepsilon_b\ \&\ \lambda)_a^{\eta_A,\eta_B} \\ &\times \text{Prob}\,(\varepsilon_b/\lambda)_b^{\eta_A,\eta_B} \\ &\times \rho^{\eta_A,\eta_B}(\lambda)\end{aligned} \tag{31}$$

where $\text{Prob}\,(\varepsilon_a/\varepsilon_b\ \&\ \lambda)_a^{\eta_A,\eta_B}$ is the conditional probability for a to have the value ε_a, given values ε_b for b and λ for the hidden variable (with settings η_A and η_B for the spin-meters), $\text{Prob}\,(\varepsilon_b/\lambda)_b^{\eta_A,\eta_B}$ is the conditional probability for b to possess the value of ε_b given the value λ for the hidden variable, and $\rho^{\eta_A,\eta_B}(\lambda)$ is the probability density for finding the value λ of the hidden variable.

In order to derive the Bell inequality, we begin by making the following *completeness* assumption

$$\text{Prob}\,(\varepsilon_a/\varepsilon_b\ \&\ \lambda)_a^{\eta_A,\eta_B} = \text{Prob}\,(\varepsilon_a/\lambda)_a^{\eta_A,\eta_B} \tag{32}$$

The significance of (32), first pointed out by Jarrett (1984), is that λ is sufficient to determine *completely* $\text{Prob}\,(\varepsilon_a/\varepsilon_b\ \&\ \lambda)_a^{\eta_A,\eta_B}$. Specification of

ε_b is not required. We shall return in a moment to discuss the significance of violating the completeness condition.

Under the completeness assumption (32), Eq. (31) reduces to

$$\text{Prob}\,(\varepsilon_a, \varepsilon_b, \lambda)_{a,b}^{\eta_A,\eta_B} = \text{Prob}\,(\varepsilon_a/\lambda)_a^{\eta_A,\eta_B} \times \text{Prob}\,(\varepsilon_b/\lambda)_b^{\eta_A,\eta_B} \times \rho^{\eta_A,\eta_B}(\lambda) \qquad (33)$$

But in order to derive the Bell inequality, it is necessary to make the following additional locality assumptions:

$\text{Prob}\,(\varepsilon_a/\lambda)_a^{\eta_A,\eta_B}$ is independent of η_B

$\text{Prob}\,(\varepsilon_b/\lambda)_b^{\eta_A,\eta_B}$ is independent of η_A

$\rho^{\eta_A,\eta_B}(\lambda)$ is independent of η_A and η_B.

Introducing these further locality assumptions into (33), we obtain finally

$$\begin{aligned}\text{Prob}\,&(\varepsilon_a, \varepsilon_b, \lambda)_{a,b}^{\eta_A,\eta_B}\\ &= \text{Prob}\,(\varepsilon_a/\lambda)_a^{\eta_A}\\ &\times \text{Prob}\,(\varepsilon_b/\lambda)_b^{\eta_B}\\ &\times \rho(\lambda)\end{aligned} \qquad (34)$$

where we have suppressed those indices on which the indicated probabilities do not depend. In particular, with ε_a and ε_b now referring to measurement results,

$$\text{Prob}\,(\varepsilon_a, \varepsilon_b, \lambda)_{a,b} = \text{Prob}\,(\varepsilon_a/\lambda)_a \times \text{Prob}\,(\varepsilon_b/\lambda)_b \times \rho(\lambda) \qquad (35)$$

The representation (35) is often referred to as 'factorizability' in the literature.

From (35)

$$\text{Prob}\,(\varepsilon_a, \varepsilon_b)_{a,b} = \int_\Lambda \text{Prob}\,(\varepsilon_a/\lambda)_a \cdot \text{Prob}\,(\varepsilon_b/\lambda)_b \cdot \rho(\lambda)\,d\lambda \qquad (36)$$

Similarly

$$\text{Prob}\,(\varepsilon_a, \varepsilon_{b'})_{a,b'} = \int_\Lambda \text{Prob}\,(\varepsilon_a/\lambda)_a \cdot \text{Prob}\,(\varepsilon_{b'}/\lambda)_{b'} \cdot \rho(\lambda)\,d\lambda \qquad (37)$$

$$\text{Prob}\,(\varepsilon_{a'}, \varepsilon_b)_{a',b} = \int_\Lambda \text{Prob}\,(\varepsilon_{a'}/\lambda)_{a'} \cdot \text{Prob}\,(\varepsilon_b/\lambda)_b \cdot \rho(\lambda)\,d\lambda \qquad (38)$$

$$\text{Prob}\,(\varepsilon_{a'}, \varepsilon_{b'})_{a',b'} = \int_\Lambda \text{Prob}\,(\varepsilon_{a'}/\lambda)_{a'} \cdot \text{Prob}\,(\varepsilon_{b'}/\lambda)_{b'} \cdot \rho(\lambda)\,\mathrm{d}\lambda \quad (39)$$

and

$$\text{Prob}\,(\varepsilon_a)_a = \int_\Lambda \text{Prob}\,(\varepsilon_a/\lambda)_a \cdot \rho(\lambda)\,\mathrm{d}\lambda \quad (40)$$

$$\text{Prob}\,(\varepsilon_b)_b = \int_\Lambda \text{Prob}\,(\varepsilon_b/\lambda)_b \cdot \rho(\lambda)\,\mathrm{d}\lambda \quad (41)$$

With the representations (36), (37), (38), (39), (40), and (41) it is possible to prove the inequality (30). This follows at once from the following inequality that holds for any real numbers x, y, x', y' that lie in the interval [0, 1]

$$-1 \leqslant xy + x'y + xy' - x'y' - x - y \leqslant 0 \quad (42)$$

Substituting

$$\begin{aligned} x &= \text{Prob}\,(\varepsilon_a/\lambda)_a \\ y &= \text{Prob}\,(\varepsilon_b/\lambda)_b \\ x' &= \text{Prob}\,(\varepsilon_{a'}/\lambda)_{a'} \\ y' &= \text{Prob}\,(\varepsilon_{b'}/\lambda)_{b'} \end{aligned}$$

and assuming

$$\begin{aligned} \varepsilon_a &= \varepsilon_{a'} = \varepsilon_A \\ \varepsilon_b &= \varepsilon_{b'} = \varepsilon_B \end{aligned}$$

yields (30), on integrating over λ and remembering

$$\int_\Lambda \rho(\lambda)\,\mathrm{d}\lambda = 1$$

From (30), as we have seen, the basic inequality (4) follows. We have thus succeeded in giving a proof of (4) that avoids counterfactuals.

The violation of the Bell inequality means that we must give up factorizability. But this circumstance can also be derived without making any use of the Bell inequality, for the singlet state correlations, by the following argument. The factorizable stochastic hidden-variable theories cannot accommodate the existence of strict anti-correlation, as in the original EPR set-up with **a** parallel to **b**. In this case we would require $\text{Prob}(+1, +1)_{a,b} = \text{Prob}(-1, -1)_{a,b} = 0$, but substituting in (36) this requires, for example,

$$\int_{\Lambda} \text{Prob}(+1/\lambda)_a \cdot \text{Prob}(+1/\lambda)_b \cdot \rho(\lambda)\,d\lambda = 0$$

Since all factors in the integrand are non-negative, this implies

$$\text{Prob}(+1/\lambda)_a = 0 \text{ or } \text{Prob}(+1/\lambda)_b = 0$$

Considering, for example, the first alternative implies $\text{Prob}(-1/\lambda)_a = 1$, but repeating the above argument with $\varepsilon_a = \varepsilon_b = -1$, then gives $\text{Prob}(-1/\lambda)_b = 0$, which in turn implies $\text{Prob}(+1/\lambda)_b = 1$. It is clear, then, that in the case of strict anti-correlation all conditional probabilities $\text{Prob}(\varepsilon_a/\lambda)_a$ and $\text{Prob}(\varepsilon_b/\lambda)_b$ are zero or one, and the theory has collapsed into a deterministic one.

It is true, as we shall see in the next section, that the experimental tests of the Bell inequality that have so far been carried out have employed systems that do not involve the strict anti-correlation of our idealized thought experiment. So in these cases we do require the violation of the Bell inequality to demonstrate the failure of factorizability.

We now want to return to the question, what would be the significance of violating the completeness condition (32)? This would mean that probability distributions of properties possessed by the A-particle would depend in an essential way on what property is possessed by the B-particle. Another way of understanding the completeness condition is that it identifies λ, the complete state of the source, as the common (stochastic) cause of a having the value ε_a and b the value ε_b. Thus Eq. (32) tells us that λ *screens off* ε_b from ε_a. If (32) is violated it is often argued that a and b must exhibit a direct stochastic *causal* link on the grounds that the correlations between a and b can only be accounted for on the basis of stochastic links to a common cause or a direct stochastic causal link. But this conclusion can be questioned if proper account is taken of necessary conditions of robustness required for a direct causal link. By robustness of a causal relation we mean the following: A stochastic causal connection between two physical magnitudes a and b pertaining to two separated systems A and B is said to be *robust* if and only if there exists a class of sufficiently small disturbances acting on $B(A)$ such that $b(a)$ screens off $a(b)$ from these disturbances.

Denoting the disturbance acting on B by d, then the first part of this condition can be rendered formally as

$$\exists D(\forall d \in D(\mathrm{Prob}(a = \varepsilon_a / b = \varepsilon_b \& d) = \mathrm{Prob}\,(a = \varepsilon_a / b = \varepsilon_b))) \quad (43)$$

A similar condition can be written down for disturbance acting on A.

The requirement of robustness as a necessary condition for a causal relation means that suitably small disturbances of either relata do not affect the causal relation. This is essentially the basis of the mark method for identifying causal processes. The processes propagate small disturbances (marks) in a local event-structure in accordance with the causal law at issue.

We can easily translate this robustness condition so as to apply to the singlet-state correlation we are discussing. Consider possible perturbations of the quantum-mechanical state $|\Psi\rangle$ by disturbances acting on the particle B. In order to make the problem tractable we shall restrict the discussion to coupling of particle B to uniform c-number fields of arbitrary strength, which are switched on for some specified interval of time to provide the perturbation. Let Ψ' be a variable ranging over these perturbed states. Then a necessary condition that a and b exhibit a stochastic causal connection for arbitrary choice of the directions **a** and **b** is:

$$\exists D \forall a \forall b (\forall |\Psi'\rangle \in D\,(\mathrm{Prob}^{|\Psi'\rangle}(a = \varepsilon_a / b = \varepsilon_b) = \mathrm{Prob}^{|\Psi\rangle}(a = \varepsilon_a / b = \varepsilon_b))) \quad (44)$$

where the superscript on Prob denotes the quantum-mechanical state, and where the class D is some non-empty set of perturbed states arising from sufficiently weak perturbing fields.

We shall now show by explicit calculation that condition (44) is violated. Since (44) is a necessary condition for a direct stochastic causal link between a and b it will follow that no such link exists.

Denote the spin-projection of σ_A and σ_B along the arbitrarily chosen Z-axis by σ_{A_z} and σ_{B_z} respectively. Then we have

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\sigma_{Az} = +1\rangle\,|\sigma_{Bz} = -1\rangle - |\sigma_{Az} = -1\rangle\,|\sigma_{Bz} = +1\rangle) \quad (45)$$

which is a vector in $\mathbb{H}_A \otimes \mathbb{H}_B$, the tensor product of the Hilbert Spaces, $\mathbb{H}_A$ and $\mathbb{H}_B$ for the particles A and B.

Consider an arbitrary perturbation acting on $\mathbb{H}_B$. It will induce a 2×2 unitary transformation on all vectors belonging to $\mathbb{H}_B$. This transformation is thus an element of $U(2)$, the group of 2-dimensional unitary transformations. It is well-known that $U(2)$ can be exhibited as the direct product of $U(1)$, the group of 1-dimensional unitary transformations, and $SU(2)$, the group of 2-dimensional unitary unimodular transformations. Formally

$$U(2) = U(1) \times SU(2) \tag{46}$$

Now an element of $U(1)$ merely induces a phase-shift which does not change the physical state (ray) associated with particle B and can be ignored in computing all probabilities. The action of an element of $SU(2)$ can always be represented as $e^{i(\sigma_B \cdot \mathbf{n})\phi/2}$, where the direction of the unit vector $\mathbf{n}$ and the magnitude of the angle ϕ range over the three parameters of the group (note that $0 \leqslant \phi < 4\pi$).

We denote $e^{i(\sigma_B \cdot \mathbf{n})\phi/2}$ by $u(\mathbf{n}, \phi)$. Then the most general perturbed state is given by

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}(|\sigma_{Az} = +1\rangle u(\mathbf{n}, \phi)\,|\sigma_{Bz}$$

$$= -1\rangle - |\sigma_{Az} = -1\rangle\, u(\mathbf{n}, \phi)|\sigma_{Bz} = +1\rangle) \tag{47}$$

In what follows we shall consider the particular choice $\varepsilon_a = \varepsilon_b = 1$.

Then we have the familiar results (cf. Eq. 1.106)

$$\mathrm{Prob}^{|\Psi\rangle}(a = 1) = \tfrac{1}{2} \tag{48}$$

$$\mathrm{Prob}^{|\Psi\rangle}(b = 1) = \tfrac{1}{2} \tag{49}$$

$$\mathrm{Prob}^{|\Psi\rangle}(a = 1/b = 1) = \sin^2 \tfrac{1}{2}\theta_{ab} \tag{50}$$

where θ_{ab} is the angle between the directions $\mathbf{a}$ and $\mathbf{b}$. We are interested now in calculating $\mathrm{Prob}^{|\Psi'\rangle}$ $(a = 1)$, $\mathrm{Prob}^{|\Psi'\rangle}$ $(b = 1)$ and $\mathrm{Prob}^{|\Psi'\rangle}$ $(a = 1/b = 1)$.

The robustness condition for stochastic casuality is simply

$$\mathrm{Prob}^{|\Psi'\rangle}\,(a = 1/b = 1) = \mathrm{Prob}^{|\Psi\rangle}\,(a = 1/b = 1) \tag{51}$$

We shall show that for any given disturbed state $|\Psi'\rangle$ there always exists directions $\mathbf{a}$ and $\mathbf{b}$ for which (51) is violated.

To calculate the new probabilities apply the unitary transformation $u(\mathbf{n}, -\phi)$ $(= u^{-1}(\mathbf{n}, \phi))$ to the space $\mathbb{H}_B$ This converts $|\Psi'\rangle$ back into $|\Psi\rangle$, but induces a rotation $R(\mathbf{n}, -\phi)$ in the operator $\boldsymbol{\sigma}_B$. $R(\mathbf{n}, \phi)$ is an element of the 3-dimensional rotation group $SO(3)$, corresponding to an (active) clockwise rotation about the direction $\mathbf{n}$. The above result is a direct expression of the famous homomorphism that exists between $SU(2)$ and $SO(3)$. ($SU(2)$ is just the simply connected universal covering group of $SO(3)$.) Succinctly

$$u(\mathbf{n}, \phi)\,\boldsymbol{\sigma}_{\mathbf{B}}u(\mathbf{n}, -\phi) = R(\mathbf{n}, \phi)\boldsymbol{\sigma}_{\mathbf{B}} \tag{52}$$

On the left of Eq. 52, u acts on the spinor indices of $\boldsymbol{\sigma}_B$, while on the right R acts on the vector indices. Thus the effect of our unitary transformation on the operators a and b is as follows:

$$a = \boldsymbol{\sigma}_{\mathbf{A}}\cdot\mathbf{a} \to a' = a \tag{53}$$

$$\begin{aligned} b = \boldsymbol{\sigma}_{\mathbf{B}}\cdot\mathbf{b} \to b' &= (u(\mathbf{n}, -\phi)\,\boldsymbol{\sigma}_{\mathbf{B}}u(\mathbf{n}, \phi))\cdot\mathbf{b} \\ &= (R(\mathbf{n}, -\phi)\boldsymbol{\sigma}_{\mathbf{B}})\cdot\mathbf{b} \\ &= \boldsymbol{\sigma}_{\mathbf{B}}\cdot(R(\mathbf{n}, \phi)\mathbf{b}) = \boldsymbol{\sigma}_{\mathbf{B}}\cdot\mathbf{b}' \end{aligned} \tag{54}$$

where

$$\mathbf{b}' = R(\mathbf{n}, \phi)\mathbf{b} \tag{55}$$

Hence we have at once the following results:

1. $$\mathrm{Prob}^{|\Psi'\rangle}(a = 1) = \mathrm{Prob}^{|\Psi\rangle}(a' = 1) = \mathrm{Prob}^{|\Psi\rangle}(a = 1) = \tfrac{1}{2} \tag{56}$$

This shows that the perturbation acting on particle B cannot be used to send signals to the location of particle A. We shall discuss this more fully in Section 4.6 below.

2. $$\mathrm{Prob}^{|\Psi'\rangle}(b = 1) = \mathrm{Prob}^{|\Psi\rangle}(b' = 1) = \tfrac{1}{2} \tag{57}$$

This is a rather surprising result that is a special property of the example under discussion.

3. $$\begin{aligned} &\mathrm{Prob}^{|\Psi'\rangle}(a = 1/b = 1) = \mathrm{Prob}^{|\Psi\rangle}(a' = 1/b' = 1) \\ &= \mathrm{Prob}^{|\Psi\rangle}(a = 1/b' = 1) = \sin^2 \tfrac{1}{2}\theta_{ab'} \end{aligned} \tag{58}$$

The robustness condition (51) thus reduces to $\sin^2 \frac{1}{2}\theta_{ab'} = \sin^2 \frac{1}{2}\theta_{ab}$, or

$$\theta_{ab} = \theta_{ab'} \tag{59}$$

since the angles all lie in the range 0 to π. (59) can be given a simple geometrical interpretation.
In Fig. 15

ON represents the unit vector $\mathbf{n}$
OB " " " " " $\mathbf{b}$
OB′ " " " " " $\mathbf{b}'$

B and B′ lie on a circle with centre O′ whose plane is perpendicular to ON. ∠BO′B′ = ϕ.

Now $\theta_{ab} = \theta_{ab'}$ is equivalent to

$\cos\theta_{ab} = \cos\theta_{ab'}$
or $\mathbf{a}\cdot\mathbf{b} = \mathbf{a}\cdot\mathbf{b}'$
or $\mathbf{a}\cdot(\mathbf{b} - \mathbf{b}') = 0$

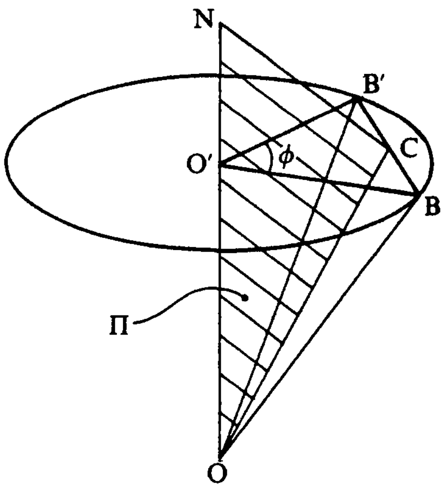

Fig. 15. Rotation of unit vector $\vec{OB}$ through angle ϕ about axis $\vec{ON}$ into new position $\vec{OB}'$. OC is the bisector of the angle between $\vec{OB}$ and $\vec{OB}'$. The plane NOC, denoted by Π, is shown shaded.

Hence **a** must line in the plane perpendicular to the line BB′. Call this plane Π. Π can equally well be characterized as the plane through ON and OC, where OC is the bisector of the angle between **b** and **b**′. The plane Π is shown shaded in the diagram. So we have the following

Theorem:

For any given perturbation on particle B that issues in a rotation of **b** to **b**′, the conditional probability $\text{Prob}^{|\Psi\rangle}(a = 1/b = 1)$ will be invariant (robust) under the perturbation if and only if the direction **a** lies in the plane defined by the axis of rotation and the bisector of the directions **b** and **b**′.

Corollaries:

(1) If **n** coincides with **a**
$\text{Prob}^{|\Psi\rangle}(a = 1/b = 1)$ is robust for all **b**.
(2) If **n** coincides with **b**
$\text{Prob}^{|\Psi\rangle}(a = 1/b = 1)$ is robust for all **a**.
(3) For any perturbation (rotation) however small there always exist directions **a** and **b** for which $\text{Prob}^{|\Psi\rangle}(a = 1/b = 1)$ is *not* robust.

It is this last corollary which demonstrates, I believe, that a and b cannot be regarded as related by stochastic causality. The correlations between a and b are a property of the particular quantum-mechanical state, viz. the singlet state, in which the particles emerge from the source. The state involves a feature of holism or nonseparability, which, lacking the necessary robustness for stochastic causality, may be termed *passion-at-a-distance* as opposed to *action-at-a-distance*. The A-particle does not possess independent properties (propensities) of its own. The conditional probability Prob $(\varepsilon_a/\varepsilon_b)\,{}_a^{\eta_A,\eta_B}$ is a candidate for an inherently relational property of the joint two-particle system. The situation should be contrasted with the result of violating the additional locality assumptions introduced below Eq. (33). If these conditions were not satisfied we would have a clear case of action-at-a-distance. Changing the setting of the apparatus B for example would alter the conditional probabilities of properties manifested at the location of the A-particle and the source, and so on.

We shall find a striking analogy to the distinction drawn here between locality in the sense of no-action-at-a-distance and separability in the sense of ascribing properties separately to the two particles, in the developments dealt with in Chapter 6.

4.5. Experimental Tests of the Bell Inequality

In this section we turn to the experimental tests of the Bell inequality that have been carried out during the past fifteen years or so.

The first point we would like to stress is that, since the Bell inequality involves experimentally accessible correlation coefficients, it can be tested directly, without any intermediate reference to QM. As we have seen, the predictions of QM do in certain circumstances violate the Bell inequality, but there are two distinct questions:

1. Is the Bell inequality violated?
2. Does the violation conform to the predictions of QM?

The broad consensus of the experimental results is that both these questions are answered in the affirmative.

There are basically three types of experiment which have, so far, been carried out. The first type measures polarization correlations between two 'visible' photons emitted in a cascade transition from the excited state of an atom such as calcium or mercury, or, in the most recent example, simultaneously from excited deuterium; the second type looks at spin correlations in low-energy proton–proton scattering; while the third type involves γ-ray polarizations from the annihilation decay of the singlet state of positronium. Before discussing the special features of these types of experiments we have listed in Table 1 the principal experiments in each class, and whether the result agrees (indicated by a tick) or disagrees (indicated by a

Table 1. Experimental tests of the Bell inequality. Violation indicated by a tick, no violation by a cross.

Date	*Experimenters*	*Result*	*Remarks*
Visible Photon Correlation Experiments			
1972	Freedman and Clauser	√	Used Ca cascade
1972	Holt and Pipkin	×	Used Hg cascade
1976	Clauser	√	Used Hg cascade
1976	Fry and Thomson	√	Used Hg cascade
1981	Aspect, Grangier, and Roger	√	Used Ca cascade with single-channel polarizers.
1982	Aspect, Grangier, and Roger	√	Used Ca cascade with 2-channel polarizers.
1982	Aspect, Dalibard, and Roger	√	Used Ca cascade with optical switches.
1985	Perrie, Duncan, Beyer, and Kleinpoppen.	√	Used simultaneous 2-photon emission by metastable atomic deuterium
Low Energy Proton–Proton Scattering:			
1976	Lamehi-Rachti and Mittig	√	Used double scattering to measure spin correlations.
γ-Ray Polarization Correlation Experiments			
1974	Faraci, Gutkowski, Notarrigo, and Pennisi	×	Used Compton polarimeter and ^{22}Na source
1975	Kasday, Ullman, and Wu	√	Used Compton polarimeter and ^{64}Cu source
1976	Wilson, Lowe, and Butt	√	Used Compton polarimeter and ^{64}Cu source
1977	Bruno, d'Agostino, and Maroni	√	Used Compton polarimeter and ^{22}Na source

cross) with the prediction of QM and the violation of the Bell inequality.

We now make some brief comments on these experiments. Only two of them, Holt and Pipkin, and Faraci *et al.*, have disagreed with QM and indeed have shown agreement with the Bell inequality where QM predicts a violation. Of course, from a physicist's point of view, any violation of the predictions of QM is looked at very critically, and in both of the discordant cases the experiments were repeated, and the discrepancies with QM could not be reproduced. On balance, it is now generally agreed that these anomalous experiments must have involved some unsuspected systematic error.

The experiments involving γ-ray polarizations suffer from the necessity of using a secondary Compton scattering as an indicator of the polarization state of the γ-ray. In analysing these experiments, it is assumed that the Klein–Nishina formula is applicable to the Compton scattering even in 'hidden' states of polarization. The same reservations apply in the case of the low energy proton–proton scattering where the scattering of the protons off a secondary target is used as an indicator of their spin orientations.

In the optical photon experiments, one has the big advantage of being able to detect polarizations directly. Typically, pile-of-plates analysers are used to transmit photons of a given linear polarization with high efficiency. With this type of analyser, the orthogonal component of polarization is detected by absence of transmission. The only experiment in which two-channel analysers have been employed is the one by Aspect, Grangier, and Roger (1982). Here a calcite crystal is used to split orthogonal polarizations into two beams, each of which has a detector placed behind it. In all of these optical photon experiments, the photomultipliers used in the detection process are relatively inefficient; and in addition the nature of the angular correlation is such that the experiment is not sensitive to all the coincidence photons (the cascade transition is essentially a three-body decay due to the recoil of the atom). For both these reasons, one is only observing a sample of all the photon pairs emitted from the source; and in assessing the violation of the Bell inequality, it is necessary to assume that the sample of pairs actually observed is not biased in such a way as to explain the violation. For example, since the detection process is 'downstream' from the analysers, it is not surprising that, with suitable *ad hoc* assumptions as to how the efficiency of detection might depend on polarization, one can 'explain' the violation of the Bell inequality in a 'local' way. Such 'explanations' have to be ruled out by auxiliary assumptions concerning the functioning of the apparatus used to analyse and detect the photons. It would be nice to design an experiment which would eliminate the need for such auxiliary assumptions. Such an experiment has been suggested by Lo and Shimony (1981), which employs the coincidence detection of the two dissociation fragments of a metastable molecule. With such a two-body decay, strong angular correlations would obtain, and with Stern–Gerlach analysers and ionization detectors very high efficiencies can in principle be achieved. This experiment appears to be quite possible, and there is no doubt it should be carried out in order to eliminate the 'auxiliary assumptions' loophole in the existing experiments.

There is, however, another and more profound problem raised by all the experiments so far described. They are *static* experiments—that is to say, the choice of analyser setting is made well in advance of the emission of the particles from the source. In principle, there is a possibility that the settings of the analysers 'communicates' itself to the source in such a way as to affect the correlations being measured with these particular settings. This would be an example of violating Bell locality but not Einstein locality, in the terminology introduced in Chapter 3. In order to check for a violation of Einstein locality, one must devise an experiment in which the settings of the analysers are decided after the photons have left the source in the photon-cascade type of experiment. Such an experiment was planned by Aspect and finally carried out by Aspect, Dalibard, and Roger in 1982. In the Aspect experiment, an 'optical' switch or commutator is introduced in the path of each photon which can deflect the photon towards one or other of two analysers with different settings. The arrangement is shown schematically in Fig. 16. The analyser settings are labelled **a**, **a**′, **b** and **b**′ to conform with the arrangement in the idealized Bell experiment shown in Fig. 9. The switch is pulsed at such a frequency that it changes the selection of analyser while the photon is in transit from the source.

Consider the light-cone structure for the source *S* and the two switches *A* and *B*, as seen from the reference frame in which *S*, *A*, and

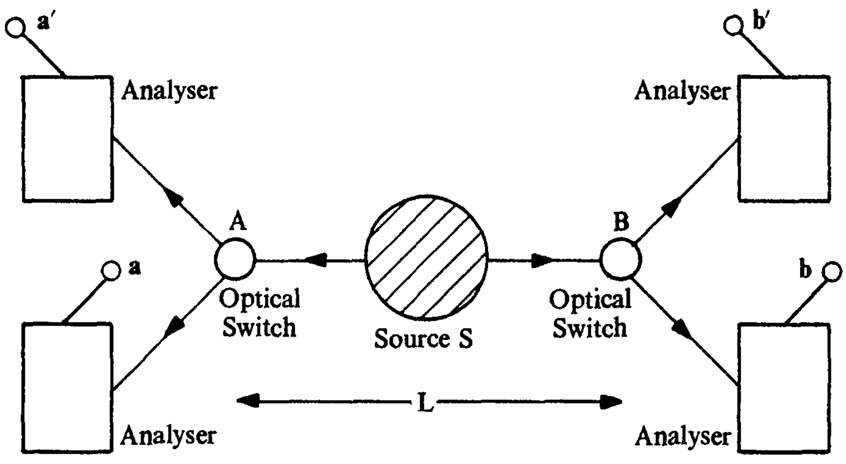

Fig. 16. The Aspect version of the Bell experiment. *A* and *B* are optical switches sending each photon to one or other of a pair of analysers, with settings denoted by **a**, **a**′, **b** and **b**′.

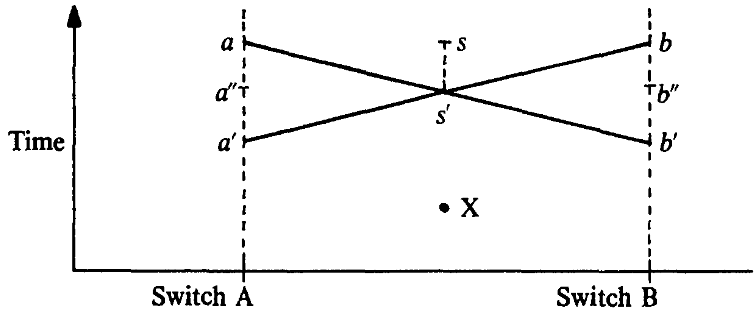

Fig. 17. Light-cone structure for the source S and the two switches A and B in the Aspect experiment. For details see text.

B are all stationary. This is illustrated in Fig. 17. The source emits the two photons at space-time point s'. The photons reach the switches at space-time locations a and b, at which time the source is at s. The part $b'b$ of the world-line of the switch B is outside the light-cone with vertex at s'. Similarly the part $a'a$ of the world-line of switch A is outside the light-cone at s'.

Let us put in some numbers. In Aspect's experiment, the distance L between A and B is 12 metres. Hence it is easy to calculate that $a'a = b'b = 40$ ns (1ns $= 10^{-9}$ seconds). Furthermore, $s's = 20$ ns. The switch is designed to change its setting every 10 ns. This is half the time (20 ns) it takes for the photons to go from the source to the switches. So the switches will certainly alter their settings while the photons are in transit, e.g. at points such as a'' and b'' outside the light-cone at s'. Indeed, in the following discussion we shall assume that a'' and b'' are the last switching operations prior to a and b respectively. Clearly, the photon moving from s' to a is always outside the light-cone with vertex at b'', and similarly the photon moving from s' to b is always outside the light-cone with vertex at a''. In other words, any influence of the switching event at a'' on the polarization state of the photon moving to b, or of the switching event at b'' on the polarization state of the photon moving to a, would be a violation of Einstein locality.

It is worth noting that the emission event involved in the cascade actually has a half-life of about 5 ns, so the location of the source at s' is 'blurred' by this amount. It is important for the above argument that this time is small compared with the 40 ns extent of the exterior region of the light-cone at s', sectioned at the switches A and B.

In the experiment devised by Aspect, the optical switches consist of a glass cell filled with water, in which ultrasonic standing waves are produced via electroacoustic transducers connected to a 25 MHz generator. The cell acts as a variable diffraction grating. When the standing wave is of maximum amplitude, the incoming photon suffers a Bragg reflection from the antinodal planes. Half a period later, when the amplitude of the standing wave is zero, the photon travels straight through the cell without any Bragg reflection (diffraction). The device is illustrated in Fig. 18. In the experiment the Bragg angle θ_B is about 1/4°, so the deflection $2\theta_B$ of the diffracted beam is approximately $\frac{1}{2}$°. With a 25 MHz generator the switching frequency is 50 MHz, since clearly the switch operates at twice the acoustic frequency. The half-period for switching, i.e. the time interval between the two 'directions' of the switch, comes out at 10 ns, as stated above.

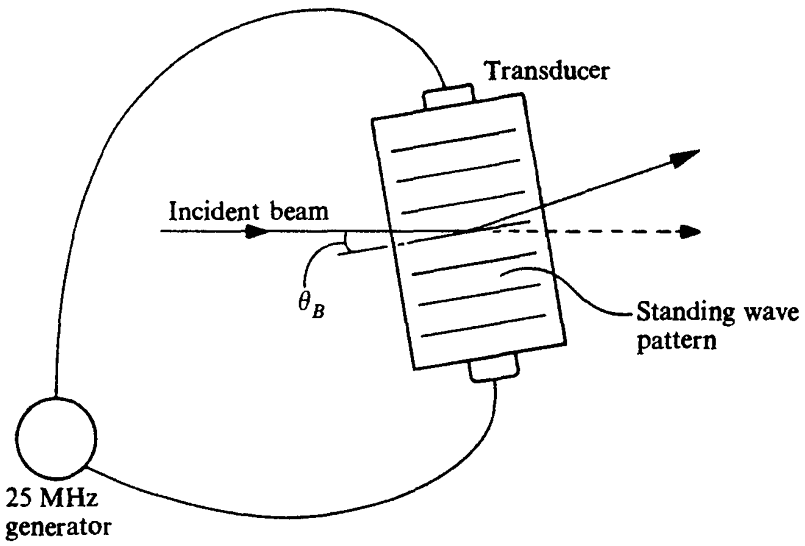

Fig. 18. Optical switch in the Aspect experiment.

The results of Aspect's experiment were a clear violation of the Bell inequality. So prima facie we have an important result here, showing a violation of the Einstein version of LOC_3. There are, however, two points to be noticed:

1. Although the two switches are run from separate independent generators, it is clear that the switches are not truly operated in a random fashion. In other words, referring again to Fig. 17, the state of the apparatus prior to b', i.e. inside the backward light-cone, with vertex at s', determines the switching operation at b''. So knowledge of

the switch setting at b, when the photon hits the switch, is available at locations inside the backward light-cone at s', and hence could be 'communicated' to s' without violation of Einstein locality. Similarly for the A-switch.

2. But even if attempts were made to randomize the switching operations, this would still be consistent with ontological determinism (compare the discussion on p. 90 above); and some event X in the overlapping backward light-cones of b'' and s' could be held responsible both for the switch-change at b'' and an effect on the source at s', so correlating the state of the switch at b with the state of the source without violation of Einstein locality.

While admitting that bizarre possibilities of this sort could circumvent the demonstration of a violation of Einstein locality in the Aspect experiment, the situation is rather like that referred to in connection with the auxiliary assumptions, which of course also have to be invoked in the Aspect experiment. Duhemian 'good sense' may dictate that we should accept the demonstration in the Aspect experiment of a violation of Einstein locality. The difference lies in the fact that experiments can in principle be designed that do not need the 'auxiliary assumptions', while no experiment can rule out the type of defence of Einstein locality referred to above.

4.6. Statistical Nonlocality

In Section 4.1 we discussed the condition under which a Bell-type experiment might lead us to argue for a violation of LOC_3—that sharp values and hence, assuming FM, measurement results at one location can be changed by altering the setting of a remote piece of apparatus. But LOC_3 is concerned with what happens on a particular occasion. It still leaves open the question of whether the statistical frequencies with which measurement results turn up at one location can be changed by performing different sorts of measurement at another remote location. To deal with this question we introduce still another sense of locality:

LOC_5:

The statistics (relative frequencies) of measurement results of a quantum-mechanical observable cannot be altered by performing measurements at a distance.

Notice that LOC_5 is formulated in terms of the statistics of measurement results, the very question which the formalism of QM, via the statistical algorithm, is designed to answer. If LOC_5 were violated, this would show that the formalism itself, with the minimal instrumentalist interpretation, exhibited nonlocal features, independently of any more comprehensive interpretation of the formalism.

It is therefore important to realize that LOC_5 is not violated in the Bell type of experiment. To see why this is so, let us consider that we decide to measure σ_{1z} on particle 1 in the state $|\Psi_{\text{singlet}}\rangle$ as given in Eq. (3.2). We investigate how this would affect the statistics of any spin-component measurements made on particle 2. We use the ideal measurement theory developed in Section 2.4.

We label the measuring device for σ_{1z} as system 3. Let the initial state of the apparatus be denoted by $|w_0(3)\rangle$ and the final state be denoted by $|w_+(3)\rangle$ if σ_{1z} has the measured value $+1$, and by $|w_-(3)\rangle$ if σ_{1z} has the measured value -1.

Then the state of the whole system 1, 2, and 3 goes from

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}}(|\alpha(1)\rangle|\beta(2)\rangle - |\beta(1)\rangle|\alpha(2)\rangle)|w_0(3)\rangle$$

before the measurement to

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}(|\alpha(1)\rangle|\beta(2)\rangle|w_+(3)\rangle - |\beta(1)\rangle|\alpha(2)\rangle|w_-(3)\rangle)$$

after measurement.

In terms of von Neumann statistical operators, it follows, from the discussion given in Section 2.4, that $P_{|\Psi_i\rangle}$ behaves like $\frac{1}{2}P_{|\alpha(2)\rangle} + \frac{1}{2}P_{|\beta(2)\rangle}$ in respect of measurements of any observables pertaining to particle 2 only (compare Eq. (2.10)). This arises because of the orthogonality of the states $|\alpha(1)\rangle$ and $|\beta(1)\rangle$. But $P_{|\Psi_f\rangle}$ also behaves like $\frac{1}{2}P_{|\alpha(2)\rangle} + \frac{1}{2}P_{|\beta(2)\rangle}$ in respect of measurements of an observable pertaining to particle 2 only. This is because the interference terms arising in any measurement statistics for particle 2 are now 'killed' twice over by the orthogonality of $|\alpha(1)\rangle$ and $|\beta(1)\rangle$, and also by the orthogonality of $|w_+(3)\rangle$ and $|w_-(3)\rangle$. So the statistics of measurement results pertaining only to particle 2 is unaffected by 'hooking on' the apparatus for measuring σ_{1z} on particle 1. This result holds *a fortiori* if we assume that the measurement actually produces a final *mixed* state for the joint system.

We are referring here of course to the nonselective stage of measurement. If we select a sub-ensemble of particle 2's with

$\sigma_{2z} = +1$, say, this will be described by the statistical operator $P_{|\alpha(2)\rangle}$, which of course gives different statistics in general from $\frac{1}{2} P_{|\alpha(2)\rangle} + \frac{1}{2} P_{|\beta(2)\rangle}$. This selection could be effected in the light of knowing which measurement results we had obtained for σ_{1z} in view of the mirror-image correlations built into $|\Psi_{\text{singlet}}\rangle$. But then the selection is made at the wrong location to provide any 'instantaneous statistical effects' at a distance.

Thus suppose we are measuring a sequence of values for σ_{2z} on successive particles emitted by the source. The sequence might be $+ - - + - + - + + - - - - \ldots$ where the limiting frequency of $+$ and $-$ is $\frac{1}{2}$. Suppose we perform measurements of σ_{1z} simultaneously on particle 1. This enables us to 'tag' each particle 2 as either $+$ or $-$ in our abbreviated notation for values of $\sigma_{2z} = \pm 1$. But the tagging information is in the wrong place to change the statistics at the location of particle 2. To do this we would have to transmit the tagging information from location 1 to location 2, with instructions, for example, to insert an absorbing screen every time a minus particle is approaching the spin-meter for particle 2, and to remove it every time a plus particle is approaching. In this way we would clearly change the sequence of observed measurement results of σ_{2z} to $+ + + + + \ldots$, but to effect this change we have to transmit information from location 1 to location 2; we cannot do it simply by 'hooking on' the apparatus to measure σ_{1z}.

A simple example can illustrate the problem. When I lecture in Oxford, the audience there learn instantaneously that my room in London is empty; but to produce a physical change in London, for example to prevent students knocking on my door, the information that I have arrived in Oxford must be transmitted back to London.

The conclusion of this discussion is that the nonlocality putatively demonstrated in the Bell type of experiment cannot be used to transmit information instantaneously between two remote locations. In brief, there is no such thing as a 'Bell' telephone! In this respect the situation is quite different from what would obtain if the operators referring to particles 1 and 2 failed to commute. In such a case LOC_5 would be violated. But the nonlocality demonstrated in the EPR and Bell arguments is subtler than this. In particular, the fact that no statistical effects get transmitted 'at a distance' means that no nonlocality problems arise in an ensemble or statistical interpretation of QM. It is only in the context of an attempt to impute states to individual systems that the difficulties are manifested.

We have discussed the no-signalling result in the context of interactions which perform a measurement on particle 1, and for the particular case of the quantum-mechanical state $|\Psi_{\text{singlet}}\rangle$. But in fact we can give a general proof that quantum-mechanical correlations cannot be used for signalling along the following lines.

Consider, quite generally, two systems A and B with associated Hilbert spaces $\mathbb{H}_A$ and $\mathbb{H}_B$. Let C be a third system which may interact in any way with system B, with associated Hilbert space $\mathbb{H}_C$. Denote $\mathbb{H}_B \otimes \mathbb{H}_C$ by $\bar{\mathbb{H}}_B$. a is any observable on $\mathbb{H}_A$, extended to $a \otimes I$ on $\mathbb{H}_A \otimes \bar{\mathbb{H}}_B$. Consider any state $|\Psi(t)\rangle$ of the triple system at time t and denote by Prob $(\lambda)^{|\Psi(t)\rangle}_{a(t)\otimes I}$ the probability that the time-dependent observable $a(t) \otimes I$ will yield the measurement result λ in the state $|\Psi(t)\rangle$. We shall work in the Dirac picture (see p. 12) so in the absence of perturbation $|\Psi(t)\rangle$ is constant, while in all cases $a(t) \otimes I$ evolves in time according to the unperturbed Hamiltonian.

Now perturb the system B in any way in the time interval (t, t') by the action of a unitary time-evolution operator $U_B(t', t)$ acting on $\bar{\mathbb{H}}_B$.

Then at time t', the state of the triple system is

$$|\Psi(t')\rangle = (I \otimes U_B(t', t))|\Psi(t)\rangle \tag{60}$$

We now compute

$$\text{Prob}(\lambda)^{|\Psi(t')\rangle}_{a(t')\otimes I} = \text{Prob }(\lambda)^{|\Psi(t)\rangle}_{a'(t')\otimes I} \tag{61}$$

where

$$\begin{aligned} a'(t') \otimes I &= (I \otimes U_B^{-1})\,(a(t') \otimes I)\,(I \otimes U_B) \\ &= (I \otimes U_B^{-1})\,(a(t') \otimes U_B) \\ &= a(t') \otimes (U_B^{-1} U_B) = a(t') \otimes I \end{aligned} \tag{62}$$

So

$$\text{Prob }(\lambda)^{|\Psi(t')\rangle}_{a(t')\otimes I} = \text{Prob }(\lambda)^{|\Psi(t)\rangle}_{a(t')\otimes I} \tag{63}$$

But the RHS of this equation is the probability of finding the result λ by measuring a on A at time t' in the absence of perturbation, since then the state vector at t' is the same as the state vector at t.

In other words, the probability of obtaining the result λ by measuring a on A at any time t' is independent of any possible perturbation of the system B between t and t'.

4.7. Summary of Conclusions

We have distinguished three basic approaches to the interpretation of QM, labelled A, B and C, and also five senses of locality: LOC_1,

LOC_2, LOC_3, LOC_4 and LOC_5. We can now answer the question 'Does QM predict a violation of locality?' in the form of a simple table showing which senses of locality can be violated, as indicated by a cross, in each of the three interpretations. A tick indicates that the corresponding sense of locality is not violated.

View	LOC_1	LOC_2	LOC_3	LOC_4	LOC_5
A	✓	✓	X	X	✓
B	X	✓	✓	✓	✓
C	✓	X	✓	✓	✓

Notes and References

Most of the material covered in this chapter is based on Redhead (1983).

The original proof of the Bell inequality was given in Bell (1964). This used the deterministic hidden-variable framework, and also assumed perfect anti-correlation when the two spin-meters measured parallel spin-components. These restrictions were relaxed in Bell (1971). See also Clauser and Horne (1974). An interesting alternative approach is provided in Wigner (1970). The Stapp–Eberhard approach to the Bell inequality discussed in Section 4.2. was initiated by Stapp (1971) and developed by Eberhard (1977). The mathematics, as distinct from the interpretation, of Eberhard's paper is used in Section 4.1, with improvements dues to Peres (1978) and Brody (1980). See also Peres and Zurek (1982).

For discussion of counterfactuals, see in particular Lewis (1973), Slote (1978), Lewis (1979), Bowie (1979), and for an approach that leads to the opposite conclusion to the one discussed in the text, see Thomason and Gupta (1980). A useful collection of reprints on this topic is given in Harper *et al.* (1981). Our own treatment is based on a simplified adaptation of Lewis (1973). For a critique of some of the assumptions involved in the proof of the Bell inequality even in the deterministic case, see in particular Fine (1974) and (1979), Brody and de la Peña-Auerbach (1979), and Brody (1980).

Stochastic hidden-variable theories and the relevance of factorizability to locality are discussed by Clauser and Horne (1974), Selleri and Tarozzi (1980), Fine (1981), Shimony (1981), Hellman (1982), Jarrett (1984) and Shimony (1984). The term 'passion-at-a-distance' is due to Shimony. The nonrobustness of the singlet state was first analysed in Redhead (1986). For the mark criterion for causal connectibility see Reichenbach (1928) and (1956) and Salmon (1984). The proof of the inequality (42) given in the text can be found in Clauser and Horne (1974), Appendix A. The fact that strict correlation combined with factorizability implies determinism was first pointed out by Suppes and Zanotti (1976).

Fine (1982a) and (1982b) has demonstrated an interesting mathematical property of the Bell inequality, viz. that it is a sufficient condition for the existence of joint distributions for all observables including incompatible ones. The fact that this does not contradict the claim made in this chapter that the Bell inequality can be derived without assuming joint distributions for incompatible observables has been argued in Redhead (1984) and Svetlichny *et al.* (1988). The basic point here is that Fine's theorem demonstrates the existence of joint distributions in a mathematical model of statistics that satisfy the Bell inequality. But this model may not reflect the frequencies arising in the real world.

Comprehensive reviews of the experiments designed to test the Bell inequality are provided by Clauser and Shimony (1978) and Pipkin (1978). The references for the more recent work of Aspect and his collaborators are Aspect, Grangier, and Roger (1981) and (1982), and Aspect, Dalibard, and Roger (1982). The most recent experiment of Perrie *et al.* is presented in their (1985). A critique of the auxiliary assumptions used in the Aspect experiment has been made in particular by Marshall *et al.* (1983). The Lo–Shimony experiment is discussed in great detail in Lo and Shimony (1981).

For the general proof that LOC_5 is not violated in QM see Eberhard (1978), Ghirardi *et al.* (1980), Page (1982) and Shimony (1984). A very simple and elegant proof is also provided by Jordan (1983). The proof given in the text follows Redhead (1986).

It can be shown that the QM violation of the Bell inequality never exceeds $2\sqrt{2}$. For an elegant demonstration see Landau (1986).

A recent general survey of the topics covered in this Chapter is provided by D'Espagnat (1984).

Bell's collected papers on the interpretation of QM are now available in Bell (1987).