Incompleteness, Nonlocality, and Realism

A PROLEGOMENON TO THE PHILOSOPHY OF QUANTUM MECHANICS

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The Einstein–Podolsky–Rosen Incompleteness Argument

IN 1935 Einstein, Podolsky, and Rosen (EPR) produced a famous argument for the incompleteness of the minimal instrumentalist interpretation of QM. In this chapter we shall present a version of the argument, and comment on its significance. We begin with a necessary condition for a theory in physics to be complete.

Necessary Condition for Completeness (*P*):

Every element of the physical reality must have a counterpart in the physical theory.

This is a straight quotation from EPR (1935).

The idea behind it is very simple. The language in which the physical theory is formulated must be sufficiently rich that every proposition signifying putative relations between the 'elements of reality' can at least be expressed in the language of the theory.

P is not a sufficient condition. A sufficient condition might involve the additional requirement that every proposition concerning the 'elements of reality' is provably true or false in accordance with the axiomatic-deductive structure of the theory (supplemented by the specification of contingent initial conditions). This sort of completeness is the subject of the famous Gödel result in mathematical logic, that every first-order theory (a first-order theory is one which does not allow quantification over predicate variables) which includes a significant fragment of arithmetic is actually incomplete according to this additional requirement. The condition *P* has nothing to do with these Gödelian complications. It is solely concerned with the expressive power of the language in which the theory is formulated. If a theory is incomplete by the failure of this criterion, it should be noted that this is consistent with the theory asserting the existence of elements of reality which in fact do not exist, not just that it fails to refer to all those elements of reality which do exist.

In order to apply the principle P in a proof of the incompleteness of QM, we must have at hand some means of identifying the 'elements of reality'. To show that QM is incomplete, the idea of EPR is simply to demonstrate an element of reality which does *not* have a counterpart in the theory. EPR proceed to introduce a sufficient condition for identifying elements of physical reality. We shall present the condition in a form that is slightly different from EPR (and will comment on the difference later).

Sufficient Condition for Element of Reality (R):

If we can predict with certainty, or at any rate with probability one, the result of measuring a physical quantity at time t, then at the time t there exists an element of reality corresponding to the physical quantity and having a value equal to the predicted measurement result.

The reality criterion R certainly seems eminently reasonable. It really depends on a version of *Inference to the Best Explanation*. The best explanation of why we make the successful measurement prediction at time t is that there exists an element of reality at time t, having that value and predictable in accordance with the physical theory at issue, which is then simply discovered by the measurement. Arguing contrapositively, if there were no element of reality at time t then we should not expect to be able to make a successful prediction of the measurement result at time t, since predictability can only be expected to arise as a result of regularities underlying the behaviour of elements of reality. Notice that we are not claiming that all elements of reality have predictable values—a thesis of determinism—only that, if it is predictable, then there must exist an element of reality to explain this possibility.

Suppose I look at a table, turn my back and predict that, if I look again, assuming no outside intervention, then I will see the table again. The predictability of my seeing the table again allows me to infer, in accordance with R, that there exists an element of reality corresponding to the table at the instant when I look again. In this case the element of reality exists both at the time t_0 the prediction is effected and the time t at which it is effective. But in order to show that an element of reality exists at the time t_0 requires another principle, that will allow us to infer that the element of reality was not brought into being at some time between t_0 and t. In the case of the table, this is provided by the assumption that there was no outside interference with the state of the table during the period when my back was turned.

To take another example, suppose I predict at t_0 that at a later instant t' some mechanical contrivance will throw a stone in a pond, and hence that at a still later time t a ripple will be observed to strike the bank, then R allows me to infer that there really is a ripple at time t, but this was brought into existence at time t' when the stone was thrown and did not exist at time t_0 when the original prediction was made.

Let us now return to the situation in QM. We employ the notation already introduced in Section 2.1. Denote by $[Q]$ the value of an element of reality corresponding to the observable Q. We shall sometimes use $[Q]$ to denote the element of reality itself. The meaning should always be clear from the context. Consider the three views introduced in Chapter 2. On view A, $[Q]$ will exist at all times in all quantum states for all observables. But views B and C claimed that in non-eigenstates of Q, $[Q]$ did not exist.

Since $[Q]$ may depend partly on what particular QM state the system happens to be in, we shall use the notation $[Q]^{|\phi\rangle}$ to indicate the value that Q possesses, on a particular occasion, in the state $|\phi\rangle$. Then we may employ R to obtain the following result which we call

$$\textit{The Eigenvector Rule:} \qquad [Q]^{|q_i\rangle} = q_i \tag{1}$$

where $|q_i\rangle$ as usual denotes the i^{th} eigenket of Q belonging to the eigenvalue q_i.

The Eigenvector Rule shows that, on any interpretation of QM, if we allow R, then in an eigenstate of Q there does always exist an element of reality corresponding to Q, and having a value equal to the associated eigenvalue. (This serves to justify the remarks made below Eq. (2.6) about the component states of a mixture as exhibiting possessed or sharp values of the relevant observables.)

Consider a QM system consisting of two spin-$\frac{1}{2}$ particles, in the singlet state of their total spin, and widely separated spatially, so that there is no significant overlap of the spatial wave functions of the two systems. Such a state can be produced, for example, in low-energy p-p scattering which is well known to proceed via the singlet (antisymmetric) state of the total spin of the two protons (since the S-wave spatial wave-function that dominates low-energy scattering is symmetric under exchange of particles, the Pauli Principle forces the spin part of the state vector to be antisymmetric). From Eq. (1.102) we know that this spin state is given by

$$|\Psi_{\text{singlet}}\rangle = \frac{1}{\sqrt{2}}(|\alpha(1)\rangle|\beta(2)\rangle - |\beta(1)\rangle|\alpha(2)\rangle) \tag{2}$$

where

$$\begin{aligned} |\alpha(1)\rangle &= |\sigma_{1z} = +1\rangle \\ |\beta(1)\rangle &= |\sigma_{1z} = -1\rangle \\ |\alpha(2)\rangle &= |\sigma_{2z} = +1\rangle \\ |\beta(2)\rangle &= |\sigma_{2z} = -1\rangle \end{aligned} \tag{3}$$

For the state $|\Psi_{\text{singlet}}\rangle$ we can easily compute the conditional probability of finding the measurement result $+1$ for σ_{2z}, given that the measurement result for σ_{1z} is -1. This is given by

$$\begin{aligned} \text{Prob}\,(\sigma_{2z} = 1/\sigma_{1z} = -1) \\ &\underset{\text{Df}}{=} \frac{\text{Prob}\,(+1, -1)^{|\Psi_{\text{singlet}}\rangle}_{\sigma_{2z}, \sigma_{1z}}}{\text{Prob}\,(-1)^{|\Psi_{\text{singlet}}\rangle}_{\sigma_{1z}}} \\ &= \tfrac{1}{2}/\tfrac{1}{2} = 1 \end{aligned} \tag{4}$$

Similarly

$$\text{Prob}\,(\sigma_{2z} = -1/\sigma_{1z} = +1) = 1 \tag{5}$$

Eqs. (4) and (5) express what are often referred as the mirror-image correlations built into $|\Psi_{\text{singlet}}\rangle$. Measuring σ_{1z} enables one to predict that a subsequent measurement of σ_{2z} will show the opposite value to the measurement result for σ_{1z}. Let us put in some times. At t_1 measure σ_{1z}, then we can predict the result of measuring σ_{2z} for any time $t_2 > t_1$ (assuming no interference with the joint system other than the measurement of σ_{1z} at time t_1). Hence we can infer, using R, that $[\sigma_{2z}]$ exists for any time $t_2 > t_1$. Suppose, to be specific, that at time t_1 we performed an ideal measurement on σ_{1z} and obtained the value $+1$. The state of the joint system after this measurement, and selected in accordance with this measurement result, is $|\Psi\rangle = |\alpha(1)\rangle|\beta(2)\rangle$. This is an eigenstate of σ_{2z} with eigenvalue -1, and this corresponds to the fact that at any time $t_2 > t_1$, we can predict the measurement result for σ_{2z} to be -1, and hence infer by R the existence of $[\sigma_{2z}]$ with the value -1. We now want to argue that we can project the existence of $[\sigma_{2z}]$ back to a time $t_3 < t_1$ when the state of the joint system was $|\Psi_{\text{singlet}}\rangle$. In order to do this we invoke a locality principle L.

Locality Principle (L):

Elements of reality pertaining to one system cannot be affected by measurements performed 'at a distance' on another system.

The locution 'at a distance' can be understood in two senses, which we distinguish as *Bell locality* and *Einstein locality*. For Bell locality, 'at a distance' means in the absence of causal influences recognized by current physical theories. For Einstein locality, 'at a distance' means at a space-like separation between the space–time locations where the element of reality pertaining to one system exists and the measurement on the other system takes place.

If we accept, provisionally, that special relativity (SR) implies Einstein locality, then we are claiming, in the Einstein version of locality, not just that no known physical causal influence is at work, but that no possible causal influence consistent with the constraints of SR could be effective in inducing the change in the element of reality.

Now apply L to the element of reality $[\sigma_{2z}]$ at the time t_1. This shows that no change in that element of reality can be effected as a result of the measurement of σ_{1z}; in other words, L enables us to project the existence of $[\sigma_{2z}]$ backwards (with the value -1 in our example) to a time $t_3 < t_1$. But since, at time t_3, the state of the system is $|\Psi_{\text{singlet}}\rangle$ which is certainly not an eigenstate of σ_{2z} (or of course of σ_{1z}), what we have demonstrated is the existence of $[\sigma_{2z}]$ in a non-eigenstate of σ_{2z}. So what we have shown is that, for a special choice of Q and $|\psi\rangle$, $[Q]^{|\psi\rangle}$ exists even when $|\psi\rangle$ is not an eigenstate of Q. And now we can employ P to conclude that the minimal instrumentalist interpretation of QM is incomplete, since in non-eigenstates of Q there is nothing in that interpretation that refers to $[Q]$—indeed, in interpretations of QM such as B and C it is denied that $[Q]$ exists at all in such states.

The argument we have given is sketched schematically in Fig. 8.

We can summarize the EPR argument in the form

$$F \wedge L \rightarrow \text{Incompleteness} \tag{6}$$

where F denotes the formalism of QM with the minimal instrumentalist interpretation, and we use the logical symbol '$\wedge$' for conjunction. Note that only a small, but very significant, fragment of F is actually used in the argument, viz. the existence of non-factorizable states for the joint Hilbert space of the two systems with strict mirror-image correlations in the way described.

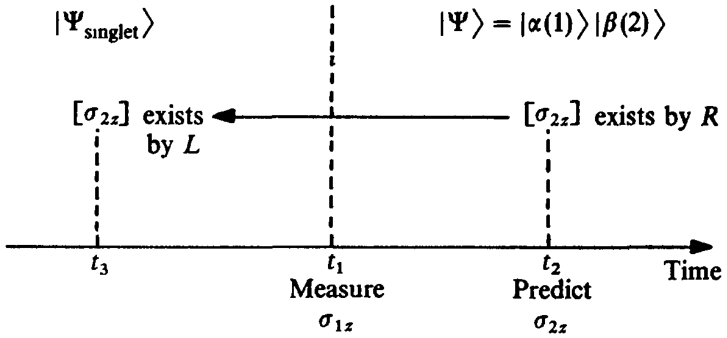

Fig. 8. Schematic illustration of the EPR argument. σ_{1z} is measured at time t_1, and this is used to predict σ_{2z} at a later time t_2. The element of reality $[\sigma_{2z}]$ is asserted to exist at t_2 by the Reality Principle R, and is then projected back to a time t_3 earlier than t_1 by the Locality Principle L. The state of the combined system before the measurement at t_1 is $|\Psi_{singlet}\rangle$ and after the measurement is $|\Psi\rangle = |\alpha(1)\rangle|\beta(2)\rangle$.

(6) can be rewritten as

$$F \rightarrow \sim (L) \vee \text{Incompleteness} \qquad (7)$$

where '$\sim$' denotes negation and '$\vee$' disjunction.

We shall refer to (7) as the *Einstein Dilemma.* It says that, if we accept the formalism F of QM as correct at the purely observational/instrumental level, then either we must give up L the locality principle or we must admit the incompleteness of F. If, with Einstein, we are not prepared to give up L, then (7) constitutes an argument for the incompleteness of F. This indeed is the conclusion of the EPR paper. On the other hand, if we assume that F is complete, then (7) constitutes an argument for nonlocality in the sense that, at the very time t_1 at which the measurement of σ_{1z} took place, an element of reality $[\sigma_{2z}]$ was brought into existence. Since the events comprising the measurement of σ_{1z} and the bringing into existence of $[\sigma_{2z}]$ are simultaneous, relative to the reference frame with respect to which $|\Psi_{singlet}\rangle$ is the appropriate spin state of the two particles, then these events are at space-like separation, and so L is being violated in the Einstein sense. This way of arguing for nonlocality we shall refer to as the EPR paradox. It is a paradox, not in the strict logical sense, but in the sense of involving a counter-intuitive conclusion, viz. a violation of Einstein locality.

So far we have discussed the EPR argument as it applies to F, the minimal instrumentalist interpretation of the QM formalism. Let us now consider how the argument would look if we first of all filled out the interpretation F with the interpretations we labelled A, B, and C in

Chapter 2 (see pp. 45 ff.). These are all intended to provide complete descriptions of reality. So, if by the EPR argument we demonstrate elements of reality *denied* to exist by any of these interpretations, and hence to show its *incompleteness*, we can conclude that the interpretation is actually false.

On view *A* it is asserted that $[\sigma_{2z}]$ exists at all times. So, there would be no need for the EPR argument to show the existence of $[\sigma_{2z}]$ in the state $|\Psi_{\text{singlet}}\rangle$ and hence the incompleteness of *F*.

In the case of view *B*, we can be more specific as to what sort of effects are denied to be possible by the locality principle *L*. Indeed, let us replace *L* by the formulation

LOC_1: An unsharp value for an observable cannot be changed into a sharp value by measurements performed 'at a distance'.

Then the EPR argument leads to the result

$$B \wedge \mathrm{LOC}_1 \rightarrow \text{Incompleteness} \qquad (8)$$

$$\rightarrow \sim (B) \qquad (9)$$

From (9) we conclude

$$B \rightarrow \sim (\mathrm{LOC}_1) \qquad (10)$$

i.e. view *B* in conjunction with the EPR argument leads to a demonstration that LOC_1 is violated in the Einstein sense. Alternatively, we can read (10) contrapositively as showing that if we assume LOC_1 then view *B* is refuted.

Let us now turn to view *C*. Here the appropriate specialization of *L* is

LOC_2: A previously undefined value for an observable cannot be defined by measurements performed 'at a distance'.

We then obtain

$$C \wedge \mathrm{LOC}_2 \rightarrow \text{Incompleteness} \qquad (11)$$

$$\rightarrow \sim (C) \qquad (12)$$

Hence

$$C \rightarrow \sim (\mathrm{LOC}_2) \qquad (13)$$

Now Bohr's response to the EPR argument, and his rejection of the charge of incompleteness, amounts essentially to the claim that LOC_2 can be violated without any physical effects being transmitted in violation of SR. This denial of LOC_2 prevents the use of (13) to argue contrapositively for the falsity of *C*, and also means from (11) that the argument for incompleteness breaks down, because one of the assumptions, LOC_2, of the argument is not subscribed to. Clearly this manoeuvre to block the EPR argument depends on our accepting the definability criteria associated with Bohr's complementarity philosophy which has already been discussed in Section 2.3.

We now want to comment briefly on the differences between the argument presented above and the original EPR paper:

1. The Reality Criterion R is given the following formulation, which we call R'.

> R': If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to the physical quantity.

R' is not explicit about the time at which the existence of the element of reality can be inferred, and furthermore it mixes up the locality or nondisturbance principle with the reality criterion, which somewhat obscures the logical structure of the argument for incompleteness.

2. EPR demonstrate the simultaneous existence of elements of reality corresponding to non-commuting observables. Thus, continuing with our spin version of the argument, having demonstrated the existence of $[\sigma_{2z}]$ in the state $|\Psi_{\text{singlet}}\rangle$, we could exploit the fact that $|\Psi_{\text{singlet}}\rangle$ can also be expanded in terms of eigenkets of σ_{1x} and σ_{2x}, as in Eq. (1.103), and the mirror-image correlations of the X-components of the Pauli spin observables for the two particles now enable us to infer the existence of $[\sigma_{2x}]$ in the state $|\Psi_{\text{singlet}}\rangle$. So the two non-commuting observables σ_{2z} and σ_{2x} have elements of reality associated with them in the same QM state. This is perfectly correct, and certainly allows us to infer the incompleteness of the minimal instrumentalist interpretation F of QM. In discussion of the EPR argument, a great deal is often made of the choice presented to the experimenter of measuring either σ_{1z} or σ_{1x}—that, while it is true that both $[\sigma_{2z}]$ and $[\sigma_{2x}]$ are predictable in accordance with the mirror-image correlations built into $|\Psi_{\text{singlet}}\rangle$, they cannot both be predicted, since σ_{1z} and σ_{1x}, being incompatible observables, cannot be measured in the same experiment. However, it should be clear that the argument for incompleteness goes through without any consideration of the alternative possibilities of measuring σ_{1z} or σ_{1x}. As we have shown, we need only consider the single component, σ_{1z} say, to establish incompleteness.

3. In their original paper, EPR do not use the spin version of the argument which is due to Bohm (see notes and references at the end of

this chapter). Instead, they consider measurements of position and momentum observables for two particles in one-dimensional motion. Thus they consider the state

$$|\Psi\rangle = \int_{-\infty}^{\infty} |p\rangle|-p\rangle e^{-ilp}\,\mathrm{d}p \tag{14}$$

where we use the convention that, in the tensor product, the first component refers to particle 1 and the second to particle 2. (14) is thus a superposition of simultaneous eigenkets of the momenta P_1 and P_2 of the two particles, with associated eigenvalues p and $-p$ respectively. Hence $|\Psi\rangle$ is itself an eigenket of $P_1 + P_2$ with the eigenvalue zero. But (14) is also an eigenket of $Q_1 - Q_2$, where Q_1 and Q_2 are the positions of the two particles.

This follows at once by expanding

$$|p\rangle = \int_{-\infty}^{\infty} \langle q|p\rangle|q\rangle\,\mathrm{d}q$$

$$|-p\rangle = \int_{-\infty}^{\infty} \langle q'|-p\rangle|q'\rangle\,\mathrm{d}q'$$

and remembering $$\langle q|p\rangle = \frac{1}{\sqrt{2\pi}}\,e^{iqp}$$

and $$\langle q'|-p\rangle = \frac{1}{\sqrt{2\pi}}\cdot e^{-iq'p}$$

So we obtain

$$|\Psi\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}q\,\mathrm{d}q'\,\mathrm{d}p\,e^{i(q-q'-l)p}|q\rangle|q'\rangle$$

$$= \int_{-\infty}^{\infty}\mathrm{d}q \int_{-\infty}^{\infty}\mathrm{d}q'\delta(q-q'-l)|q\rangle|q'\rangle$$

$$= \int_{-\infty}^{\infty} |q\rangle|q-l\rangle\,\mathrm{d}q \tag{15}$$

From (15)

$$(Q_1 - Q_2)|\Psi\rangle = l|\Psi\rangle$$

showing that $|\Psi\rangle$ is indeed an eigenket of $Q_1 - Q_2$ with the eigenvalue l.

Clearly, the state $|\Psi\rangle$ as specified in (14) or (15) has built into it similar correlation features to the spin example. Thus (14) shows that measurement of P_1 enables us to predict the result of measuring P_2, viz. the negative of the measurement result for P_1, and similarly (15) shows that measuring Q_1 enables us to predict that the result of measuring Q_2 will be the same value displaced through the fixed distance l. EPR are thus able to conclude that, in the state $|\Psi\rangle$, $[P_2]$ and $[Q_2]$ simultaneously exist, and hence to infer the incompleteness of F.

It is instructive to consider how the state $|\Psi\rangle$ might be prepared. Consider a thin rigid diaphragm with two parallel slits which are very narrow compared with their separation, and through each of which one particle, defined by a wave-packet of dimensions large compared with the width of the slits, passes independently of the other. We suppose that the initial momentum of each particle is sharply defined about some average value which specifies the motion of the incoming particles as being directed perpendicular to the diaphragm. Let the diaphragm be suspended by weak springs from a solid yoke which is rigidly connected to the rest of the spatial frame. In these circumstances we are in a position to 'survey' the transverse momentum communicated between the two particles and the diaphragm, providing that we renounce any control of the precise location of the diaphragm relative to the spatial reference frame. If we now select pairs of particles emerging from the two slits with zero transfer of momentum to the diaphragm, and if Q_1, P_1 and Q_2, P_2 refer to the transverse locations and momenta of the two particles, then the state vector for describing such pairs of particles in respect of their transverse motions is effectively just the EPR state $|\Psi\rangle$ we have been discussing.

Two points should be noticed. First, the tight correlation between measurements of Q_1 and Q_2 will only be true at the instant at which the particles emerge from the slits. The time evolution of $|\Psi\rangle$ will result in a progressively increasing decorrelation between measurement results for Q_1 and Q_2, due to the usual diffraction effects behind a narrow slit. Secondly, $|\Psi\rangle$ is a state for which $Q_1 - Q_2$ has the sharp value l, but for which Q_1 and Q_2 separately have unsharp values, ranging in the limiting situation we are considering from $-\infty$ to $+\infty$. From the point of view of Schrödinger time evolution, the EPR state acts as an infinite line source, which is 'incoherent' in the sense that no interference effects arise between the Schrödinger waves originating at different points of the source.

Notes and References

The original EPR argument was given in Einstein, Podolsky, and Rosen (1935). Bohr's response is to be found in Bohr (1935). Bohm (1951) presents the spin version of the argument. The locality principles LOC_1 and LOC_2 were introduced in Redhead (1983). The thought experiment for producing the EPR state is given in Bohr (1935). For further discussion see Redhead (1981).