PROOF DEFINED*

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In this chapter I shall analyse and compare two parallel passages in Sextus (*PH* II 134–43, and *M* VIII 300–15), which chronicle the efforts made by the Hellenistic philosophers to arrive at a satisfactory definition of proof (*apodeixis*). I shall make occasional use of a third passage in Sextus (*M* VIII 411–23) which contains closely related material. This third text has the advantage of explicitly guaranteeing the Stoic origin of the doctrines it expounds,[1] whereas the other two make no ascription to any particular author or school;[2] its point of view, however, is a little different: it aims simply to classify the various types of argument, *logos* (cf. *M* VIII 411, 424), whereas the other two texts undertake that task in the hope of thereby reaching a definition of a particular species of argument, viz. proof (cf. *M* VIII 300, 315; *PH* II 135, 143). That difference has been neglected by the few commentators who have offered a close analysis of these familiar texts:[3] Urs Egli regards the passages as strictly comparable accounts of two distinct theories about the same subject matter ([41], 61); Benson Mates treats the texts bearing on the theory of proof as though their primary aim was to classify the different types of argument, and he finds them less interesting than those passages which testify to another, and equally Stoic, classification of arguments—

* Translated by Jennifer Barnes.

[1] See *M* VIII 425 (*τοῖς ἀπὸ τῆς στοᾶς*). The Stoic origin of the other two texts is widely accepted. *SVF* II, p. 89.3–10 prints no more than eight lines of extracts from *M* VIII 310 and 314. Other texts have preserved a different Stoic definition of proof from the one I am going to analyse: according to that condensed definition, a proof is 'an argument which concludes to what is less well apprehended by way of things better apprehended' (cf. DL VII 45: *λόγον διὰ τῶν μᾶλλον καταλαμβανομένων τὸ ἧττον καταλαμβανόμενον περαίνοντα*; *Acad.* II 26: 'itaque argumenti conclusio, quae est graece *ἀπόδειξις*, ita definitur: ratio quae ex rebus perceptis ad id quod non percipiebatur adducit'.) I shall return later to this definition and to its relation to the definitions which will occupy us (see below, n. 22 and p. 146).

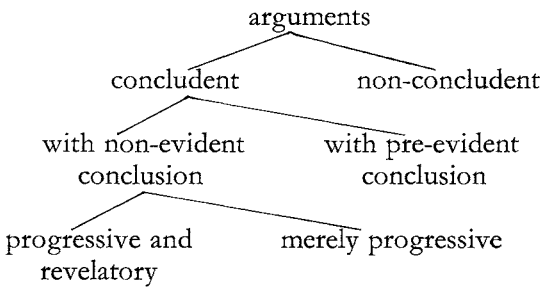
[2] *M* VIII 300–15 uses the first person (*καλοῦμεν*, 302; *φημί*, 304); but near the end it uses a third-person expression (*ὑπογράφουσιν* 314). In *PH* II 134–43 Sextus regularly refers to an unnamed school (*τί φασιν, ὡς φασίν, ὃ λέγουσιν*, 135; *εἰώθασί*, 143).

[3] See esp. Mates [44], 58–63, 110–11; Egli [41], 61–4; Frede [42], 118; Gould [92]; Barnes, Chapter 7 below, pp. 161–81

a classification which involves the celebrated notion of an 'indemonstrable' (*anapodeiktos*) argument.[4] That latter classification gives a different sense to the word '*apodeixis*' from the one to be investigated here: it has been the subject of many penetrating studies,[5] and I shall leave it alone.

The two texts which I consider central, *PH* II 134–43 and *M* VIII 300–15, each present a definition of *apodeixis* by way of a carefully elaborated dichotomous analysis of its genus, *logos*;[6] and they do so in terms which are superficially similar. But there are numerous differences between the two texts which oblige us to analyse them separately, following, as far as possible, the inner logic of each of them.[7] For reasons which will emerge later, I shall begin by looking at the *M* version—though I shall not scruple to compare it to the *PH* version when the need arises.

The *M* version can be divided schematically as follows: (1) argument to justify locating proof in the genus *logos* (*M* VIII 301); (2) definitions of *logos* and of the elements of a *logos* (301–2); (3) dichotomous division of the genus *logos* along these lines (303–9):



(4) first recapitulation of the series of divisions, in the form of a definition of proof (310); (5) supplementary elucidations of that

[4] Basic texts: DL VII 76 f.; *PH* II 156 f.; *M* VIII 223 f.; cf. Mates [44], 63 ('another classification of valid arguments which seems more important than that just discussed').

[5] Cf. Mates [44], 63 f.; Kneale [43], 162 ff.; Frede [42], 127 f.

[6] In several passages Sextus also says that proof is a species of the genus sign (*σημεῖον*); cf. the references given by Barnes, Chapter 7 below, p. 179 n. 22. Later on we shall see how this double allegiance is to be explained (below, nn. 14, 22).

[7] The differences between my analysis and Egli's derive in the main from the fact that he does not follow this method. Thus ([41], 62–4), he arranges in parallel columns pieces extracted from each of the three texts, thereby suppressing bridge passages and recapitulations as important as *M* VIII 310 and 314.

recapitulation (311–13); (6) second recapitulation, introducing two new definitions of proof (314).

Let us look in turn at each of the parts of that schematic plan.

(1) Sextus offers the *M* version as an explication of the notion of proof (cf. 300), and he begins by locating it within the generic notion of *logos*. He gives the following justification for that inclusion: a proof 'is certainly not a perceptible object, but a sort of movement and assent of thought (*dianoias tis kinēsis kai sunkatathesis*)—and those are logical things (*logika*)' (301). That argument does not amount to much: it seems to presuppose that everything that is not perceptible is 'logical', a presupposition falsified, in Stoic doctrine, by time, the void, and space, which are incorporeal without therefore being 'logical'; further, everything 'logical' is not necessarily a *logos* in the strict sense of the word, i.e. an argument: for example, a proposition (*axiōma*) is certainly something 'logical', but it is not a *logos*. Note also that the *PH* version does not offer this feeble argument—or indeed any argument at all—for the inclusion of the notion of proof in the generic notion of *logos*: it thinks that the point is self-evident (*PH* II 135).

(2) In 301–2, the *M* version defines a *logos* as 'what is composed of premisses (*lēmmata*) and conclusion (*epiphora*)'. This definition, with its typically Stoic terminology, appears again in practically the same form in the *PH* version; the latter, however, is more particular about the formal accuracy of its definition: it uses a noun for the proximate genus of the *definiendum*, stating that a *logos* is 'a *system* composed of premisses and conclusion' (135).

The two versions differ more sharply when it comes to defining what a premiss is. The *M* version has this (302): 'We call *lēmmata* not certain assumptions which we wrest [sc. from our interlocutor], but those which our interlocutor grants (*didōsi*) and concedes (*parachōrei*) because they are obvious (*tōi emphanē einai*).' By insisting on the goodwill and bona fides of the dialectician, and on the ease with which his interlocutor will grant him the 'obvious' premisses which he submits for his approval, this definition runs a clear risk from a logical point of view: it implicitly supposes, or tempts us to suppose, that a premiss is always a *true* proposition—for it is granted by virtue of its being

'obvious'.[8] Now that presupposition strongly discourages the development of logical theory; for it leads us to neglect the interesting case where we reason correctly from false premisses—the case which enables us to distinguish clearly between the genuinely logical question of the validity or invalidity of an argument, and the extralogical question of the truth or falsity of its component propositions. And in point of fact, as we shall see, the *M* version will not introduce an explicit distinction among logically concludent arguments between those that may be described as true (because their premisses and conclusions are true), and the rest.

Here now is the definition of a premiss in the *PH* version: 'We call *lēmmata* the propositions (*axiōmata*) which are assumed by agreement (*sumphōnōs lambanomena*) for the establishment of the conclusion' (136). This is a more restrained and a more abstract definition, in which the psychological and dialectical connotations are far less apparent. Admittedly, it does not expressly state that one can examine 'by agreement' what would follow logically from false premisses, but neither does it exclude that possibility, and its difference from the *M* definition might well be inspired by a desire to leave room for false premisses. At any rate, the *PH* version, unlike the *M* version, draws, as we shall see, a careful distinction between the logical concludency of an argument and the truth of the propositions it contains; and we may properly regard this difference as deriving from the difference between the definitions of a premiss in the two versions.

As far as the definition of the conclusion is concerned, the two versions are again in agreement, except that once more *PH* alone refers to the proximate genus: according to *M* (302) the conclusion is 'that which is established (*to kataskeuazomenon*) from the premisses'; according to *PH* (136), it is 'the proposition (*axiōma*) established from the premisses'. Given that a *logos* is not

[8] I assume that *ἐμφανής* ('obvious') implies *ἀληθής* ('true'), both because of the context, which contrasts propositions wrested by guile or by force with premisses asked for in good faith, and because of the word itself, which seems to refer to the evidence of the thing itself. The proposed definition leaves no room for false premisses—neither false premisses recognized as such both by the dialectician and by his partner and required by the former for the purposes of his argument, nor false premisses recognized as such by the dialectician but not by his partner. If the premiss is false but not recognized as such by either of the interlocutors, then matters proceed just as if it were true.

always concludent, we must surely give a proleptic sense to '*to kataskeuazomenon*'—'what is to be established from the premisses', and not 'what is actually established from the premisses'. The criterion for picking out the conclusion of a *logos*, whether concludent or non-concludent, is probably its position—it comes last in the *logos*—and the fact that it is introduced by way of the particle 'therefore' (*ara*).

M 302, like *PH* II 136, gives as an example of a *logos* the celebrated argument: 'If it is day, it is light; it is day: therefore, it is light.' *M* explains carefully, *PH* briefly, which are the premisses of the argument and which the conclusion.

(3) After those preliminary definitions of a *logos* and its constituent elements, the *M* version launches out on the series of dichotomies which I summarized in the diagram above. I shall now comment on each of its successive stages.

(3A) The first dichotomy (303–5) divides *logoi* into concludent (*sunaktikoi*) and non-concludent arguments. Sextus describes the criterion for this distinction in the following way: 'Concludent arguments are those in which, once the premisses are agreed to hold (*huparchein*), the conclusion also clearly follows from the agreement to the premisses (*para tēn toutōn sunchōrēsin akolouthein phainetai kai hē epiphora*).'[9] This definition, which is in some respects reminiscent of Aristotle's definition of the syllogism[10] (and, indeed, also of his definition of the 'perfect' syllogism[11]), seems to me to muddle psychological considerations of clarity ('*phainetai*') and logical considerations of formal validity ('*akolouthein*'). If we take it literally (at least on the interpretation I have

[9] Two notes on the translation. (i) *καί* ('also') connects the conclusion to the premisses: if the premisses are true, it too is true. Cf. the *καί* in 304 (*καὶ τὸ δεύτερον*). (ii) One might hesitate between 'It clearly follows' and 'It seems to follow' as a translation of *ἀκολουθεῖν φαίνεται*. The second version, which may be better favoured by the ordinary rules of grammar, seems to me to be excluded by the context, which aims at giving a definition of a *concludent* argument. If the conclusion 'seems to follow', without actually doing so, will we have an argument which is, purely and simply, concludent?

[10] See *APr.* I 1 24b 18–20. Note in particular the parallel between *τεθέντων τινῶν* and *τῷ ταῦτα εἶναι* in Aristotle, and *συγχωρηθέντων ὑπάρχειν τῶν λημμάτων* and *παρὰ τὴν τούτων συγχώρησιν* here. On the point of these qualifications, which are intended to exclude superfluous or 'redundant' premisses, see Barnes, below pp. 168–9.

[11] See *APr.* I 1 24b 22–4. Compare the 'clarity' (*φαίνεται*: above n. 9) of the inference in the Stoic case with the clarity of the logical necessity in Aristotle's 'perfect' syllogisms (*πρὸς τὸ φανῆναι τὸ ἀναγκαῖον*).

just given), we shall have to classify as non-concludent 'Lewis Carroll' arguments—arguments which are logically valid but so complex that the logical connection between premisses and conclusion is no longer evident.

The following portion of the text does not make this much clearer. Non-concludent arguments are defined only in a negative way ('those which are not of this type': *hoi mē houtōs echousin*, 305); and that really tells us nothing more. As an example of a concludent argument, the same passage (303–4) gives us the well-worn: 'If it is day, it is light; it is day: therefore, it is light'; but the accompanying commentary merely underlines the muddled thinking. The argument is concludent, the text reports, because

> If we grant the truth of the conditional (*dothentos men alēthous einai tou sunēmmenou*) . . . and if we also grant that its antecedent holds (*dothentos de huparchein kai tou prōtou tōn en autōi*) . . . its consequent will of necessity be concluded (*kat' anankēn suneisachthēsetai*) because of the holding of the premisses (*dia tēn toutōn huparxin*).

There is no mention here of the clarity of the link between premisses and conclusion, and we note the appearance of the idea of necessity which was absent from the initial definition of concludency. But these little changes make the whole thing tautological: the commentary boils down to saying that the argument is concludent (*sunaktikos*) because it is necessary to conclude (*suneisachthēsetai*) to its conclusion.

Note finally that a detail of this commentary seems to confirm that the *M* version presupposes that only *true* premisses are to be reckoned with: we are told that once the premisses are allowed to be true (*dothentos men alēthous einai . . . dothentos de huparchein*), the conclusion necessarily follows, not because of the agreement we gave to the premisses which were hypothetically taken as true, but categorically, because of the actual truth of the premisses (*dia tēn toutōn huparxin*).[12] This slide from agreement about truth to

[12] If he did not presuppose the truth of the premisses, the author ought, I think, to say 'because of the agreement given to the premisses' and not, as he does, 'because of the truth of the premisses'. For the identity between ὕπαρξις and truth cf. the parallel descriptions of the premisses: δοθέντος μὲν ἀληθοῦς εἶναι τοῦ συνημμένου, and δοθέντος δὲ ὑπάρχειν καὶ τοῦ πρώτου (the difference is no doubt due to a desire to distinguish between the types of truth possessed by the two premisses: the major is a complex conditional proposition, whose truth is not given by perceptual evidence; the minor is simple and its truth rests immediately on perception).

actual truth shows that for the author of the *M* version a premiss worthy of the name is always a true proposition.

When the *PH* version comes to the problem of the criterion for concludency, it differs importantly from the *M* version, and suggests a criterion which is famous for a rigour and a precision which have only been fully appreciated since the modern rebirth of logic (cf. Mates [44], 74). For we now read (137) that an argument is concludent 'when the conditional which has as its antecedent the conjunction of the premisses of the argument and for its consequent the conclusion, is sound (*hugies*)';[13] in other words, an argument '*p*, *q*; therefore, *r*' is concludent if and only if the conditional proposition 'If *p* and *q*, then *r*' is true.[14] Doubtless the problem is shelved rather than solved by this definition: the concludency of an inference depends on the truth of its associated conditional, and we have still to discover the criterion for the truth of a conditional—a problem which, as we know, gave rise to interminable disputes in the schools of the period.[15] But at least the problem of concludency was clearly reduced to another well-formed problem, and it was freed from any intrusive psychological elements. The continuation of the text of *PH* (137) shows how the new criterion for concludency is satisfied by the argument 'If it is day, it is light; etc.' (the text here has now been correctly understood and restored after centuries of distortion[16]); but unfortunately the author does not make clear by reference to

[13] *ὑγιής* (from which the modern term 'valid' originates) is used by the Stoics both of a logically valid argument and also of a true proposition. Mates [44], 136, says that in the latter sense the word is 'interchangeable with *ἀληθής*', citing *M* VIII 125–8, 244 ff. It is worth observing that these examples only concern *complex* propositions (conditionals, conjunctions). See Barnes, below, p. 169, n. 11.

[14] The criterion of concludency given by *M* VIII 411–23 is first expressed in an elliptical and scholastic manner (an argument is concludent 'because it is formulated in a sound form', *διὰ τὸ ἐν ὑγιεῖ ἠρωτῆσθαι σχήματι*—I do not know why Egli [41], 64, finds in this 'the exact formulation'); but it is then explained in the same terms as in *PH* II (cf. 416–17). This connection between the form of an argument and the form of an associated conditional enables us to understand how a proof can be called a species of sign (see above, p. 126 n. 6). For a sign is 'the antecedent in a sound conditional, which reveals the consequent' (*M* VIII 245). Thus a proof can be considered as a species of sign; 'for it makes clear the conclusion, and the conjunction of its premisses will be a sign of the truth of the conclusion' (*M* VIII 277).

[15] Sextus himself makes this point in a polemical context (cf. *M* VIII 426–8).

[16] Read, with Mates and against the manuscripts and various editorial proposals: *εἰ ‹ἡμέρα ἐστί, καὶ εἰ ἡμέρα ἐστί, φῶς ἐστι›, φῶς ἐστιν*. Cf. Mates [44], 110–11 and [110].

which criterion the associated conditional is held to be true.[17]

(3B) If we now return to the *M* version, a new dichotomy appears (305–6): it consists in distinguishing, among concludent arguments, between those which conclude to something pre-evident (*prodēlon*) and those which conclude to something non-evident (*adēlon*).[18] It is thus a property of the conclusion alone that makes the difference. We may ask two questions about this property. The first is whether the non-evidence of the conclusion, in arguments of the second class, belongs specifically to one or other of the three categories of *adēla* distinguished by the Stoics. Those categories are: (a) things that are non-evident once and for all (*kathapax*), like the exact number of grains of sand in Libya; (b) things that are non-evident by nature (*phusei*), like the invisible pores of the skin; and (c) things which are non-evident for the time being (*pros kairon*)—things evident by nature, but temporarily prevented from being so by external circumstances, like the city of Athens for someone who is not there.[19] We may certainly exclude the first category: no argument could conclude to something non-evident *kathapax*—for such things can never fall under human understanding (*anthrōpinē katalēpsis*), whereas demonstrated conclusions are objects of rational understanding (cf. *M* VIII, 147; DL VII 52). But there is no reason in the text of the *M* or the *PH* versions to exclude either of the two remaining categories: the fact that the arguments in the second class of our dichotomy are described as having a non-evident conclusion, without any further qualification, leads us to suppose that this

[17] The parallel text at *M* VIII 411–23 is fuller on this point. It reports that a conditional is true if it is never the case that its antecedent is true and its consequent false. That seems to be the criterion ascribed to Diodorus (cf. *PH* II 110; *M* VIII 115–17).

[18] On the notions of ἄδηλον and πρόδηλον cf. *PH* II 97–9; *M* VIII 144–7.

[19] If an argument allows us to conclude to a thing which is by nature non-evident, that thing does not thereby become evident; its ἄδηλον character is not annulled by the knowledge which the argument allows us to acquire about it (see, correctly, Barnes, below, pp. 177–8. But exactly the same goes for a conclusion which is temporarily non-evident: if you prove at Oxford that the city of Athens exists, its existence remains for the moment ἄδηλον; only by going there can you put an end to its non-evidence. In the same way, a microscope will annul the non-evidence of the pores in the skin; but the Stoics never envisaged that possibility (as is shown by their use of the word νοητοί, literally 'intelligible', for those invisible pores). That is so despite the use of the word δηλωτική at *M* VIII 277, and the imperfect 'percipiebatur' in Cicero (above, p. 125 n. 1).

non-evidence can be either temporary or permanent, either accidental or natural.

The second question about the pre-evidence or non-evidence of the conclusion is whether we are dealing with an *absolute* property of the conclusion, to be determined simply from its own content, or rather with a *relative* property which the conclusion has in relation to the premisses or to one of the premisses. The two examples in the *M* version of arguments with pre-evident conclusions are accompanied by comments somewhat lacking in coherence in this respect. In the analysis of the first ('If it is day, it is light; etc.'), it is stated that the conclusion 'It is light' is evident to the same degree (*ep' isēs phainomenon*) as the premiss 'It is day', that is to say, as the minor premiss alone.[20] (Nothing is said of the relationship between the conclusion and the conditional major premiss, whose self-evidence—whether it is seen as rational or as inductive—cannot be granted quite the same status as the perceptual self-evidence which belongs to the minor premiss and to the conclusion.) On the other hand, the second example ('If Dion walks, Dion moves; the first; therefore the second'[21]), it is said that the conclusion 'is one of those things patent in itself (*tōn autophōratōn hupērchen*)'. The specific property of the conclusion is therefore sometimes presented as relative and sometimes as absolute. The *PH* version is simpler and clearer on this point; it only gives one example of an argument with a pre-evident conclusion (the first of the two examples above), and it is content to state that its conclusion ('It is light') is *prodēlon* (140).

Of the two classes distinguished in the present dichotomy, it is the class of arguments with a non-evident conclusion that is relevant to the theory of proof. The ancient idea of *apodeixis* implies, as is plain enough in Aristotle, that the premisses are 'better known' than the conclusion (cf. *APo.* I 2 71^{b} 21 f.). But in Aristotle's view the premisses, at least in the paradigm cases, must be better known 'by nature' or 'in themselves' than the conclusion; and that implies that they must be further removed than the latter from the immediate data of perception. For the Stoics, on the other hand, and in the empiricist climate of the

[20] Read *τῷ ἡμέρα ἐστι*, for the MSS reading *τῷ εἰ ἡμέρα ἐστιν* (305).

[21] For the sake of brevity I here use (as Sextus often does elsewhere) the hybrid formulation which the Stoics called *λογότροπος* (cf. DL VII 77).

Hellenistic schools in general, an argument will only be probative if it allows us to establish a conclusion which it is impossible (*de facto* and/or *de jure*) to verify by direct observation, and to establish it by means of premisses which themselves, it seems, must be immediate.[22] While Aristotelian knowledge is knowledge of the cause, Stoic proof, we might say in a Cartesian vein, is proof from effects.[23]

However, it would be wrong to reduce the self-evidence required of the premisses to perceptual self-evidence. Let us look at the two examples in the *M* version that illustrate the idea of an argument with a non-evident conclusion. In the first example ('If sweat flows through the skin, there are invisible (*noētoi*) pores in the flesh; the first; therefore the second') we can say that the non-evident conclusion rests on the one hand on the rational self-evidence of the major conditional premiss, and on the other hand on the perceptual self-evidence of the minor premiss. The second example ('That whose separation from the body causes men to die is the soul; it is separation of the blood from the body that makes men die; therefore, the blood is the soul') has a different logical structure; it seems nevertheless that the epistemological standing of the premisses is the same as in the previous example: a rational or semantic type of self-evidence for the major premiss, and an empirical kind of self-evidence for the minor.

I have one last comment to make on this section of the dichotomy in the *M* version—the last comment, but perhaps not the least, even though it is negative in form: whereas the *PH* version, as we shall see, divides concludent arguments into *true* and *non-true* arguments and only then distinguishes, among true arguments, between those that have a pre-evident conclusion and

[22] For the moment I say that it is a necessary condition, since the *M* version identifies proof with a particular species of argument with non-evident conclusion; but note that, according to the abbreviated definition of *ἀπόδειξις* which I cited above (p. 125, n. 1), it is a sufficient condition (and Egli [41], 62, calls this 'the old Stoic definition of proof'). I shall return to this problem later. Again, I say that the conclusion must be empirically unverifiable *de facto* and/or *de jure* in order to leave open the two possibilities of *ἄδηλον πρὸς καιρόν* and *ἄδηλον φύσει*. In any case, this condition on probativeness explains perfectly why a proof is included in the genus sign (see above, nn. 6, 14). As for the premisses, they are in principle *πρόδηλα* ; but see below, p. 152 and n. 45.

[23] Or, in the more up to date vocabulary of Barnes (below, p. 181), 'an inference to the best explanation'.

those that have a non-evident conclusion, the *M* version ignores this intermediate stage. This is certainly not mere chance, nor a textual aberration—we shall have a proof of that later on when we examine what I have called the first recapitulation. Rather, the 'omission' is a direct consequence (and, conversely, a confirmation) of that presupposition which the *M* version has already made, both in its notion of concludency and in its definition of the premisses: since the author has implicitly admitted, from the very beginning, that a premiss is always a true proposition, he has no reason to make the distinction among concludent arguments which the author of the *PH* version introduces.

(3C) The next dichotomy (307–9) divides the class of arguments which have a non-evident conclusion. It consists of discriminating between those that 'lead us from the premisses to the conclusion in a merely progressive (or: processive) way (*ephodeutikōs monon*)' and those that lead us 'at the same time in a progressive way and in a revelatory way (*ephodeutikōs hama kai ekkaluptikōs*)'. In order to understand this distinction, reasonably described by Mates ([44], 61), as 'by no means clear', we must take a close look at the definitions and the examples which accompany it.

We read that the first category contains 'the arguments which clearly depend on faith and memory (*hoi ek pisteōs kai mnēmēs ērtēsthai dokountes*)'. These are illustrated by the following example: 'If some god has told you that this man will be rich, he will be rich; this god (I point to Zeus, say) has told you that this man will be rich; therefore this man will be rich.' In this kind of case, we are told, we accept the conclusion 'not because it is established by the force of the argument put forward, but because we have faith in the god's assertion (*ouk ek tēs tou protathentos logou dunameōs kataskeuasthen, alla tōi pisteuein tēi tou theou apophansei*)'. To illustrate the second category, that of arguments 'at the same time progressive and revelatory', the text returns to the example that proves the existence of pores in the skin,[24] and explains that the premisses of such a proof teach us, by their very nature (*ek tēs autōn phuseōs*) to draw the conclusion 'in accordance with a

[24] According to Egli [41], 64, this example is 'an alien replacement for the original example, which we still find at *M* VIII 422 [*sic*; read: 423]'. The example of a probative argument in that passage is this: 'If this woman has milk in her breasts, she has conceived; the first; therefore, the second.' Egli does not say, and I cannot see, why we should suppose that this example is more 'original' than the other.

progress of something like this sort (*kata tina toiautēn ephodon*)' (the phrase betrays some mild embarrassment): 'It is impossible for a liquid to flow through a solid and non-porous body; but sweat runs through the body; therefore the body cannot be solid but must be porous.' Our author's embarrassment is readily explained; for his 'justification' of the original argument amounts in effect to replacing it by another argument, in which the original major premiss, of the form 'If *p*, then *q*', gives way to its contrapositive, qualified by a modal operator, 'If not-*q*, then necessarily not-*p*.' It is hard to see how this new argument could be an adequate foundation for the original argument of which it is merely a transformation.[25]

I will now return to the dichotomy as a whole. Leaving on one side the special problems raised by the possibility of our pointing to Zeus,[26] and hearing his words, it seems to me that the real difficulty of this passage lies in the fact that the reasons the author gives for distinguishing between the two types of argument are not those that actually distinguish them. According to our author, we do not accept the conclusion of an argument such as the one about the oracle because it is established 'by the force of the argument' or 'by the force of the premisses'.[27] (Incidentally, that should impel him to deny that such an argument is concludent, if he bears in mind his own earlier definition of a *logos sunaktikos*.) Rather, we accept it because 'we have faith in the god's assertion', and probably also because we recall times when we have been able to establish the veracity of the gods in general and/or of Zeus in particular. (However, memory, though linked with faith in the abstract description of this type of argument, receives no further mention in the commentary on the example.) In the argument of the oracle, we might say, pure reason is subordinate to or dependent upon (*ērtēsthai*) something else; but scientific proof, as illustrated by the argument of the pores, is put forward as a

[25] To be quite fair, I should point out that the major premiss of the first argument talks of sweat and flesh, whereas that of the second talks in general of liquids and solids; thus the author hopes to justify the first argument by showing it to be a particular case of a general law. But it is left for the author of *PH* to bring out clearly how the proof is rooted in the conceptual content of the notions of solidity and liquidity (see below, p. 153).

[26] On this see Frede [42], 55; Lloyd [101], 286. On the general problem of δεῖξις see also P. Pachet [113].

[27] Barnes (below p. 178), cites, with references, various parallel expressions.

strictly autonomous process in which the conclusion, fully deserving the name '*epiphora*' (which reminds us of garnering and the harvest home), grows like a ripe fruit on the intertwining branches of the premisses.

It is therefore in the relation between the premisses and the conclusion that our author finds the distinction between the two types of argument: he writes (cf. 308) as if this relation were intrinsic and rational in the one case, extrinsic and fideistic in the other; and as if faith in the god's announcement played exactly the same role in the latter case as the logical force of the argument does in the former. In this, it seems clear to me, he is at best confused. In order to accept the conclusion of the oracle argument (if we do accept it) we must both recognize the logical force of the argument, which rests on the formal structure of the relation between its premisses and its conclusion, and also grant the truth of the premisses, and in particular of the conditional major premiss, which demands an act of faith in the veracity of the gods: we must be both good logicians and good pagans. At the logical level, the relation between premisses and conclusion is exactly the same in the oracle argument and in the pore argument: it is that of the 'first indemonstrable' of Stoic theory (cf. *PH* II 157; *M* VIII 224; DL VII 80), later christened *modus ponens*: if *p*, then *q*; *p*; therefore, *q*. It goes without saying that the logical force of *modus ponens* remains the same, whatever we may substitute for the variables '*p*' and '*q*'; as soon as the conditional, together with its antecedent, are presented as true, the truth of the consequent follows necessarily, by virtue of the meaning of the connective 'If . . . then . . .'; it really does not matter what kind of criteria we choose—rational, inductive, mystical, or Heaven knows what—to test or verify the conditional premiss.

So what is different in the two cases distinguished by the present dichotomy is not the logical relation between the premisses and the conclusion of the argument; it is rather the epistemological relation between the antecedent and the consequent of the conditional major premiss; in particular, it is, at least implicitly, the nature of the criterion that we use when judging that premiss to be true. The celebrated squabble about the truth-conditions of the conditional thus crops up again in the theory of inference (as long, of course, as we limit ourselves to arguments in *modus ponens* form; and in practice that is what our author does here,

although he knows, as we saw above,[28] that there are other kinds of arguments with non-evident conclusions): just as we distinguish different truth-conditions for the conditional, graded from the weakest to the strongest, so we can distinguish classes of inferences with conditional major premisses, according to how many or how few of those criteria are satisfied by that premiss.

That is surely the reason why the division that interests us now is not, as before, a simple dichotomy of the genus by way of the presence or absence of a specific feature (like, for instance, the distinction between concludent and non-concludent arguments); rather it is a complex dichotomy, using two characteristics of which one (progressiveness) belongs to both the classes distinguished while the other (revelatoriness) belongs to one only. Since the division is tacitly put forward as being exhaustive, we must deduce that there are no arguments which are revelatory without being progressive; in other words, that revelatoriness implies progressiveness—but not vice versa.

Although our text only distinguishes two classes of arguments with conditional major premisses, it is tempting to compare it with the celebrated list, given elsewhere in Sextus, of the four distinct criteria of truth for the conditional, each stronger than its predecessor and weaker than its successor,[29] and to ask by reference to which of those criteria the major premisses of the oracle argument and of the pore argument are judged to be true. The former ('If some god has told you that this man will be rich, he will be rich') must be put fairly low on the scale: it is acceptable to a Philonian, for (if you have 'faith') you will agree that it is not the case that its antecedent is true and its consequent false; but it must be held false by a Diodoran, for there will be a time (beginning from the death of the Croesus in question) when its antecedent will be true and its consequent false.

[28] Cf. the argument of 306 about blood and the soul (cited above, p. 134). There is nothing to indicate how we should locate an argument of this sort in the present dichotomy. Thus our author has not divided the genus he says he has.

[29] Cf. *PH* II 110 f.; *M* VIII 112 f.: the criterion of Philo (a conditional is true if it is not the case that its antecedent is true and its consequent false); the criterion of Diodorus (if it has not been and is not possible that its antecedent is true and its consequent false); the criterion, usually ascribed to Chrysippus, of 'connectedness' or *συνάρτησις* (if the contradictory of the consequent conflicts, *μάχεται*, with the antecedent); the criterion of 'implication' or *ἔμφασις* (if the consequent is contained potentially, *περιέχεται δυνάμει*, in the antecedent).

As for the conditional major, 'If sweat flows through the skin, there are invisible pores', that is no doubt thought of as satisfying a criterion similar to that of 'connectedness' (*sunartēsis*): our author supports it by saying that 'through a solid and non-porous body [negation of the consequent], it is impossible (*adunaton*) for a liquid to flow [generalization of the antecedent]'; and according to the criterion of *sunartēsis*, a conditional is true when the negation of its consequent is incompatible (*machetai*) with its antecedent (*PH* II 111). Although he does not explicitly say so, the author of the *M* version tends, I think, to distinguish between *a priori* and empirical impossibility: I assume that he would classify an argument as 'revelatory' when the antecedent, '*p*', of its conditional major, and the consequent, '*q*', are such that the conjunction '*p* and not-*q*' is *a priori* impossible, in the sense that its impossibility can be seen merely by inspecting its terms. (Thus a non-porous body through which sweat flowed would qualify, independently of any experience, as both compact and non-compact.[30]) On the other hand, he would classify as non-revelatory an argument where the conjunction '*p* and not-*q*' could not be ruled out merely by inspection of its terms, but could be excluded empirically, by appeal to 'memory' of cases in which *q* has been experienced in association with *p*, and by an act of 'faith' in the unfailing repetition of that association.

I have two further notes to make on the present dichotomy.

(1) It is certainly closely connected with the Stoic theory of 'signs' (*sēmeia*), and with the distinction which the Stoics, beginning perhaps with Zeno, made between a 'commemorative' (*hupomnēstikon*) and an 'indicative' (*endeiktikon*) sign.[31] A commemorative sign is one which signals the existence of something temporarily non-evident (*pros kairon adēlon*); it owes its value as a sign, and its name of 'commemorative' sign, to our *memory* of the occasions on which it has been present in *observable* association

[30] In 309 the word ἀδύνατον must, I think, be interpreted in the sense of *a priori* impossibility. In the same way, the possibility that Zeus is lying, or had made a mistake, is a purely *a priori* possiblity, and not one guaranteed by actual experience.

[31] On signs see *PH* II 99–102; *M* VIII 151–5; and the account by Verbeke [122]. Zeno wrote Περὶ σημείων (DL VII 4); cf. Rist [119], esp. pp. 389–90 (Rist translates σημεῖα by 'signals' in order to avoid possible confusion with σημαίνοντα, 'signs' or 'signifiers').

with that of which it is a sign. Thus smoke can be considered a sign, or a signal, of a fire which, though bright enough in its own nature, is temporarily invisible. An indicative sign, on the other hand, is one which signals something naturally non-evident (*phusei adēlon*); thus it cannot owe its signalling capacity to our observation of a constant conjunction between it and what it signals, for the latter is in principle unobservable; it must rather be 'by its own nature and constitution' (*ek tēs idias phuseōs kai kataskeuēs*) (*M* VIII 154) that it makes us 'conclude rationally' (*logizometha*) (155) to the existence of what it signifies; for example, bodily movements are indicative signs of the existence of the soul, which is itself always and in principle invisible.

Those characteristics fit perfectly with the dichotomy we are at present examining. For, on the one hand, the future wealth of our *protégé* of Zeus is quite certainly a temporary *adēlon*, since we shall have to wait at most a few decades before it is manifest to every eye; and faith in the words of the god is, as we have seen, implicitly founded on our memory (*mnēmē*) of the many occasions on which we have been able to see for ourselves the efficacy of his prophecies. On the other hand, the existence of invisible pores in the skin is naturally *adēlon*, as we know, and the flow of the sweat is a revelatory sign of them by the very nature of the premisses (*ek tēs autōn phuseōs*) which serve to prove it. Thus we may suggest that the author of the dichotomy before us divided the genus he had reached (the genus of concludent arguments with a non-evident conclusion) by applying the distinction between *adēlon phusei* and *adēlon pros kairon*, and by adapting to the theory of proof the distinction between the corresponding species of sign.

(2) We might try to delineate more nearly the class of 'merely progressive' arguments, and at the same time to determine more precisely the nature of 'revelation', by asking the following question: in the conditional, 'If some god has said to you that this man will be rich, he will be rich', which are the elements without whose presence we should be obliged to place the argument in a different category? A series of imaginative modifications to the example will enable us to separate the essential from the accidental here.

(a) The asymmetry between a major premiss with an indefinite subject ('some god') and a minor premiss with a definite subject ('this god') is an interesting feature of many examples of Stoic

arguments;[32] but our argument would remain in the same class, I think, if we were to replace the indefinite expression 'some god' by a definite expression ('this god'), or even by what the Stoics called an 'intermediate' expression ('Zeus').[33]

(b) It would surely still belong to the same class if the god had declared himself on some subject other than the economic prospects of a given individual: every argument beginning 'If some god says to you that *p* . . .' will be 'merely progressive', provided only that *p* is *adēlon* (the argument 'If some god has said to you that it is day . . .' would have a pre-evident conclusion). But note that *p* need not be a temporary *adēlon*: if a god decided to tell us that there were $\iota\rho^{853946}$ grains of sand in Libya, we should have to take his word for it, even though we had no hope, in this case, of testing it; and if he decided to tell us that there are invisible pores in the skin, that would not make his words into an 'indicative' sign of the existence of those pores. A mistaken utterance is not an *a priori* impossibility—not even from divine lips; and whatever a Stoic god may say, his word is never a 'revealed truth', and his message is no apocalypse. Thus the fact that the proposition *p* in the oracle example refers to the future is not essential: the *prophetic* aspect of the example is accidental.

(c) We can place in the class of 'merely progressive' arguments every argument beginning 'If *x* has said to you that *p*', provided that the range of *x* is restricted to speakers who are known empirically to be always reliable[34]—the divinity of a speaker is simply the limiting case of this, a case peculiarly favourable to the 'faith' which we can have in his words.[35] The *religious* aspect of our example is no more essential to it than its prophetic aspect.

(d) We might perhaps think that we can go no further, and

[32] On this see Imbert [95], esp. 240–1. This functional difference between major and minor premiss is sometimes reflected in Stoic terminology: *λῆμμα* is reserved for the major and *πρόσληψις* is used to name the minor premiss (see e.g. DL VII 76).

[33] Cf. *M* VIII 96–7, and the analogous classification, using a different vocabulary, at DL VII 70 (discussed by Goulet [93]).

[34] It is true that the Sage, who possesses knowledge of all truths, sometimes utters falsehoods (cf. *PH* II 80–3; *M* VII 38–45; and the discussion by Long [108]); but if it is true of the Sage, it must be true, *a fortiori*, of God—and if we admit such a possibility, the major premiss of the argument will be (empirically) unacceptable.

[35] Oddly enough, the manuscripts omit the word '*θεῶν*' in the *PH* version of the oracle argument (141); the editors restore it, adducing the parallel in the *M* version. The opposite solution, proposed by Mutschmann, is to rely on *PH* and expel the word from *M*; but that is not very convincing.

that the *verbal* aspect of the example cannot be suppressed without removing it from the category of 'merely progressive' arguments. If that is so, we shall have to say that 'revelatory' arguments are those whose conclusions rest on the sort of evidence proper to signs of the type later called 'natural' (the flowing of sweat would be a 'natural' sign of the porousness of the skin), and that 'merely progressive' arguments are those whose conclusion rests on the trust we place in certain producers of verbal signs. The distinction will thus approximate to the later distinction between arguments from authority and arguments from reason. But if my remark (1) is correct, that cannot be right. The distinction between indicative and commemorative signs is not at all the same as that between natural and verbal signs: thus smoke, a typical example of a natural sign, is taken by the Stoics to be a *commemorative* sign of the temporarily invisible fire which causes it, because the association between smoke and fire is supported, in their view, by no more than an empirical regularity (*PH* II 100; *M* VIII 152). Hence, we must classify as 'merely progressive' and not as 'revelatory' every argument in which the major premiss is a conditional whose antecedent is a commemorative sign of its consequent, even if that sign has nothing to do with the making of any verbal statement (e.g. 'If there is smoke, there is fire; the first; therefore the second').

Thus we see that the author of the *M* version perhaps took as essential to his example certain features which are not in fact so—I mean the aspects of prophecy, of religion, and of language. He underlines the part played by man's faith in the word of god; but, as I have already noted, he pays little attention to the role of memory: he mentions it alongside faith in his definition of non-revelatory arguments (*ek pistēōs kai mnēmēs*: 308), but he does not comment on it, and indeed he never mentions it again in his discussion (*pistis* reappears alone at the end of 308: *alla tōi pisteuein tēi tou theou apophansei*). Now memory and faith seem to be indissolubly linked in the functioning of commemorative signs: memory of past experiences would do us no good unless it bred a faith that the future would see them repeated in similar fashion; and faith in signs would be blind if it did not rest on the memory of those observations which support it.[36] We know,

[36] This functional complementarity of memory and faith prevents me from following a suggestion made by Jonathan Barnes, who wondered if μνήμη refers to

moreover, that memory gave the commemorative sign its name, and that the role of memory is regularly emphasized in descriptions of the functioning of this sort of sign (*PH* II 100; *M* VIII 152–3); we know also that when Sextus concentrates his sceptical polemic on the indicative sign, and spares the commemorative sign, that is because the former is the tool *par excellence* of the dogmatic philosophers, who claim with its help to transcend the level of the phenomena, whereas the latter is something we must all of us make use of and have confidence in it if we are to live our daily lives (*to gar hupomnēstikon pepisteuetai hupo tou biou*: *PH* II 102; *touto gar para pasi koinōs tois ek tou biou pepisteuetai chrēsimeuein*: *M* VIII 156).[37] In these circumstances, it is perhaps not too rash to suppose that in trying to apply the distinction between the two types of sign to his classification of arguments, the author of the *M* version mistook the meaning of the word '*pistis*'—he took it in a religious sense, although in his sources it referred only to the 'trust' which we spontaneously place in the regularity of natural phenomena.

Let me end this long commentary on the dichotomies of the *M* version by setting out the definition of proof which they implicitly contain. Admittedly, it is never stated in the paragraphs we have analysed (301–9) that a proof is an argument 'at the same time progressive and revelatory'; but that must surely be the case, given that the goal of the whole series of dichotomies is the definition of proof, and that the series culminates with arguments that are 'at the same time progressive and revelatory'. Thus we may conclude that the dichotomies of *M* imply the following definition of proof: *a concludent argument, with a non-evident conclusion, which is at the same time progressive and revelatory*. I propose to call this *Definition R*, to mark the particular role which revelation plays in it; that characteristic, as we shall see, will be what differentiates it from the other definitions that we shall encounter.

(4) In 310 Sextus himself proceeds to give a recapitulation of

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'reliance on one's own experience' and *πίστις* to 'reliance on that of others'. If that were so, I think we should have to say that non-revelatory arguments depend on faith *or* on memory; but our author says that they depend on faith *and* on memory.

[37] One might object that it is a matter of trust in commemorative signs and not, properly speaking, of trust in natural phenomena. But if we place trust in a commemorative sign, surely that is only because we have a basic trust in the regularity of nature and because we assent to what will later be called the 'principle of Induction'.

the sort which I have just presented on my own account. That he is speaking here *in propria persona*, drawing the moral from the texts he has just reproduced or paraphrased, is shown clearly enough by his mode of expression ('That being so, proof *must be* . . .': *opheilei einai*). But we notice at once that there is an important difference between the definition he claims to draw from the series of dichotomies and the one I drew from the text a moment ago. This is what he says: 'This being so, a proof must first of all be an *argument*, secondly *concludent*, thirdly *true* (*triton kai alēthēs*), fourthly *having a non-evident conclusion*, fifthly *revealing this by the force of the premisses*.' This new definition includes the notion of revelatoriness which *R* contains; but it also includes the notion of truth which is absent from R. I shall call it *Definition C*, since it contains the conjunction of these two notions.[38]

It is tempting to explain the difference between Sextus' recapitulation and the text he professedly recapitulates by supposing either that the dichotomies in 303–9 mentioned truth in a paragraph which has been accidentally lost, or that the notion was later added to the recapitulation in 310 by some *lector eruditus*. But the next part of the text in my view excludes each of those hypotheses.

(5) For in 311–13 Sextus tries to justify the addition of the notion of truth by giving an example of an argument which is 'concludent but not true', i.e. which is valid from the point of view of formal logic but which has a false premiss and leads to a false conclusion. This discussion is connected to the previous paragraph by the particle '*goun*' ('However that may be, this much

[38] Definition *C* has been subject to very different assessments. Barnes refers to it, in a general way, as 'the Stoic analysis of proof' (below, p. 165 n. 6); Egli ([41], 62), on the other hand, thinks it is 'not Stoic in origin'. His general thesis about the three texts on the definition of proof is that they incorporate two distinct definitions: (i) an 'old Stoic' definition containing three features (an argument that is concludent, true, and demonstrative): the main text for this is *M* VIII 411–23, and it is the same as the abbreviated definition preserved by Diogenes Laertius and Cicero (cf. above, p. 125 n. 1); that will appear later as my Definition *T*. (ii) There is a definition which is 'not Stoic in origin', consisting of five points (an argument which is concludent, true, with non-evident conclusion, revealing its conclusion by the logical force of the premisses—but it is illogical to count the feature 'argument' as one of the five points in this definition but not to count it in the other): this is attested by *M* VIII 310 and *PH* II 143, and it is my Definition *C*. According to Egli, Sextus mixed up these two theories of proof in the two expositions which are not explicitly called Stoic (i.e. *PH* II 134–43 and *M* VIII 300–15); and his source was a treatise of Clitomachus which attacked the Stoic theory of proof.

is clear . . .'); and that, I think, proves both that the present case was not adverted to in the earlier dichotomies, and also that Sextus took it upon himself to introduce the notion of truth into the recapitulation—as he must if he is to take the new case into consideration.[39]

Sextus' example (311) is this: '(When it is day), "If it is night, it is dark; it is night; therefore, it is dark" '. No doubt Sextus borrowed the example from some other source—we find it performing the same function at *PH* II 139 and also at *M* VIII 413. Nevertheless, it is notable that when Sextus explains why such an argument must be called concludent, he here refers to the criterion of concludency of the *M* version (*dothentōn gar autou tōn lēmmatōn huparchein, sunagetai kai hē epiphora*—compare that with the expressions of 303–4). In the *PH* version his explanation invokes the rigorous criterion of concludency which, as we have seen, is characteristic of that version:[40] Sextus is a conscientious and an intelligent worker; and he does not ignore the differences between the various sources he uses.

And here is a further indication of that professional conscientiousness: having inserted an additional chapter into the story, Sextus tidies things up by listing once again all the chapters which follow the one he has added. Thus he next gives an example of an argument which, though concludent and true, is not probative, since its conclusion is pre-evident (312: the example, again, is the celebrated 'If it is day, it is light . . .'—stated, presumably, in the daytime; the example has already been adduced, to the same end, in 305). Next, he gives (313) an example of an argument which, though it is concludent and true, and has a non-evident conclusion, is not probative, since it does not reveal the conclusion (the example of the oracle).

(6) After this reworking of the material, with its complementary elucidations, we feel the need for a new recapitulation which will finally set our thoughts in order. A second recapitulation does indeed appear in 314; but it appears in a most unexpected

[39] Egli [41], 64, considers 311–14 to be an addition by Sextus himself; he notes that if you pass directly from 310 to the second half of 314 (*ἔνθεν καὶ οὕτως . . .*), you get a sequence of thought entirely comparable to that in *PH* II 143. But he has not seen that the addition was made necessary by Sextus' decision to introduce *τρίτον καὶ ἀληθής* into the definition of proof (310).

[40] Cf. *PH* II 139 (where we should read, with Mates [44], 111 n. 30: *εἰ νύξ ἐστι, καὶ εἰ νύξ ἐστι σκότος ἐστί, σκότος ἐστίν*); *M* VIII 415–17.

form. For Sextus begins by laying down a new definition of proof, which tallies exactly neither with *R* nor with *C*. Here it is: 'When all these characteristics are found together, and the argument is *concludent and true and establishing* (*parastatikon*) *a non-evident conclusion*, there is a proof.' Like *C* and unlike *R*, this definition includes the notion of truth; but unlike both *C* and *R*, it does not explicitly contain the notion of revelation: to mark those peculiarities, I shall call it *Definition T*.

Definition *T* is, according to Egli (above, p. 144, n. 38), the old Stoic definition; even so, I must defend its independent reality against possible objections by Barnes (below, p. 178, n. 21) and his readers. I entirely agree with Barnes that the notion of revelation is 'a crucial feature of the Stoic notion of proof' (see the many references he cites); moreover, the abbreviated Stoic definition (cited above, p. 125, n. 1) basically comes down to just that. But in my view this notion has neither the same sense nor the same function in all the relevant texts. We may indeed say that all the Stoic authors who applied themselves to the subject of proof agreed that the conclusion of an *apodeixis* is 'revealed' by its premisses; but that did not mean the same thing for all of them: some took the word 'revelation' in a narrow sense, others in a broad sense, perhaps without being quite aware of the difference.

(i) The narrow sense goes along with the idea that there are several ways of establishing a non-evident conclusion, and that only one of those ways can properly be called 'revelatory': every 'revealed' conclusion is non-evident, but a non-evident conclusion can be reached without being 'revealed'. In this narrow sense of the word, it will be stated that the conclusion of a proof is revealed 'by the force of the premisses' (and not simply 'by the premisses'). From this point of view, the notion of revelation must be introduced into the definition of proof as a specific difference, independent of the others, allowing us to subdivide the class of arguments with non-evident conclusion. That is what happens in Definitions *C* and *R*.

(ii) The broad sense of the word 'revelation' goes along with the idea that a non-evident conclusion is revealed merely by the fact that, though non-evident, it is nevertheless reached: the conclusion of a true concludent argument is revealed if and only if it is non-evident. The existence of this broad sense is, I think,

clearly attested: it is plainly to be seen at *M* VIII 422–3, where it is said that in a probative argument 'the conclusion, being non-evident, is revealed by the premisses (*tēn epiphoran adēlon ousan ekkaluptesthai hupo tōn lēmmatōn*), and that the argument about lactation (see above, p. 135, n. 24) 'having as a conclusion the non-evident proposition "This woman has conceived", reveals it through its premisses (*adēlon gar echōn to sumperasma, to kekunēken ara hēde, touto dia tōn lēmmatōn ekkaluptei*).' (On the same note, observe that the idea of revelation figures in the definition of sign in general (cf. *M* VIII 245; *PH* II 104); for, although it appears in the definition of the indicative sign, it does not, despite its position, do so as a specific difference of that sort of sign (*PH* II 101). If it is thus true that proof is a species of sign (cf. above, nn. 6, 14, 22), and that a proof is 'revelatory of its conclusion' (*PH* II 131), it would be wrong to infer that the type of sign in question is only the indicative sign.) In the broad sense of the word, revelation need not appear in the definition of proof as an independent condition: it is implied by the condition that the conclusion be non-evident. And that is just what happens in Definition *T*.

In Definition *T*, as it is formulated at the beginning of 314, '*parastatikos*' is indeed, as Barnes says (p. 165, n. 7), a synonym for '*ekkaluptikos*'—but only in the broad sense of the latter term. The use of '*parastatikos*' here may well be specifically intended to avoid any confusion with the narrow sense of '*ekkaluptikos*'. Thus it is not surprising that Sextus can immediately present another definition, which contains the word '*ekkaluptōn*', as though it were an alternative formulation of Definition *T*

For at this point Sextus clearly wants to work his way towards yet another new definition of proof—a definition which he professedly borrows from a written source. This is how he presents it (314): 'That is why they also describe it (*hupographousin*) as follows: A proof is *an argument which, from agreed premisses, reveals in a concludent fashion a non-evident conclusion* (*apodeixis esti logos di' homologoumenōn lēmmatōn kata sunagōgēn epiphoran ekkaluptōn adēlon*).' This definition contains, in an arbitrary order but in more or less literal form, most of the constituents of the definitions we have already met: the generic notion of argument (*logos*); concludency (*kata sunagōgēn*); perhaps truth (*di' homologoumenōn lēmmatōn*—assuming that a premiss is only agreed to if

it is true); non-evidence of the conclusion (*epiphoran adēlon*); revelation (*ekkaluptōn*). The most remarkable thing about this definition, in my view, is that it has evidently not been obtained by way of dichotomous divisions.[41] Grammatically speaking, it does not possess the conjunctive structure which comes from a succession of divisions (and which Definition *C* for example, characteristically does possess); it has what might be called an organic structure: its several components are centred about the participle '*ekkaluptōn*' which is completed by an object ('*epiphoran adēlon*') and by various qualifications introduced by different prepositions (*di' homologoumenōn lēmmatōn*; *kata sunagōgēn*). To mark that structure, let us call this formula *Definition S*. Its author is plainly concerned neither to set its constituents in a branching logical order, nor to give any precise idea of those classes of arguments which lack one or other of the characteristic marks of probative argument.

Thus the reader of version *M* has come across no less than four definitions of proof: *R, C, T,* and *S*. But there is still a surprise in store for him; for if he was at least able to recognize in all four definitions a common Stoic inspiration, he now (314) finds the last of them illustrated by an example which is none other than the famous Epicurean demonstration of the existence of the void from the existence of motion.[42] On that final note of astonishment, let us end our analysis of this tortured text, the genesis of which was evidently complicated in the extreme.

I turn now to *PH* II 134–43. That version is simpler and more straightforward than *M*, and I have already anticipated some of its essential features: thus I shall be able to proceed—as you may well hope—with a little more dispatch.

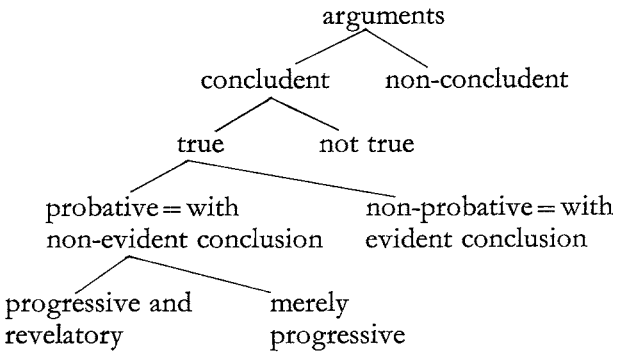
Sextus himself indeed evinces the same desire for brevity. At the beginning of the chapter he promises to expound shortly (*suntomōs*) the theory of proof, 'trying first to explain in a few words (*dia bracheōn*) what they say (*ti phasin*) proof is'. As if to give an immediate example of his wish to be brief, he does not bother to say that *apodeixis* belongs to the genus *logos*, nor—*a fortiori*—to explain why it does.

His account can be put schematically as follows: (1) opening

[41] I do not think that it can have been 'put together' from the five points listed at *M* VIII 310, as Egli [41], 62, suggests.

[42] Cf. Epicurus *apud* DL X 40; *M* VIII 329 (=Usener [13], no. 272).

statement of a definition of proof (135)—this is in fact Definition *S*, and all that follows is offered as a series of elucidations designed to make the sense of that definition more intelligible; (2) definition of *logos* and of its elements (135–6); (3) dichotomous division of the genus *logos* (137–42) along the following lines:



(4) Recapitulation, and *reprise* of Definition *S* which the whole discussion was designed to justify (143).

As in the case of the *M* version, I shall now consider in a little more detail each of the successive parts of this text.

(1) Definition *S* is brought onto the stage as a classic definition (*hōs phasin*). What follows is designed to clarify the content of the definition: 'What they say will be more clear from what follows.' But these elucidations are probably not Sextus' own handiwork; rather, I think that he takes them from a different source—at least the final sentence of the chapter (143) suggests such a conjecture: 'It is in this way, then, that they are accustomed (*eiōthasi*) to explain the notion of proof' (cf. Egli [41], 64).

(2) While the definition of *logos* is practically the same as that in *M*, the definition of the premisses is significantly different; but I have nothing to add to what I have already said on the matter.

(3) The sequence of dichotomies contains several important points which demand our attention:

(3A) As we have seen, the division of *logoi* into concludent and non-concludent arguments (137) is grounded on a different criterion of concludency from the one used in the *M* version—a criterion which has won the admiration of modern logicians.

(3B) Concludent arguments are next subdivided into 'true' and

'non-true' arguments (138–9). A concludent argument is true when 'not only is the conditional which has the conjunction of its premisses as antecedent and its conclusion as consequent sound (so that, on the new criterion, the argument is concludent), but also both the conclusion and the conjunction of its premisses, which forms the antecedent of that conditional, are true.' For the conjunction to be true, each of the premisses must of course be true; and if it is so, then the conclusion too will be true.[43]

There is no need to comment on the change in sense of the word 'true', which applies to an argument in the *definiendum* and to a proposition in the *definiens*—the point has often been noticed (cf. Mates [44], 132). Rather let us remember that the distinction between concludent arguments which are true and those which are not true—a distinction which is fundamental from a logical point of view—was absent from the dichotomies of *M*. As if to underline its importance and its novelty, *PH* carefully explains (139) how an argument can be concludent without being true; he uses an example ('If it is night, it is dark; etc.', stated in daylight) which we have already met in the elucidations which Sextus added to the first recapitulation in the *M* version (311).[44]

(3C) In 140 true arguments are, in their turn, subdivided into two classes: some are 'probative' (*apodeiktikoi*), viz. those which 'from pre-evident premisses (*dia prodēlōn*) lead to a conclusion which is non-evident (*adelon ti sunagontes*)'; others are 'non-probative', viz. those which 'are not of that sort'. With regard to this dichotomy, I confess that at first it seemed to me evident that there was an absolute identity between probative argument (*logos apodeiktikos*) and proof (*apodeixis*); but since Barnes holds the contrary opinion (below, p. 178, and n. 21), I must find some arguments to justify my intuitive identification of the two things.

First, and from a general point of view, let me say that in a division of the genus *logos* which is designed to produce a definition of the species *apodeixis*, it would be very odd if the

[43] Strictly speaking, the truth of the conclusion should not appear in the definition of a true argument, on pain of redundancy. That is why Mates [44], 111 n. 29, proposes to excise the words *καὶ τὸ συμπέρασμα* in 138. He has not noticed that at the end of 139 Sextus quotes another Stoic definition of a true argument, which proves that the Stoics did not shrink from such a redundancy: 'They also say that a true argument is one which establishes a true conclusion from true premisses.'

[44] Here again the text has only recently been rightly understood and emended; cf. Mates's correction, above, p. 145 n. 40.

differentia *apodeiktikos* were introduced at any level other than that at which the species *apodeixis* is reached: would you tolerate a classification of animals in which the notion of *human animal* was introduced at an earlier level than that of *man*? That would surely be more than 'infelicitous'.

But I can also adduce a textual argument, drawn from *M* VIII 411–28: that long polemical passage is designed to test whether the Stoics (named at 425) can, in their logical theory, fulfil 'the claim of proof' (*hē tēs apodeixeōs huposchesis*) (411). How does Sextus conduct the test? He explains in detail that there are three classes of argument, each included in its predecessor; viz, concludent arguments, true arguments, and probative arguments (*apodeiktikoi*: 411–12, 424); and he shows that, given the relations of inclusion among those classes, if we can prove that concludent arguments are 'undiscoverable' (*aneuretos*), we shall thereby have proved that neither true nor probative arguments can be discovered (425). Thus it appears that in order to rebut 'the claim of proof', it is necessary and sufficient to rebut the claim of probative argument: nowhere is there the least suspicion of a distinction between proof and probative argument.

Hence it is, I believe, legitimate to hold that in the series of dichotomies in the *PH* version, the paragraph which introduces the notion of probative argument (140), at the same time marks the stage at which the notion of proof is reached. Thus proof—and here we have an important innovation compared to the *M* version—is no longer a subdivision of the species *argument with a non-evident conclusion*; rather it is identical with that species itself.

The dichotomy raises a second question. Probative arguments are there defined as 'leading from pre-evident premisses to a non-evident conclusion'. If we take the definition literally, we shall have to countenance three distinct types of non-probative argument: those whose premisses and conclusion are pre-evident; those whose premisses and conclusion are non-evident; and those whose premisses are non-evident and whose conclusion is pre-evident. The text does not, however, make that complicated move. The author of the *PH* version is content to give two examples: one of a non-probative argument (that of the day and its inevitable light), which is explicitly classified as such because its conclusion is pre-evident; and one of a probative argument

(that of the pores and their ceaseless sweat), which is explicitly classified as such because its conclusion is non-evident.

Thus it appears that it is not essential to the *PH* definition of proof that the premisses be pre-evident. That might perhaps encourage us to expel from the text the words '*dia prodēlōn*' (the words are not echoed in the later appearances of the definition at 141 and 143, and they have no parallel at the corresponding stage in the dichotomies of *M*, 305–6); and it seems in particular to show that the Stoic concept of proof is not strictly limited to inferences founded directly upon the data of sense-experience: the Stoic theory *can*—even if it rarely *does*—recognize as proofs arguments whose premisses are not pre-evident but have themselves been proved by a sequence of one or more pro-syllogisms in which the only pre-evident propositions are the initial premisses of the probative chain.[45]

(3D) The next dichotomy (141–2) divides arguments with non-evident conclusions into those which are 'merely progressive' and those which are 'at the same time progressive and revelatory'. This section reproduces for the most part the corresponding section of the *M* version, which I have analysed at length. The essential difference is that here (although the text does not explicitly say so[46]) we are dealing with a distinction between two types of proof; for proof has already been identified as argument to a non-evident conclusion. The author of *PH* thinks that the argument about the oracle is no less a proof than the argument about the pores, whatever other differences there may be between them. That apart, the text of *PH* is here very similar to that of *M*: the definitions, the examples, and the comments, are pretty well identical in the two versions—though the author of *PH* is both more concise and more precise.

That general similarity sets in higher relief two differences to

[45] According to Barnes, below, p. 181, 'linked series of syllogisms, such as constitute an Aristotelian science, will not be found among the Stoics.' However, they allow that the premisses of an argument can be proved by means of another argument (cf. the 'third rule' of their metalogic, Alexander, *in APr.* 278.6 ff.: 'If from two propositions a third is deduced, and if there are propositions from which one of the premisses can be deduced, then the other premiss together with these propositions will yield the conclusion').

[46] As Barnes rightly remarks (below, p. 178 n. 21), Sextus never says that he is distinguishing two types of ἀπόδειξις, and he offers 143 as a summary of the whole of 134–42 (including the last dichotomy, 141–2). I shall suggest later on a possible explanation for this state of affairs (below, p. 159 n. 53).

which I must draw attention. (i) Commenting on the example of the oracle, the author of *M* said (308) that we accept its conclusion 'not because it is established by (*ouk ek*) the force of the argument put forward, but (*alla*) because we have faith in the god's assertion'. In his note on the same example (141), the author of *PH* significantly weakens the contrast and says that 'we give our assent to the conclusion not so much (*ouch houtōs*) because of the necessity of the premisses as (*hōs*) by our faith in the statement of the god.'[47] That modification seems to indicate that the author of *PH* discreetly removed the error committed by *M*: acceptance of the conclusion, in the example of the oracle, certainly depends on our admitting the premisses, and hence on our 'faith' in divine veracity; but it also depends, essentially, on the formal validity of the reasoning.

(ii) The second nuance is no less significant: in attempting to show why the argument of the pores is 'revelatory', the author of *M*, as we have seen, found himself a trifle embarrassed, and he tried to ground it on another piece of reasoning which had no very plausible claim to serve as a foundation for the original argument. Now the author of *PH* expresses himself with the assured vigour and technical accuracy of a competent logician (142): 'The flowing of the sweat is revelatory of the existence of pores because we grasp in advance (*dia to proeilēphthai*) that a liquid cannot pass through a solid body.' In the word '*proeilēphthai*' we can hear an echo of the celebrated Stoic notion of *prolēpsis* (preconception, or prenotion);[48] and we thus see that the author of *PH*, instead of courting the danger of an infinite regress by resting one argument on another, justifies the argument of the pores by grounding it on the criterion of preconception, which forms the absolute basis for the dogmatists' attempt to transcend the phenomena.[49] It is because we read directly, from our preconceptions of solidity and liquidity, that a liquid cannot pass through a solid, that we are able to understand the phenomenon of the flowing of the sweat as a revelatory sign of the existence of pores in the skin, even though those pores are irreducibly invisible.

[47] I am not sure that the strange phrase *οὐχ οὕτως . . . ὡς . . .* should be translated like this; but I do not see any plausible alternative.

[48] I owe this point to Gisela Striker.

[49] On the 'canonical' role of *πρόληψις* see Goldschmidt [128]; Schofield, Chapter 11 below.

(4) Now that the series of dichotomies is at an end, we expect the author of *PH* to set out the definition of proof which they implicitly contain. Given our analysis of the preceding text, that definition should be formulated as follows: a proof is *an argument which is concludent and true, and which has a non-evident conclusion* (i.e. is probative). That definition is simply Definition *T*; and the account of *PH*, up to this point, can be considered a clear and coherent explication of it. But things become a little perplexed in Sextus' last paragraph (140), where he proceeds, as he did in *M*, to offer on his own account a recapitulation of the sort I have just made. I translate the whole of the paragraph:

> Thus a proof must be both an argument (*kai logos*), and concludent (*kai sunaktikos*), and true (*kai alēthēs*), and having a non-evident conclusion (*kai adēlon echōn sumperasma*) and [this word is omitted in one of our sources for the text[50]] revealed by the force of the premisses (*kai ekkaluptomenon*—or simply *ekkaluptomenon—hupo tēs dunameōs tōn lēmmatōn*). And that is why it is said that a proof is an argument which, from agreed premisses, reveals in a concludent fashion a conclusion which is non-evident.

The last sentence of that paragraph poses no problems: we recognize Definition *S*, which was announced at the start of the chapter and which the remainder of the chapter was designed to elucidate. But the first part of the paragraph causes difficulty. If we keep the text transmitted by the Greek manuscripts (i.e. including the '*kai*'), we must acknowledge that Sextus has extracted from the dichotomies a definition different from the one which they imply: they imply Definition *T*, as we have just seen; but Sextus seems to extract from them Definition *C*.

But that difficulty can be overcome with the help of the variant reading offered by one of our sources for the text. For we have seen that the conjunction 'and' (*kai*), which the recapitulation regularly places before each component of the definition, is exceptionally missing, in one of the authorities for the text, before '*ekkaluptomenon*'—i.e. before the very term which causes the difficulty. Now a variant of that sort standardly indicates that a gloss has probably been interpolated into the text—the syntactical

[50] Viz. the Latin translation of Sextus, preserved in the MS Parisinus Lat. 14700. Mutschmann dates this translation to the thirteenth century; he thinks that it represents an archetype independent of that of the Greek manuscripts, and he ascribes to it an importance equal to that of the manuscripts for the establishment of the text.

hiatus created by the interpolation being left empty by some copyists and filled up by others. In the circumstances I do not hesitate to suggest that we athetize the whole expression '*kai ekkaluptomenon hupo tēs dunameōs tōn lēmmatōn*'.[51] Thus emended, the text, I think, no longer presents any difficulty. The recapitulation gives Definition *T*, which is just what can be properly extracted from the dichotomies. Moreover, Sextus is right in thinking that the definition obtained in the recapitulation allows him to explain Definition *S*, which he goes on to quote; for even though the former no longer contains '*ekkaluptomenon*' (after the emendation I propose), whereas the latter contains '*ekkaluptōn*', nevertheless the property of having a non-evident conclusion, which is included in Definition *T*, implies, as we have seen, that the conclusion is 'revealed' in the broad sense of the term.

Having analysed and compared the two texts of *M* and *PH*, I shall now give a summary in the form of a table containing the different definitions of proof we have encountered, together with their constituent elements:

		logos	*sunaktikos*	*alēthēs*	*adēlon sumperasma*	*ekkaluptikos*
		argument	concludent	true	with non-evident conclusion	revelatory (narrow sense)
R	*M* VIII 301–9 (implicit in the dichotomies)	+	+	−	+	+
C	*M* VIII 310 (first recapitulation)	+	+	+	+	+
T	*M* VIII 314 (second recapitulation)	+	+	+	+	−
	PH II 137–40 (implicit in the dichotomies)					
	PH II 143 (emended text)					
	M VIII 411–24					
S	*M* VIII 314	+	+	?	+	?
	PH II 135		(*kata sunagōgēn*)	(*di' homologoumenōn lēmmatōn*)		(*ekkaluptōn*)
	PH II 143					

[51] Kayser and Mutschmann preserve the text as transmitted by the Latin translation, i.e. they keep ἐκκαλυπτόμενον . . . λημμάτων but do not print the καί; that

Now let us ask the following question: what are the most conservative hypotheses by means of which we can give an historical explanation of the complex state of affairs I have tried to describe? We can, I think, make a few plausible conjectures.

(1) Of the four distinct definitions of proof which our texts present, the oldest must be Definition *S*. Unlike all the others, its structure, as I have already noted, does not reflect a dichotomous division. It is hard to imagine that it was formulated on the basis of the dichotomies of *M* or of *PH* (*pace* Egli [41], 62); and it is very easy to imagine that those dichotomies were elaborated after the event, in order to justify and explain Definition *S*. Remember too, that in the *PH* version the chapter on proof opens by citing this definition, and presents a unitary attempt to elucidate it.

Moreover, we should note that the other definitions preserve, with hardly any disagreement, those elements of *S* which contain no ambiguity: that is true for the defining characteristics *argument*, *concludent*,[52] *having a non-evident conclusion*. On the other hand, the characteristics over which Definitions *R*, *C*, and *T* disagree are precisely those which *S* expresses in a relatively obscure manner. One might well have hesitated, for example, over the phrase '*di' homologoumenōn lēmmatōn*': did the author of *S* presuppose that a premiss is 'agreed to' if and only if it is true? According to your answer to that question, you will be led in one of two directions: either you will explicitly specify that the premisses of a proof, unlike those of certain arguments which are concludent but not probative, must be true (that is what *C* and *T* do); or else,

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is unconvincing, given the distinction I make above: it amounts to preserving the notion of revelation in the narrow sense (cf. *ὑπὸ τῆς δυνάμεως τῶν λημμάτων*) without giving it the status of an independent condition in the definition. My surgical solution is, despite appearances, less rash. Sextus' text has certainly been interlarded with interpolations of this sort, produced by the action of competent, or semi-competent, readers. Myles Burnyeat kindly drew my attention to two of them, whose author he wittily proposes to baptize 'Lector Sublogicus': *M* VII 243 (where the manuscripts give *ἢ ἀπιθάνων*, rightly rejected by von Arnim, Mutschmann, and Bury); and *M* VII 158 (where the manuscripts give *οὐ περὶ πάντων ἐπέχων*, which is generally and rightly corrected to *ὁ περὶ πάντων ἐπέχων*).

[52] *συνακτικός* is a correct gloss on *κατὰ συναγωγήν*, as *PH* II 170 shows (*κατὰ συναγωγήν, τουτέστι συνακτικῶς*).

presupposing that every 'agreed' premiss is true, you will omit the specification as being superfluous (that is what *R* does).

Again, you might wonder if the word '*ekkaluptōn*' in Definition *S* describes a relation which holds between an argument and its conclusion whenever that conclusion is non-evident; in that case you will hold that a non-evident conclusion can only be reached by 'revelation', and you will take that word in a broad sense. Or does the word rather describe a particular form of the relation, which must be considered as specifically characterizing the notion of proof? In that case you will hold that there are several ways of reaching a non-evident conclusion, only one of which can properly be called 'revelatory'; and you will take that word in its narrow sense. In the former case, there is no need explicitly to introduce the notion of revelatoriness into the definition of proof, since it is already implied by the non-evidence of the conclusion (that is what Definition *T* does); in the latter case you must put 'revelatoriness' into the definition as an independent feature (that is what *R* and *C* do). Thus the differences between *R*, *C*, and *T* have their roots in the least clear parts of *S*.

So we may posit the existence of an original author whose contribution to the mass of material we have studied consisted simply in supplying us with Definition *S*—he left to others the tasks of justifying that definition, and of elucidating its obscurities. Let us call this author Mr S. We can issue the following identikit picture of him: Mr S is a pioneer; he likes phrases that are well-turned and pregnant with meaning; he enjoys a prestige so great that his assiduously pious heirs will produce volumes of systematic and elucidatory commentary on his words.

(2) The earliest of the attempts to deduce Definition *S* by way of a series of dichotomies must have been that of *M* VIII 301–9, which implicitly contains Definition *R*. For we find there many indications that its author has not reached the same level of logical competence as the author of the *PH* version, who seems to have conscientiously corrected his mistakes. The author of the *M* dichotomies defines the notion of a premiss so as to exclude the possibility of arguing validly from false premisses; consequently, he does not feel the need to distinguish between a class of concludent arguments with true premisses and a class of concludent arguments with false premisses. He defines concludency in such a way that psychological clarity appears as a

component of logical validity. Having introduced a fairly complex distinction between 'merely progressive' arguments and arguments which are 'at the same time progressive and revelatory', he seemed to us incapable of giving an exact description of their difference: failing to grasp that the logical structure of the two examples he analyses is the same, he confuses an epistemological difference, which bears on the type of truth possessed by one of the premisses, with a logical difference, which bears on the type of relation between premisses and conclusion. I attempted to show that in making this distinction he tried to adapt to the theory of proof a distinction which had its origin in the theory of signs; and I ventured to suggest that he wrongly interpreted the notion of 'faith' in a religious sense.

For this second character in our story, whom I shall call Mr R, we can construct the following identikit: Mr R. is a zealous commentator on Mr S; a fairly mediocre logician; a man preoccupied with religious matters to the point of giving a religious twist to notions which originally had no such connotation.

(3) The *PH* version of the dichotomies, and Definition *T* which it implies (and which it explicitly states, if my textual suggestion for 143 is accepted), are chapters of our story due to the intervention of a third character, Mr T. Mr T took up again the task undertaken by Mr R; but he possessed resources of an incomparably superior kind to devote to the service of Mr S. Mr T can, I think, be given a later date than Mr R; for some of the differences we see between the work of the two men can only be properly explained if we allow that Mr T knew the work of Mr R, approved of it in principle, and used his own talents as an accomplished logician to improve upon it.

We noted, you will remember, the following modifications: Mr T adopts a new definition of the notion of a premiss, which dissipates Mr R's confusion between an 'agreed' and a true proposition; he distinguishes clearly between concludent arguments with true premisses and concludent arguments with false premisses, and he thereby stresses that the logical validity of an inference is independent of the truth of its constituent propositions; he formulates a criterion of concludency that is admirable in its rigour. He has enough respect for Mr R not to sacrifice the latter's distinction between 'merely progressive' arguments and 'progressive and revelatory' arguments, even

though, in his view, the distinction no longer bears upon the definition of proof as such.[53]

For these reasons (if we may continue our little game of identikit portraiture) we shall describe Mr T as follows: he was a logician of genius, no less attached than Mr R to the memory of Mr S; he undertook the same task as Mr R, but with distinctly superior resources; he had sufficient respect for Mr R to try to preserve whatever could be preserved of his work, but he did not shrink from correcting his errors with consummate self-assurance—and sometimes, perhaps, with a touch of irony.

(4) We had four definitions of proof; and thus far only three characters have been introduced into our hypothetical history of their origins. But three characters are enough: now that we have found fathers for definitions *S*, *R*, and *T*, only *C* remains parentless; but we need not look further than Sextus himself to file our fourth paternity suit. For Definition *C* was put forward as a recapitulation, probably composed by Sextus himself, of the series of dichotomies in *M* VIII 301–9, i.e. of the series composed by Mr R; but instead of extracting from that series Definition *R*, which it implies, Sextus proposed a definition which preserved all the characteristics of *R* but added to them the notion of truth, and he thus obtained Definition *C*; that addition then obliged him to produce some complementary commentary. Now the notion of truth is the mark of Definition *T*, which Sextus knew, and which he had in front of him not only when he wrote up the *PH* version but also when he wrote up the *M* version—for he cited it there (to the reader's surprise) in what I have called the second recapitulation (314). Thus it is highly probable that Sextus himself completed *R* by adding to it the notion of truth which he borrowed from *T*. Definition *C*, then, will simply be a hybrid produced by Sextus from his two originals; and it will be the only one of our definitions which is not properly Stoic.

(5) My investigations lead me, finally, to a conclusion which I neither foresaw nor desired: plainly, Mr S is very like Zeno (he is the founder of a school, and he has a taste for pithy expression—cf. DL VII 18, 20, 23, 24); Mr R makes us think of Cleanthes (he lacks intellectual finesse, he is interested in religious affairs, he is devoted to his predecessor's work—cf. DL VII 37, 170, 174;

[53] That will be why this dichotomy is retained in the *PH* version (cf. above, p. 152 n. 46).

and, of course, the *Hymn to Zeus*); and Mr T summons up Chrysippus (he is a brilliant logician; and he adopts towards his immediate predecessor a complex attitude compounded of respect, impatience, and mild irony[54]). Those similarities of character and circumstance, however, are not enough in themselves to warrant the identification of the three masters of the Old Stoa with the three logicians who seem to have turned their minds to the problem of the definition of proof. But a small detail gives substance to our hypothesis. We have seen that Mr T was the first to use the criterion of preconception to underpin the argument of the pores—the example of a revelatory probative argument. Now we know from Diogenes Laertius (VII 54) that Chrysippus added preconception to the criteria of truth. And if Chrysippus is Mr T, who are Mr S and Mr R if not Zeno and Cleanthes, his two predecessors at the head of the school? But enough of such speculation: for further information, please apply to your local clairvoyant.[55]

[54] Cf. DL VII 179: Chrysippus often said to Cleanthes that Cleanthes need only teach him the doctrines and he would find the *proofs* for himself; clearly, then, he thought himself better equipped than his master from that point of view.

[55] This is a revised version of the paper delivered at the Conference at Oriel College; I have been greatly benefited by remarks made by several participants, in particular by Gisela Striker, Myles Burnyeat, Michael Frede, Geoffrey Lloyd, and especially (both during and after the Conference) by Jonathan Barnes. I thank them all; and I also thank Jennifer Barnes for kindly undertaking the English translation.

PROOF DESTROYED

Jonathan Barnes

I

Sceptical philosophers customarily attack both perception and ratiocination; they question the salubrity of the sources from which we hope to draw our knowledge, and they cast doubt upon the reasonings whereby we expect to purify, canalize, and extend our understanding of the world. Scepticism of the reason is less discussed than scepticism of the senses, but it is no less menacing.

Rustic sceptics, who pretend to doubt everything, may, as Aristotle thought, deserve punishment rather than counter-argument; but urbane sceptics, whose doubt is both systematic and circumscribed, pose a sensible threat to our intellectual aspirations. And when the urbane sceptic turns his attention to matters of reason, one of his prime targets is the science of formal logic. John Locke, that prince of sceptical urbanity, is nowhere more vigorous than in his attack on syllogistic (which for him constituted the sum total of formal logic): the syllogism, he alleges, is not 'the great Instrument of Reason' (*Essay* IV xvii 4, p. 670, 34 Nidditch); formal logic fuddles our native wits and obfuscates the clear lines of natural deduction; 'to an ingenuous Searcher after Truth, who has no other aim, but to find it, there is no need of any such Form' (ibid. 675, 6); prudent scientists will ignore the artifices of the Schoolmen, and construct instead 'a Chain of *Ideas* . . . visibly link'd together in train' (ibid. 673, 19). The Lockean view, which in truth was largely cribbed from Descartes,[1] has never lacked support.

Lockean sceptics who doubt the utility of formal logic will train their guns on the theory of formal proof or demonstration. For the citadel of proof stands at the confluence of the two great streams of logic and epistemology; and it is therefore a natural object for sceptical siege. So, at least, it was for the ancient

[1] See J. A. Passmore, 'Descartes, the British Empiricists, and Formal Logic', *Philosophical Review* 62 (1953), 545–53; on Locke's attitude to logic see esp. W. S. Howell, *Eighteenth Century British Logic and Rhetoric* (Princeton, 1971), 264–98.

sceptics. And the major part of this chapter will narrate one clever manoeuvre in their campaign. Having scrutinized the attacking force, I shall turn briefly to the defenders and ask what damage and what losses they suffered from the sceptical investment. But first let me set down a few facts about the campaign as a whole.

Aristotle, whose *Posterior Analytics* is the first essay on the theory of proof, took part in a dispute over the nature, and the very possibility, of formal demonstration. Galen, the most passionate advocate among the ancients of the demonstrative art, tells us that his contemporaries were still engaged in the same debate.[2] And we know from the evidence of Sextus Empiricus that the centuries between Aristotle and Galen witnessed similar disputations.

The sceptical philosophers of the Hellenistic age gave the theory of proof a sound drubbing: there is no such thing as a demonstrative argument, they said; and if there were, it would be of no utility. Sextus patches together a lengthy account of their reasoning at *PH* II 134–92, and again at *M* VIII 300–481 (see also *PH* I 60, 122–3; II 113–14; *M* II 106–12). His immediate sources will doubtless have included Agrippa, the Pyrrhonian, whose arguments against proof are summarized by Diogenes Laertius (IX 88–91), and Clitomachus, the star pupil of Carneades, whom we know to have written *Refutations of Proof* and who was well versed in Peripatetic and Stoic philosophy.[3]

According to tradition, the sceptics were not the only sect who scorned proof: Epicurus despised logic and the paradigmatically demonstrative science of geometry, and he managed to corrupt the mathematician Polyaenus into sharing his view; and a later Epicurean, Zeno of Sidon, conducted an assault on Euclid. But Epicurus was himself no logical ignoramus; and Zeno may have argued, not that Euclidean proof is a mug's game, but rather that Euclid himself did not show sufficient logical rigour. In any

[2] For Aristotle see *APo.* A 3 (cf. J. Barnes, 'Aristotle, Menaechmus and Circular Proof', *Classical Quarterly* 26 (1976), 278–92); for Galen see *Ord. lib. prop.* XIX 52 K= *Scripta Minora* II 82, 3–10 (there is a list of Galen's writings on proof in *Lib. prop.* XIX 39–45 K=*Scr. Min.* II 115, 19–121, 4; the Galenian material on proof is collected and discussed by Müller [137]; see also Egli [41], 90–1).

[3] Several scholars have speculated on Sextus' immediate sources (see esp. Natorp [51], 258–64; dal Pra [27], 185–7, 385–7; Egli [41], 48–50, 61–73); none is wholly convincing, and the matter awaits a detailed examination of the whole Sextan context.

event, Epicureanism had its logicians: Sextus reports an anonymous Epicurean defence of demonstration (*M* VIII 337–336a); and he rehearses an argument in favour of the possibility of proof which he ascribes to Demetrius of Laconia (*M* VIII 348–66). Demetrius is known from other sources to have taken a logical interest in the theory of signs, and to have written at some length on the subject of geometry.[4]

But it was the Stoics in whom the theory of proof found its Hellenistic champions. Like Aristotle, the Stoics were gifted logicians; unlike Aristotle, they were profoundly concerned with epistemological issues: it is only to be expected that they should have devoted some thought to the nature of formal proof. In their view, we are told, 'the matter of logic is argument; but its aim is knowledge of demonstrative procedures—for everything else culminates in this, viz. the giving of scientific demonstrations' (Ammonius, *in APr.* 9.26–9). And 'they say that they embarked on the art of logic not simply in order to know what follows from what, but in particular in order to know how to distinguish truths and falsehoods by means of demonstrative arguments' (*PH* II 247).

Sextus reports that 'the Stoics seem to have given particular precision to the forms of demonstration' (*M* VIII 396); he purveys the complex Stoic analyses of the concept of proof; and he indicates that the Stoics made some attempt to repel the sceptical attack on demonstrative argument (*PH* II 185–6; *M* VIII 463–9). 'All things', according to the Stoics, 'are seen by consideration through arguments' (DL VII 83); and demonstration 'contributes greatly to the correcting of our beliefs' (DL VII 45). Those general remarks are given particular content in several accounts of Stoic doctrine—Cicero's Stoic texts, for example, are marked by a special attention to demonstrative rigour.

If we turn to individual Stoics, the documentation is less rich. At least four of Chrysippus' numerous books were, to judge from

[4] Epicurus 'totam dialecticam et contemnit et irridet' (*Acad.* II 97); and the Epicureans *τὴν διαλεκτικὴν ὡς παρέλκουσαν ἀποδόκιμαζουσι* (DL X 31—note the verb *παρέλκειν*; cf. Cic. *Fin.* I 22); Polyaenus 'qui magnus mathematicus fuisse dicitur . . . Epicuro adsentiens totam geometriam falsam esse credidit' (Cic. *Acad.* II 106; cf. *Fin.* I 20); on Zeno see Proclus, *in Euc.* 214–18; for Demetrius on signs see Philodemus, *Sign.* XXVIII 13–XXIX 15, on geometry see the papyri in V. de Falco, *L'epicureo Demetrio Lacone* (Naples, 1923). For Epicurus, see Long [20], 29–30; for Zeno see Vlastos [125]. Further details, references, and bibliography in Sedley [132], 23–6.

their titles, collections of demonstrations (DL VII 197, 201, 202); and Diogenes of Babylon 'says that music is useful with regard to knowledge; for there are very many definitions and divisions and demonstrations in the science of harmony' (Philodemus, *Mus.* 89, 24–7 = *SVF* III Diog. 87). For the other older Stoics there is no explicit evidence.

Matters are different in the case of Posidonius. He, according to Galen, 'was brought up on geometry and was more accustomed than the other Stoics to follow out proofs' (*Plac.Hipp.Plat.* V 390 K = 362.5 M = T 83 Edelstein–Kidd); and Strabo calls him 'demonstrative'—in the logical sense (II 3.5 = T 46 E–K). Posidonius undertook to defend Euclid against Zeno of Sidon (Proclus, *in Euc.* 199, 3–200, 6 = F 46 E–K): his researches led him to develop a branch of the logic of relations, and to reflect upon the proper canons of demonstrative rigour (Galen, *inst.log.* XVIII = F 191 E–K). Moreover, he was a polymath, who attempted to supply proofs in the various sciences he studied (e.g. Simplicius, *in Phys.* 291. 21–292. 31 = F 18 E–K); and his concern to give explanations of things fits well with that probative purpose (Strabo, II 3.8 = T 85 E–K). Admittedly, Posidonius was something of an eclectic—Strabo calls him an 'Aristotelizer' (ibid.)—but there is little reason to doubt that his interest in proof was a part of his Stoic no less than of his Aristotelian heritage.[5]

The theory of proof was a talking-point and a matter of controversy in the Hellenistic schools. To what extent the history of that controversy can be reconstructed I do not know; and I shall imitate Sextus' usual practice by speaking impersonally of 'the sceptics' and 'the Stoics': my interest is in the substance rather than in the historical course of the Hellenistic disputes.

II

A: LOGICAL REDUNDANCY

Sextus offers Stoic accounts of proof in three separate passages (*PH* II 135–43; *M* VIII 301–15, 411–23), from which it emerges that a demonstration is an argument (*logos*) which is concludent

[5] On the matter of this paragraph see esp. Kidd [97].

(*sunaktikos*) and true (*alēthēs*), which has a non-evident (*adēlon*) conclusion, and which reveals (*ekkaluptein*) that conclusion. The analysis calls for a few brief comments.[6]

An argument or *logos* is 'a system composed of premisses and a conclusion' (*PH* II 135); and we may represent a Stoic *logos* as an ordered pair, $<\pi, \sigma>$, where π is a set of propositions, $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ (the putative premisses), and σ is a single proposition (the putative conclusion). An argument is *sunaktikos* if 'the conclusion follows the conjunction of the premisses' (*M* VIII 415); i.e. $<\pi, \sigma>$ is concludent if σ follows from the conjunction of the members of π. (I shall return to concludency shortly.) And a concludent argument is true if 'both the conclusion and the conjunction of its premisses are true' (*PH* II 138).

A true argument is only a demonstration if its conclusion, σ, is something 'non-evident' or *adēlon* (e.g. *M* VIII 305), and if it is 'revelatory' or *ekkaluptikos* of that conclusion.[7] A demonstration is thus an *illuminating* argument: its conclusion is a dim truth which its torch lights up for the eye of knowledge to see. But to say that is only to exchange one metaphor for another; and I shall have to return later to the matter of revelation or illumination.

So much for the Stoic analysis: let us now turn to the sceptical attack. At *M* VIII 429–447 Sextus adverts to 'the formal theory of conclusive and non-conconclusive arguments': 'conclusive (*perainōn*)' is synonymous with '*sunaktikos*', and Sextus is about to sketch a theory which classifies non-concludent arguments or fallacies. The theory is explicitly ascribed to the Stoics (435) who 'say that non-conclusive arguments come about in four ways: either in virtue of disconnectedness, or in virtue of redundancy,

[6] The Sextan passages have been analysed in detail by Brunschwig in the preceding Chapter. He discovers *four* distinct definitions of proof in them; but all four contain all and only the elements of what, in the text, I loosely call 'the' Stoic analysis. (Brunschwig's Definitions R and S do not *explicitly* contain the notion of truth; but they both implicitly assume that a demonstration has true premisses—and that is enough for my purposes. Brunschwig's *T* contains the notion of revelation, but only in a weak form: I shall return to that point later.) For the major part of my paper, what I say, generally, about 'the' Stoic concept of proof is compatible with everything in Brunschwig's subtle analysis. I may be allowed to add that my understanding of all these issues has been greatly influenced and improved by many discussions with Jacques Brunschwig.

[7] Revelation is a crucial feature of the Stoic notion of proof: see *PH* II 131, 134, 135, 143, 177, 178; *M* VIII 140, 277, 299, 310, 314, 422, 423. In the first of the two definitions at *M* VIII 314, the word '*ἐκκαλυπτικός*' does not appear; but the definition contains the synonymous '*παραστατικός*' (see *PH* II 178~*M* VIII 392).

or in virtue of being propounded in an incorrect form, or in virtue of a deficiency' (429).

Sextus proceeds to explain the four types of fallacy (430–4), and he then employs each in turn to attack the theory of proof. The second type of fallacy appears on the stage at 438–43: the sceptics argue that the first of the five Stoic 'indemonstrables' is guilty of redundancy; that it is therefore non-concludent; and that no demonstration can properly be couched in its form.

At *PH* II 146–67 the same material is more generously reproduced: 146–51 recount the fourfold typology of fallacy, which is now ascribed to 'the Dialecticians'; and 156–67 consider redundancy. Here Sextus argues that redundancy infects more than the first indemonstrable—it infects all five of the indemonstrables, and also the 'perfect' syllogisms of the Peripatetics. Since 'the Dialecticians lay these down as the foundation of all inferences' (166), it will follow that all formal arguments are guilty of redundancy, that none are concludent—and hence that none can be used to formulate proofs.

The Argument from Redundancy is thus very simple: all formal inferences are redundant; no redundant arguments are concludent; hence no formal inferences are concludent, and no proofs can be expressed by way of formal logic.

Sextus' word for redundancy is '*parolkē*', and the verb '*parelkein*' means 'be redundant'. The term is not restricted to logical contexts but has a perfectly general sense: Apollonius Dyscolus, for example, frequently uses it in a semantic context (observing, say, that the prefix '*en-*' in '*enantios*' and '*enanchios*' *parelkei*; for those adjectives are respectively synonymous with '*antios*' and '*anchios*', and in each case the prefix is semantically idle (*Adv.* 183.25–184.2)).[8]

Logically speaking, an argument is said to be non-concludent 'by virtue of redundancy' if one or more of its premisses are redundant; that is to say, if a premiss is logicallv idle, or makes

[8] For other occurrences of '*παρέλκειν*' see e.g. A. D. *Pron.* 3.6; 27.19; 38.3; Alex. Aphr. *in Top.* 428.20; 430.21; 431.1; *PH* II 77, 255; III 265; cf. Stephanus, *Thesaurus Linguae Graecae*, s.v.—'*παρέλκειν*' contrasts with '*συμβάλλειν*' (*PH* II 175), '*συντείνειν*' (A.D. *Synt.* I.4), '*εὐχρηστεῖν*' (*M* I 209); '*παρέλκων*' with '*ἀναγκαῖος*' (A.D. *Synt.* I.3), '*χρηστός*' (Alex. Aphr. *in APr.* 278.29). One of the literal senses of '*παρέλκειν*' is 'draw out'; hence 'spin out' or 'be prolix'; from which it is a short step to 'be redundant'. In another sense, a prudent rider who takes a spare mount with him may be said *παρέλκειν* it (see Hdt. II 102; Suid. s.v. *ἄμιπποι*); but it is, I fear, a little fanciful to see an equestrian metaphor in our use of the word.

no contribution to the inference. Sextus gives the following example:

(A) (1) If it is day, it is light
(2) It is day
(3) Virtue is beneficial
Therefore: (4) It is light

'For that virtue is beneficial is superfluously assumed along with the other premisses; since it is possible to exclude it and to draw the conclusion, "It is light", from the two remaining premisses, "If it is day, it is light", and "It is day" ' (*M* VIII 431; cf. 439). In short, proposition (3) is redundant in (A) because the *logos* $<\{(1), (2)\}, (4)>$ is concludent; and (A) is a redundant argument because premiss (3) is redundant in it.

The general idea of logical redundancy is not difficult to grasp; and those with a penchant for precision may be encouraged by Sextus' illustration to offer the following formal definition:
an argument, $<\pi, \sigma>$, *is redundant* iff there is some α_i in π which is redundant in $<\pi, \sigma>$; and *a member of* π, α_i, *is redundant in an argument* $<\pi, \sigma>$ iff there is a proper subset, ρ, of π, not containing α_i, such that $<\rho, \sigma>$ is concludent.

Now our Sceptical Argument from Redundancy assumes that no redundant argument is concludent; indeed, it claims to borrow that assumption from the Stoics themselves. To most modern logicians the assumption will seem outrageous: redundancy is inelegant and aesthetically displeasing, but it does not amount to a *logical* fault. On the contrary, all redundant arguments are, logically speaking, impeccable; for all such arguments are in fact concludent. If ρ is a proper subset of π, and $<\rho, \sigma>$ is concludent, then $<\pi, \sigma>$ is concludent; if σ follows from ρ, then it follows from π—for π is simply ρ with an addition or two, and the addition of premisses to a concludent argument cannot produce a non-concludent argument.

Thus the sceptical assumption that redundant arguments are inconcludent is totally false; and its falsity is so obvious and so elementary that we should hesitate to ascribe it to the subtle logicians of the Porch.[9]

[9] 'By all the usual tests these [redundant arguments] would be perfectly valid arguments, though inelegant. Perhaps Sextus made a mistake here, or perhaps he was following an inferior handbook' (Mates [44], 83); Egli [41], 48–52, develops that view in detail; there is a more sympathetic account in C. L. Hamblin, *Fallacies* (London, 1970), 92–3.

B: REDUNDANCY AND CONCLUDENCY

That depressing conclusion has been reached too hastily; and the sceptics deserve a better hearing. Let us begin by looking briefly at Aristotle; for Sextus, in *PH* at least, implicitly ascribes the assumption that redundancy breeds inconcludency to the Peripatetics as well as to the Stoics.

The term '*sullogismos*' is the Aristotelian counterpart to the Stoics' '*logos sunaktikos*'. According to Aristotle, a *sullogismos* is a *logos* in which 'certain things being posited, something other than the things posited follows of necessity by their being the case' (*APr.* A 1 24^b 18–20; cf. *Rh.* A 2 1356^b 16–17). The phrase 'by their being the case (*tōi tauta einai*)' is glossed as 'because of them': if $<\pi, \sigma>$ is a *sullogismos* then if the members of π are true, σ follows *by virtue of their truth*. Now if σ holds because of ρ, and if ρ is a proper subset of π, it is plausible to infer that σ does *not* hold because of π; if σ holds in virtue of ρ alone, then it does not hold in virtue of π. Of course, if π's members are all true, then σ is true; but σ will not follow *by virtue of* the truth of π's members, but rather by virtue of the truth of a proper subset of π's members.

It follows that if $<\rho, \sigma>$ is a *sullogismos*, then $<\pi, \sigma>$ is not a *sullogismos*; that is to say, it follows that a redundant argument cannot be an Aristotelian *sullogismos*.

That interpretation of Aristotle may seem pedantic to the point of madness; and it is not suggested by the *Prior Analytics*' gloss on the phrase 'because of them'.[10] But it was the interpretation of no less a commentator than Alexander of Aphrodisias, who remarks that 'by this additional phrase [sc. 'by their being the case'] arguments which contain a redundant premiss are ruled out' (*in APr.* 22.30–23.2; cf. *in Top.* 13.25–14.2; 432.2–3; 568.18–23).

Moreover, the interpretation has Aristotle's own authority behind it. In *Topics Θ* 11 he asserts that 'there are five ways of criticizing an argument in itself . . . Again, <you may criticize an argument> on the grounds that a *sullogismos* would come about even if some of the premisses were removed—for sometimes people assume more than the necessary premisses, so that

[10] 'To follow "because of them" is for there to be no need of any external term in order for the necessity to come about' (*APr.* A 1 24b 21–2).

the *sullogismos* does not come about by their being the case (*tōi tauta einai*)' (161^{b} 28–30). A little later there is an illustration of this deductive fault, after which Aristotle comments: 'What is the trouble? Is it that it makes obscure the cause on which the argument depends?' (162^{a} 32–4).

Suppose that $<\rho, \sigma>$ is a *sullogismos*, in which ρ contains 'the necessary premisses'; and imagine that a man assumes more than those premisses and tries to infer σ by way of the argument $<\pi, \sigma>$. Aristotle says that we should criticize $<\pi, \sigma>$ on the grounds that 'the *sullogismos* does not come about by their being the case', or that 'it makes obscure the cause on which the argument depends'. The context of the criticism is dialectical, but its content comes from the official definition of a *sullogismos*: Aristotle is plainly saying that a redundant argument, $<\pi, \sigma>$, offends against the *tōi tauta einai* condition; and he must be taken to imply that no redundant arguments are *sullogismoi*.

Aristotle thus held that redundancy was a logical flaw, in that it debars an argument from the title of *sullogismos*; and the later Peripatetics did not dispute their master's view. The sceptics were therefore right to ascribe to the Peripatetics the assumption that a *sullogismos* cannot be redundant. Were they also right in ascribing the corresponding assumption to the Stoics? To answer that question we must look more nearly at the Stoic notion of concludency.

According to Sextus, a Stoic argument is concludent iff 'the conditional having as antecedent the conjunction of the premisses and as consequent the conclusion is sound (*hugies*)' (*PH* II 137). Let '*p*' express the conjunction of the members of π, and let '*q*' express σ; then $<\pi, \sigma>$ is concludent iff 'if *p*, then *q*' is sound. The word 'sound' may here be replaced by 'true';[11] so that $<\pi, \sigma>$ is concludent iff 'if *p*, then *q*' is true.

But when is a Stoic conditional true? Sextus records four ancient analyses of the conditional (*PH* II 110–11; cf. *M* VIII 112–18). The third account is stated thus: 'Those who introduce connectedness (*sunartēsis*) say that a conditional is sound whenever

[11] In logical contexts '*hugiēs* (sound)' is contrasted with '*sathrós*' (*PH* II 42), '*mochthērós*' (ibid. 105), '*phaûlos*' (ibid. 150), '*pseudḗs*' (ibid. 200). It is often coupled with '*alēthḗs*' *PH* II 42), and Sextus uses '*hugiḗs*' and '*alēthḗs*' interchangeably when talking of conditionals (*PH* II 110; *M* VIII 417, 426). Of course, '*hugiḗs*' does not mean 'true' (Sextus explains it by way of reliability: a conditional is *hugiḗs* if it 'preserves the consequence': *M* VIII 112; cf. VII 78); but 'If *p*, then *q*' is *hugiḗs* iff it is true.

the contradictory of the consequent conflicts (*machētai*) with its antecedent' (*PH* II 111). Diogenes Laertius expressly ascribes that analysis to the Stoics (VII 73); and there is good reason, of an inferential kind, to attribute it to Chrysippus in particular (see Cic. *Fat.* VI 12). It may well be that some Stoics flirted with rival analyses of the conditional; but it is beyond serious doubt that Sextus' third account was official Stoic doctrine.[12]

Alas, that account is not pellucid: in particular, the two notions of 'connectedness' and 'conflict' cry out for elucidation. As for conflict, various sources tell us that two propositions conflict if they are incompatible (e.g. Apollonius, *Conj.* 218, 22–3); but that is hardly helpful. A compressed text in Alexander promises better: 'A thing is consequential (*akolouthon*) if it is necessary for it to be the case by something else's being the case (*tōi heteron einai*); a thing is conflicting (*machomenon*) if it is necessary for it not to be so' (*in Top.* 93, 10). Alexander's '*tōi heteron einai*' is to be taken in the same way as Aristotle's '*tōi tauta einai*'; and his comment may be paraphrased as follows: 'One proposition, σ, is consequential <upon another, σ',> iff <if σ' holds then> necessarily σ holds because σ' holds; and one proposition, σ, is conflicting <with another, σ',> iff <if σ' holds then> necessarily σ fails to hold <because σ' holds>'. Thus σ and σ' conflict iff, assuming that σ' holds, σ fails to hold because σ' holds. Alexander does not attribute that account of conflict to the Stoics; but '*machesthai*' is not a term of Peripatetic logic, and it is not implausible to suppose that Alexander is transcribing a Stoic view.

Thus 'If *p*, then *q*' is true iff π and the negation of σ conflict, i.e. iff if *p*, then not-not-*q* because it is the case that *p*, i.e. iff if *p*, then *q* because *p*. That harmonizes reasonably well with Sextus' account of Stoic concludency in *M* VIII 302 and 304, if we overlook the psychologism which insinuates itself into those texts.

For information on connectedness or *sunartēsis* we might turn to a passage in Philodemus' tract *On Signs*. The question there at issue is how we can establish that 'this is connected (*sunērtēsthai*) to that by necessity' (XXXV 5); and it emerges that mortality

[12] Full references, discussion, and bibliography in Frede [42], 80–93. DL reports that, according to the Stoics, arguments are inconclusive 'if the contradictory of the conclusion does not conflict with the conjunction of the premisses' (VII 77): that fits exactly with Sextus' account of Stoic concludency provided that we assume the συνάρτησις analysis of the conditional.

say, is 'connected' to man only if 'man, according as (*katho*) and in so far as (*hēi*) he is man, is mortal' (ibid. 15).[13] Connectedness here is a relation between properties: *F*-ness is connected to *G*-ness iff anything *G* is *F* in so far as it is *G*. But Philodemus' account is easily adapted to the case of propositions: σ is connected to π iff if the members of π are true, then σ holds in virtue of their truth, i.e. iff if *p*, then *q* because *p*.

And now Stoic concludency has taken on a Peripatetic aspect: $<\pi, \sigma>$ is an Aristotelian *sullogismos* only if σ holds in virtue of the truth of π; and $<\pi, \sigma>$ is a concludent argument for the Stoics only if 'If *p*, then *q*' is true, and hence only if σ holds in virtue of the truth of π. Redundant arguments cannot be Peripatetic *sullogismoi*; no more can they be Stoic *logoi sunaktikoi*. Thus the Stoics did indeed hold, as the sceptics allege, that no redundant arguments are concludent.[14]

But why should any self-respecting logician suppose that if $<\pi, \sigma>$ is a concludent argument, then σ must hold in virtue of the truth of π? And what, in any case, is the force of 'in virtue of' in such a supposition?

A further passage in Sextus may help. The first of his four types of fallacy happens 'by virtue of disconnectedness (*diartēsis*)'; and Sextus says, by way of explanation, that disconnectedness occurs

> when the premisses have no communality (*koinōnia*) or connectedness (*sunartēsis*) with one another and with the conclusion; e.g. in the argument 'If it is light, it is day; but corn is being sold in the market: therefore it is light.' For we see that in this argument 'If it is day it is light' has no agreement (*sumpnoia*) or connection (*sumplokē*) with 'Corn is being sold in the market', nor does either of them with 'It is light'; but each is disconnected from the others. (*M* VIII 430)

That text contains some puzzling features;[15] but it shows that

[13] Philodemus later adds '*παρό*' to '*καθό*' and '*ᾗ*' (XXXII 33); and he offers four different interpretations of those three terms (XXXII 33–XXXIV 24). But he explicitly says that the Stoics failed to discern the finer distinctions he himself makes.

[14] Sextus' ultimate source was perhaps Chrysippus' treatise *On Redundant Arguments* (DL VII 195).

[15] But surely '*εἰ ἡμέρα ἔστι, φῶς ἔστιν*' has some *κοινωνία* with '*φῶς ἔστιν*'? The parallel text in *PH* gives a different illustration: *εἰ ἡμέρα ἔστι, φῶς ἔστιν · ἀλλὰ μὴν πυροὶ ἐν ἀγορᾷ πωλοῦνται · Δίων ἄρα περιπατεῖ* (II 146). There each proposition plainly lacks *κοινωνία* with *each* of the other two (*PH* simply says that they lack *ἀκολουθία*, and fastidiously avoids the rich vocabulary of *M*). It is tempting to

*dis*connectedness is a matter of lack of communality, or irrelevance; and it thus suggests that connectedness is a matter of communality or relevance. In that case, it will be a necessary condition for the truth of an implication that its antecedent has something in common with, or is relevant to, its consequent; and we can give at least some account of 'in virtue of': σ will hold in virtue of π only if π (i.e. only if each α_i in π) is relevant to σ.

That view is not obviously silly. Consider again argument (A). If I produce that argument, claiming to infer (4) from (1)–(3), you may well demur: I can infer (4) from (1) and (2); but proposition (3) has nothing whatever to do with the inference—it is otiose, inert, logically idle. If (1)–(3) are all true, then so too is (4); but it does not follow that I can infer (4) from (1), (2), *and* (3): the inferential relation holds only between (4) and (1)–(2).

Some modern logicians have attempted to systematize the thoughts vaguely expressed in the last paragraph: they hold that σ' entails σ only if σ' and σ are tied together by a bond of relevance, and that an inference is valid only if its premisses are relevant to its conclusion. I suggest that Aristotle and the Stoics anticipated that theory: their logic is at bottom a 'logic of relevance'; and their objection to redundant arguments is based upon the notion, however imperfectly grasped, of 'relevant implication'.[16]

In the major part of this section I argued laboriously that the

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introduce the appropriate *PH* example into *M* by emending the text; and *M* certainly is corrupt here (for we must read: οὔτε τὸ εἰ ἡμέρα ‹ἔστι, φῶς› ἔστιν ἔχει . . .). But *M*'s bad example is repeated at 435. Perhaps Sextus used a poor source for *M*; or perhaps his copy of his source was already textually corrupt.

[16] But consider this argument:

(1) Chrysippus was a Hellenistic philosopher
(2) Epicurus was a Hellenistic philosopher
Therefore: (3) There was at least one Hellenistic philosopher

On the one hand, the argument seems to be redundant; for $<\{(1)\}, (3)>$ is surely concludent. On the other hand, neither (1) nor (2) is irrelevant to (3)—(3) holds *both* in virtue of (1) *and* in virtue of (2).

To escape from that difficulty, we need to modify our earlier definition of redundancy: a redundant premiss is not one which merely *happens* to be superfluous to the argument, but one which *cannot* pull any logical weight or lend its force to the inference. Christopher Kirwan has suggested the following definition:

α_i is redundant in $\langle\pi, \sigma\rangle$ iff α_i is a member of π and α_i is a member of no minimal adequate subset in $\langle\pi, \sigma\rangle$

(ρ is a minimal adequate subset in $\langle\pi, \sigma\rangle$ iff ρ is a subset of π, and ρ strictly implies σ, and no proper subset of ρ strictly implies σ).—For a comprehensive account of modern studies in the 'logic of relevance' see A. R. Anderson and N. D. Belnap, *Entailment* (Princeton, 1975); they briefly examine Stoic συνάρτησις at pp. 435–52.

Stoics (and the Peripatetics) did indeed regard redundant arguments as non-concludent; and that their view of redundancy rests on a central feature of their notion of concludency. I have just suggested that there are connections between the ancient notion of concludency and certain modern views about 'relevant implication'. I do not claim that the Stoics were right (and in any case, my interpretation of their view is sketchy); but I do claim that their attitude to redundancy is worth taking seriously, both for philosophical and for historical reasons. For we cannot hope to understand the fundamental ideas of logic if we merely ignore such things as relevant implication; and we shall not comprehend any ancient system of logic unless we reconstruct it upon the implicational foundations laid down by its original builders.[17]

C: REDUNDANCY AND FORMAL LOGIC

So much for one premiss of the Argument from Redundancy. I now move to the other premiss, which states that all formal arguments are redundant. I shall restrict myself to the first of the Stoic 'indemonstrables', as Sextus does in *M* VIII: like Sextus, I assume that the first indemonstrable is of the form $<\{\ulcorner$ if σ', then $\sigma \urcorner, \sigma'\}, \sigma>$, and I use the hackneyed example:

(B) (1) If it is day, it is light
(2) It is day
Therefore: (3) It is light

According to the sceptics, argument (B) is redundant; and it is redundant because the informal argument:

(C) (1) It is day
Therefore: (2) It is light

is concludent. For (C) stands to (B) as $<\rho, \sigma>$ to $<\pi, \sigma>$. Here are Sextus' words:

These, then, are the celebrated indemonstrables; and they all seem to me to be nonconcludent by virtue of redundancy. For instance—to begin with the first—either it is agreed that 'It is light' follows 'It is day', which is its antecedent in the conditional 'If it is day, it is light', or it is unclear. But if it is unclear, we shall not grant the conditional as agreed; but if it is pre-evident that, given 'It is day', of necessity 'It

[17] For some discussion of ancient concludency see Frede [88], 6–10.

is light' is the case, then if we say that it is day, it is inferred that it is light—so that an argument of the sort 'It is day; therefore it is light' suffices, and the conditional 'If it is day, it is light' is redundant (*PH* II 159).

For they will say either that its being light follows its being day, or that it does not follow. And if it follows, once it is agreed that 'It is day' is true, 'It is light' is immediately inferred—and that was the conclusion. But if it does not follow, it will not follow in the case of the conditional either, and for that reason the conditional will be false, since its consequent does not follow its antecedent (*M* VIII 441–2).

The passages offer parallel but distinct arguments. Each considers whether, in argument (B), (3) follows (2); and each presents the Stoic with a dilemma. The dilemma of *PH* is this: either it is evident that (3) follows (2), or it is unclear; the dilemma of *M* is simpler: either (3) follows (2), or it does not.

The first horn of *M*'s dilemma is dangerous. For if (3) follows (2), then argument (C) is concludent—so that argument (B) is redundant. In general, if σ follows σ', then $<\{\sigma'\}, \sigma>$ is concludent; and hence $<\{$[if σ', then σ], $\sigma'\}, \sigma>$ is redundant. And the first horn of *PH*'s dilemma points in the same direction; for if it is evident that (3) follows (2), then (3) follows (2).

But the second horn of each dilemma is a blunt prong. Suppose, as in *M*, that (3) does not follow (2): premiss (1) of (B) will then be false, as *M* asserts; but it certainly does not follow that (B) is redundant—an argument with a false premiss is not for that reason a redundant argument. Equally, if, as in *PH*, it is unclear whether (3) follows (2), we may well hesitate to grant (1); but that does not show that (1) is redundant.

In short, each dilemma fails because its second horn is powerless to impale us. And in *M* at least Sextus recognizes the failure; for although he introduces the dilemma, as in *PH*, by saying that 'the argument propounded in the first mode [i.e. the first indemonstrable] is inconclusive', he ends it as follows: 'Thus as far as the logical theory mentioned above goes, one of two things results: either arguments propounded in the first mode are found to be inconclusive, since their hypotheticals are redundant; or they are wholly false, since their hypotheticals are false' (VIII 442). The conclusion of *M*, in other words, is not that all first indemonstrables are redundant, but that all first indemonstrables are either redundant or false.

Now that disjunctive conclusion does in fact follow from the argument in *M*, and it is equivalent to the proposition that all true first indemonstrables are redundant. Since Stoic proofs are, by definition, *true* arguments, that conclusion is no less damaging than the simpler one that all first indemonstrables are redundant.

The dilemma of *PH* yields the disjunctive conclusion that all first indemonstrables either are redundant or have a non-evident premiss; or, equivalently, that all first indemonstrables with pre-evident premisses are redundant. According to *PH* II 140, an argument is only a proof if 'it concludes to something non-evident by way of things that are pre-evident.' If that is so, then the dilemma of *PH* is as lethal as the dilemma of *M*.

The sceptical dilemmas do not show what they pretend to show; but their failure is trivial—for they actually show something equally fatal to the art of formal demonstration.

The sceptics' opponents squirmed a little. Some probably urged that argument (C), and in general arguments of the form $<\{\sigma'\}, \sigma>$, are not concludent, on the grounds that no single-premissed arguments or *logoi monolēmmatoi* are concludent. Aristotle and the Peripatetics rejected *logoi monolēmmatoi*, and so did Chrysippus; but their view is not worth tracing out here.[18] Others maintained that argument (C) is enthymematic—an elliptical version of (B) (see Alexander, in *APr.* 17.18–24; *in Top.* 9.8–17); but in order to do so they were obliged to produce a new analysis of concludency.

The plain fact is that the sceptics were right: given the Stoic analysis of concludency, the first indemonstrable is demonstratively useless. That is the central core of the Argument from Redundancy. Of course, the Argument does not establish the vacuity of formal logic or justify Locke's predilection for native intuition over artificial syllogizing; for it bears, as I have expounded it, upon only one principle of formal inference. Still less will the Argument show that 'there is no demonstration'. But for all that it has a certain significance: it points to an incoherence at the heart of Stoic theorizing about logic and knowledge.

[18] For Aristotle see e.g. *APr.* A 15 34a 17; A 25; for the Peripatetics, see esp. Alex. Aphr. *in APr.* 17.10–18.7; *in Top.* 8.16–9.19; cf. Ammon. *in APr.* 27.14–33; 32.10–2; Philop. *in APr.* 33.10–24; for Chrysippus see *PH* II 167; *M* VIII 443. The Stoic Antipater rejected this orthodoxy (*PH* II 167; *M* VIII 443; Apuleius, *De int.* VII 184.20–3; Alex. Aphr. *in Top.* 8.16–18); but we hear no details of his argument. See further Mueller [112].

III

How distressed should the Stoics have been by the sceptical attack on their formal logic? Was their theory of proof entirely overthrown by the Argument from Redundancy? Or could they have settled in a Lockean posture, admiring demonstration but rejecting as otiose the claims of formal proof? To answer that question we must look at the purpose of Stoic proof.

According to Locke, reason has four degrees: 'The first, and highest, is the discovering, and finding out of Proofs; the second, the regular and methodical Disposition of them, and laying them in a clear and fit Order, to make their Connexion and Force be plainly and easily perceived; the third is the perceiving their Connexion; and the fourth, the making a right conclusion' (IV xvii 4, 669.27–32). And Locke holds that formal logic cannot serve the first of those rational ends: it is not an instrument of reason, nor a device to be employed in the discovery of scientific truths.

Locke was attacking his own contemporaries; and they could appeal to a tradition running back to antiquity. Among the ancients, the most vigorous advocate of their view was undoubtedly Galen. Galen was an amateur of logic; and he made a special study of *apodeixis*. He recommended that no scientist should start work until he had mastered the technique of formal demonstration; and he held explicitly that that technique provided a method, and the only true method, of discovery. He maintained that every science must proceed by first assembling a set of axioms, and then deducing, by way of formal inferences, its body of theorems. Geometry provided Galen with his model; but he believed that all sciences, including his own science of medicine, should apply the geometrical method.[19]

Now Aristotle, as I have argued elsewhere,[20] did not hold the Galenian view of *apodeixis*. Rather, he ascribes an expository and didactic function to demonstration: the theory of proof provides an intellectual showcase in which our pieces of knowledge can be

[19] See Müller [137] 417–19, and e.g. Galen, *Meth.med.* X 30, 10–39, 4 K; *Simp. med.* XI 462, 5–12 K; *Ord.lib.prop.* XIX 52–3 K=*Scr.Min.* II 82, 21–83, 6. Note that Galen himself thought that neither Aristotelian nor Stoic logic provided an adequate grounding for a theory of proof, which he sought rather in geometry (*Lib.prop.* XIX 39 K=*Scr.Min.* II 116, 19–117, 16; *Plac.Hipp.Plat.* V 226 K=184, 11–185, 8 M).

[20] See Barnes [136].

displayed to their best advantage, and it gives a pedagogical procedure whereby science may be transmitted from teacher to pupil. Aristotle, like Descartes, had observed that, 'as for logic, its syllogisms . . . serve rather in explaining to others the things one knows . . . than in learning them' (*Discours*, pt. II).

Of course, the connection between proof and knowledge is not merely extrinsic. To know that q is, in many cases, precisely to possess a proof $<\pi, \sigma>$; and if it is demonstrable that q, then a man knows that q iff he has a demonstration that q (*APo.* A 2 71^b 28–9). But that does not show that proof is a heuristic device: it is one thing to hold that acquiring knowledge that q is coming to grasp a proof of σ, another to recommend researchers to sit at their desks and syllogize.

Should we ascribe to the Stoics an Aristotelian or a Galenian conception of proof? Galen had a great admiration for Posidonius; and there is little doubt that he attributed his own view of the purpose and importance of proof to his Stoic forebear. Is that attribution correct? And did the Stoics in general believe that proof was a valuable, or even an indispensable, heuristic tool?

If $<\pi, \sigma>$ is an Aristotelian demonstration, then the members of π are more familiar than, and explanatory of, σ (*APo.* A 2 71^b 21); and it is those features which give a proof its epistemological character. The corresponding feature of a Stoic demonstration is that the members of π must be 'revelatory' of the non-evident (*adēlon*) conclusion, σ. At first sight, that appears to show that a Stoic proof is an instrument of discovery; for it is natural to suppose that a 'revelation' is a making plain, and that a 'revelatory' inference makes clear what was previously unclear—that it promotes the conclusion from a post of obscurity to a position of light.

But a closer examination of Stoic theory shows that to be mistaken. The Stoics distinguished three kinds of non-evidence: some things, they held, are 'once and for all (*kathapax*) non-evident, if they are not of a nature to fall under our apprehension'; others are 'temporarily (*pros kairon*) non-evident, if they have a clear nature but are made temporarily non-evident for us by some external circumstances'; others are 'naturally (*phusei*) non-evident, if they are not of such a nature as to fall under our clear perception' (*PH* II 97–8; cf. *M* VIII 144–7). Plainly, non-evidence is not a characteristic that can be alleviated or cured by a proof:

no argument can make evident what is non-evident; and hence 'revelatory' inference cannot be a matter of raising a proposition from a state of obscurity to a state of lucidity.

What, then, is revelation? Our sources give no very satisfactory answer to the question. Sextus implies that if σ' reveals σ, then 'by attending to the former, we get an apprehension of the latter' (*M* VIII 253); and we must grasp σ' *before* we grasp σ 'in order that, by being known beforehand, it may lead us to a conception of the object that becomes known by way of it' (*PH* II 119). Elsewhere he adds that the revelation is achieved by the 'power (*dunamis*)' of the premisses (*PH* II 143; *M* VIII 310) or of the argument (*M* VIII 308); or, more informatively, that the premisses 'by their own nature teach us to establish' the conclusion (*M* VIII 309).

In a proof there must be a 'natural' connection between premisses and conclusion, such that knowledge of the former permits us to grasp the latter. If we consider the standard examples of Stoic demonstrations, we may conjecture what that natural connection was meant to consist in.

Take the following inference (*PH* II 142; *M* VIII 309):

(D) (1) If sweat flows through the skin, there are imperceptible pores in the flesh
(2) Sweat flows through the skin
Therefore: (3) There are imperceptible pores in the flesh

It is clear that in argument (D) the conclusion *explains* the premisses; or rather, that the non-evident state of affairs described by the conclusion accounts for the evident fact set down in premiss (2). Other illustrative proofs exhibit the same feature: a punctured lung explains bronchial discharge (*M* VIII 252); pregnancy accounts for lactation (ibid. 423); motion is possible because there is void (ibid. 314); bodily movements depend on the soul (*PH* II 101; *M* VIII 155). In any concludent argument, the conclusion holds in virtue of the premisses; in a demonstrative argument, the conclusion will also express the state of affairs underlying and explaining the overt facts recorded in the premisses.[21]

[21] Brunschwig (above, p. 146) agrees with me (above, p. 165 n. 7) that revelation is an essential feature of Stoic proof; but he distinguishes between two senses of 'reveal' ('ἐκκαλύπτειν'). His *narrow* sense corresponds to the sense I sketch in the

Sextus' account of the different types of *adēla* prefaces his discussion of the Stoic theory of signs (*sēmeia*). Now he says more than once that proof is a species of sign,[22] and he asserts that that is so because a proof 'is revelatory of the conclusion' (*PH* II 131; cf. *M* VIII 140, 277). The Stoics distinguished two sorts of sign: 'commemorative' signs signify what is temporarily non-evident; 'indicative' signs signify what is naturally non-evident; and an indicative sign is defined as 'an antecedent proposition in a sound conditional, revelatory of the consequent' (*PH* II 101; cf. *M* VIII

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text; his *broad* sense is simply this: $\langle \pi, \sigma \rangle$ reveals σ iff $\langle \pi, \sigma \rangle$ is a true argument and σ is non-evident. Since Brunschwig holds that only *broad* revelation is invoked in Definition *T* (the last of the three Stoic definitions, the one tentatively associated with Chrysippus—and, presumably, the official Stoic line), he in effect maintains that revelation in *my* sense forms no part of the most polished Stoic account of proof.

No ancient author, to my knowledge, explicitly distinguishes between two senses of '*ἐκκαλύπτειν*' in that fashion; and certainly Sextus' long discussions of proof, to which the distinction should be of the last importance, betray no knowledge of it. Nor can I find the distinction implicit in the text which Brunschwig cites, *M* VIII 422–3. (On revelation and signs, see below, p. 180 n. 24.) Thus I am not convinced of the reality of Brunschwig's distinction; and in consequence, I incline to doubt the existence of his Definition *T*.

But, as Brunschwig clearly shows, there is a grave difficulty with my own view. *PH* II 134–43 proposes to explain what *ἀπόδειξις* is; by 140 Sextus has set out all the ingredients of Definition *T*, and he says that they together constitute a *λόγος ἀποδεικτικός*. We naturally suppose that '*λόγος ἀποδεικτικός*' is a synonym for '*ἀπόδειξις*': thus Brunschwig argues that the *definition* of *ἀπόδειξις* is completed by 140, with Definition *T*, and that 141–2 introduce a distinction between two *types* of *ἀποδείξεις*, one of which is characterized by being *ἐκκαλυπτικός* in the narrow sense.

I am obliged to reject that interpretation, and to suppose that, here at least, Sextus does not use '*λόγος ἀποδεικτικός*' as a synonym for '*ἀπόδειξις*'; rather, '*λόγος ἀποδεικτικός*' designates—most infelicitously—a genus of which *ἀπόδειξις* is one species. Now that, I readily admit, is not very palatable; but I believe that Sextus at least was able to swallow it. For his text, as Brunschwig allows (p. 152, n. 46), gives no hint that 141–2 introduces a distinction between types of *ἀποδείξεις*; and since he explicitly offers the definitions of 143 as a summary or recapitulation of 134–42, he must have considered 141–2 as an integral part of the *analysis* of *ἀπόδειξις*. (In 143 I retain the phrase which Brunschwig excises.) And if Sextus can tolerate the infelicity, cannot we do so too?

The issue requires further discussion. Those readers who share Brunschwig's view may take my text to be talking about only *some* of the Stoic analyses of proof.

[22] e.g. *PH* II 96, 122, 131, 134; *M* VIII 140, 277, 299. Of course, the sign is not, strictly speaking, the genus of demonstration: a demonstration is a species of *logos*. At *M* VIII 277 Sextus explains: 'Demonstration is agreed to be by genus a sign; for it is clarificatory of its conclusion, and the conjunction of the premisses will be a sign of the holding of the conclusion.' That is, if $\langle \pi\ \sigma \rangle$, is a demonstration then π reveals σ, so that π is an indicative sign of σ. On the theory of signs see Verbeke [122].

245).[23] Thus if proofs are signs, they are indicative signs; and the conclusion of a proof will be something *naturally* non-evident. Just as, in a proof, the premisses 'by their own nature teach us to establish' the conclusion (*M* VIII 309), so an indicative sign 'of its own nature and constitution is said to all but cry aloud and signify that of which it is indicative' (*M* VIII 154).[24]

That connection between proof and the indicative sign has an important corollary. What a sign signifies must be non-evident; but the sign itself must be pre-evident (*M* VIII 172–3). It follows that the premisses of a proof must be pre-evident; and that is just what Sextus says: a proof 'concludes to something non-evident by way of things that are pre-evident (*dia prodēlōn*)' (*PH* II 140). But if that is right, then the Stoics cannot allow *sequences* of proofs, in which the conclusion of one argument is used as a premiss for the next; for any proved proposition will be non-evident, and therefore ineligible to appear as a premiss. Stoic proofs, unlike Aristotelian proofs, are strongly individualistic: they do not club together to form systematically concatenated demonstrative sciences.[25]

All that is, I fear, somewhat speculative and rather vague: a full account of the function and importance of Stoic proof will demand greater philosophical precision and more scholarly textual analysis. But if what I have said is roughly correct, it has several consequences for our understanding of the history of proof; and I end by indicating three of them.

First, Stoic and Aristotelian proofs differ sharply in their

[23] Brunschwig (above, p. 147) argues that the notion of revelation is not specific to the *indicative* sign, since it appears in the definition of sign *in general* at *PH* II 104 and *M* VIII 245. But those two texts refer exclusively to *indicative* signs: each occurs in the sceptical critique of the indicative sign, after Sextus has explicitly said that his discussion will be concerned *solely* with that species of sign (*PH* II 102; *M* VIII 156).

[24] Sextus' language here, and in the texts cited on p. 178, calls to mind the fourth of his accounts of the conditional: 'Those who judge by ἔμφασις say that a conditional is true if its consequent is included δυνάμει in its antecedent' (*PH* II 112). There is some evidence, admittedly slight, for connecting ἔμφασις with the Stoics (see Frede [42], 92): perhaps the Stoics suggested that if σ' is an indicative sign of σ, then ⌜if σ', then σ⌝ satisfies the condition of ἔμφασις.

[25] That is a surprising—even an outrageous—conclusion. Brunschwig (above, p. 152) hopes to evade it by suggesting that the phrase 'διὰ προδήλων' at *PH* II 140 is an interpolation. But even if his suggestion is right, we are left with the association between signs and proof. If we are to restore sequences of proofs to the Stoics, we must suppose that Sextus' account of their views is badly garbled.

attitude to explanation. Each type of proof is, in a loose sense, explanatory; but whereas in an Aristotelian demonstration the premisses supply material which explains the conclusion, in Stoic demonstrations the order of explanation is reversed—an ideal Stoic demonstration is, in a modern jargon, an inference to the best explanation.

Secondly, Stoic proofs serve to advance our knowledge: a demonstration provides a route (presumably the sole route) to knowledge of the non-evident explanations of the phenomena. It does not follow that such proofs are, in the Galenian sense, instruments of research; and it may be that a Stoic proof is no more than a formal medium for the presentation of indicative signs.

Finally, inferences to the best explanation are unlikely to require complex deductive techniques: linked series of syllogisms, such as constitute an Aristotelian science, will not be found among the Stoics. In that case, the Stoic formal logic will be largely irrelevant to Stoic theory of proof;[26] and the Stoics need not have been dismayed by the sceptical contention that their five indemonstrables were demonstratively useless.

[26] Mueller [112], 185, suggests that the Stoics actually recognized this: 'The Peripatetics . . . insisted on the claim, believed for many centuries after them, that their logic was the instrument of science. We do not know the Stoic response to this claim, but it is reasonable to suppose that they retreated to the view that the theory of deductive inference was a technical discipline, studied for some ethical end, perhaps, but not as the method of scientific discovery.' But see the texts cited above, p. 163; and cf. Frede [88], 23–4.