VAGUENESS (A)

I. In contemporary philosophy, the term 'vague' has a narrower meaning than in everyday language: vagueness in philosophy is a matter of indeterminate or fuzzy boundaries. First of all, the term is used for many predicates and the concepts they express. A predicate is vague iff (if and only if) it admits of borderline cases. For example, there are people who are definitely bald, people who are definitely not bald, and people who are neither definitely bald nor definitely not bald. The latter group constitutes the borderline cases for the predicate 'bald'. The term 'vague' in contemporary philosophy is also used for names and even for things such as mountains and cities, if they exhibit the same kind of indeterminate boundaries.

If X is a borderline case for 'bald', we sometimes say 'X is and is not bald'. I think this is just an idiom for borderline cases and does not constitute a contradiction; that is, we are not claiming that a situation is real (X is bald) and that the exact same situation is not real.

II. The sorites paradox:

1 grain of sand does not constitute a heap.
If 1 grain of sand does not constitute a heap, 2 do not constitute a heap either.
⋮
If 9,999 grains of sand do not constitute a heap, 10,000 do not constitute a heap either.
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Therefore, 10,000 grains of sand do not constitute a heap.

Here, 'do not constitute a heap' means 'do not constitute a heap no matter how they are arranged'. The 'if' here and throughout our discussion expresses material implication. The reasoning consists of 9,999 steps, each of which has the form of modus ponens.

We have a paradox because each premiss seems true, each step seems valid, but the conclusion is false. A view on vagueness must explain why the reasoning is not sound. If we deny any one of the hypothetical premisses (those of the form 'If **A** then **B**'), we accept for a specific number j that j grains do not form a heap but j + 1 do. It sounds absurd to accept such a thing. The vague predicate that gives rise to the paradox here is 'do not constitute a heap'; every vague predicate leads to a form of the paradox.

Let's consider a long sequence of contiguous numbered bands on a wall such that the first is clearly red, the last is clearly orange, and any two consecutive bands appear identical. We can formulate a version of the paradox using the predicate 'appears red' instead of 'do not constitute a heap'. In this example, it will be even more difficult to deny any one of the premisses.

Views about vagueness are of two types: according to the epistemic theory, a vague word draws a sharp boundary, but we cannot know where that boundary is; according to all other views, a vague word draws no sharp boundary.

III. One may consider that statements such as 'X is bald', where X is a borderline case, are neither true nor false, but indefinite; that is, they have a third truth-value. In this case, it is quite natural to characterize the connectives of propositional logic in accordance with Kleene's so called strong three-valued logic. This logic accepts all classical principles about how the truth-value of a compound statement is determined by the truth-values of its components, and characterizes a compound statement as indefinite if the classical principles are not sufficient for considering it true or considering it false. Thus, ¬**A** is true if **A** is false; false if **A** is true; and indefinite if **A** is indefinite. **A** ∧ **B** is true if both **A** and **B** are true; false if at least one of **A** and **B** is false; and indefinite in the remaining cases (i.e., if one of **A** and **B** is indefinite and the other is indefinite or true). **A** → **B** is true if **A** is false or **B** is true; false if **A** is true and **B** is false; and indefinite in the remaining cases (i.e., in the cases '**A**: true; **B**: indefinite', '**A**: indefinite; **B**: indefinite' and '**A**: indefinite; **B**: false').

This approach (developed by S. Körner in the 50s and M. Tye in the 90s) tackles the sorites by saying that some hypothetical premisses (those around the middle) are indefinite and so not true (and that is why the reasoning is not sound).

Problems with this approach: (a) It treats the hypothetical premises in the same way as their converses:

If 2 grains of sand do not constitute a heap, 1 does not constitute a heap either.

⋮

If 10,000 grains of sand do not constitute a heap, 9,999 do not constitute a heap either.

Here, too, the first implications, as well as the last, turn out true, while those in the middle turn out indefinite. But these converse implications do not lead to paradox and all appear to be true. (b) When **A** is a statement that concerns a borderline case and is therefore considered indefinite, then **A** → **A** and **A** ∧ ¬**A** also turn out indefinite. However, they appear to be true and false respectively (they are the most typical tautology and the most typical contradiction).

Those problems lead to the conclusion that if we accept that statements such as 'X is bald', where X is a borderline case, are neither true nor false, we should not try to provide truth-tables for the logical connectives. Rather, we should say that whether a compound statement is indefinite does not depend solely on the truth values of its components. (For, e.g., **A** → **B** may be indefinite while **C** → **D** is not, even though **A** has the same truth-value as **C**, and **B** has the same truth-value as **D**.) But if we say such things, we have strayed from what is commonly called 'three-valued logic'.

IV. The theory of degrees of truth (developed by J. Goguen and others in the 60s):

We replace the concept of truth with the concept of degrees of truth, which are all the real numbers from 0 to 1. If John is definitely bald, the statement 'John is bald' has a truth degree of 1; if he is definitely not bald, the statement has a truth degree of 0; if he is equally far from being definitely bald and from being definitely not bald, the statement has a truth degree of 0.5; and so on. '[**A**]' means 'the degree of truth of **A**'. Falsehood also comes in degrees: the degree of falsehood of a statement **A** is 1 – [**A**]. Degrees of truth are not probabilities: the concept of a degree of truth has no relation to what we know (as does the concept of probability) and degrees of truth do not need to follow the probability calculus.

Here is one way to extend the concept of degrees of truth to compound statements: [¬**A**] = 1 – [**A**]. The idea here is that the degree of truth of ¬**A** coincides with the degree of falsehood of **A**. [**A** ∧ **B**] is the smaller of [**A**] and [**B**], whereas [**A** ∨ **B**] is the larger of those two numbers. (And if [**A**] = [**B**], then the common degree of truth of **A** and **B** is also the degree of truth of their conjunction and their disjunction.) The idea here is to maintain an analogy with the classical truth-tables for ∧ and ∨ (understanding truth in the classical tables as a 'greater degree' than falsehood). [**A** → **B**] = 1 – ([**A**] – [**B**]) if [**A**] > [**B**], and [**A** →

B] = 1 if [**A**] ≤ [**B**]. The idea here is that if there is a decrease in truth between the antecedent and the consequent, then the greater the decrease, the smaller the degree of truth of the implication; and if there is no decrease, then the implication has a truth degree of 1. Finally, statements that have the form of a universal quantification ('every F is G') or an existential quantification ('there is H') are treated like conjunctions or disjunctions respectively. For example, the statement 'There is a student who is bald' is treated like 'George is bald, or John is bald, or ...' (where George, John, etc. are all the students) and takes as its degree of truth that number r among ['George is bald'], ['John is bald'], etc. which is such that, for every other r′ among those numbers, r ≥ r′.

Here is also a degree-theoretic concept of a valid argument: validity admits of degrees from 0 to 1; if [**P**] ≤ [**C**], where **C** is the conclusion and **P** is the premiss with the lowest degree of truth, then the argument has a validity degree of 1; if [**P**] > [**C**], then the degree of validity is 1 – ([**P**] – [**C**]). The idea here is similar to the idea behind the rule for →.

According to this approach, if we consider the statements '1 grain of sand does not constitute a heap', ..., '10,000 grains of sand do not constitute a heap', we see that there is an initial group where the degree of truth is 1, then there is a large group where the degree of truth decreases very gradually, and finally there is a group where the degree is 0. So the problem with the sorites is that, although the initial hypothetical premisses have a truth degree of 1, and the last ones also have a degree of 1, those in the middle have a truth degree slightly less than 1. Furthermore, the first among the 9,999 applications of modus ponens have a validity degree of 1, and the last ones also have a degree of 1, but the middle ones have a validity degree slightly less than 1.

The theory of degrees of truth leads to a rejection of classical logic for statements involving vague concepts. Of the axioms and theorems of classical logic, it retains only those all of whose instances have a truth degree of 1. Thus, it retains the principle p → p, but not the principle p ∨ ¬p. ('Instances' of e.g. the principle p → p are the statements that have that form.)

V. Problems with the theory of degrees of truth:

(a) It assigns a validity of less than 1 to some arguments that have the form of modus ponens. However, modus ponens is considered by almost all philosophers to be beyond doubt.

We can overcome this problem by replacing the definition of degrees of validity which we gave earlier with the following: if the degree of falsity of the conclusion is less than, or equal to, the sum of the degrees of falsity of the premisses, then the argument has a validity degree of 1; but if the degree of falsity of the conclusion is greater than the sum of the degrees of falsity of the premisses, and the difference between the two is k, then the argument has a validity degree of 1 – k. It can be proved that, for any statements **A** and **B**, the degree of falsity of **B** is less than, or equal to, the sum of the degree of falsity of **A** and the degree of falsity of **A** → **B**. The new degree-theoretic definition of validity is due to D. Edgington.

(b) It assigns many statements degrees of truth that are not intuitively correct. First of all, the statement 'There exists a (natural) number j such that j grains of sand do not constitute a heap, but j + 1 grains do' gets a truth value of 0.5 (why? apply the rules). However, intuitively it should get 0 or close to 0.

Next, let's assume that the sentence 'John is tall' has a truth degree of 1, while 'John is bald', 'George is tall' and 'George is bald' all have a truth degree of 0.5. Then the theory assigns the same degree of truth (0.5) to the conjunctions 'John is tall and bald' and 'George is tall and bald'. Intuitively, however, we would expect the latter to have a lower degree. One can tackle that example by replacing the rule for conjunction with the following: [**A** ∧ **B**] = [**A**] × [**B**]. Then the sentence 'John is tall and bald' will get 0.5, whereas 'George is tall and bald' will get 0.25. However, that replacement does not help much with the sentence 'John is bald, and it is not the case that John is bald'. Intuitively, it does not seem right to give it either 0.5 or 0.25, since it is a contradiction and should therefore get 0.

Finally, let's assume that the degree of truth of the sentence 'Eve is a child' is 0.5, and let's take the word 'girl' with the sense in which it runs parallel to childhood, so that if person X is female, the degree of truth of the statement 'X is a girl' is identical with that of 'X is a child'. Then, contrary to our intuitions, the theory assigns the same degree of truth, 1, to 'Eve is a girl iff Eve is a child' and to 'Eve is a girl iff Eve is not a child'. ('**A** iff **B**' is equivalent to '(if **A** then **B**) and (if **B** then **A**)'.) Such examples are due to K. Fine.

The example of the contradiction and the example of Eve lead to the conclusion that we cannot find satisfactory rules for calculating the degree of truth of compound statements from the degrees of their constituent statements. For, even if we accept that truth admits of degrees, we must recognize that sometimes compound statements that have been constructed, through the same connective of propositional logic, from components that pairwise coincide in degree of truth nevertheless have different degrees. If truth admits of degrees, the degree of a compound statement does not depend solely on the degrees of its components; it also depends on their content.

(c) Higher-order vagueness is a problem for existing versions of the theory of degrees of truth. What is higher-order vagueness? Let's take a vague predicate, e.g., 'red'. It seems that 'definitely red' is also vague. Just as there is no clear boundary between red and non-red things, so there is no clear boundary between those that are definitely red and those that are borderline cases for 'red'. There are things that are borderline for 'definitely red' — this is best seen in the long sequence of bands on the wall which we imagined earlier. And the predicate 'definitely not red' is also vague.

Why is higher-order vagueness a problem for the theory we are discussing? If X is borderline for 'definitely red', then the sentence 'X is red' does not definitely have a truth degree of 1, nor does it definitely have a truth degree less than 1. Hence, the predicate 'has a truth degree of 1' is a vague predicate of sentences; it admits of borderline cases, such as the sentence 'X is red'. Similarly, the predicates 'has a truth degree of 0', 'has a truth degree of 0.5', etc. are vague. Now, existing versions of the theory of degrees of truth introduce a non-classical logic for vague concepts, but develop the study of these concepts and their logic by applying classical logic, because they presuppose that the concepts they use, unlike those they study, are precise. Wrong! They should apply the logic they introduce.