III.—A NEW 'LAW OF THOUGHT' AND ITS IMPLICATIONS.

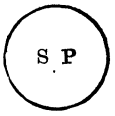
By E. E. Constance Jones.

"I am the pillars of the house,
The keystone of the arch am I;
Take me away, and roof and wall
Would fall to ruin utterly."—K. Tynan.

It will not be disputed that assertions of the forms

S is P, *S is not P*

are possible, actual, significant, useful and necessary. They *can* be used, they *are* used, they *must* be used. I agree with Prof. Frege in holding that propositions of the form *S is P*



are correctly analysed as asserting identity of extension or denotation (*Bedeutung*) in diversity of intension or signification (*Sinn*),[1] and from this analysis[2] I obtain the principle that—

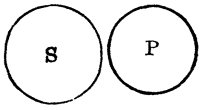
Every subject of affirmative Predication is an identity-in-diversity (*i.e.* denotational unity in intensional difference). This applies absolutely without exception to every Proposition of the form *S is P*.

A corresponding analysis applies to propositions of the

[1] By Extension or Denotation of a name I mean the things to which the name applies; by Intension or Signification of a name I mean the properties of the things to which the name applies. Extension gives the 'existential' aspect, Intension gives the qualitative aspect. The things in question may be material or immaterial; they may have a fixed and definite position in space and time, or be, on the other hand, ideal, imaginary, or merely suppositional. The Extension or Denotation of *e.g.* Quadruped is: Lion, Tiger, Horse, Dog, Cat, Mouse, etc.; the Intension is: Animality and Four-footed-ness.

[2] This is the most elementary, and the only absolutely general analysis of Propositions (as distinguished from sentences) of the form *S is P*. It is as general as the form *S is P* itself, and from that most abstract form, this universally applicable analysis can be obtained. Compare $a=b$ as symbolic of equations.

form *S is not P*. Every proposition of this form asserts difference of Denotation (Otherness) in difference of Intension (Diversity)



I will examine cases in illustration of the above analysis of *S is P* and *S is not-P* further on, and briefly consider the relation of *S is P*, *S is not P*, to 'Relative' Propositions, such as *A is equal to B*, *C is father of D*, etc.

Propositions of the forms *S is P*, *S is not P*, are indispensable for significant assertion; and we need them for a satisfactory statement of the 'Laws of Thought'. Without them we are in strictness limited to

A is A, *A is-not not-A*, *A is either A or not-A*,[1]

forms which, though they have at first sight a dazzling appearance of self-evidence, are not only unnatural and difficult to interpret, but are also separated by an impassable chasm from *A is B* (*S is P*).

Granted that we can assert *A is B*, *A is not B* (*S is P*, etc.), and further that we can explain and justify this form, we can proceed to a straightforward, effective and applicable statement of the Laws of Contradiction and Excluded Middle, thus:—

S is P } cannot both be true (Law of Contradiction).
S is not P } cannot both be false (Law of Excluded Middle).

It follows from these two Laws that of any Subject of Predication (S), any Predicate (P) is affirmable or deniable; and that of any Subject of Predication (S), either P or not-P can be affirmed. And so from *S is P*, *S is not P* (analysed as above) we obtain the principle that—

Every Subject of Predication is an identity-in-diversity. (It is the above analysis of Categoricals and its implications that I desire to expound and advocate in this paper.)

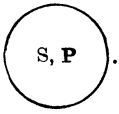
It follows further from the above that every Predicate (P) is necessarily incompatible with *not-P*, (*absence of intension P*,) and necessarily compatible with *not-not-P*. (This suggests a principle of necessary connexion of attributes.)

The learner[2] who is informed that

S is P

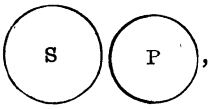
[1] *Everything is A or not-A* is of the form: *S is P or not-P*.

[2] The thought-process of the teacher (speaker, writer, etc.) is always prominently a process of analysis—he has a *whole* before him and sets it forth to his audience—(pupils, hearers, readers). (Compare Bradley's

is thereby entitled to make a construction to which the designations S and P both belong, thus: 

. Having this before his mind, he is entitled to say, not only that S is P, but also that P is S, that S is-not not-P, that not-P is not-S, etc. If he is informed that

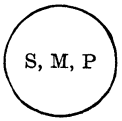
S is not P

he is similarly entitled to make a construction in which S is separated from P, 

, and to say that P is not S, not-S is P, etc.

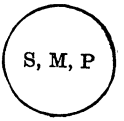
Again, if he accepts the statements—

M is P
S is M

he is entitled to the construction of something which is S, M, P, thus: 

; and this entitles him to the further assertion *S is P;* for the thing which S and M and P taken in Extension denote, has in the *one extension* common to them all the *diverse intensions* signified by S, M, and P.

The gist of the last paragraph may be expressed in a self-contained Hypothetical thus:—

If M is P and S is M, then S is P 

Principles of Logic, bk. ii., pt. i., ch. iii., § 4; Stout's *Analytic Psychology*, ii., 71; Sidwart's *Logic*, English translation, i., 25, 26.)

The thought-process of the learner, listener, reader, seeker, is always emphatically synthetic.

But no one can ever be permanently—hardly even momentarily—altogether in *one* of these attitudes. The teacher, in setting out his material, must be constantly getting fresh *aperçus*, grasping new connexions, annexing fresh facts. The learner or seeker who can consciously learn or seek to any purpose, must already have, and use, some store and background of knowledge.

This distinction of attitude, and corresponding divergences in past interpretations of Categoricals, is not only interesting but highly important—especially perhaps in connexion with the meaning of Inference and its place in logical theory.

And other Hypotheticals are reducible to a similar statement, *e.g.* :—

(*a*) If A is B
and B is C
and C is D
and D is E
then A is E

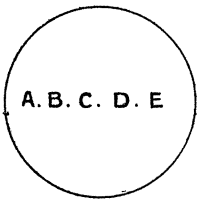


FIG. 1.

(*b*) If all A is B
and all B is C
and all C is D
and all D is E
then all A is E

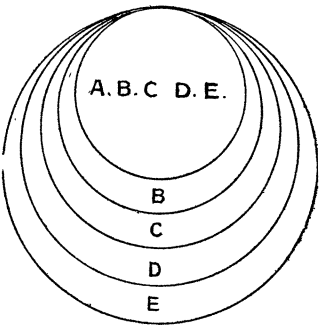


FIG. 2.

(*b*) may be represented diagrammatically by Fig. 1 or by Fig. 2 or by some combination of the two.

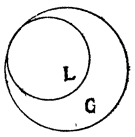
In Inference the identity-in-diversity which is inferred is given directly not in the premisses, but in the construction to which the premisses entitle the learner or seeker.

As much of the denotation of B, C, D and E as are predicable of A are in denotation identical with A; and of this one denotation or extension, the diverse intensions signified by *A*, *B*, *C*, *D*, *E taken in intension*, are predicated.

In accepting *S is P* as an appropriate symbolic expression of all affirmative Categoricals—the most general and 'abstract' expressions of such Categoricals, as $a=b$ is of equations—it is of course taken for granted that S stands for the whole of the Subject, and P for the whole of the Predicate, whatever the Subject and the Predicate may be, and that *is* signifies identity of denotation between them, without offering or attempting any *explanation*, by reference to origin or otherwise, of the co-existence in one denotation of the intensions concerned, or any discrimination of the differences by which one kind of Categorical may be distinguished from another. When *S is P* is used as symbolising Class-Propositions,—All R is Q, No R is Q, etc.—S stands for the explicitly quantified subject All R, and P for the implicitly quantified Predicate All or any Q, some Q. By *implicitly quantified* I mean that there is no explicit quantification, but that explicit quantification is justified.

If *S is P* stands for—

All Lions are carnivorous



S symbolises *All Lions*, P symbolises [*some*] *carnivorous*. Unless C were implicitly quantified, by *some*, thus limiting the 'distribution,' *Lions* must be understood to be coincident in denotation or extension with *carnivorous*, and thus to be also Tigers, Panthers, Wolves, Vultures, etc. Similarly with negative Categoricals. In: No Hellebores are fragrant, ⓗⓕ, (= All H are not F) Hellebores is explicitly quantified by No (= All not), fragrant is implicitly quantified by All or Any, and

All Hellebores = *S*, *All fragrant* = *P*.

If this were not so, we should not be justified in inferring from No H are F, that No F are H.

In: Some beeches are not green-leaved, *beeches* is quantified explicitly by *some*, green-leaved is quantified implicitly by *Any*.

Some beeches = *S*
Any green-leaved = *P*.

The reason why O is inconvertible is not because there is any question about implicit quantification of the Predicate, but because when the [explicitly] quantified converse of O has been reached (No green-leaved things are some beeches), in deference to common usage (and therefore to ordinary thought) the quantification of its Predicate has to be dropped and the converse becomes: No green-leaved things are beeches. This of course involves an illegitimate extension of the denotation of *beeches*.[1]

In: All Planets move in elliptical orbits,
Jupiter is a Planet,
∴ Jupiter moves in an elliptical orbit,

moving in elliptical orbit must be understood to be implicitly quantified by *some*, otherwise Planets would be coincident with the things, whatever they are, which move in elliptical orbits—that is, with the whole extension of *moving in elliptical orbit*. In the conclusion, the extension of *moving in elliptical orbit* is restricted to the one-planet-extension of Jupiter, as, in the Minor Premiss, the extension of *Planet* is restricted to the extension of the Minor Term, Jupiter.

It is the Identity-in-diversity of affirmative Categoricals which justifies their conversion, with the implied quantification, and the pivot of Mediate Inference is a denotational identity of whole or part of the Middle Term in one Premiss with the whole or with part of its denotation in the other.

[1] Conversion of A and I, with the implied quantification, would be impossible unless there were denotational identity between Subject and Predicate. It is to be noted also that in many languages an adjective predicated agrees in gender and number with its Subject.

In the Planet-instance above, the denotation of the Middle Term in the Minor Premiss is identical with part of the denotation of the Middle Term in the Major Premiss. We may compare Thackeray's story of the priest and his first penitent quoted by Dr. Bosanquet (*Essentials of Logic*, pp. 140, 141) as an instance of what Dr. Bosanquet calls "inference from mere identity":—

"An old Abbé talking among a party of intimate friends happened to say: 'A priest has strange experiences; why, ladies, my first penitent was a murderer'. Upon this the principal nobleman of the neighbourhood enters the room: 'Ah, Abbé, here you are; do you know, ladies, I was the Abbé's first penitent, and I promise you my confession astonished him!'"

Here an unambiguous Middle Term—*the Abbé's first penitent*—unexpectedly reveals the horrifying fact that the principal nobleman of the neighbourhood is a murderer.

In this we have certainly an inference from 'identity'—not however 'mere' identity but denotational identity in intensional diversity. It would be interesting to be shown precisely how, in any case *without* identity of this sort,—without denotational or extensional identity—any inference whatever could be drawn.

The same principle of identity in diversity applies in the case of concrete Hypotheticals. Take *e.g.* this example: If Ferdinand marries Henrietta, he will be ruined. This may be expanded as follows:—

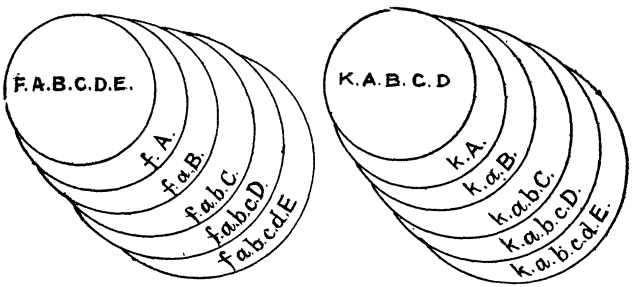
If F marries H (A), he will be responsible for her debts (B).
If B, he will be responsible for double his income (C).
If C, he will be unable to meet his responsibilities (D).
If D, he will be financially ruined (E).

It is the identity of Henrietta with a person who will spend double Ferdinand's income, and of Ferdinand with a person who marries Henrietta, with a person who will be responsible for her debts, and for double his income, and therefore unable to meet his responsibilities, that leads inevitably to the regrettable conclusion. If Ferdinand were a minor and his father a millionaire, F might not be B; if Henrietta were herself a millionaire, or if her expenditure would be only half Ferdinand's income, he would not be C; and so on.

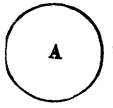
Again:—

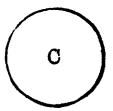
If Kate marries Peter, she will be wretched, may mean—
If Kate marries Peter (A) she marries an old-fashioned miser (B).
If B, she will be half-starved (C).
If C, she will be wretched (D).

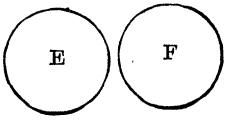
It is on the identity (in diversity) of Kate with a person who marries Peter, and therefore (the denotation of Peter being the denotation of a miser and therefore the denotation of a man who will half-starve his wife) with a person who marries a miser, and thus with a person who will be half-starved, that ensures her identity with a person who will be wretched. The two examples may be illustrated diagrammatically thus:—



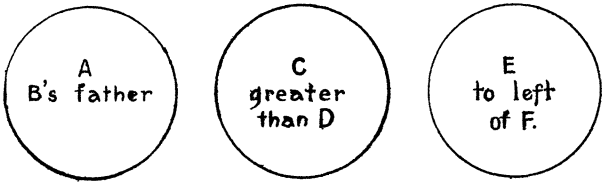
It would be easy but tedious to multiply examples.

How do the propositions which are what is called 'Relative' —*i.e.* propositions which state the relation to each other of two or more objects connected as members of a system, *e.g.* A is father of B, C is greater than D, E is to the left of F—how do these compare with assertions of the form *S is P*, *S is not P*? What *S is P* gives us is intensional diversity in identical denotation; in all Relatives we deal with two such denotations, which are correlated, and neither of which can be predicated of the other. Obviously in the above instances A *is not* B 

; C *is not* D 

; E *is not* F 

; but A, although he is not B, is B's father, C although not D, is greater than D, E although not F, is to the left of F:—



We are constantly using Relatives in common speech in conjunction with the non-relative *S is P* form, and this form is easily imposed on Relatives when desired (as in the above examples). I do not regard the denotation assigned to Subjects or Predicates as implying existence in space or time, or indeed any particular kind of existence; no such implication could possibly attach to S and P in *S is P;* to admit the generality of the form *S is P* is to bar the implication—but intension cannot be, or be thought of, imagined or supposed, except as the intension of *something*, of some *that*, which has just as much (or as little) 'reality' as the qualities the intension, the what-ness, which it holds together in a denotational unity. We must be able to use propositions, and to have some general theory of import—*i.e.* of what propositions in general mean—before we can proceed to settle what precise kind or measure of 'existence' or 'reality' our Subjects and Predicates have.

Dr. Keynes, in the fourth edition of his *Formal Logic* allows that 'logical equations,' such as

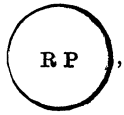
Equilateral triangles = equiangular triangles,

may be understood to assert *Identity of denotation in diversity of connotation.* It seems obvious that on this basis nothing but the recognition of implicit quantification is necessary in order to make acceptable my analysis of affirmative Categoricals in cases in which the terms are connotative. And then the way seems clear to an acceptance of it as quite general. *Ce n'est que le premier pas qui coûte*, and there is no witchcraft about connotation as distinct from intension. I may point out that in the alternative interpretations on page 178 of *Formal Logic*,[1] and in the passage of Mrs. Ladd Franklin mentioned in note 1 on page 179, the force of the copula is not referred to; in other words, we are not told exactly *how* the two aspects of Terms are to be 'taken account of' in the proposition, and this is the very point of my analysis; unless the *is* of the copula in *S is P* signifies denotational identity (intensional diversity is signified by the terms) S and P cannot be held together in the proposition, affirmative Class-propositions cannot be converted, there is no link to connect the premisses in mediate inference, we must lapse into the disintegration of Lotze's analysis, and say that

$$\text{S is P} = \begin{cases} \text{S is S} \\ \text{P is P} \\ \text{S is not P} \end{cases}$$

[1] Compare MIND, 1893, p. 452, etc.

The copula is sometimes stigmatised as a 'verbal device' of the most objectionable kind, and it is asked: What is there in the subject-matter of an assertion which corresponds to it, is it not irredeemably artificial? I admit of course that if, *e.g.*, I am eating a ripe peach, and say: This peach is ripe,

(1) the whole, *ripe peach*, 

, which is present to me,

is not a matter of words, and (2) that in particular there does not seem to be anything in that whole which corresponds to the copula to as great an extent as its being and qualities correspond to *this peach* and *ripe*. But if we admit words as a necessary device for the recording and communicating of knowledge, it must be allowed that the copula fulfils an important function very modestly and economically. Mill (quite naturally) placed disproportionate stress on connotation, but it is noticeable that he lays it down that the most common meaning which propositions of the form *S is P* are ever intended to convey is that whatever is denoted by (or has the Attributes connoted by) the Subject, has the Attributes connoted by the Predicate (Mill, *Logic*, bk. i., ch. v., § 4). This gives us identity of denotation in diversity of connotation: but Mill does not live up to this—it seems indeed as if he had hardly realised its force. It occurs to him when he is asking: *Between what* is connexion asserted in a Proposition? When he goes on to the further question: *What is the connexion* asserted? he enumerates *five* ultimate kinds of predication—afterwards reduced to *four, viz.:* Simple Existence, Order in Time, Order in Place, and Resemblance; and, so far as I remember, he makes no subsequent use of or reference to his one almost general analysis. The present analysis of *S is P* into *identity in diversity* is fundamentally similar to Mill's, but has a wider scope. It was first, I believe, put forward in print in a little book of mine in 1890. A view which I understand to be the same as mine was published by Prof. Frege in 1892; and in Mr. Bertrand Russell's *Principles of Mathematics* (1903), Frege's view is adopted with some reservations. On this view the science of Logic *is* the science of the "Laws of Thought" (if we choose to call them so). I should however like instead of Laws of Thought, and Law of Identity (1), Law of Contradiction (2), Law of Excluded Middle (3), to speak of Laws of Logic, to substitute for (1) a "Law of Identity in Diversity," of the form given above, to call (2) the Law of Consistency (since the Law of Contradiction excludes inconsistency), and to call (3) the Law of Coherence.

(since it formulates a principle of Subject-Predicate connexion between all terms).

For affirmation, extension of S and P must be identical—otherwise the copula cannot be *is*—S, in intension, is different from P in intension. For significant affirmation, P must be intensionally different from S.

If the P and S of any *S is P* were taken purely in extension or denotation, we should have no use for Predicates that differed from their Subjects—S alone, or P alone, would be sufficient. If S and P are taken one in extension or denotation and the other in intension, it is clear that we can never say that one *is* the other, that the intension of one *is* the extension of the other.

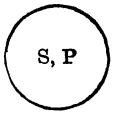
And the attempt to take the S and P of an affirmative assertion in intension only, can lead to nothing but confusion and disaster—witness Lotze's reduction, referred to above, of *S is P* into—

S is S
P is P
S is not P

(See Lotze's *Logic*, ch. ii., book i.; also MIND, 1893, pp. 449, etc.) Lotze's application of the so-called Law of Thought *A is A* to propositions of the form *S is P* is a *reductio ad absurdum* of a purely conceptual Logic. What *A is A* apparently means for him would be better expressed by *A-ness is A-ness*, for *A-ness* is never any-other-ness, it is no *not-A-ness*, it cannot be *B-ness* or *C-ness*. If this is taken into account, it becomes clear that *A is A* reduces us to a deadlock. If we begin with *A-ness is A-ness*, there we must end, and Lotze's conclusion above quoted is inevitable. But the moral I deduce is, not that we must end there, but that we must never begin there; we must recognise, with Locke, that "all affirmation is in concrete," and this brings us inevitably to the identity-of-extension-in-diversity-of-intension interpretation of *S is P*. We cannot assert one 'concept' (or intension) of another, but only that a denotation characterised by some intension (S) has another intension (P)—an intension which is compatible with, which co-exists with, the intension of S in one subject. To take Locke's example, we can say, *Man is mortal*, but we cannot say *Humanity is mortality*.

Lotze's difficulty about the interpretation of S is P, and similar difficulties felt by earlier logicians, seem to involve a failure to distinguish between—

(1) predicating of S an intension P which is incompatible with the intension of S and would involve its denial (presence of P is taken to imply absence of S);

(2) assigning to denotation S an intension P, which, while it is not intension S, is compatible with that intension and can co-exist with it in one denotation, 

The difficulties above referred to vanish on the identity-of-extension-in-diversity-of-intension view of the interpretation of *S is P*, while on a purely conceptual or intensional view they are fatal to any coherent doctrine of propositional import.

It is only *identity of extension* that can hold together the diverse intensions in affirmation; it is only identity of extension that can give the necessary connexion in Inference, Immediate and Mediate (if not, it would not, in Mediate Inference be necessary to 'distribute' the Middle term—intensional sameness and an 'undistributed Middle' would suffice, and "Substitution of *Similars*" would be a valid principle of Inference). This is of course entirely compatible with the fact that intension may be, and constantly is, a guide to extension; *e.g.* it is because of the inseparable connexion of equality of sides in a triangle with equality of angles at the base, that I can affirm: *All* equilateral triangles have the angles at the base equal.

According to my use of Terms, *S is-not P* (*S is not-P*) asserts that the intension of P is absent from what is denoted by S (not that the intension of S is diverse from the Intension of P—*that* goes without saying and applies in *S is P*)—the presence together with the absence of P in one Subject, *S is P* and *S is not P*, cannot be asserted, P and not-P as assigned to one denotation are incompatible. Thus in every assertion *S is P*, the presence of P is necessarily accompanied by the absence of not-P. And similarly, the presence of not-P is necessarily accompanied by the absence of P, while Assertion and Inference, as I contend, depend upon Identity of Extension (or Denotation) in Diversity of Intension. The principle here affirmed—that there is a formal and necessary connexion of Attributes that is predicable of every Subject of predication—may possibly be regarded as a formal Principle of Inductive Inference: it asserts (not only that the presence and the absence of P are incompatible but also) *that the presence of P* and the *absence of not-P* are inseparably conjoined. And of every subject (S) the presence or absence of any predicate (P) is predicable (L. of Excluded Middle). Thus of every subject (S) the presence or absence of every Predicate in the world (every P) may be asserted. This

measure of uniform connexion and uniform incompatibility is self-evident.

What inseparable connexions and incompatibilities there actually are beyond these purely formal, or most general, ones must be learned by appeal to experience. We cannot say *S is P* or *A is B* until (directly or indirectly) we have found some case in which S *is* P, or A *is* B.

My contention is that my Law of Identity in Diversity first makes (theoretically) possible a satisfactory statement, in *S is P*, *S is not P* form, of the Laws of Contradiction and Excluded Middle, and that it, together with them, does furnish a real and adequate basis and starting-point of syllogistic Logic. Granted propositions of the form *S is P* with the identity-in-diversity analysis and the corresponding analysis of *S is not P*, together with the already accepted Laws of Contradiction and Excluded Middle, the whole traditional scheme of Immediate and Mediate Inference can be built up systematically and explicitly, from the foundations. In Logic, as in all thinking, propositions of forms *S is P*, *S is not P*, have of course always been used. Thought cannot live and move without propositions of this form; but so far as I know they have not hitherto received a satisfactory and commonly accepted general analysis, an explicit recognition by logicians that they are the primary and fundamental forms of significant assertion, needed even for a satisfactory expression of the Law of Contradiction and the Law of Excluded Middle. It is the Law of Identity *A is A* which has stood in the way. And it is impossible really to get rid of this tautology, posing as the self-evident and significant basis, until it is seen not only that we must admit *A is B*, not only that 'mere identity' is our undoing, not only that for significant assertion we must have an identity in difference, must recognise that *A is B* is preferable to *A is A*. All this does not avail until we can give a clear account of what *exactly* is meant by the identity-in-difference of *A is B*. There are, it is admitted, no more ambiguous words in Philosophy than Identity and Difference, and there are none of which the meaning has been more elusive, none more misleading. Even some of the acutest thinkers do not seem to have escaped the snare. The source of the ambiguity is not far to seek, for of the two fundamental kinds of Sameness, (1) extensional or denotational sameness, and (2) qualitative sameness, (2) is very constantly (though by no means always) a sign of (1). *E.g.* if a stowaway is observed to have all the published characteristics of an escaped criminal, the similarity is regarded as an indication of 'identity'. It may

however turn out to be a case of 'mistaken identity'. For recognition of *likeness* there must be a comparison of two, though the two may be only *one* thing at *two* times. It is partly because all this is so simple, that it has proved so insidious. But though simple, it is absolutely primary. The *A is A* difficulty has been with us since, at any rate, the appearance of the Eleatic Stranger in the *Sophistes*, who reports the view of certain 'tyros' that of *man* we can only assert *man*, of *good* we can only predicate *good*. Neither the Eleatic Stranger nor any one else in the *Sophistes* provides a satisfactory solution of the puzzle, though the Stranger shows both common-sense and logical insight when he lays down the principle that those who deny the possibility of the assertion, concerning any subject, of a predicate different from itself, are confuted out of their own mouths, they "are obliged to admit it implicitly and involuntarily in their common forms of speech. They cannot carry on a conversation without it, and they thus serve as a perpetual refutation of their own doctrine." From that day to this the solution of the puzzle has it seems been still to seek; though from the time when the distinction between Extension (Denotation) and Intension, That-ness and What-ness, was clearly drawn, it ought to have been easy. Jevons, I believe (like Mill and many other able thinkers), came in view of it—but slipped aside into hopeless, because concealed, confusion in his "great rule of inference," the "Substitution of *Similars*" (*Principles of Science*, p. 9, 3rd edition).

Lotze has the merit of having seen that propositions of form *S is P* needed to be accommodated with the Laws of Thought; but as he could not reconcile *S is P* with *A is A*, he gave up (professedly) *S is P*. The Eleatic Stranger could have taught him better.