Physics and reality II: Geometry and physics

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What is non-Euclidean geometry?
Starting from Euclidean geometry, we have Euclid's postulates:

Postulate 1: To draw a line from any point to any point.
Postulate 2: To extend any line indefinitely in either direction.
Postulate 3: To draw a circle of any radius about any point.
Postulate 4: All right angles are congruent.
Postulate 5: The parallel postulate.

Kant (1781): The postulates are synthetic a priori truths, whose truth we understand through the form of spatial intuition.

Bolyai (1823): The first four postulates constitute "the absolute science of space." They are the (something like) the conditions of the possibility of geometrical reasoning. Geometries compatible with these are all possibiy true. Only experience can distinguish among them.

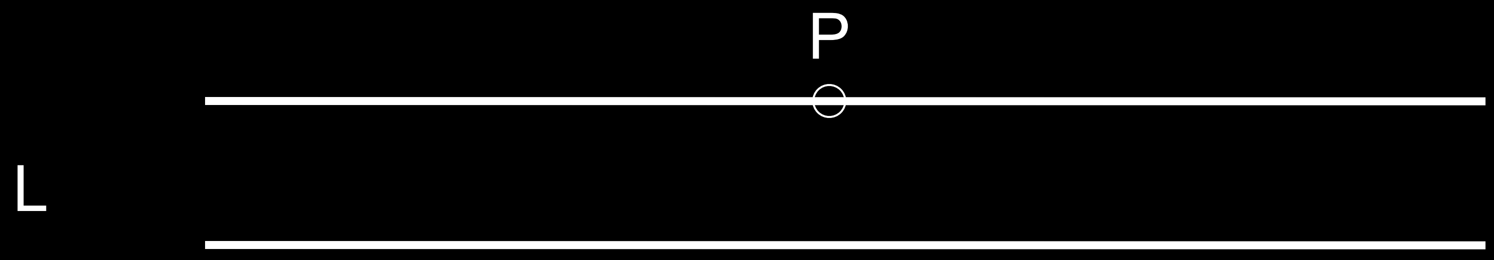
K.F. Gauss on Bolyai:

“I regard this young geometer Bolyai as a genius of the first order.”

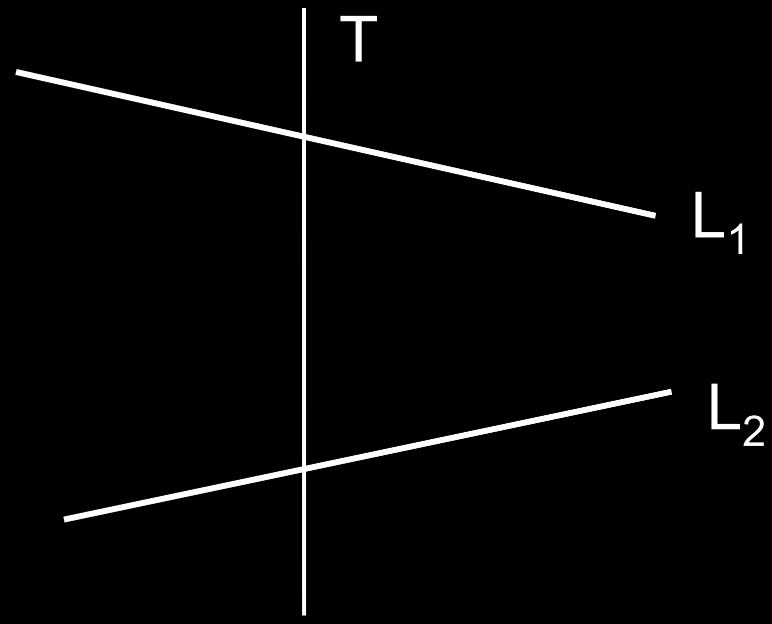
Gauss to Bolyai’s father:

“To praise it would amount to praising myself. For the entire content of the work...coincides almost exactly with my own meditations which have occupied my mind for the past thirty or thirty-five years.”

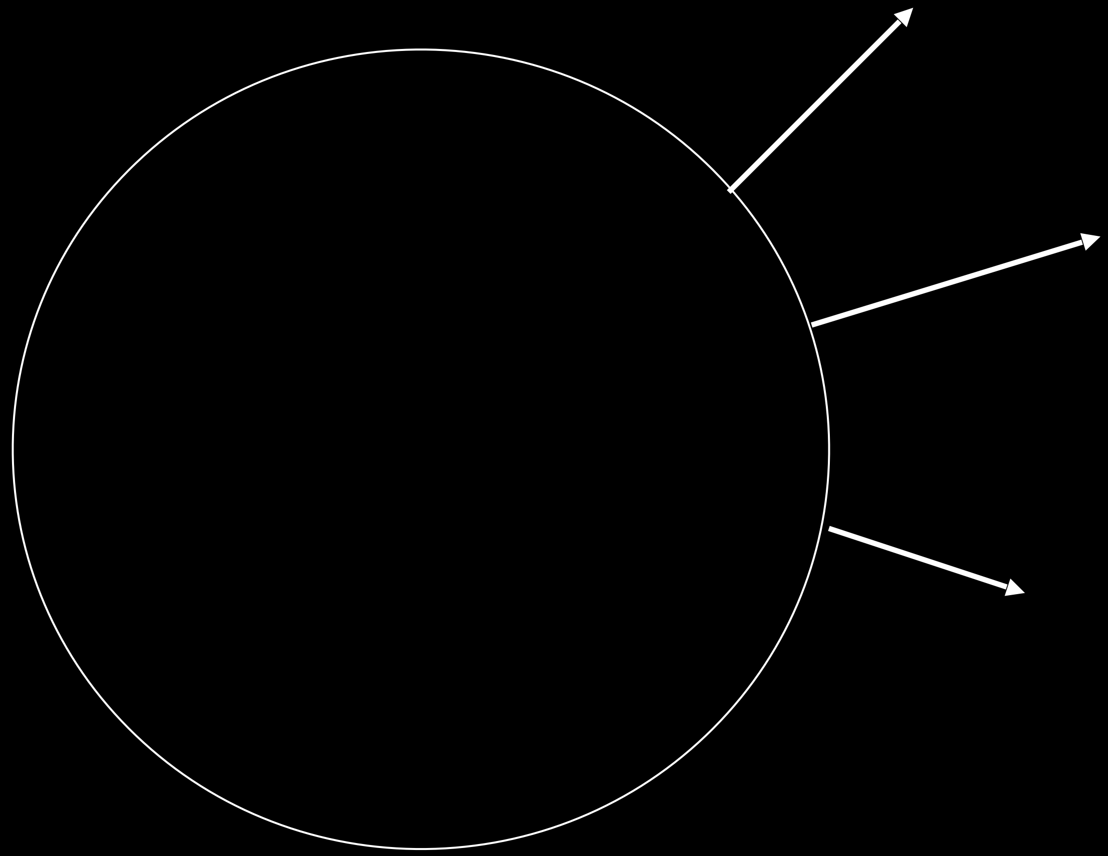
Parallel postulate: Given a line L and a point P not on L, there is exactly one line through P that does not intersect L. (From Proclus, 500-something C.E.)



Equivalently, if lines L_1 and L_2 cross a line T, L_1 and L_2 will meet on that side of T where their internal angles with T are less than two right angles. (This is Euclid's version.)



Measure of curvature: Extrinsic curvature is measured by the relation of the space to the ambient space. E.g. perpendiculars to the surface of a sphere or cylinder are not parallel.



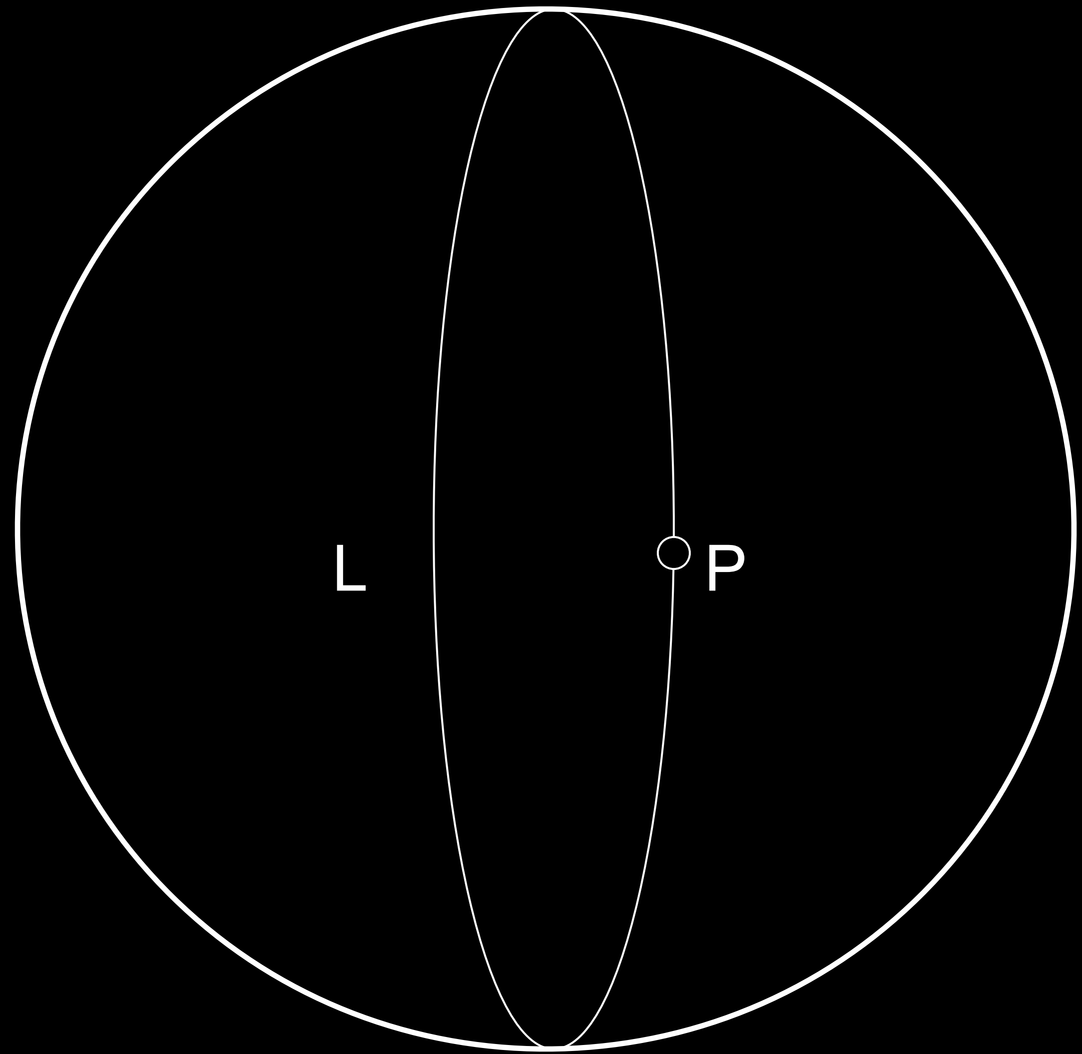
But if you roll a Euclidean surface into a cylinder, Euclidean figures remain Euclidean. The cylinder has no *intrinsic* curvature.

Intrinsic curvature is measured by features of the surface, or space, itself without regard to the ambient space. Such features are introduced by the failure of Euclid’s parallel postulate.

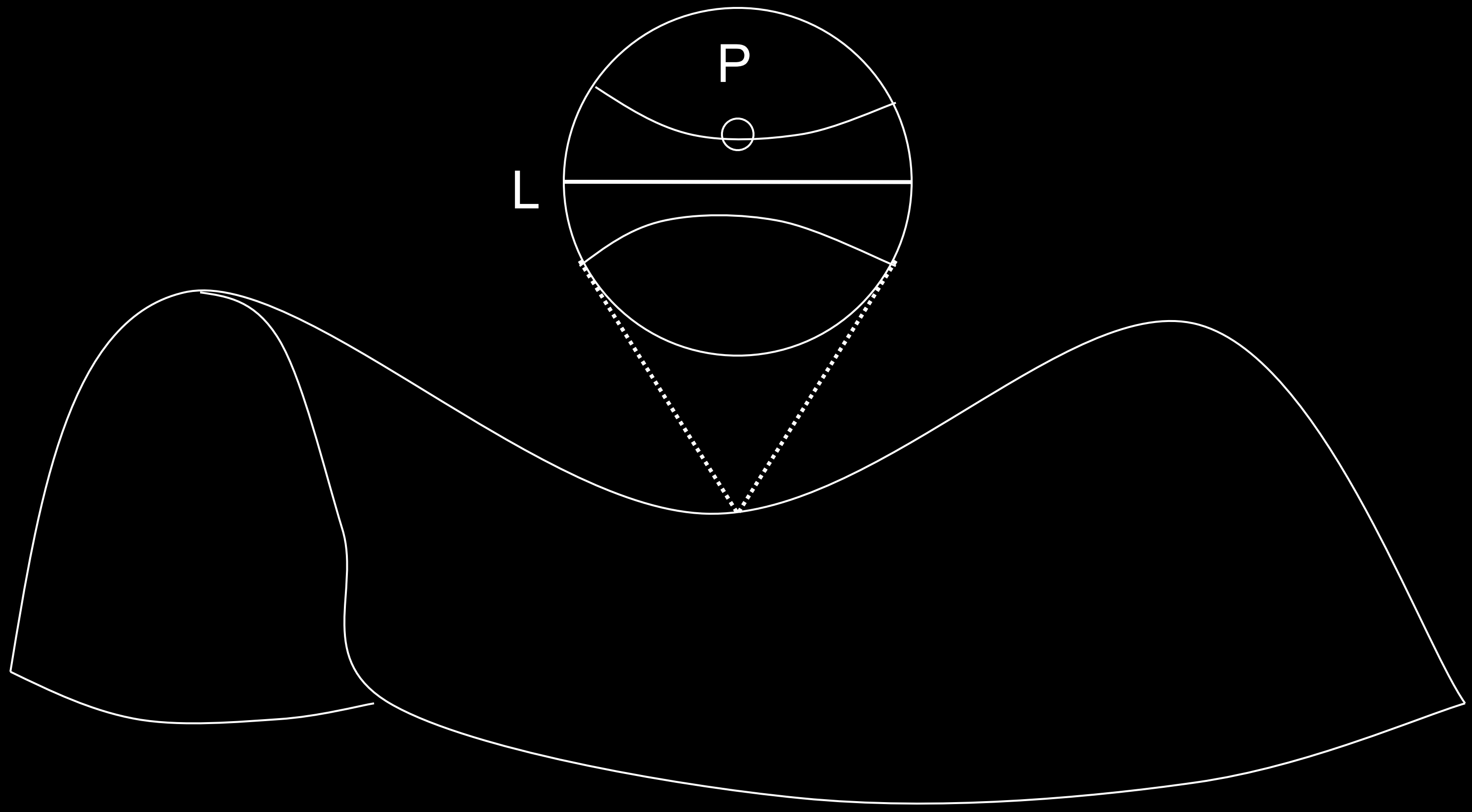
Positively curved space (spherical, elliptical, or “Riemannian” space”): Given a line L and a point P not on L, there is no line through P that fails to meet L.

Negatively curved space (Bolyai-Lobatchevsky geometry, or “pseudo-spherical” space: Given a line L and a point P not on L, there is more than one line through P that fails to meet L.

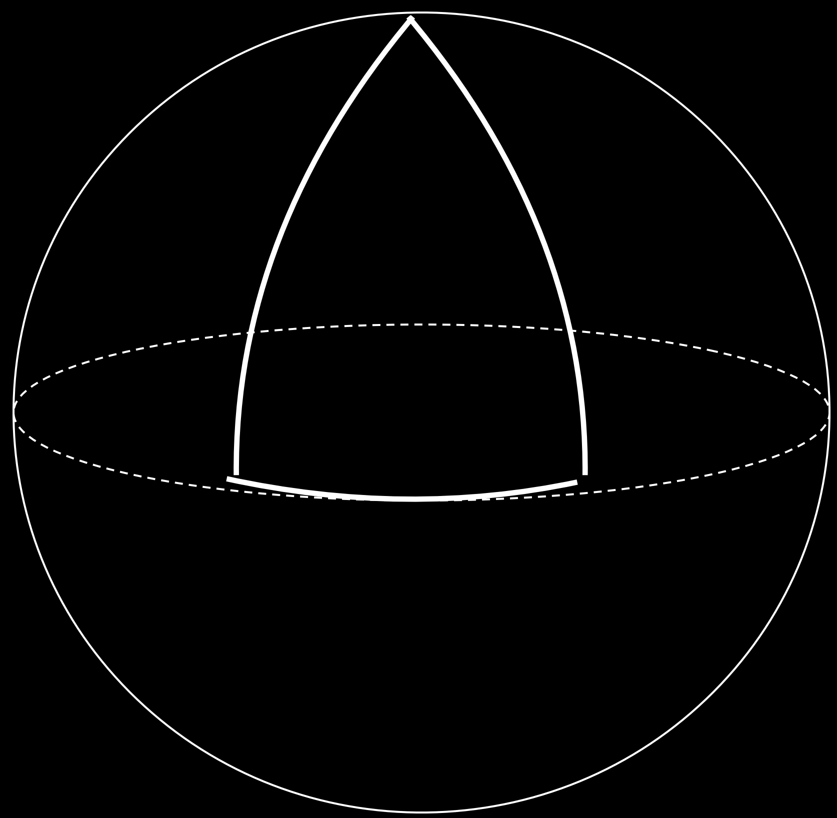
On a spherical surface, every line ("great circle") through P will intersect L.



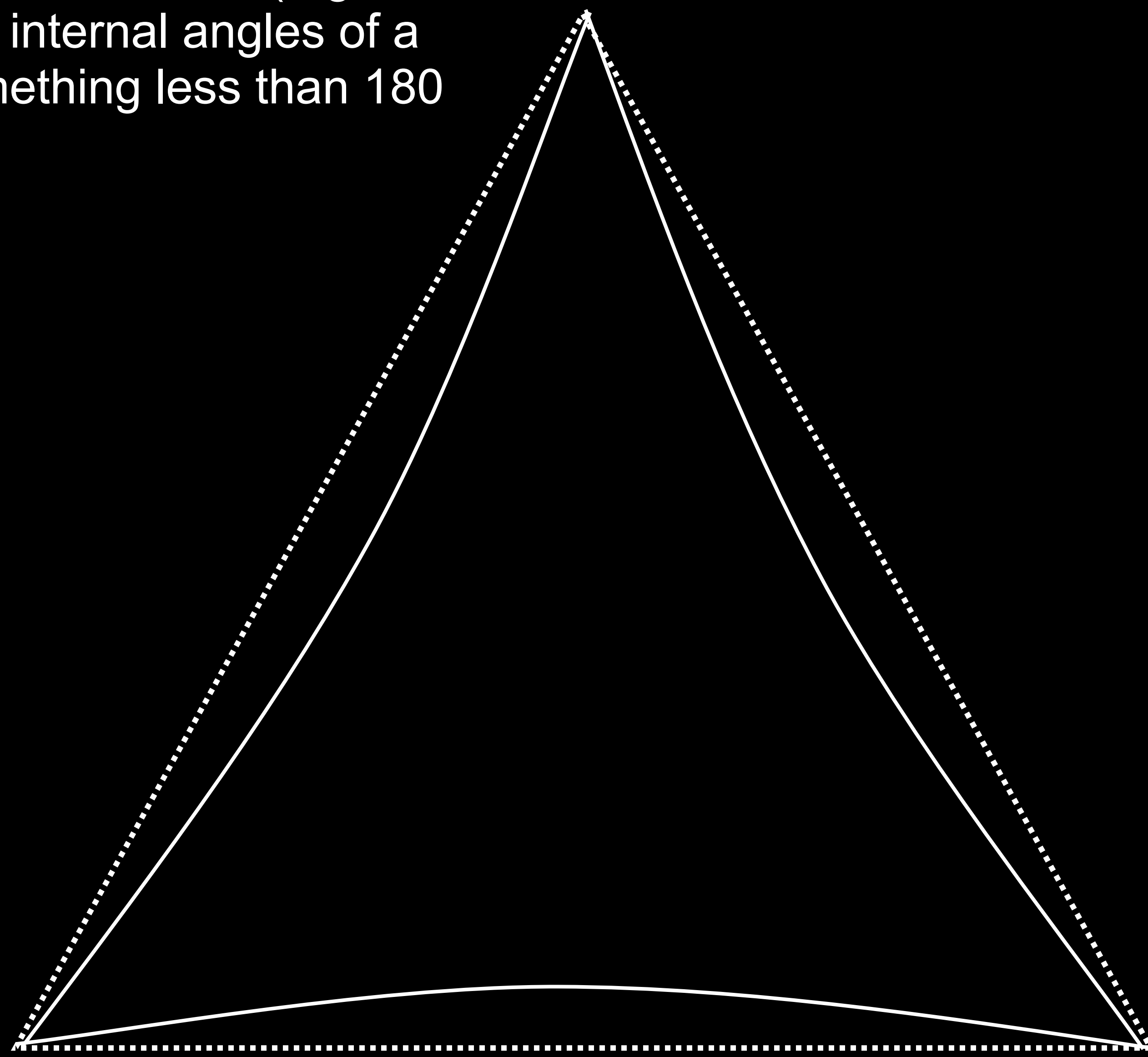
On a saddle surface, there may be infinitely many lines through P that do not intersect L.



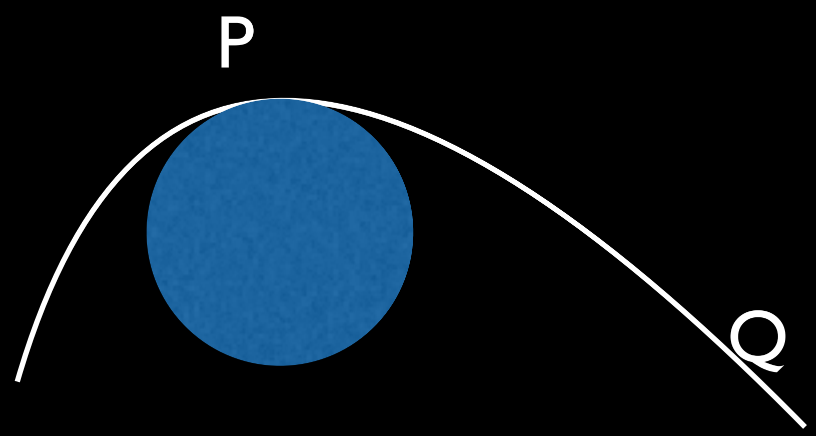
Comparison: On the surface of the sphere, the internal angles of a triangle sum to more than 180 degrees, and the excess depends on the size of the triangle. Each line of longitude forms a right angle with the equator.



On a negatively curved surface (e.g. a saddle surface, the internal angles of a triangle sum to something less than 180 degrees.

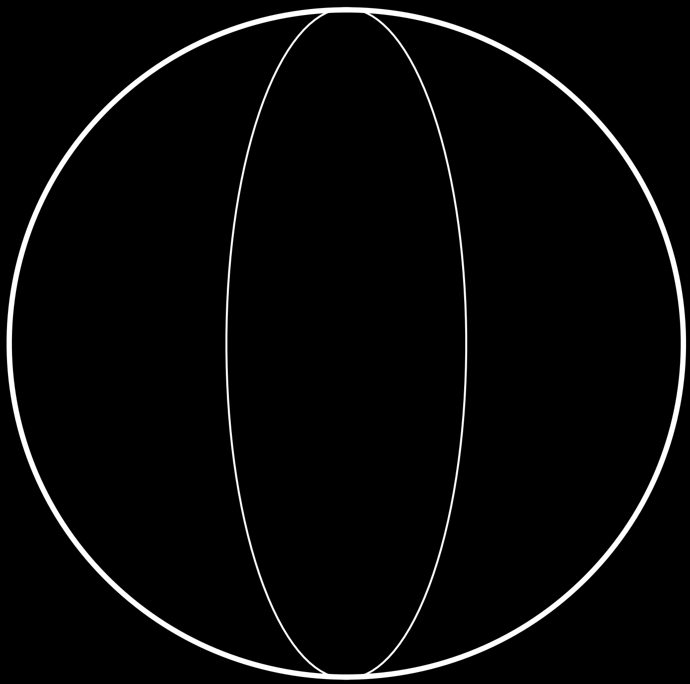


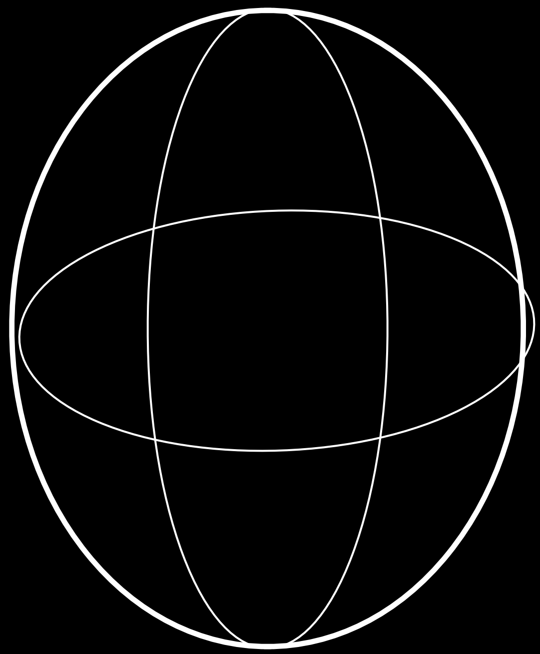
Intrinsic curvature is determined by the product of the ***greatest and least curvatures*** at a point. Each of these is determined by the radius of curvature, or the radius of the circle that best approximates the surface at a point (the “osculating circle”).



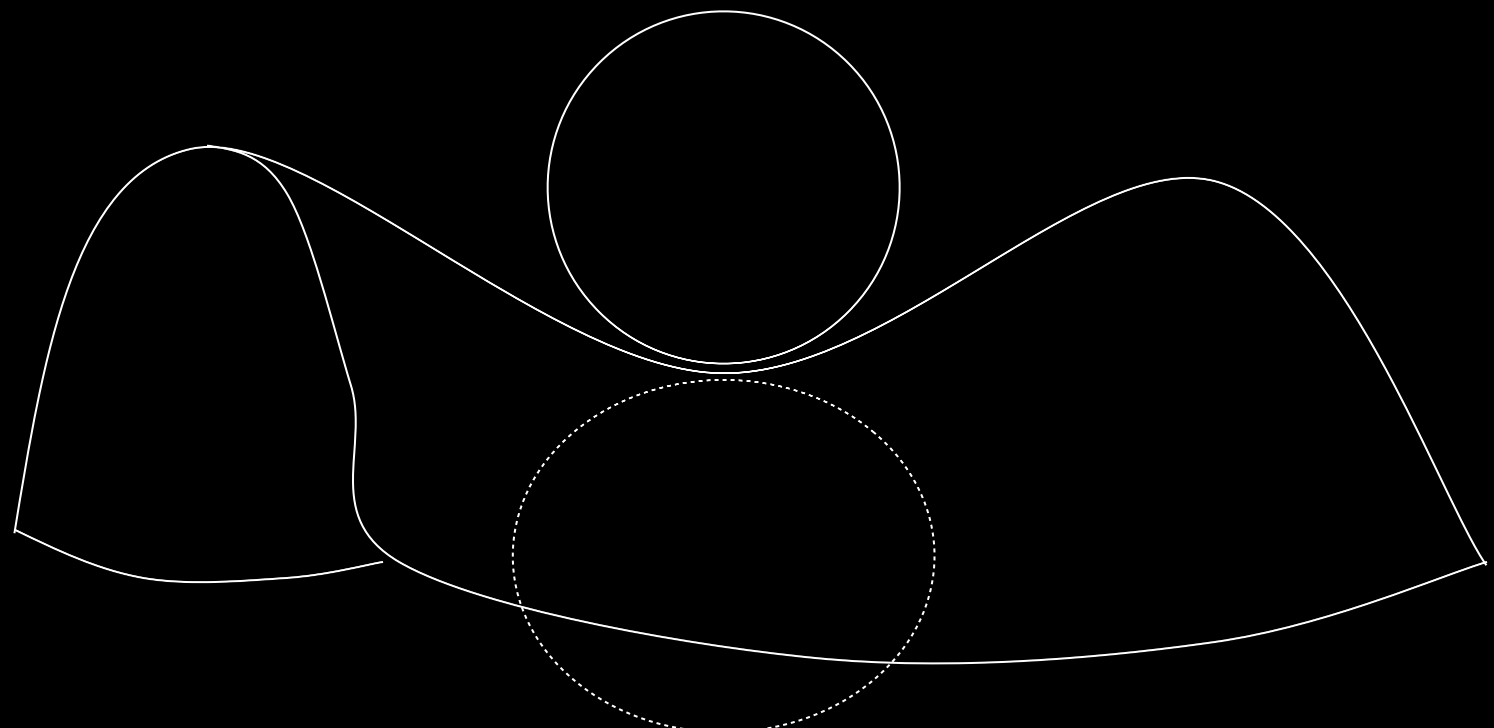
Evidently the osculating circle at Q must be greater than that at P. But the product of their radii will be a positive number.

On a spherical surface, the radius of curvature is the radius of the sphere itself, and it is the same everywhere on a perfect sphere, varying on (e.g.) an ellipsoidal surface.





On a saddle surface, at any point there are curvatures in opposite directions. Hence their product is negative.



Homogeneous geometry: The geometry of spaces of constant curvature.

Helmholtz-Lie theorem (***"free mobility"***): If a figure may be moved freely through space without changing its dimensions, then there is a quadratic function of the coordinates that is unchanging over space, and the curvature of the space is constant. (In the special case of Pythagoras's theorem, the quadratic function takes its simplest form, as the square root of the product of the squares of the coordinate-differences.)

Spaces characterized by Euclid's postulates, excluding the parallel postulate, are spaces of constant curvature. Classical proofs with compass and straight-edge are possible.

Free mobility implies constant curvature, and vice-versa.

What about ***inhomogeneous geometry*?**

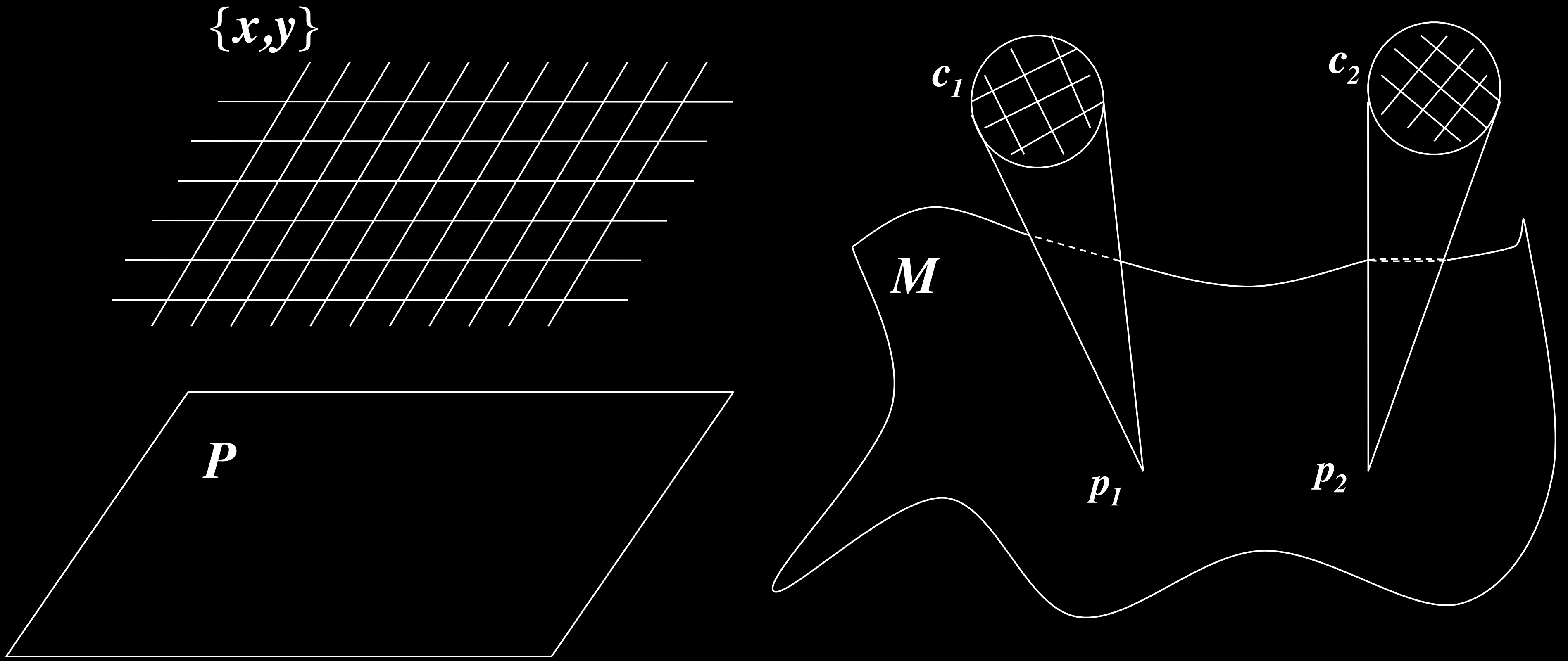
How do we describe spaces in which the curvature varies from point to point?

Bernhard Riemann (1826-66) recognized that geometry of constant curvature, in which free mobility is possible, is just a special case of a more general kind of geometry.

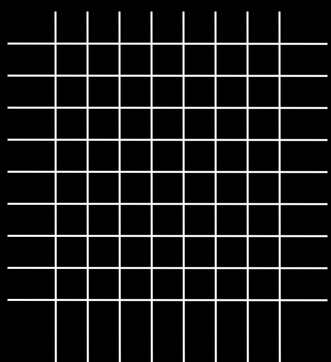
“Riemannian geometry” is the study of spaces which, at any point, have (“infinitesimally”) the structure of the Euclidean plane, but at different points, have variable curvature.

Over a vanishingly small region, a Cartesian coordinate system may be constructed. But the Cartesian coordinates at one point can’t be assumed to be extendible to any finite distance. (An irregular surface such as an apple can only be covered by a large number of very small stickers.)

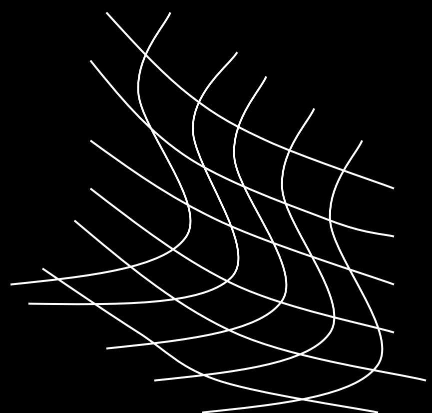
Spaces that are inhomogeneous:



The Cartesian rigid reference-frame

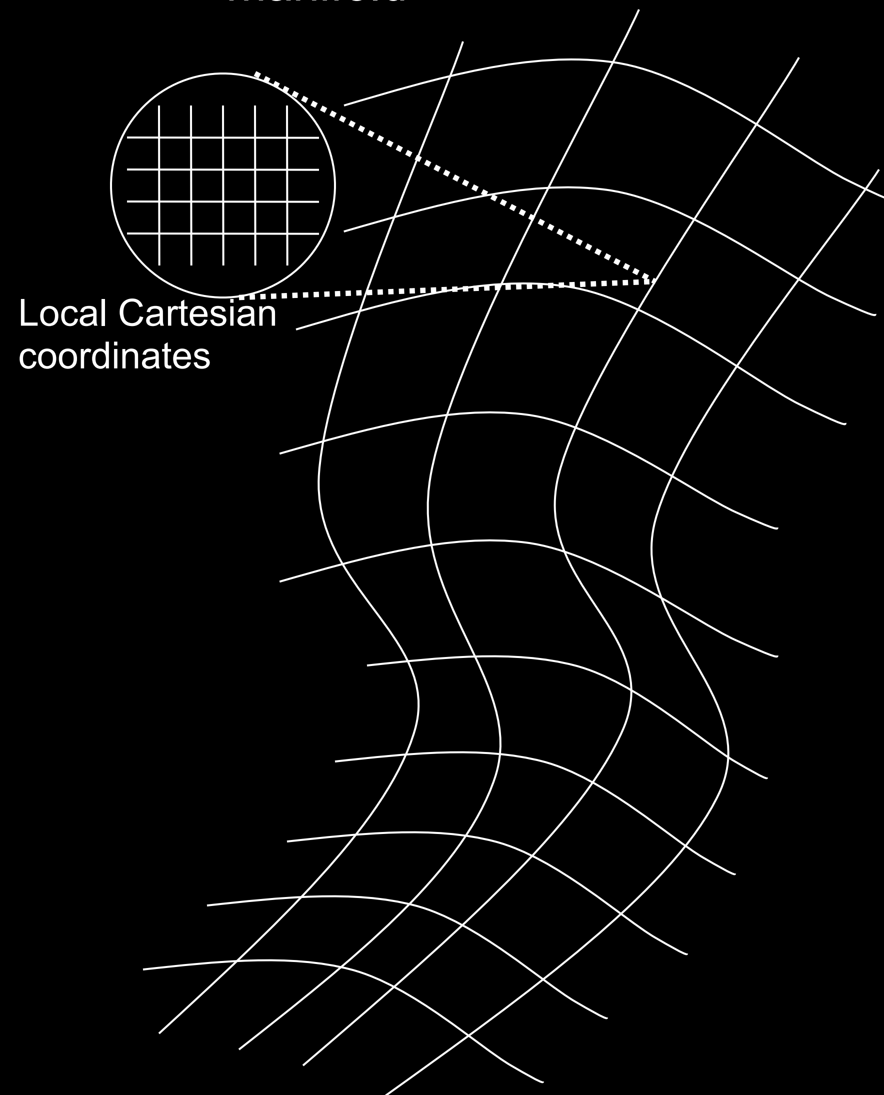


The Einsteinian "reference-mollusc"*



*Actual mollusc may differ

Large-scale and local structure of a space-time manifold



Two things to note about non-Euclidean geometry and Einstein's General Theory of Relativity:

The theory, as we will see, crucially relies on ***differential geometry***— the theory of very small variations in curvature from space-time point to space-time point, as developed by Riemann— because the variation of curvature is essential to the connection between curvature and gravitation.

The theory is not fundamentally about the curvature of ***space***, though spatial curvature plays an important role. The more fundamental notion is the curvature of ***space-time.*** The features of ***space-time geodesics*** provide the chief motivations to connect gravity with space-time curvature.

What is the most general concept of space?

Helmholtz-Poincaré: The most general concept is what is common to all spaces in which classical geometry is possible, i.e. all spaces in which it is possible to carry out classical Euclidean constructions, using a compass and straight-edge. (These assumptions characterize what Bolyai called “the absolute science of space”.)

Geometrically: These are the spaces of constant curvature (homogeneous spaces) in which there is an invariant measure of length.

Physically: these are the spaces in which a measuring-stick may be displaced in any way without changing its dimensions. Metric invariance in spaces of constant curvature (like Euclid’s0) corresponds to the free mobility of rigid bodies.

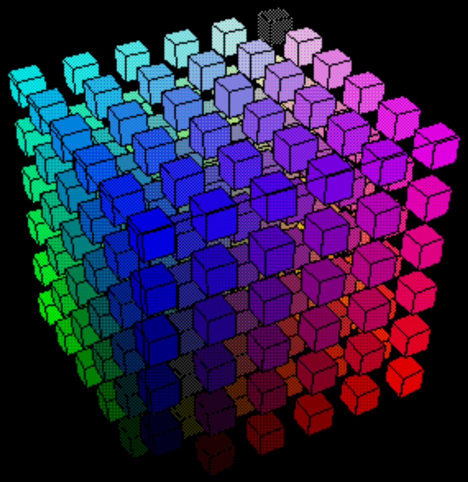
Riemann: Space as understood by Helmholtz-Poincaré is a very special case of a much more general concept.

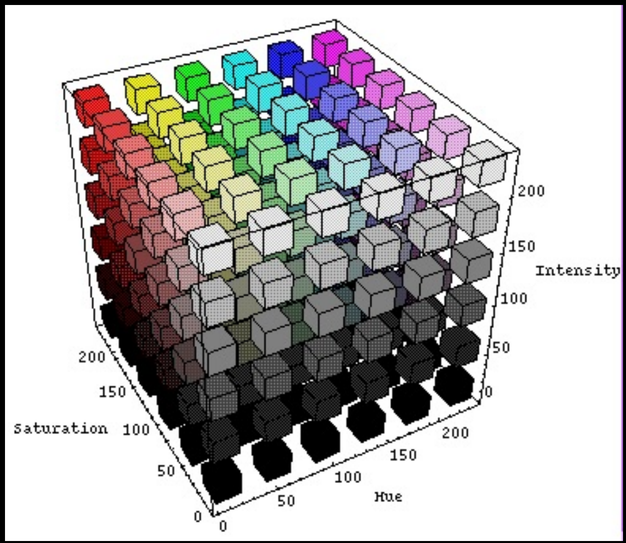
Riemann's general conception of space ("manifold"):

An n-dimensional space is an "n-fold extended aggregate"

—i.e. any aggregate in which ***n*** values are required to specify an individual. 3-D space is a three-fold extended aggregate in which three values are required to specify an individual.

"Colour space": Every colour lies in a 3-D space whose dimensions are (e.g.) RGB, or HSI ("hue, saturation, intensity")

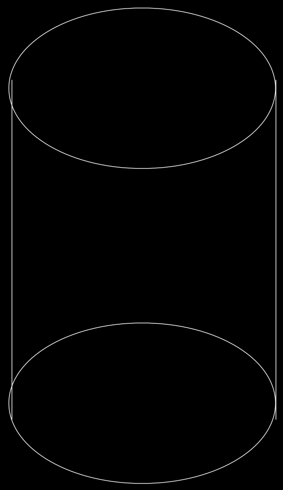


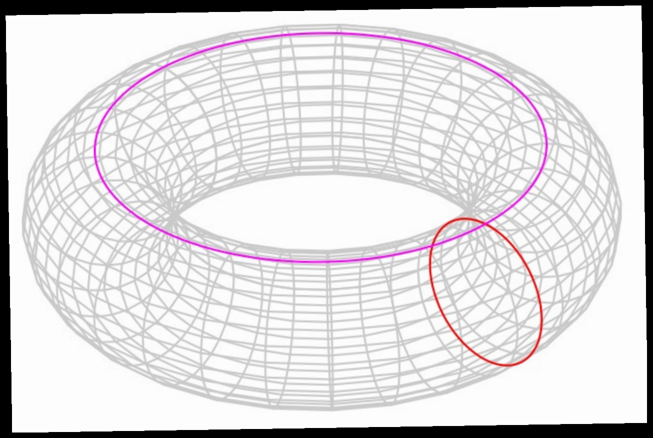


Euclidean space is a manifold whose elements can be thought of as ordered triples of real numbers, i.e., as the space $\mathbb{R}^3$ or the Cartesian product $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$.

A **cylinder** is a manifold $\mathbb{R} \times \mathbb{S}$, where $\mathbb{S}$ is the circle.

A **torus** is a manifold $\mathbb{S} \times \mathbb{S}$, so that every point lies somewhere on one circle and somewhere on another circle.





To consider 3-D space as “geometrical” space, we need to make the extra assumption that **lengths can be compared**.

To consider a 3-D homogeneous geometrical space, we need to add the further assumption of **free mobility of rigid bodies**.

In a **non-homogeneous space**, i.e. a space of variable curvature, we assume only that “infinitesimal” lengths can be compared, and that the curvature varies from point to point.

A **“differentiable manifold”** is a manifold on which all derivatives are defined, i.e. on which calculus is possible.

A **“Riemannian manifold”** is any continuous manifold that is “locally flat,” or locally Euclidean, but whose geometry varies continuously from point to point.

A **“Lorentzian manifold”** is one that is 4-D and **locally Minkowskian**, rather than Euclidean.

Scalar quantity: one that can be specified by a simple magnitude. An example of a **scalar field** is the distribution of heat on the surface of a frying pan. At every point on the disc, there is a value for the temperature at that point.

Vector quantity: one that must be specified by both by a magnitude and by a direction. An example of a **vector field** is the flow of heat in a convection oven. At every point inside the oven, there is a value for both the temperature at that point and the direction in which heat is flowing.

Tensor quantity: one that is a function of some number of vectors and yields a real number. An example of a tensor is the inner product, which takes two vectors and yields a real number. An example of a **tensor field** is the stress on a body that is subject to multiple forces (e.g. sagging shelf in a gravitational field). At every point there are stresses pulling in three independent directions.

$$ds^2 = \sum_{a,b} g_{ab}\,dx_a dx_b = \begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{12} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} dx_1dx_1 & dx_1dx_2 & dx_1dx_3 & dx_1dx_4 \\ dx_2dx_1 & dx_2dx_2 & dx_2dx_3 & dx_2dx_4 \\ dx_3dx_1 & dx_3dx_2 & dx_3dx_3 & dx_3dx_4 \\ dx_4dx_1 & dx_4dx_2 & dx_4dx_3 & dx_4dx_4 \end{pmatrix}$$

$$\begin{aligned} &= g_{11}dx_1dx_1 + g_{12}dx_1dx_2 + g_{13}dx_1dx_3 + g_{14}dx_1dx_4 \\ &+ g_{21}dx_2dx_1 + g_{22}dx_2dx_2 + g_{23}dx_2dx_3 + g_{24}dx_2dx_4 \\ &+ g_{31}dx_3dx_1 + g_{32}dx_3dx_2 + g_{33}dx_3dx_3 + g_{34}dx_3dx_4 \\ &+ g_{41}dx_4dx_1 + g_{42}dx_4dx_2 + g_{43}dx_4dx_3 + g_{44}dx_4dx_4 \end{aligned}$$

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We can arrive at the Minkowski metric as the special case where the matrix is:

$$g_{ac} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

so that all but the diagonal terms cancel, and we have the familiar Minkowski formula:

$$ds^2 = dx_1^2 - dx_2^2 - dx_3^2 - dx_4^2$$

Mathematical representation vs. *conceptual* representation:

“Any reasonable formulation of the problem of how mathematic represents reality must be predicated on the assumption that we can provisionally take for granted what is meant by *conceptually representing* reality, and that we can also take for granted that a conceptual representation does not reduce to or presuppose a *mathematical* representation. Otherwise we would be forced to reject Frege’s celebrated solution to the problem of how arithmetic *applies* to reality. For Frege, this is explained by the fact that our judgments of cardinality rest on relations between concepts, and concepts sometimes apply to reality.” (Demopoulos 2013)

Hume’s Principle (after Frege):

For any concepts *F* and *G*, the number of *F*s is identical with the number of *G*s if and only if the *F*s and the *G*s are in one-one correspondence.

“This analysis captures a central feature of our notion of number and reveals the assumptions on which our conception of their infinity may be based….

….The plausibility of the idea that Hume’s principle is analytic of the concept of numerical identity depends on the plausibility of a conceptual analysis; but the truth of the principle that expresses this analysis depends on the presuppositions of the framework of which the analysis *is* an analysis.” (Demopoulos 2000)

If we use the term *spatial to* designate those relations which we can alter directly by our volition but whose nature may still remain conceptually unknown to us, an awareness of mental states or conditions does not enter into spatial relations at all….

From this point of view, space is the necessary form of outer intuition, since we consider only what we perceive as spatially determined to constitute the external world. Those things which are not perceived in any spatial relation we think of as belonging to the world of inner intuition, the world of self-consciousness.
(Helmholtz 1878)

Poincaré:

1. In the first place, we distinguish two categories of phenomena: — The first involuntary, unaccompanied by muscular sensations, and attributed to external objects—they are external changes; the second, of opposite character and attributed to the movements of our own body, are internal changes.

2. We notice that certain changes of each in these categories may be corrected by a correlative change of the other category.

3. We distinguish among external changes those that have a correlative in the other category — which we call displacements; and in the same way we distinguish among the internal changes those which have a correlative in the first category.

Thus by means of this reciprocity is defined a particular class of phenomena called displacements. *The laws of these phenomena are the object of geometry.*

(*La science et l'hypothèse*, 1902)

Definition of a mathematical group:

A set $\boldsymbol{G}$ of elements together with a binary operation satisfying four properties:

1. Closure: If f and g are two elements in $\boldsymbol{G}$, then the product fg is also in $\boldsymbol{G}$.
2. Associativity: for all f, g, h, in $\boldsymbol{G}$, $(fg)h = f(gh)$.
3. Identity: There is an identity, $\boldsymbol{I}$, such that for every f,

$$f\boldsymbol{I} = \boldsymbol{I}f = f.$$

4. Inverse: Every element f has an inverse $-f$, such that

$$-f(f) = f(-f) = \boldsymbol{I}.$$

Think of the image of the world in a convex mirror.... the images are diminished and flattened in proportion to the distance of their objects from the mirror.... Yet every straight line or every plane in the outer world is represented by a straight line or a plane in the image. The image of a man measuring with a rule a straight line from the mirror would contract more and more the farther he went, but with his shrunken rule the man in the image would count out exactly the same number of centimetres as the real man. And, in general, all geometrical measurements of lines or angles made with regularly varying images of real instruments would yield exactly the same results as in the outer world, all congruent bodies would coincide on being applied to one another in the mirror as in the outer world, all lines of sight in the outer world would be represented by straight lines of sight in the mirror. (Helmholtz 1870)

In short I do not see how men in the mirror are to discover that their bodies are not rigid solids and their experiences good examples of the correctness of Euclid's axioms.

(Helmholtz 1870)

We can even go a step further, and infer how the objects in a pseudo-spherical world, were it possible to enter one, would appear to an observer, whose eye-measure and experiences of space had been gained like ours in Euclid's space. Such an observer would continue to look upon rays of light or the lines of vision as straight lines, such as are met with in flat space....He would think he saw the most remote objects round about him at a finite distance....But as he approached these distant objects, they would dilate before him, though more in the third dimension than superficially, while behind him they would contract. He would know that his eye judged wrongly. In short, pseudo-spherical space would not seem to us very strange, comparatively speaking; we should only at first be subject to illusions in measuring by eye the size and distance of the more remote objects. (Helmholtz 1870)

“If it were useful for any purpose, we might with perfect consistency look upon the space in which we live as the apparent space behind a convex mirror with its shortened and contracted background....Only then we should have to ascribe to the bodies which appear to us to be solid, and to our own body at the same time, corresponding distensions and contractions, and we should have to change our system of mechanical principles entirely.... Thus the axioms of geometry are not concerned with space-relations only but also at the same time with the mechanical deportment of solid bodies in motion.”

In short I do not see how men in the mirror are to discover that their bodies are not rigid solids and their experiences good examples of the correctness of Euclid's axioms.

But if they could look out upon our world as we can look into theirs, without overstepping the boundary, they must declare it to be a picture in a spherical mirror, and would speak of us just as we speak of them; and if two inhabitants of the different worlds could communicate with one another, neither, so far as I can see, would be able to convince the other that he had the true, the other the distorted, relations. Indeed I cannot see that such a question would have any meaning at all, so long as mechanical considerations are not mixed up with it. (Helmholtz 1870)

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“Since the mathematicians have invaded the theory of relativity, I do not understand it myself anymore.”

(Einstein, 1908?)

“These inadequate remarks can give the reader only a vague notion of the important idea contributed by *Minkowski*. Without it the general theory of relativity, of which the fundamental ideas are developed in the following pages, would perhaps still be in its infancy [‘*in den Windeln*,’ or ‘in diapers’].”

(Einstein, 1916)

A philosophy of science guy on Minkowski space-time:

“Not long after Einstein’s creation of this new theory, Minkowski recognized that Einstein had in effect displayed an elegant new mathematical entity....Light paths are to be represented by curves in this space along which the space-time interval equals zero...motions of bodies by paths on which points have time-like separation...and so forth. That is to say, according to the new theory, we are to use this mathematical object in that way to represent the natural phenomena in this domain….

…At this point observers and frames of reference are left behind. Neither perception nor individual cognition is a salient topic of inquiry in the context of use of Minkowski space....If someone is to use Einstein’s theory to predict...choice of coordinate system correlated to a defined physical frame of reference is required. The user must leave behind the God-like reflections on the structure of space-time in order to apply the implications of those reflections to his or her actual situation. (2008)

Geometry as formalism, interpreted or uninterpreted:

Mathematical formalism as pure uninterpreted structure, vs. physical theories as formal calculi requiring an interpretation:

Carnap: “The development of physics in recent centuries, and especially in the past few decades, has more and more led to that method in the construction, testing, and application of physical theories which we call formalization, i.e., the construction of a calculus supplemented by an interpretation. It was the progress of knowledge and the particular structure of the subject matter that suggested and made practically possible this increasing formalization. In consequence it became more and more possible to forego an ‘intuitive understanding’ of the abstract terms and axioms and theorems formulated with their help.” (Carnap, 1939).

“It is clear that the system of concepts of axiomatic geometry alone cannot make any assertions as to the behavior of real objects of this kind, which we will call practically-rigid bodies. To be able to make such assertions, geometry must be stripped of its merely logical-formal character by the coordination of real objects of experience with the empty conceptual schemata of axiomatic geometry….

Geometry thus completed is evidently a natural science; we may in fact regard it as the most ancient branch of physics. Its affirmations rest essentially on induction from experience, but not on logical inferences only. We will call this completed geometry ‘practical geometry,’ and shall distinguish it in what follows from ‘purely axiomatic geometry.’ The question whether the practical geometry of the universe is Euclidean or not has a clear meaning, and its answer can only be furnished by experience….

I attach special importance to the view of geometry, which I have just set forth, because without it I should have been unable to formulate the theory of relativity.” (Einstein, 1921)

A contemporary view of **interpretation**: What would the world be like if the theory were true?”

A better question: what are the aspects of our experience that tell us that the world is such as the theory says it is?

How do we know that we live in a Newtonian, or in Minkowski space-time, or in a general-relativistic space-time? Or in a quantum world?

What aspects of experience give us insight into these structures?

Minkowski's spacetime diagram:

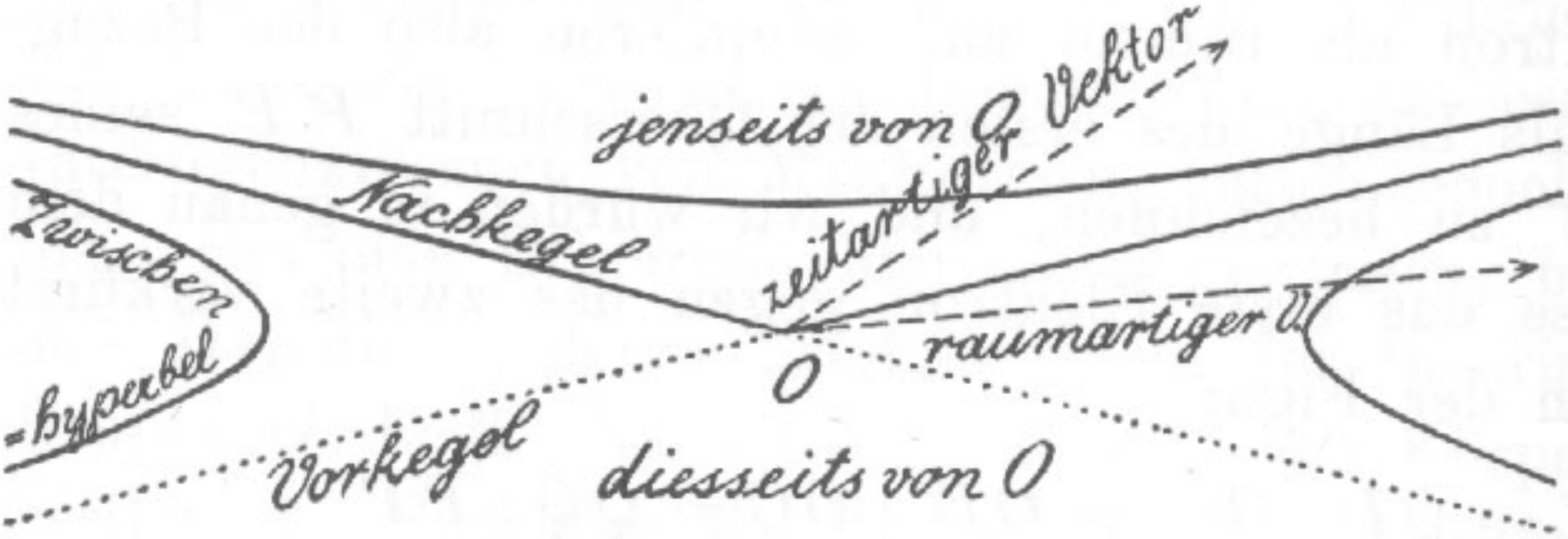
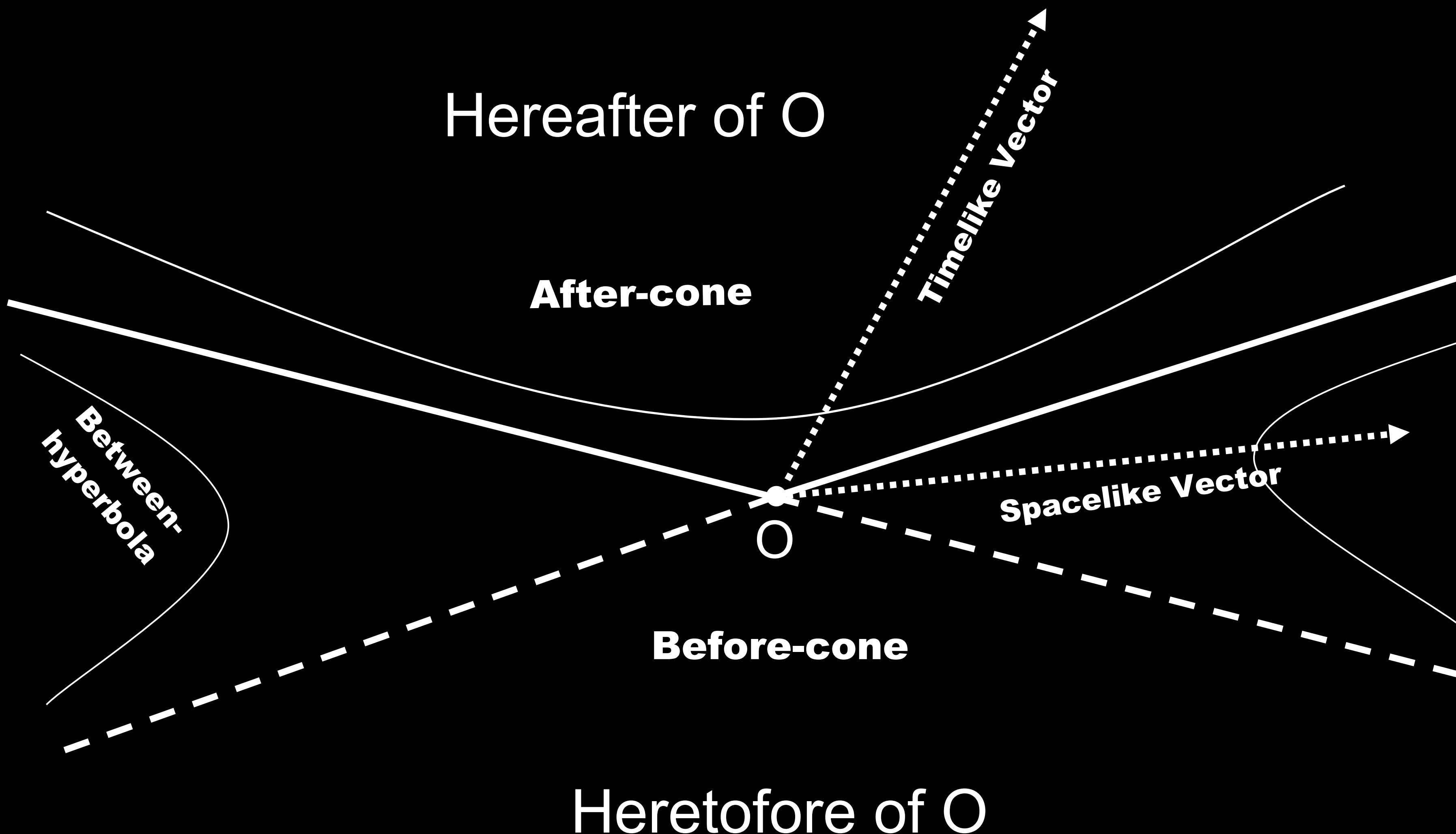


Fig. 2.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Timelike Vector
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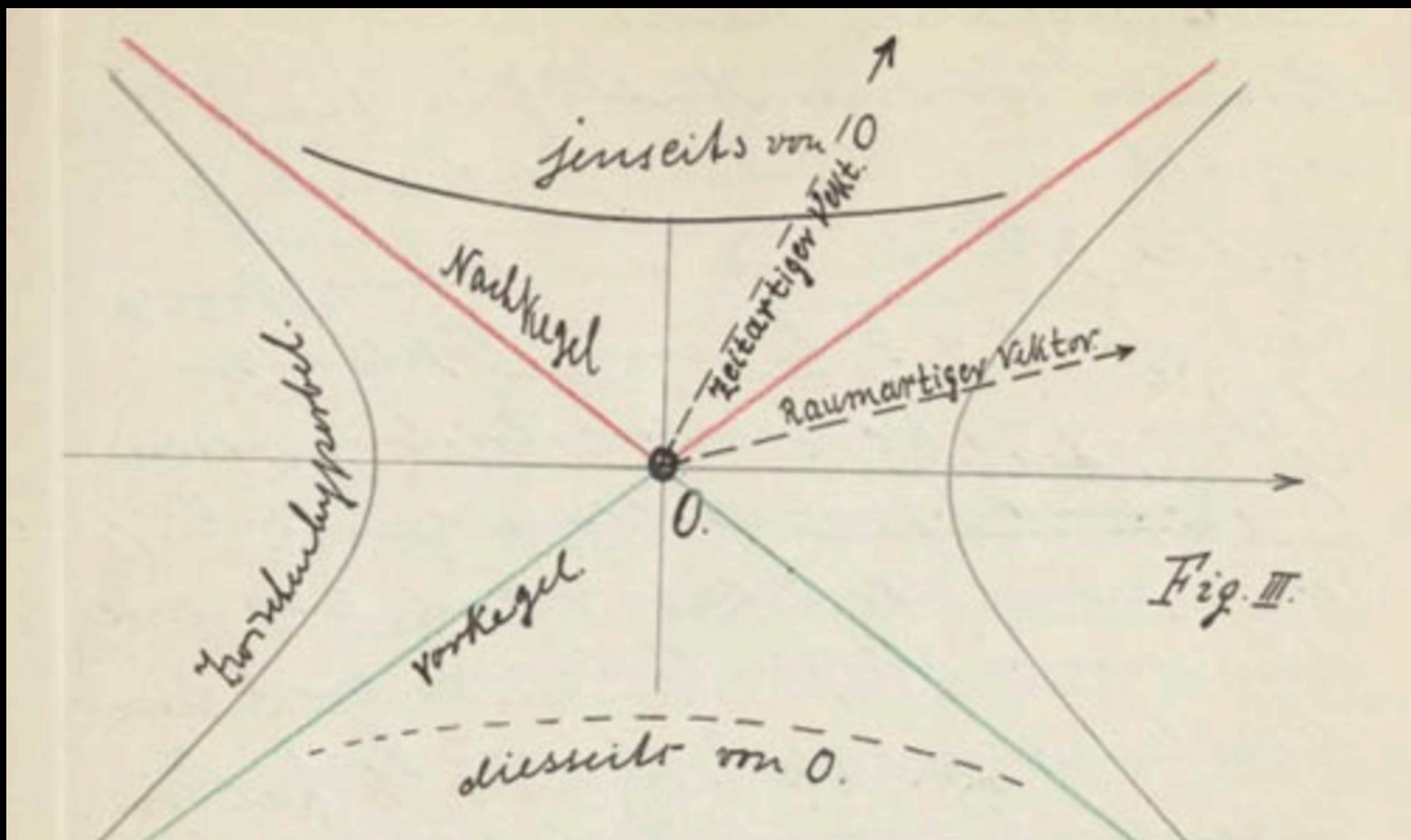
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Fig. III.

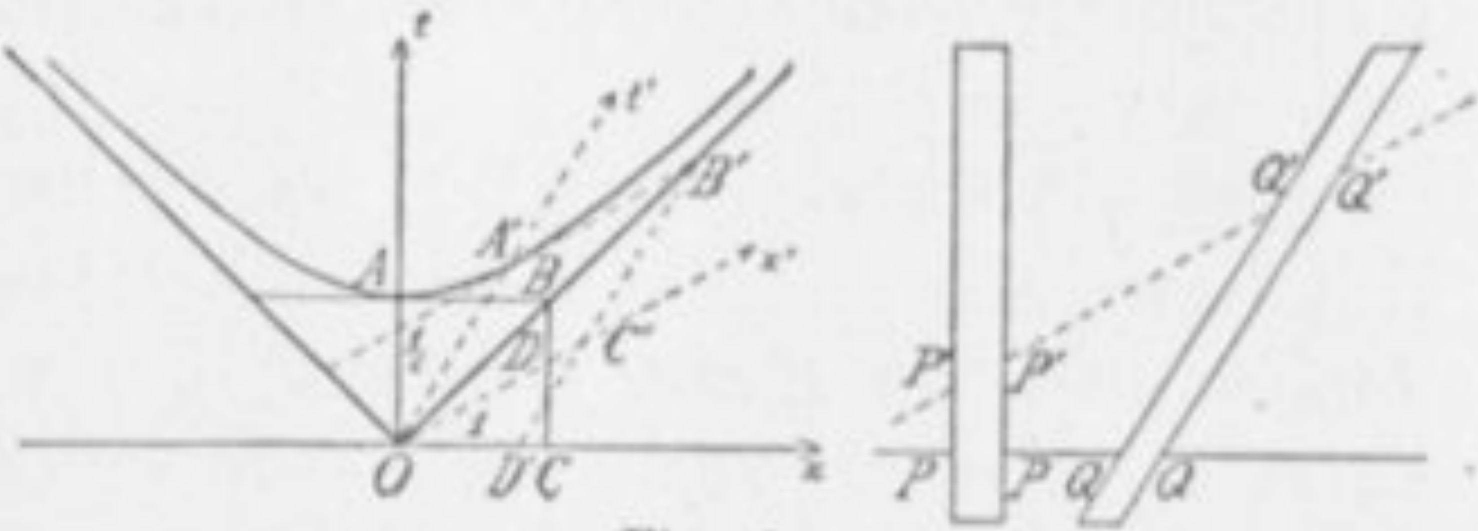


Fig. 1.

Now we allow ***c*** to increase to infinity, so that ***1/c*** converges to zero, and then the figure shows that the branch of the hyperbola ever more closely approaches the ***x*** axis, the angle of the asymptote expands to a straight angle, and that special transformation transforms in the limit to one in which the ***t*** axis can have any arbitrary upward direction, and ***x'*** ever more closely approximates ***x***.

....It is clear that the group $\mathbf{G_c}$, in the limit where $c = \infty$, thus the group $\mathbf{G_\infty}$, becomes just that complete group that belongs to Newtonian mechanics. From this situation, and from the fact that $\mathbf{G_c}$ is mathematically more intelligible than $\mathbf{G_\infty}$, the free imagination of a mathematician might well have stumbled upon the thought that at bottom, natural phenomena have an invariance not with the group $\mathbf{G_\infty}$, but with the group $\mathbf{G_c}$, where c is finite and determinate, but, in ordinary units, extremely large. Such a premonition would have been an extraordinary triumph for pure mathematics. Now that mathematics can only display its staircase wit, there remains the satisfaction that it can...draw out the far-reaching consequences of this transformation in our conception of nature. (Minkowski 1909, p. 105)

For example, in correspondence with the previous figure, we can also denote time as ***t'***, but then it would become necessary to define space by the manifold of the three parameters ***x', y', z'***, and then physical laws would be expressed in exactly the same way by ***x', y', z', t'***, as by ***x, y, z, t***. In that case we would have, in the world, no longer space, but infinitely many spaces, just as there are in three-dimensional space infinitely many planes.Three-dimensional geometry becomes a chapter in four dimensional physics. Now you will perceive why I said, at the beginning, that space and time would fall away into the shadows, and only a world in itself persists.

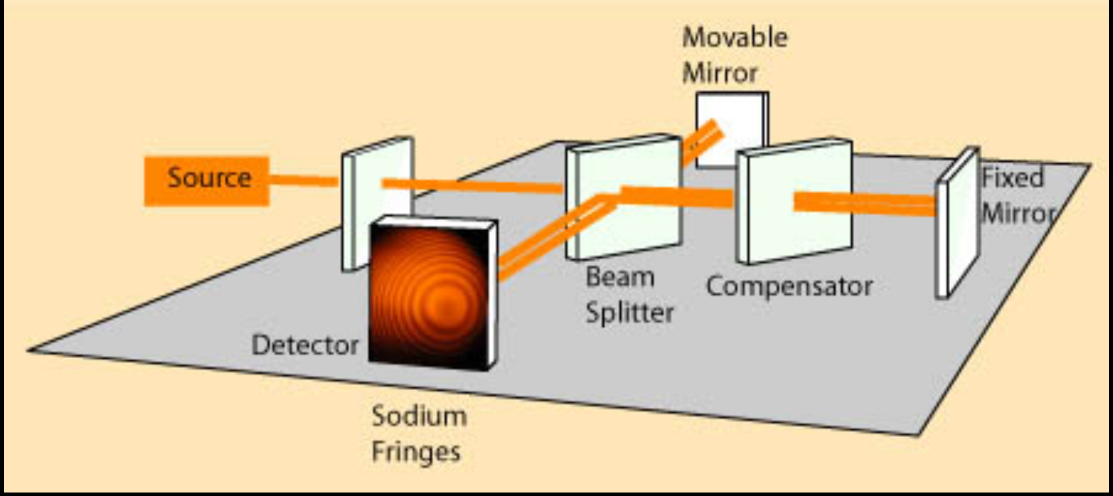
The Michelson-Morley experiment: An ingenious experiment that was delicate enough to detect the minute differences in the velocity of light in different directions, even though they are proportional to

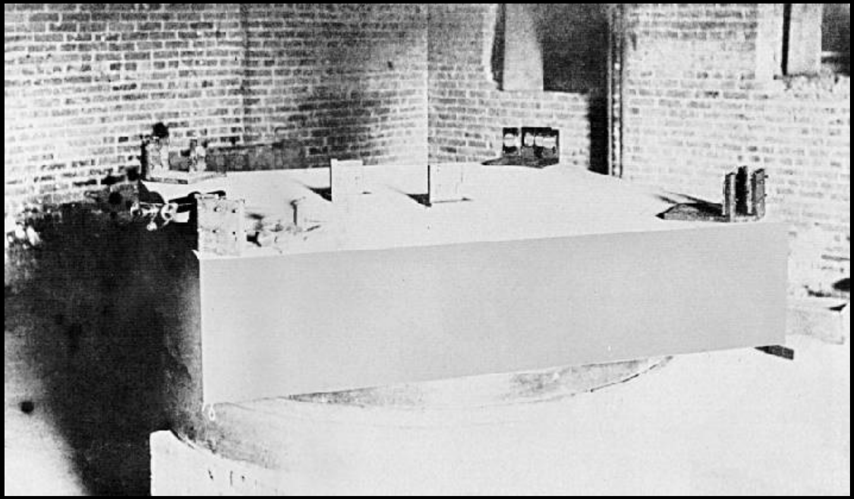
$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where ***v*** is the velocity of the moving system and ***c*** is the velocity of light. Evidently this is a minute quantity for velocities that are very small compared to the velocity of light.

Michelson split a beam of light into two parts, moving in two perpendicular directions, and then reflected back to the source. He realized that if the beams traveled with different velocities, they would not return at the same time, and that the slightest difference in timing would cause a detectable effect of interference. (If the beams did not return at the same time, they would be "out of phase.")

Michelson's interference experiment





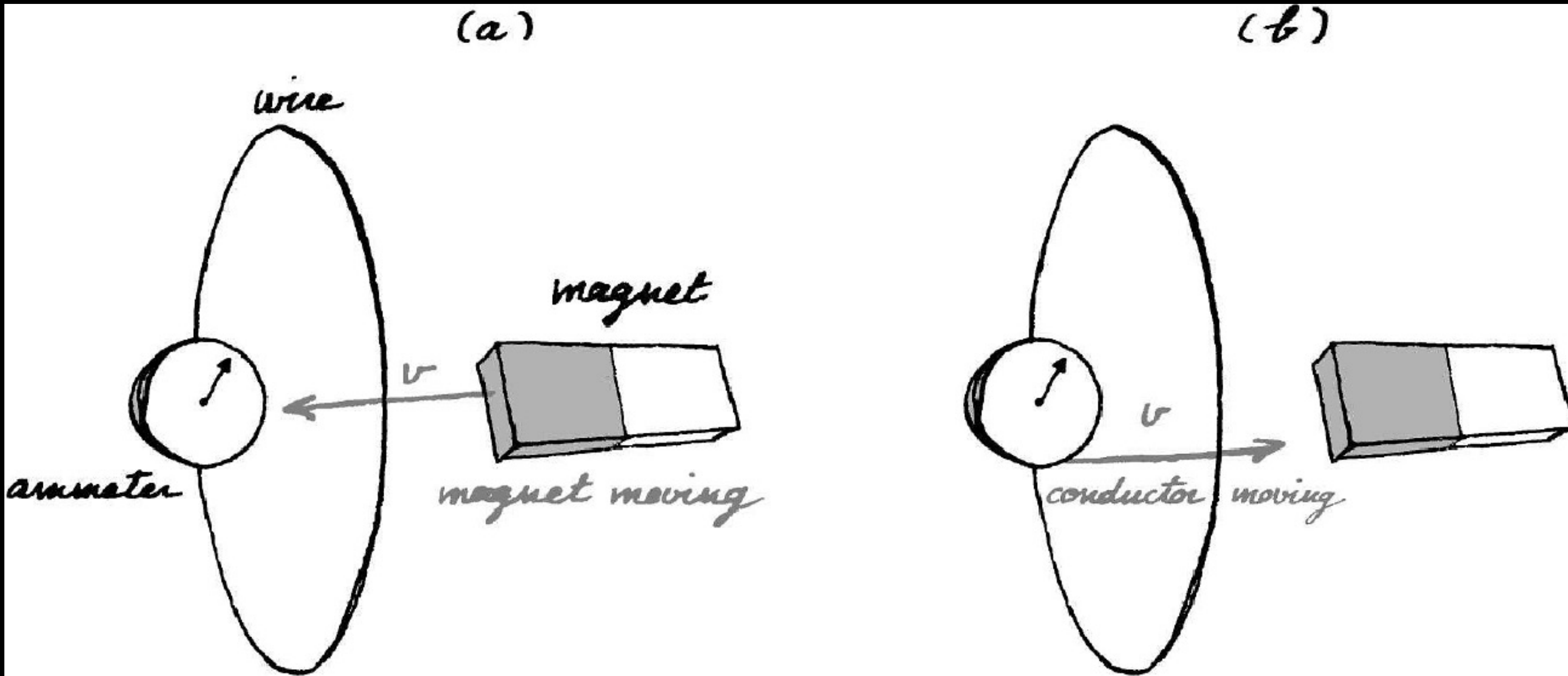
Einstein: Light really does travel at the same speed in all directions, no matter what the motion of the source or or the observer. Nature is clearly telling us this through experiments like Michelson’s.

If this sounds contradictory to us, then the problem must be hidden in the assumptions that lie behind the experiments.

Einstein was about the only physicist who was ***not*** surprised by the null outcome of the Michelson-Morley experiments.

Even before the Michelson Morley experiment, Einstein was struck by theoretical distinctions in electrodynamics that made *no physical difference.*

The relative motion of a magnet and a conductor:



If the magnet moves and the conductor rests, an electric field is created. If the conductor moves and the magnet rests, an "electromotive force" is created. But both cases ***the same observable phenomenon***: an electric current whose value (measured by the ammeter) depends ***only on the relative motion***.

Einstein's new principle of relativity:

"Examples like these, together with the unsuccessful attempts to discover any motion of the earth relatively to the 'light medium', suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest.

They suggest, rather, that…the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good." (1905)

But, as we've already seen, this seems to lead to a contradiction. In Newtonian relativity, velocity is relative, and only ***change*** of velocity (acceleration) is invariant. How can there be an invariant velocity? (Think of the example of the trains in relative motion.)

Einstein's solution: We only ***think*** there is a contradiction because of our ***unexamined assumptions about time***.

We have to analyze what we mean by time, and how we measure time.

This analysis starts with the concept of *simultaneity*.

Einstein on the philosophical basis of special relativity:

“The theory that is to be developed rests—like all electrodynamics—on the kinematics of the rigid body, since the assertions of any such theory concern the relationships between rigid bodies (systems of coordinates), clocks, and electromagnetic processes. Insufficient consideration of this circumstance lies at the root of the difficulties which the electrodynamics of moving bodies presently has to struggle.” (1905)

“A mathematical description [of the motion of a material point] has no physical meaning unless we are quite clear as to what we understand by ‘time.’” (1905)

Einstein on the concept of time:

“If we wish to describe the motion of a material point, we give the values of its coordinates as a function of time. Now we mist bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by ‘time’.” (1905)

“We encounter the same difficulty with all physical statements in which the conception " simultaneous " plays a part. The concept does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case. We thus require a definition of simultaneity that supplies us with the method by means of which, in the present case, he can decide by experiment whether or not both the lightning strokes occurred simultaneously. As long as this requirement is not satisfied, I allow myself to be deceived as a physicist (and of course the same applies if I am not a physicist), when I imagine that I am able to attach a meaning to the statement of simultaneity.” (1917)

Why did Einstein think that simultaneity is the source of the problem?

Moments of time seem like numbers in the number line. One of the following relations is true for any numbers $\boldsymbol{x}$, $\boldsymbol{y}$:

$\boldsymbol{x} > \boldsymbol{y}$ OR

$\boldsymbol{x} < \boldsymbol{y}$ OR

$\boldsymbol{x} = \boldsymbol{y}$

If you don't understand this, perhaps you don't understand how numbers work. Why isn't it the same for moments of time $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$?

$\boldsymbol{t}_1$ is later than $\boldsymbol{t}_2$, OR

$\boldsymbol{t}_1$ is earlier than $\boldsymbol{t}_2$ OR

$\boldsymbol{t}_1$ is simultaneous with $\boldsymbol{t}_2$.

This seems equally obvious, but is it equally true? Is the relation of simultaneity equally objective?

Einstein's reconciliation of the relativity principle with the light principle (1905):

At the time $t = \tau = 0$, when the origin of the co-ordinates is common to the two systems, let a spherical wave be emitted therefrom, and be propagated with the velocity c in system K. If (x, y, z) be a point just attained by this wave, then

$$x^2 + y^2 + z^2 = c^2t^2.$$

Transforming this equation with the aid of our equations of transformation we obtain after a simple calculation

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2.$$

The wave under consideration is therefore no less a spherical wave with velocity of propagation c when viewed in the moving system. This shows that our two fundamental principles are compatible.

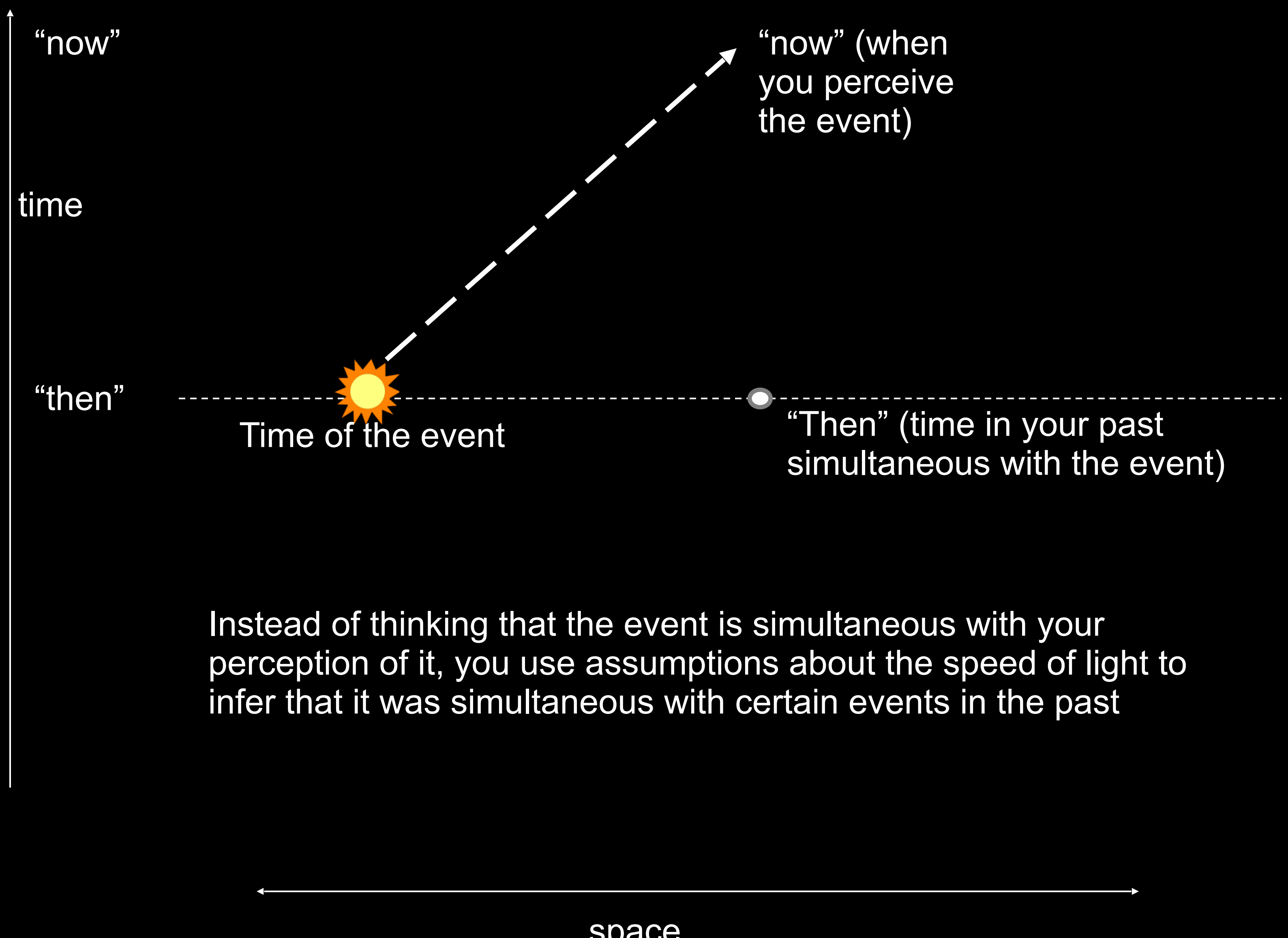
What has simultaneity to do with experiments on the speed of light?

Einstein: All measurements of the speed of light assume the objectivity of simultaneity. Subjectively, we can tell if we sense two events simultaneously in our own sense organs. But for an objective view events at a distance, we need to make some assumptions about the signals that those events send us.

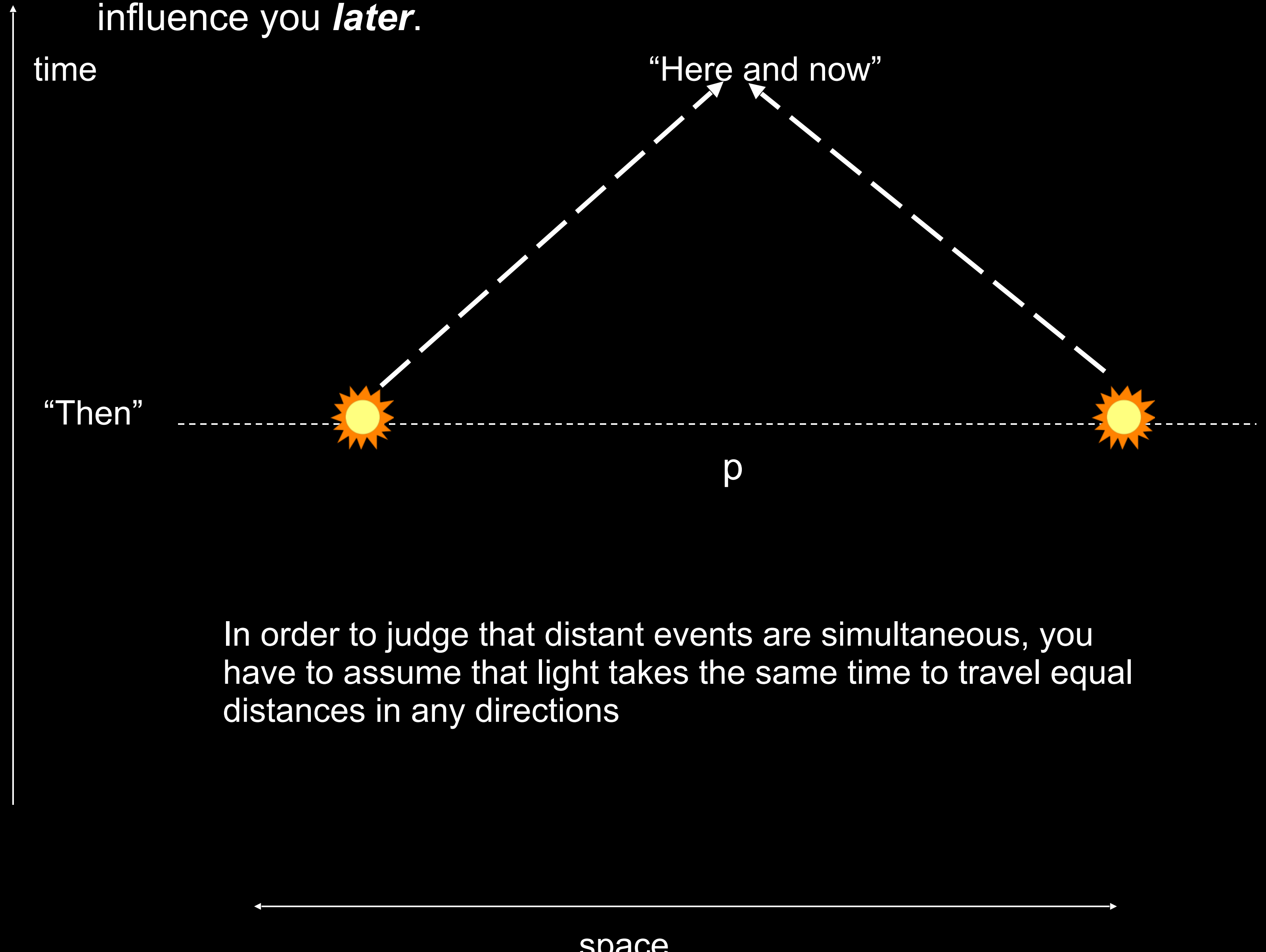
We can tell if light has the same velocity in two directions if they propagate from the source, are reflected at equal distances at points A and B, and then return to the source ***at the same time***.

But this requires us to stipulate that light takes the same time to travel from A to B as from B to A.

“There is only one demand to be made of the definition of simultaneity, namely, that in every real case it must supply us with an empirical decision as to whether or not the conception that has to be defined is fulfilled. That my definition satisfies this demand is indisputable. That light requires the same time to traverse the path A to B as for the path B to A is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own freewill in order to arrive at a definition of simultaneity.”

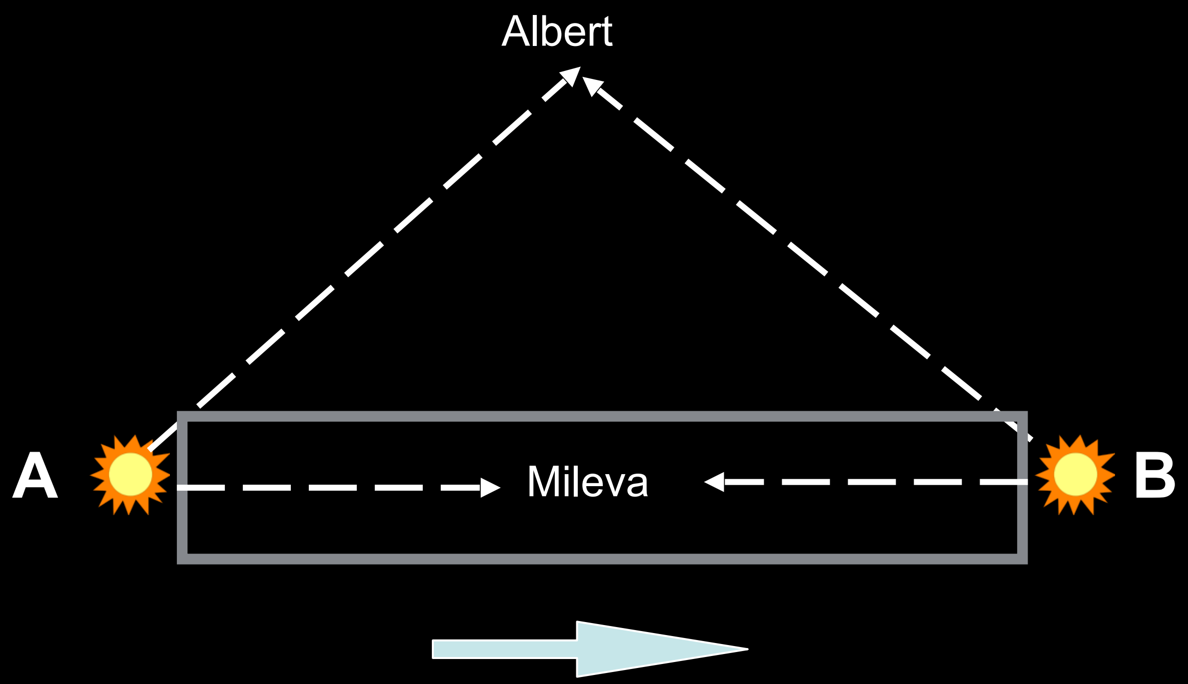

“now”
time
“then”
“now” (when you perceive the event)
Time of the event
“Then” (time in your past simultaneous with the event)
Instead of thinking that the event is simultaneous with your perception of it, you use assumptions about the speed of light to infer that it was simultaneous with certain events in the past
space

If you can't travel faster than light, then what's happening "now" can only influence you ***later***.



In order to judge that distant events are simultaneous, you have to assume that light takes the same time to travel equal distances in any directions

If a resting observer and a moving observer (relatively speaking) apply Einstein's criterion of simultaneity, they will not agree on which events are simultaneous.



Albert, at rest on the platform, will see light signals from A and B at the same time. Mileva, moving toward B, will see the light signal from B before the one from A.

Einstein on the train

(Actual train may differ)

Einstein’s reconciliation of the relativity principle with the light principle (1905):

At the time $t = \tau = 0$, when the origin of the co-ordinates is common to the two systems, let a spherical wave be emitted therefrom, and be propagated with the velocity c in system K. If (x, y, z) be a point just attained by this wave, then

$$x^2 + y^2 + z^2 = c^2t^2.$$

Transforming this equation with the aid of our equations of transformation we obtain after a simple calculation

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2.$$

The wave under consideration is therefore no less a spherical wave with velocity of propagation c when viewed in the moving system. This shows that our two fundamental principles are compatible.

Why do we have to admit the relativity of simultaneity?

Why can't we say:

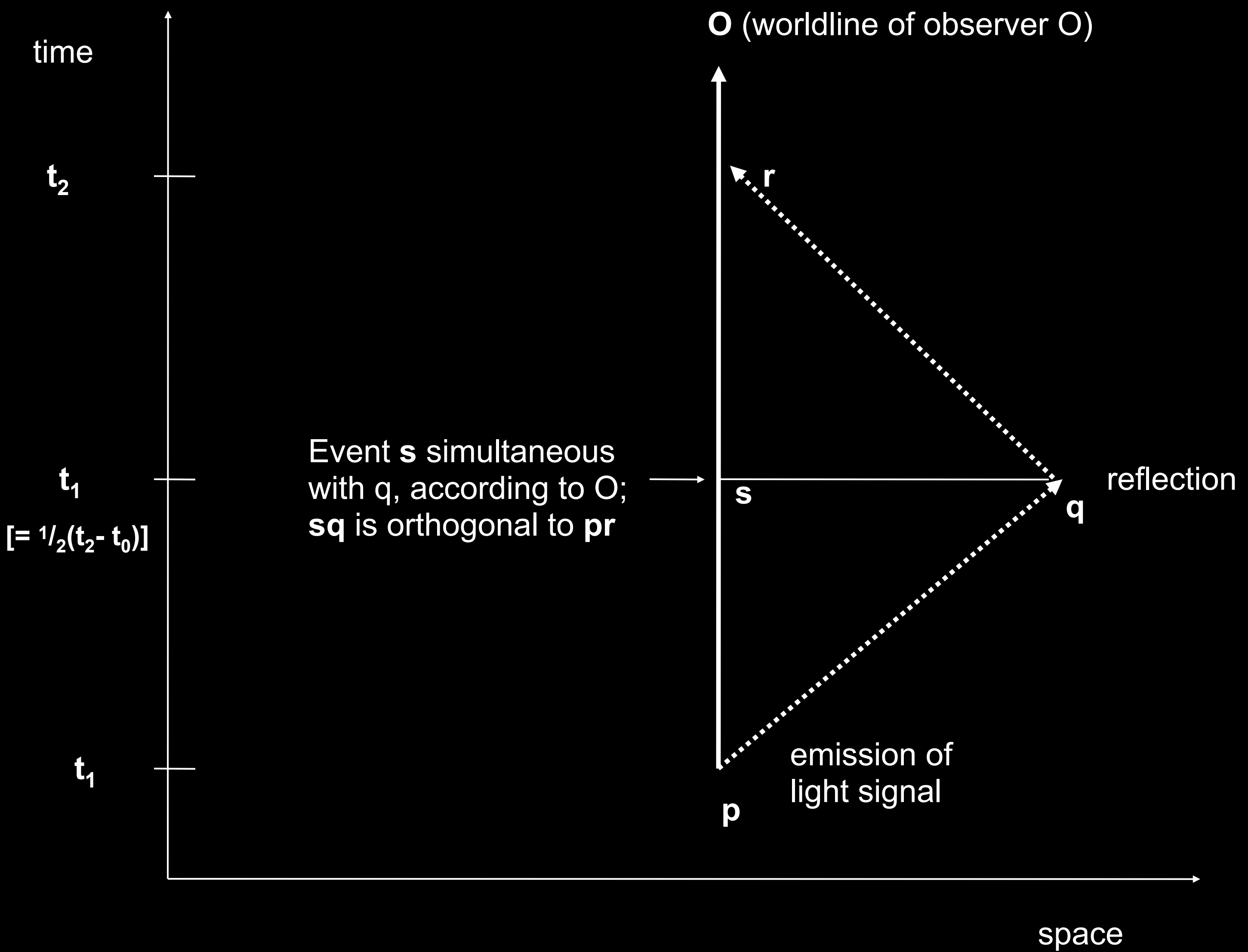
"The person on the train is wrong. Because she is moving, things that ***really*** happened simultaneously seem to happen one after the other, ***relative to her moving reference frame***. There is an objective relation of simultaneity, but she can't determine it because of her state of motion."

We ***can't*** say this, because ***there is no objective way to determine which of the two observers is really moving***.

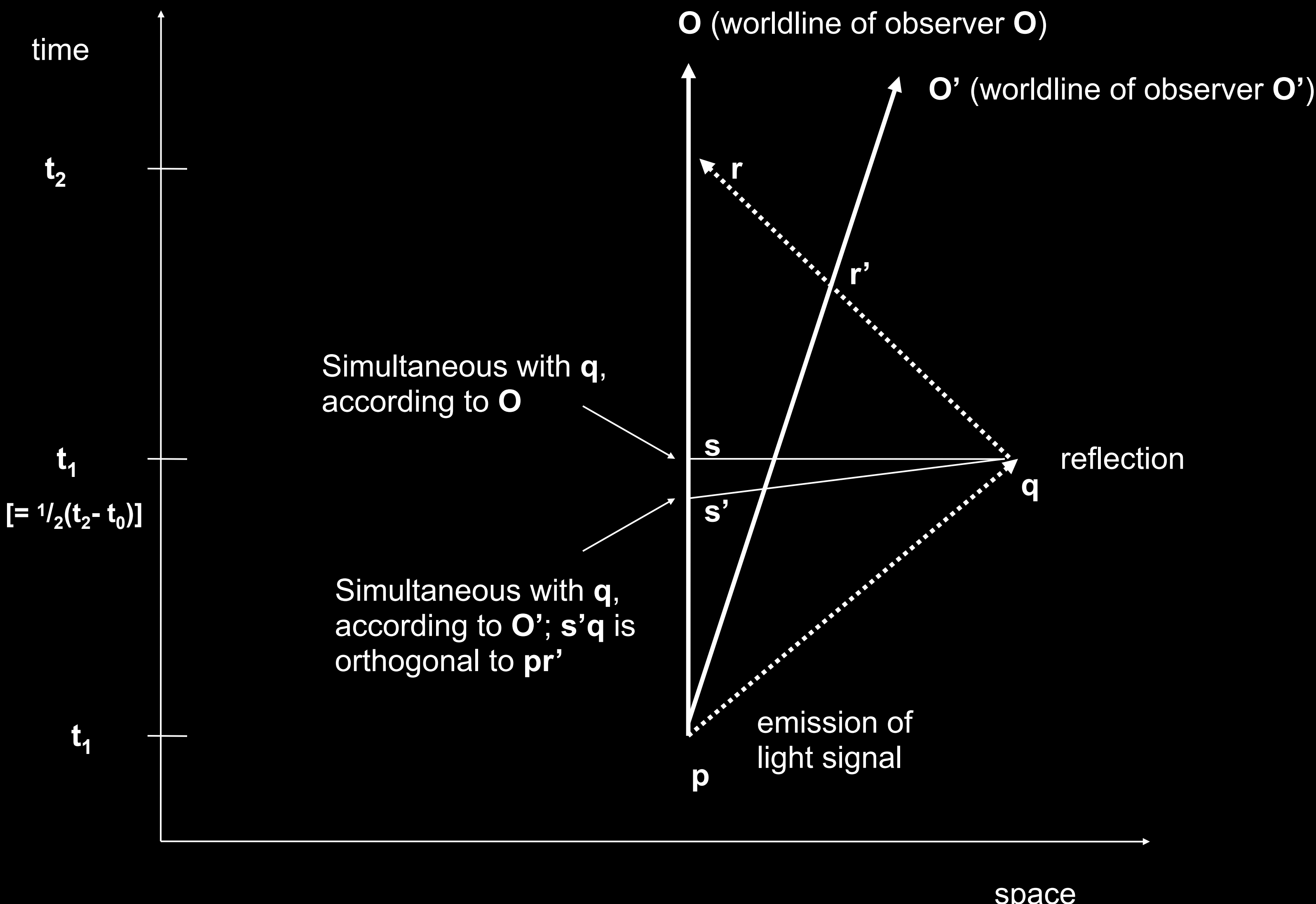
Any experiment done in a uniformly moving frame will have the same result as in a resting frame. If there is a Michelson-Morley apparatus on the train, and another on the embankment, both will give a null result. ***There is no test of whether one is uniformly moving or at rest***.

Therefore, ***there is no objective fact*** about whether the two events really were simultaneous. ***Simultaneity is a relative relation***. Observers in relative motion will not agree on which events are simultaneous.

Einstein's definition of simultaneity, in space-time geometry

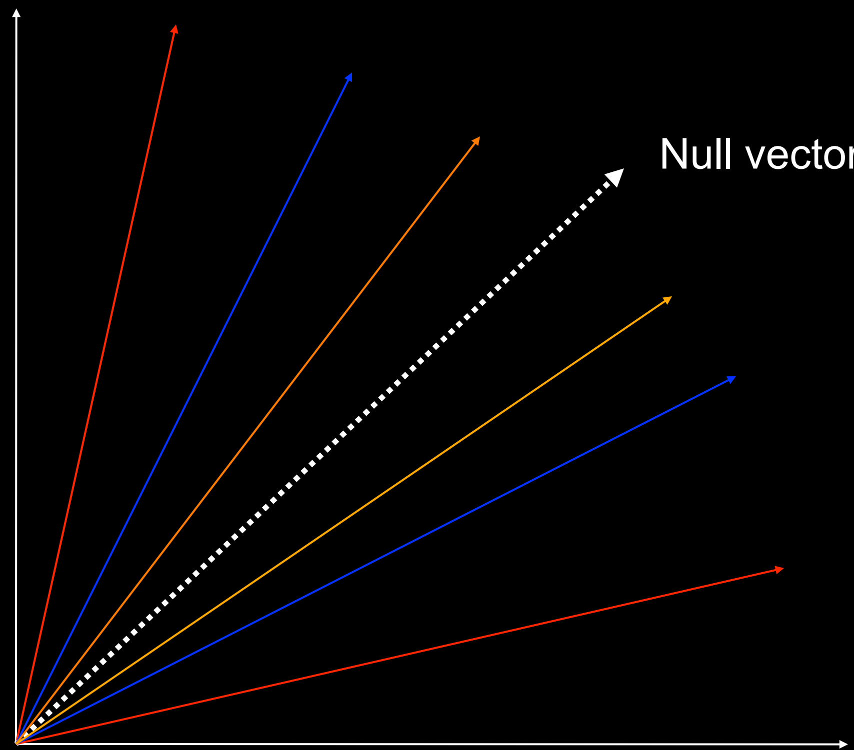


The relativity of simultaneity, in space-time geometry



Orthogonality and simultaneity

Timelike worldlines of inertial observers

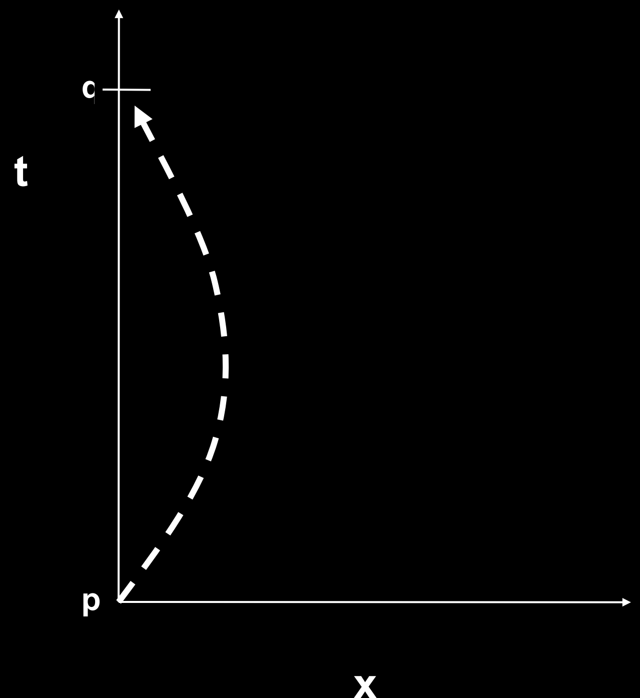


Null vector (self-orthogonal)

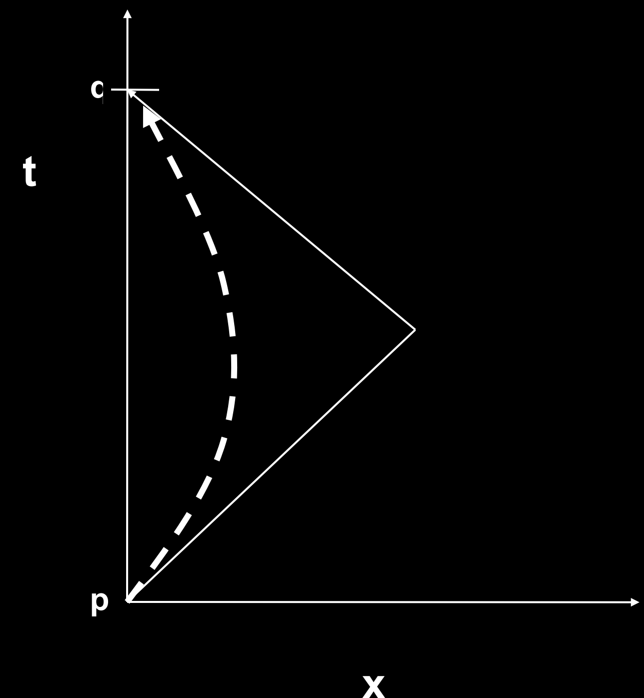
Corresponding orthogonal surfaces
(simultaneity slices)

The indefinite metric, I = t - x2, implies that a nonzero vector can have zero length, and can be orthogonal to itself.

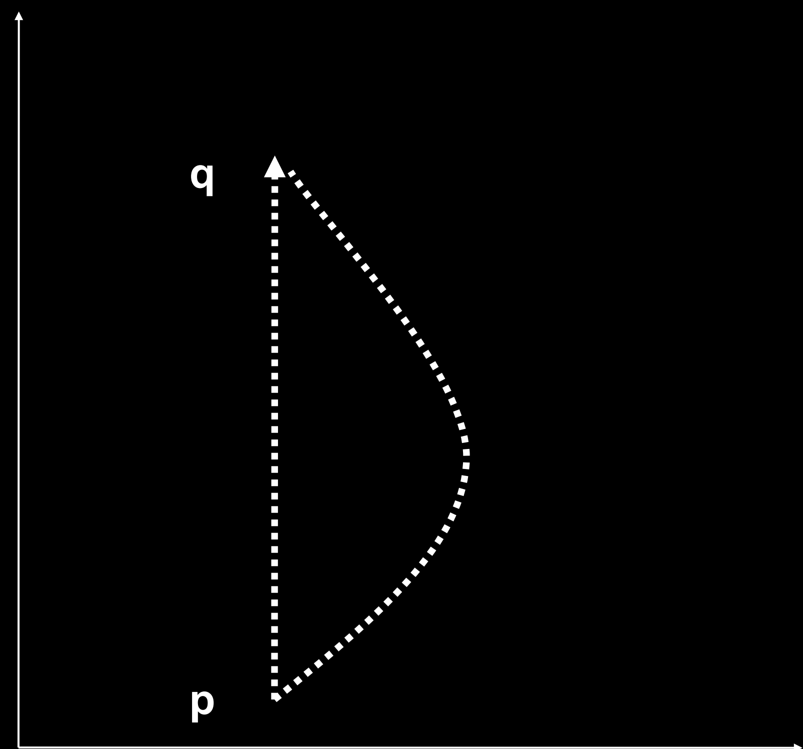
If space-time were Euclidean, with metric $t^2 + x^2$, the straight line would be the shortest distance from **p** to **q**.



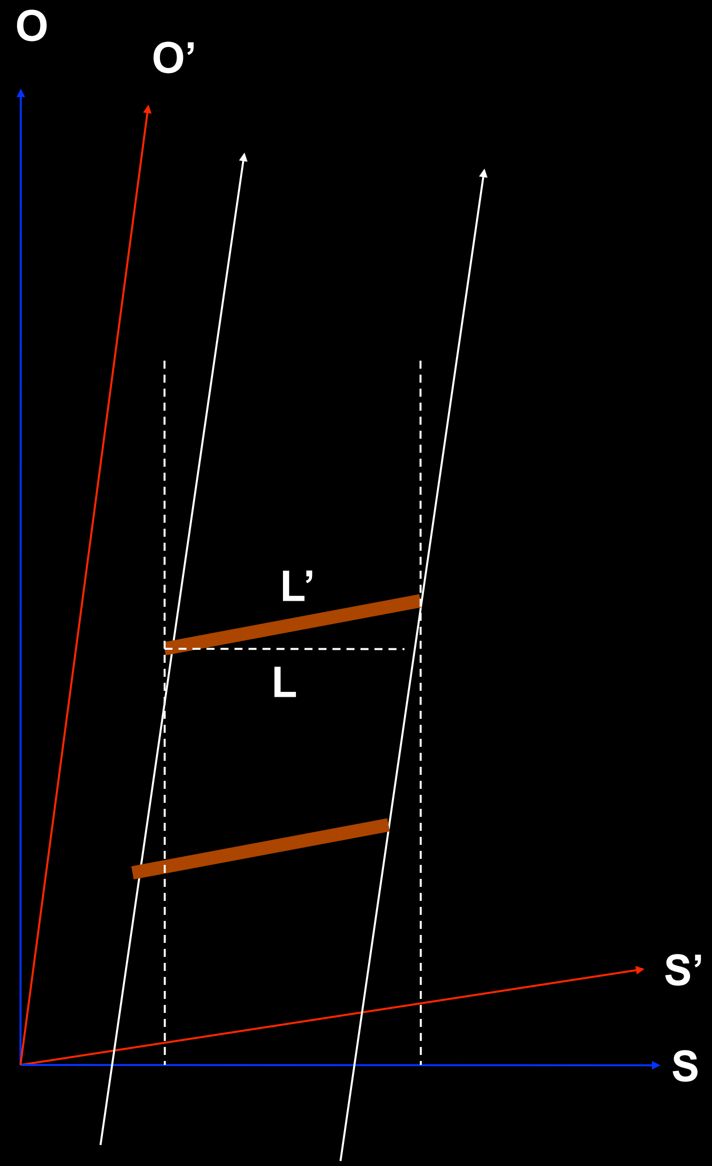
In Minkowski space-time, with metric $t^2 - x^2$, the straight line is necessarily the *longest* distance from **p** to **q**. The light-path has length *zero*.



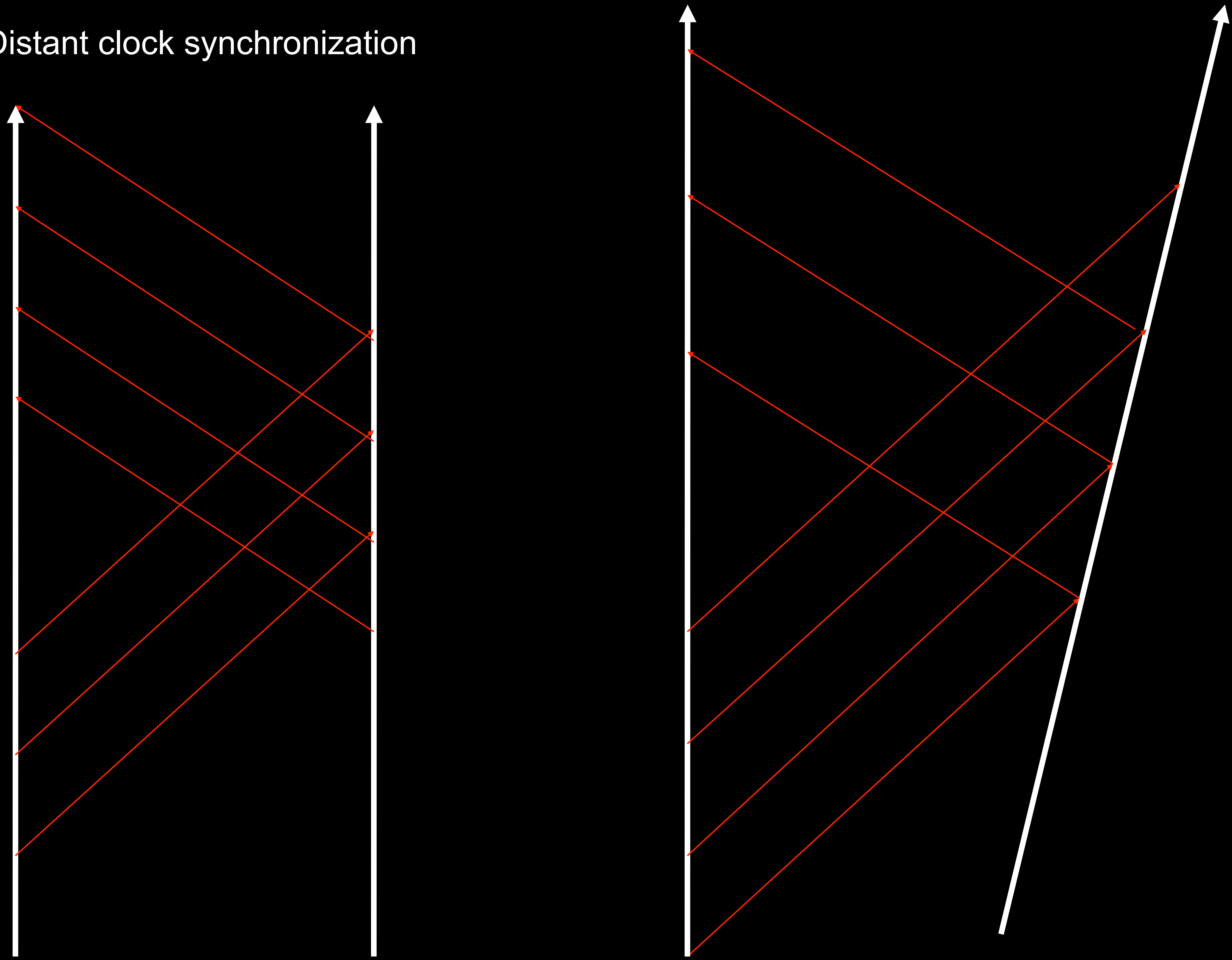
The twin paradox: A pair of twins separates, one remains at rest, the other goes away at a great velocity and returns. If time is relative, why is it that the resting twin is now older?



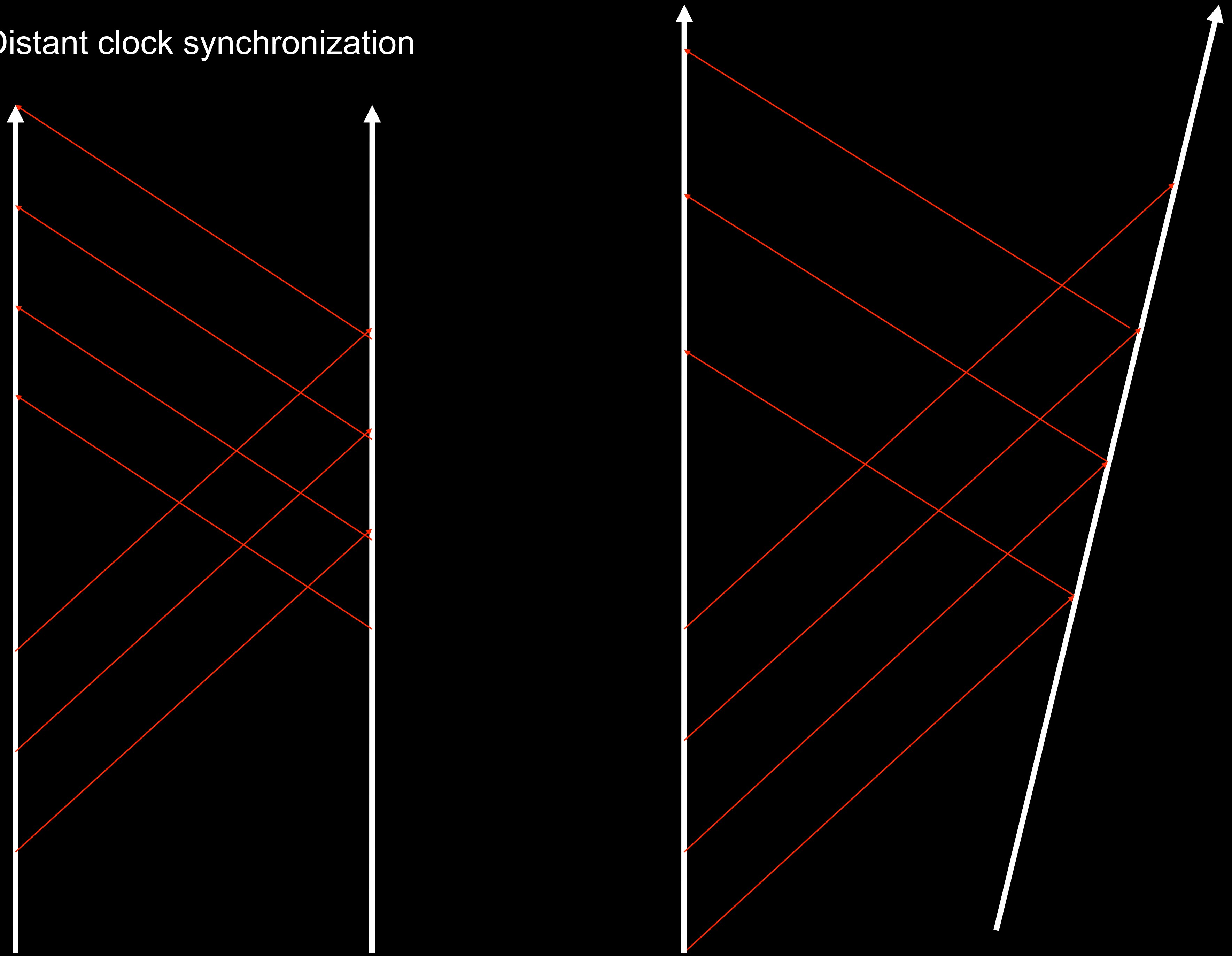
Comparison of length in relatively moving frames



Distant clock synchronization



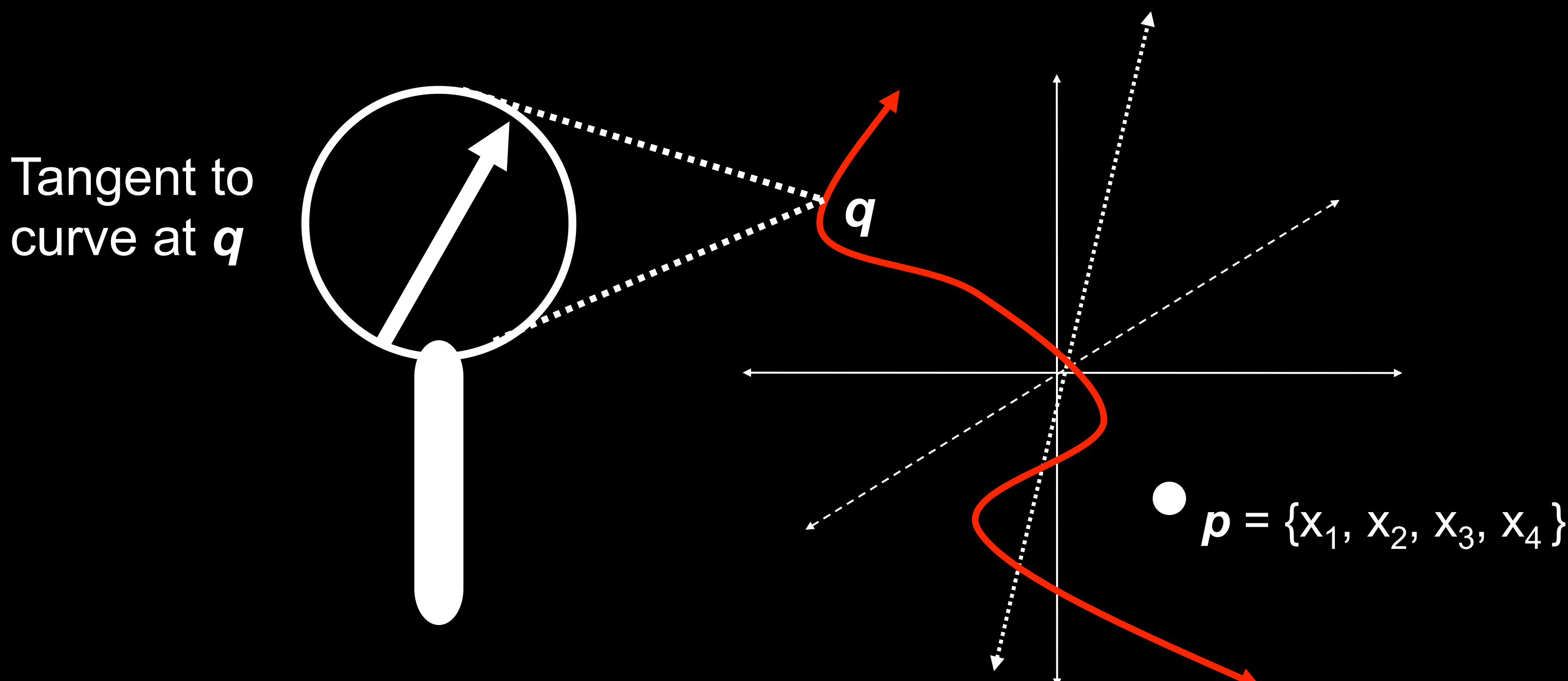
Distant clock synchronization



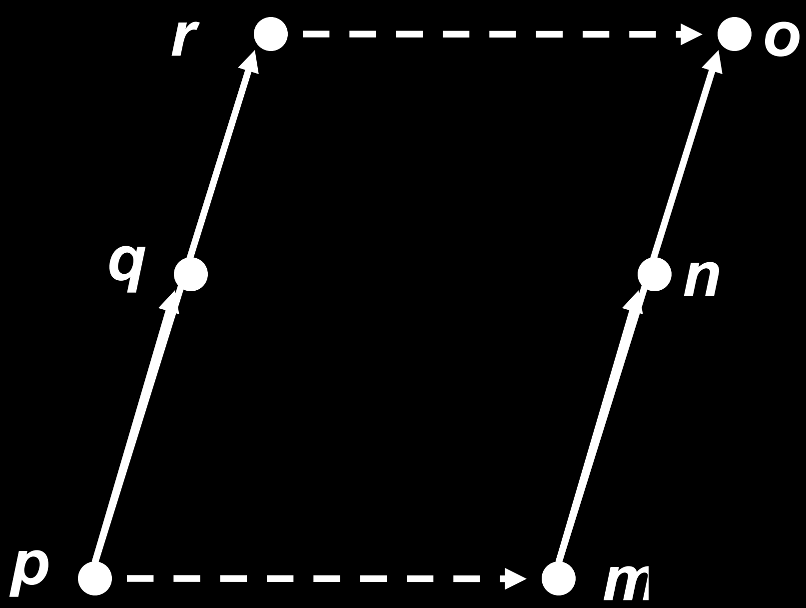
Minkowski space-time as a vector space: A real-number space constituted by the Cartesian product

$$\Re\times\Re\times\Re\times\Re$$

Each point is characterized by an ordered quadruple of real numbers, x_1, x_2, x_3, x_4.



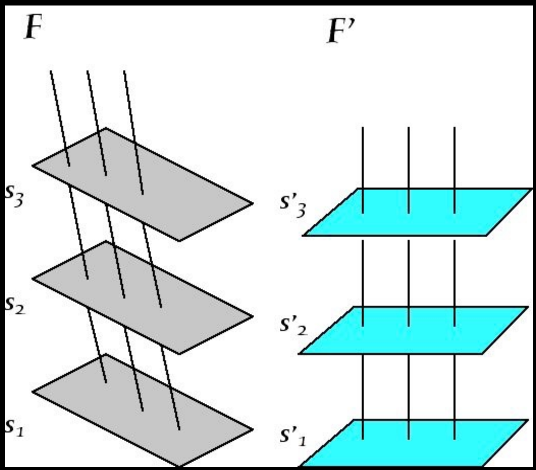
Minkowski space-time as an affine space: Any two points are connected by a displacement, and any two displacements can be characterized as parallel or not.



pr = ***k(pq)***

pr* || *mo

pr + ro = pm + mo

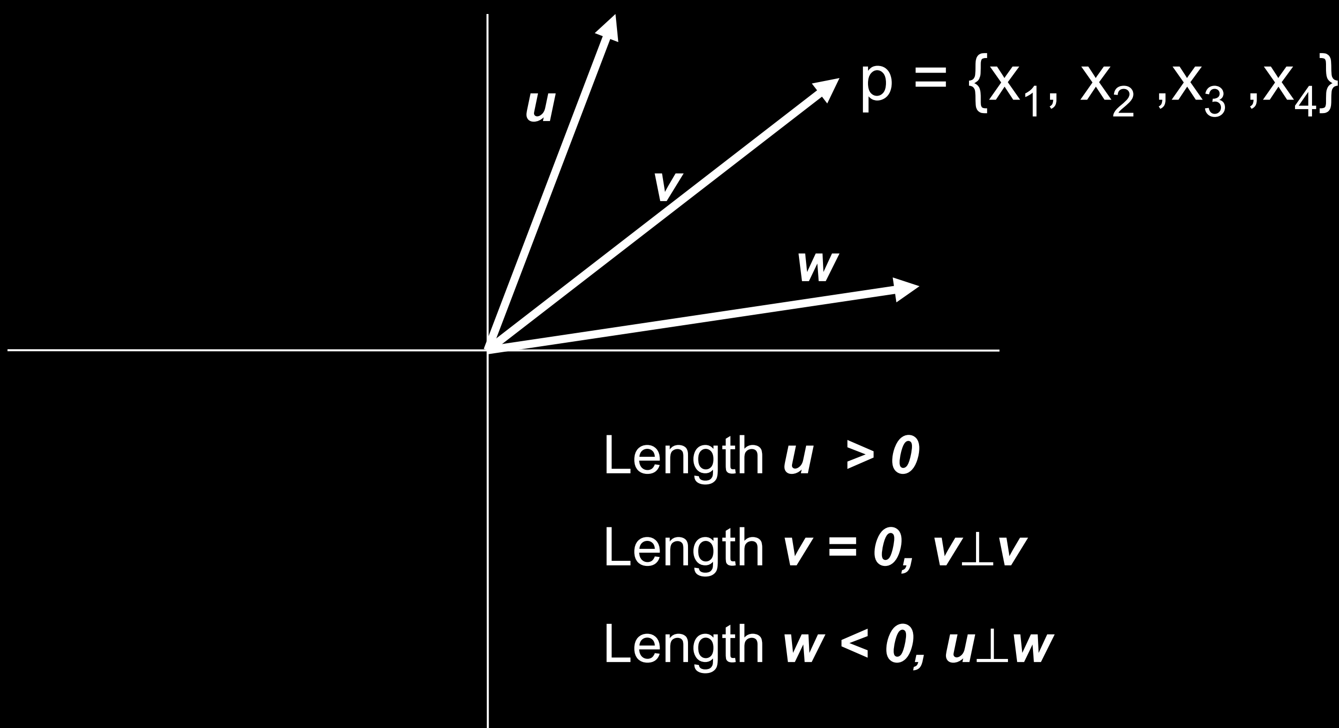


Affine equivalence

Minkowski space-time as a metric space: Between any two points there is a unique space-time interval,

$$s^2 = x_1^2 - x_2^2 - x_3^2 - x_4^2$$

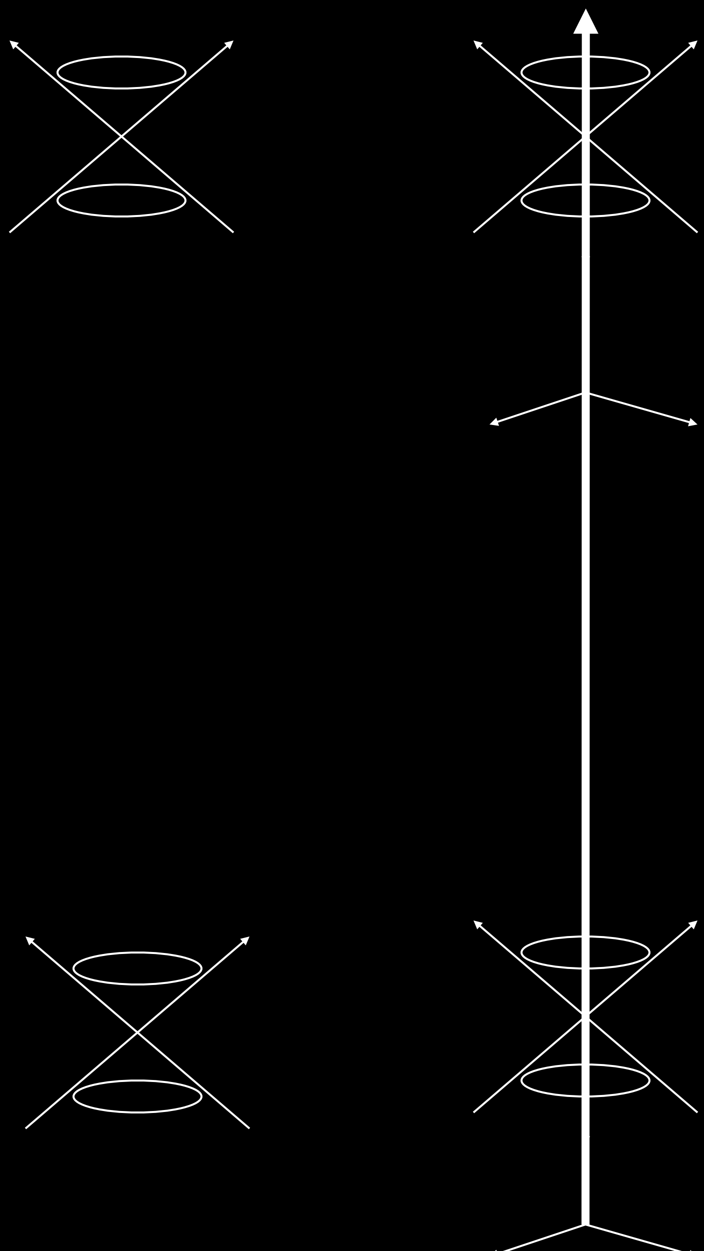
Minkowski space-time as a conformal space: at any point the invariant light-cone is well-defined.



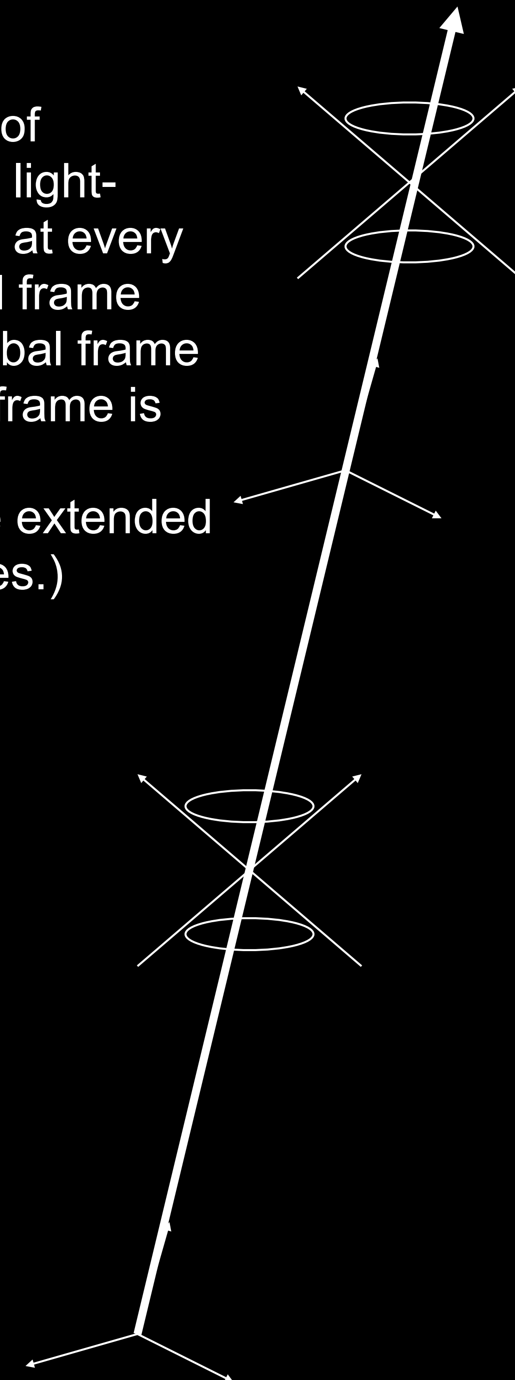
Length ***u* > 0**

Length ***v* = 0, *v*⊥*v***

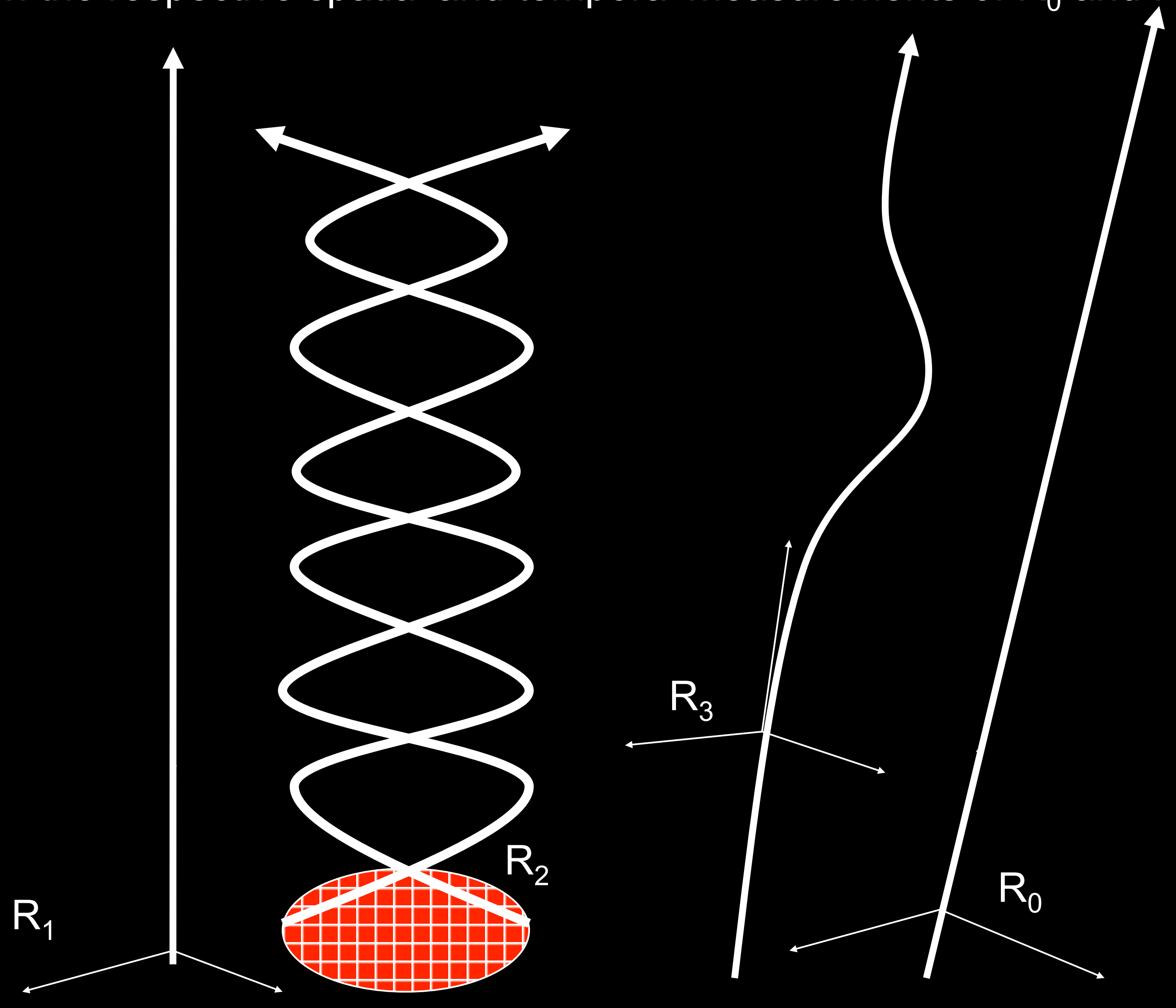
Length ***w* < 0, *u*⊥*w***



Local and global structure of Minkowski space-time: the light-cone structure is the same at every point, and any local inertial frame can be extended into a global frame in which any other inertial frame is also inertial. (Local inertial coordinates can always be extended to global inertial coordinates.)



The rotating frame R_2, and the accelerating frame R_3, are non-inertial with respect to R_0 and R_1, though the magnitude of rotation and acceleration will depend on the respective spatial and temporal measurements of R_0 and R_1.



Two views of the electrodynamics of moving bodies:

Einstein:

- The invariance of the velocity of light is real.
- Simultaneity is relative: whether two events occur at the same time depends on the frame of reference.
- Therefore the Lorentz contraction and time-dilatation are mere frame-dependent appearances.

Lorentz:

- The contraction and dilatation are real.
- Simultaneity is absolute: it is an objective fact whether two events occur at the same time.
- Therefore the invariance of the velocity of light must be mere appearance.

Minkowski on the foundation of special relativity:

“A. Einstein has up to now expressed most clearly that this postulate [the postulate of relativity] is not an artificial hypothesis, but rather a novel understanding of the time-concept that is forced upon us by the appearances.” (Minkowski, 1907)

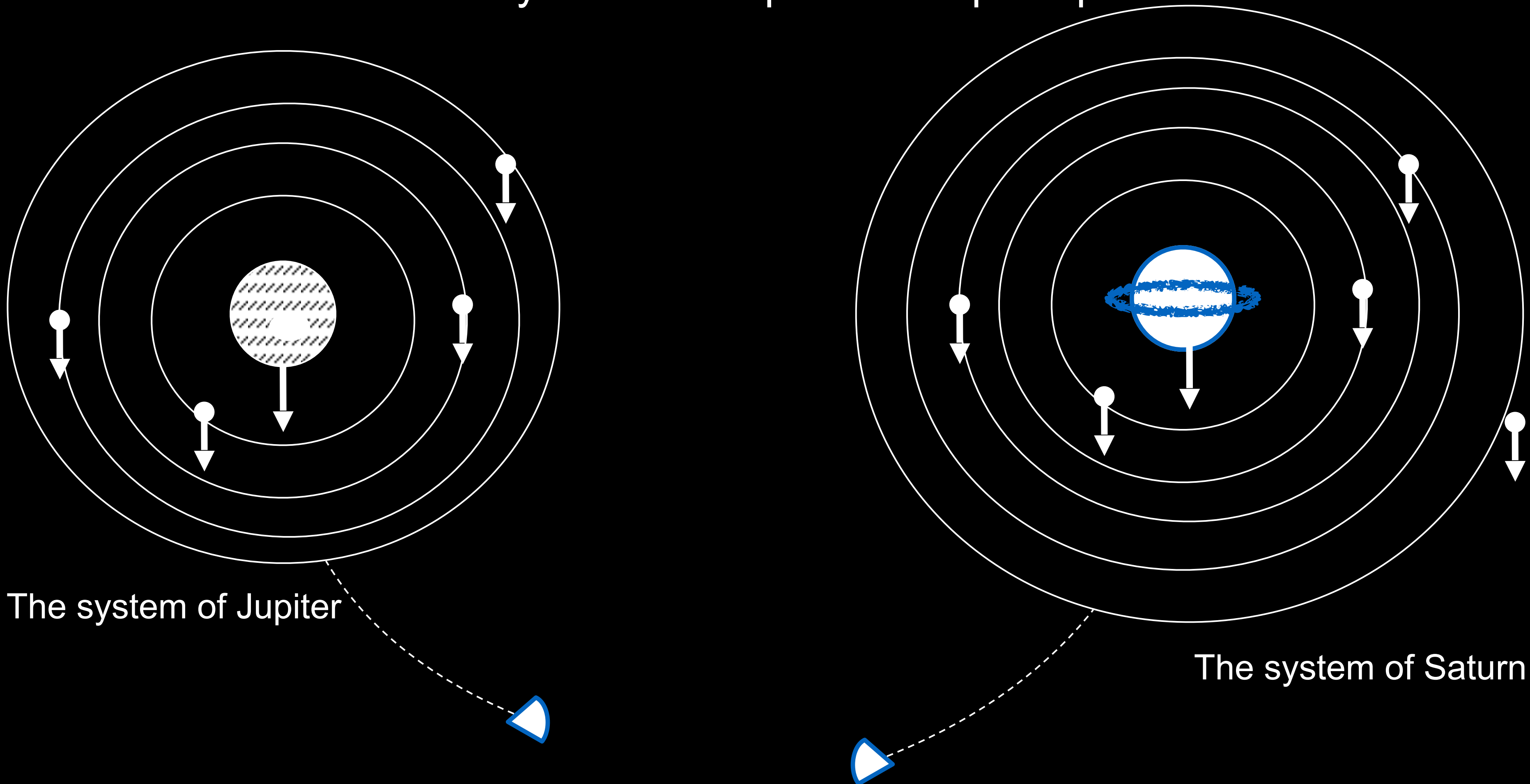
“Lorentz called the *t’* combination of *x* and *t* the local time of the electron in uniform motion, and applied a physical construction of this concept, for the better understanding of the hypothesis of contraction. But the credit of first recognizing clearly that the time of the one electron is just as good as that of the other, that is to say, that *t* and *t’* are to be treated identically, belongs to A. Einstein.” (Minkowski, 1908)

Origins of the general theory of relativity:

Special relativity: The 1905 theory takes the velocity of light ***c*** to be invariant. It follows also that the velocity of electromagnetic radiation is a ***limiting velocity*** for massive objects. The invariance group of electrodynamics becomes the invariance group of space-time, and therefore all physical interactions (in principle) ought to conform to Lorentz invariance. In particular, there should be a Lorentz-invariant theory of the gravitational field, in which gravitational influence propagates at the speed of light (or less).

The "equivalence principle": Newton had shown that, to high accuracy, gravitational and inertial mass are equivalent, and so weight in any given gravitational field is always proportional to mass. All objects will accelerate at the same rate in the same gravitational mass, independent of their mass and composition. In Einstein's time, the results of Newton's pendulum experiments were confirmed to still greater accuracy by Eötvös (1906).

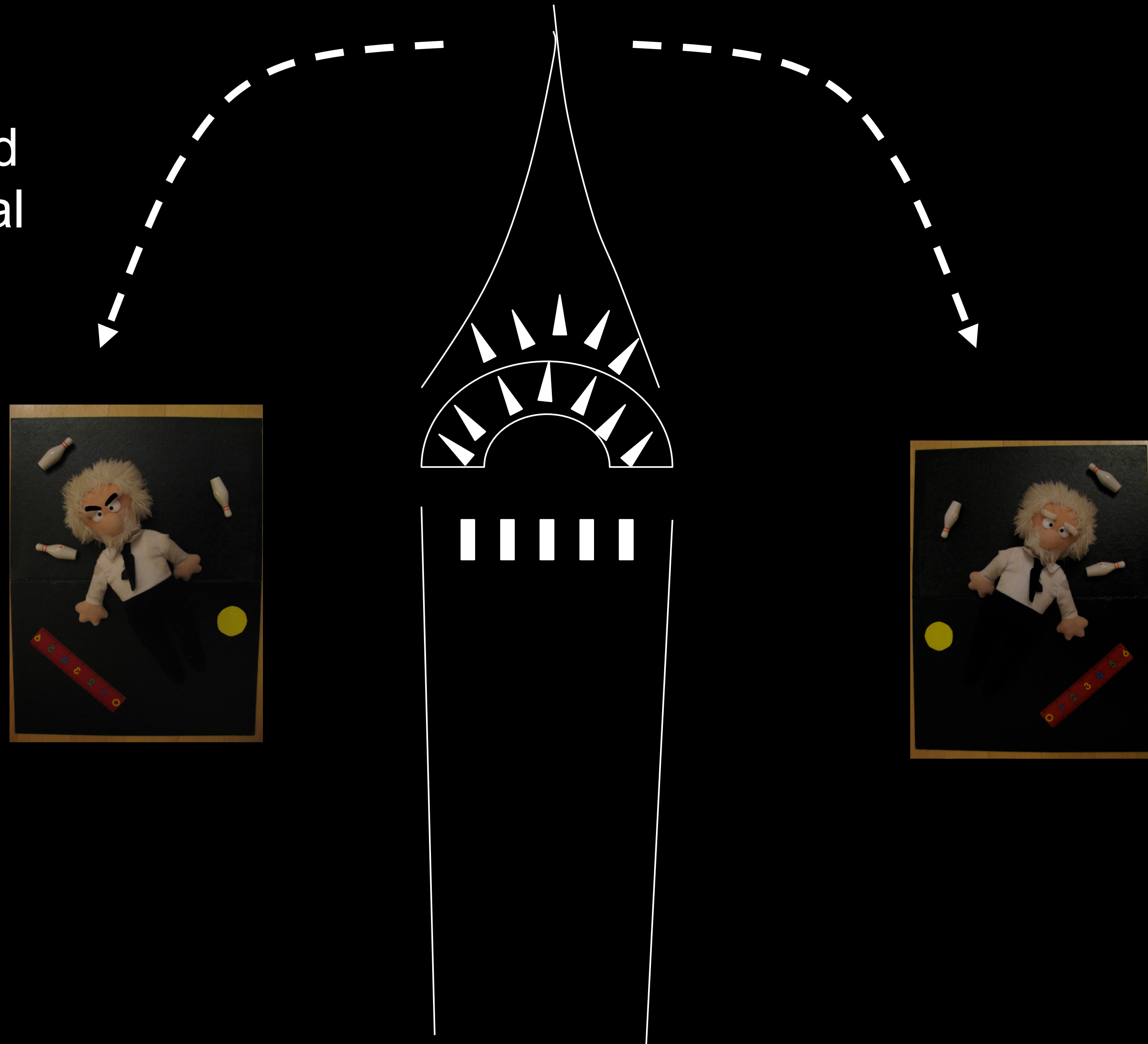
From Newton's Corollary VI to the equivalence principle



Because gravity acts equally on all bodies at a given distance from the source, the systems of Jupiter and of Saturn may both be regarded ("locally") as isolated from external forces, and the centre of mass frame of each may be regarded as an inertial frame. No local experiment will distinguish either from an inertial frame. Yet with respect to one another, they are accelerating. Neither is moving inertially in the reference frame of the other.

The other observer in another local inertial frame

Relatively accelerated local inertial frames



Three paths to general relativity.

I. **From special relativity (the historical path):** The 1905 theory, Newton's first law still holds, insofar as the path of a particle not subject to forces is uniform and rectilinear. Whereas in Newton's theory, Newton's three laws of motion determine a privileged class of reference-frames (the inertial frames), now the invariance of ***c*** defines the inertial frames.

There is a philosophical objection in principle to a theory with a privileged class of reference frames— a "restricted" or "special" relativity theory. Special relative shares with Newtonian relativity this "epistemological defect". Eliminating this defect means "generalizing" the relativity principle to include "all possible frames of reference."

At the same time, there is a ***prima facie*** need for a new theory of gravitation that is compatible with special relaivity. Preliminary attempts to adapt Newton's theory to Lorentz invariance are unsuccessful

The "general theory of relativity" solves both of these problems. The "equivalence principle" undermines the distinction between inertial and non-inertial frames. Gravity and inertia are seen to be aspects of the same physical field. Freely falling particles follow the "straight lines" of a curved space-time.

II. **From special relativity (the ideally reconstructed path):** The 1905 theory takes the velocity of light ***c*** to be invariant, which implies that simultaneity is relative, and therefore temporal and spatial measurement are individually relative. Newton's first law still holds, insofar as the path of a particle not subject to forces is uniform and rectilinear. Whereas in Newton's theory, Newton's three laws of motion determine a privileged class of reference-frames (the inertial frames), now the invariance of ***c*** defines the inertial frames.

Given special relativity, there is a ***prima facie*** need for a new theory of gravitation that is compatible with invariance of the velocity of light. Preliminary attempts to adapt Newton's theory to Lorentz invariance are unsuccessful

The "general theory of relativity" solves this problem by starting from the most striking fact about gravity discovered by Newton. The "equivalence principle" undermines the distinction between inertial and non-inertial frames. Gravity and inertia are seen to be aspects of the same physical field. Freely falling particles follow the "straight lines" of a curved space-time.

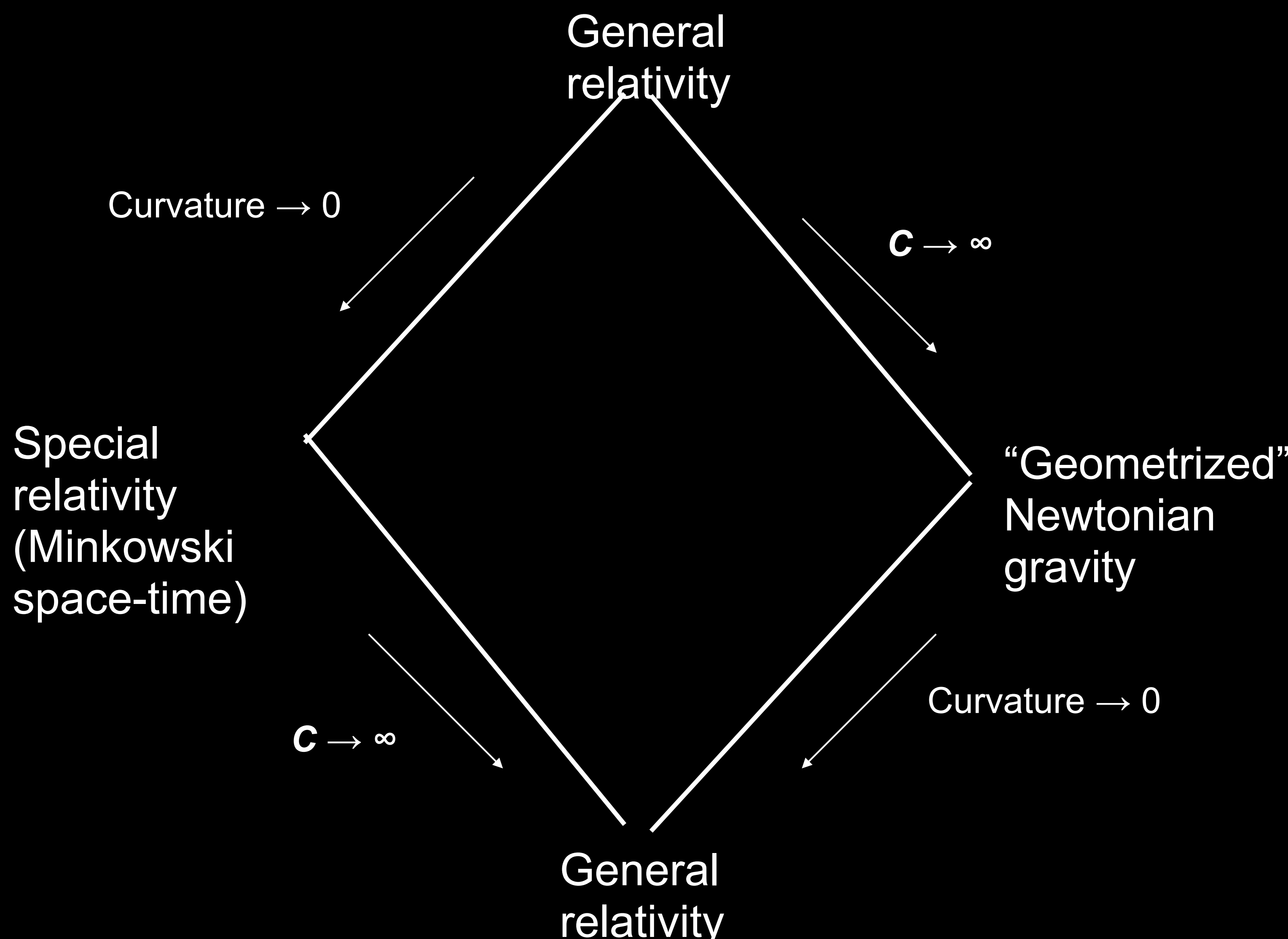
Instead of extending "relativity" generally, General Relativity takes the Minkowski structure to be the local structure of a space-time that will be variably curved, depending on the distribution of matter and energy.

III. From Newtonian gravity (counterfactual history): Newton's tests of the equivalence principle, and use of Corollary VI, already undermines the distinction between an inertial motion and the path of a freely falling body, and therefore the distinction between inertial and non-inertial frames. Riemann lives long enough to formulate Newtonian mechanics in a four-dimensional affine space. Then he infers from the equivalence principle that the geodesics of the affine space are indistinguishable from the paths of freely falling bodies, and have equal right to be treated as geodesics— but of a curved space-time. Space remains flat, but the affine structure of space-time is variably curved according to the distribution of matter and energy.

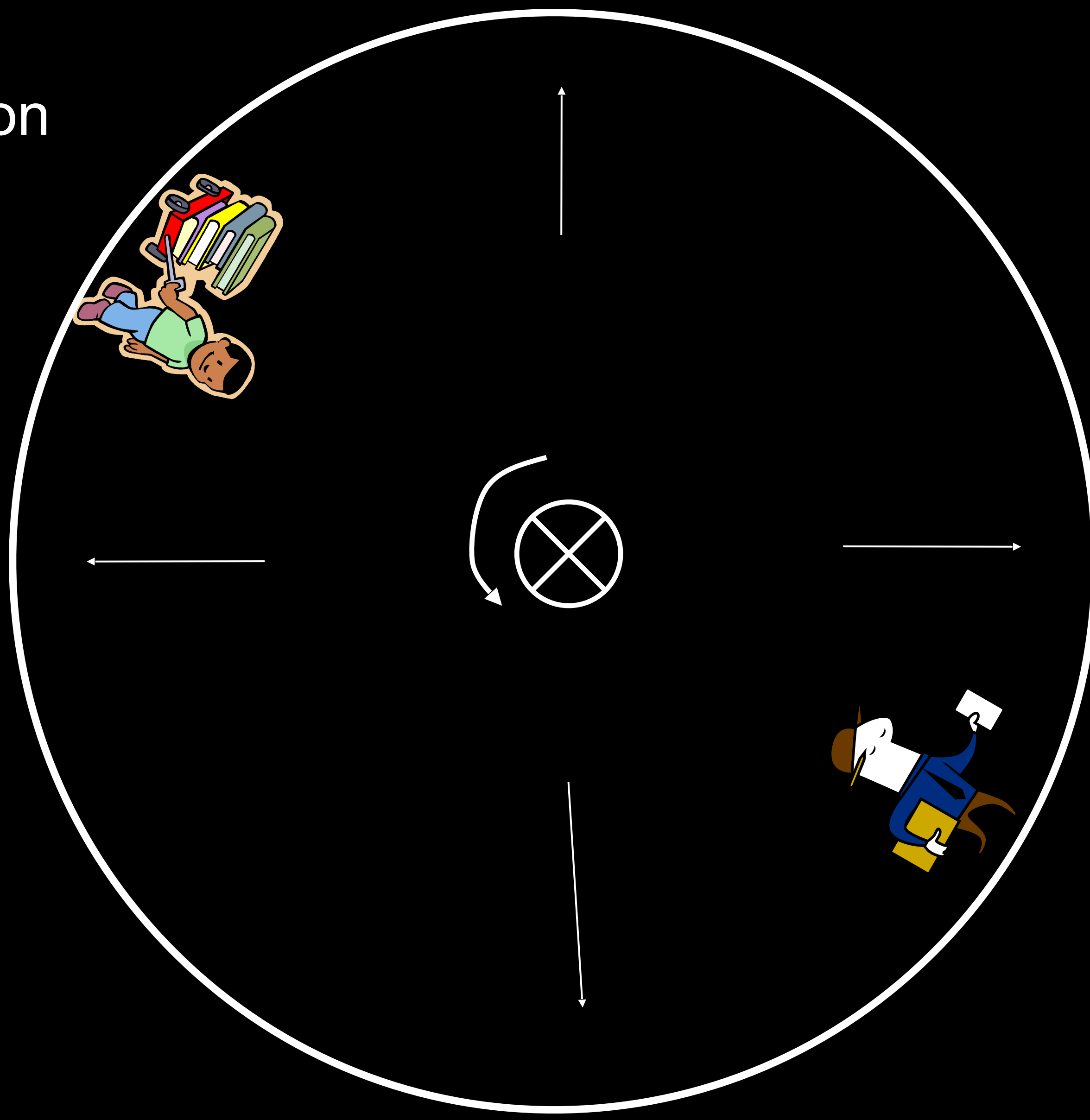
Afterwards, experiment confirms the invariance of the velocity of light. Einstein (or Minkowski) infers that the local structure of space-time is not Galilean,with hyperplanes of absolute simultaneity, but Lorentzian, with a light-come structure.

The "general theory of relativity" emerges as the revision of Newtonian curved space-time geometry to conform to local Lorentz invariance.

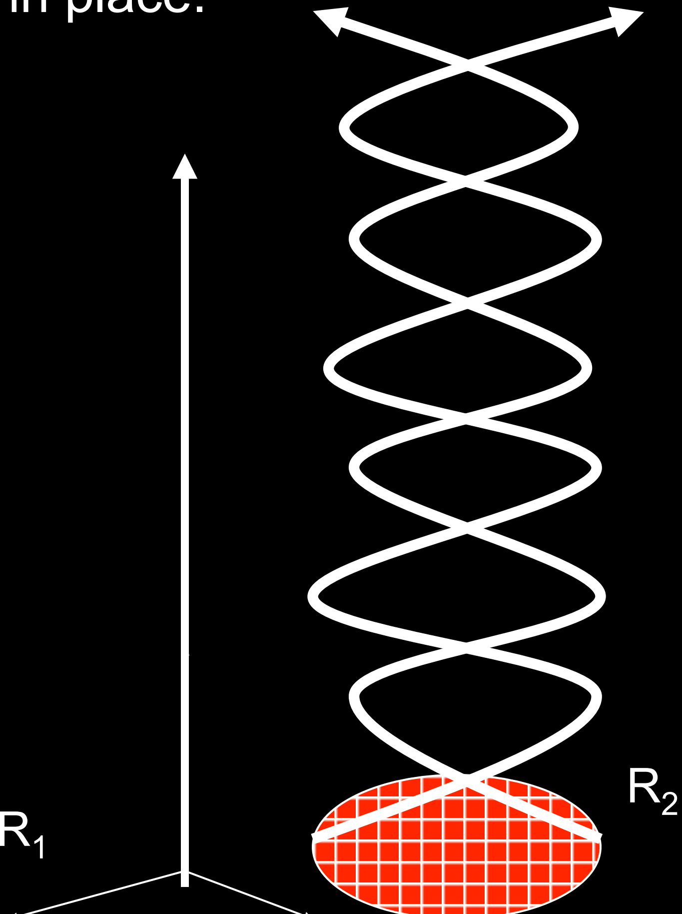
Limiting-case relations among classical space-time theories

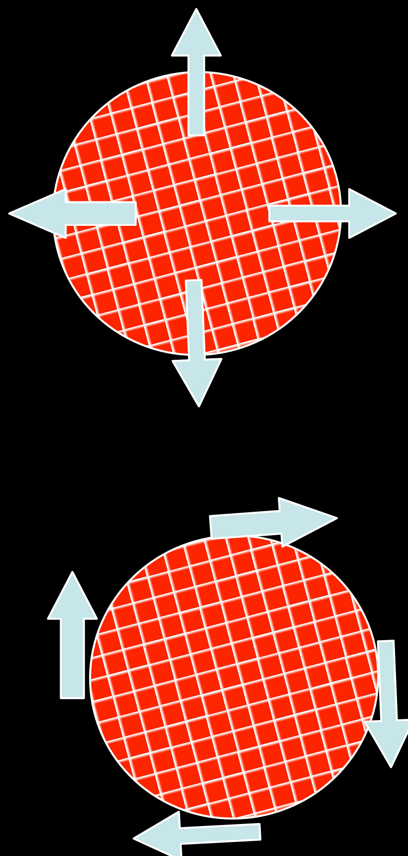


Centrifugal force mimics gravity: a centrifugal acceleration of 9.8 m/sec^2 will feel like your "weight" toward the perimeter

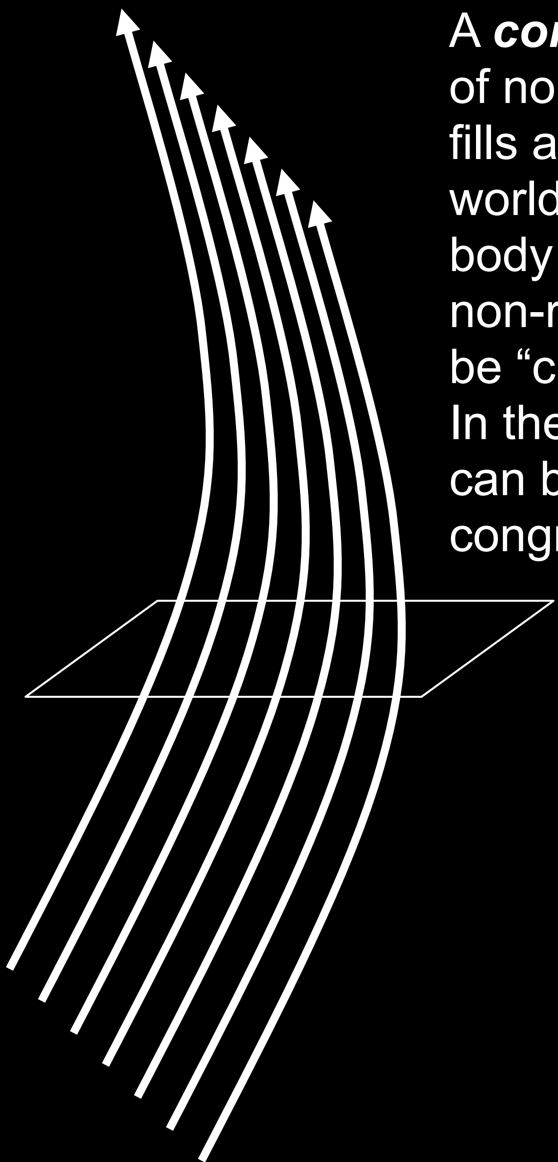


Why are the rotating frame R_2 and the inertial frame R_1 not equivalent? Why isn't R2 just an inertial frame with a force field acting within it? Because, while "centrifugal force" is real, it is neither centrifugal nor a force: it is an inertial effect that results when inertial motion (tending along the tangent) is resisted by forces acting on the parts of the disc, or the objects lying on it, to keep them in place.

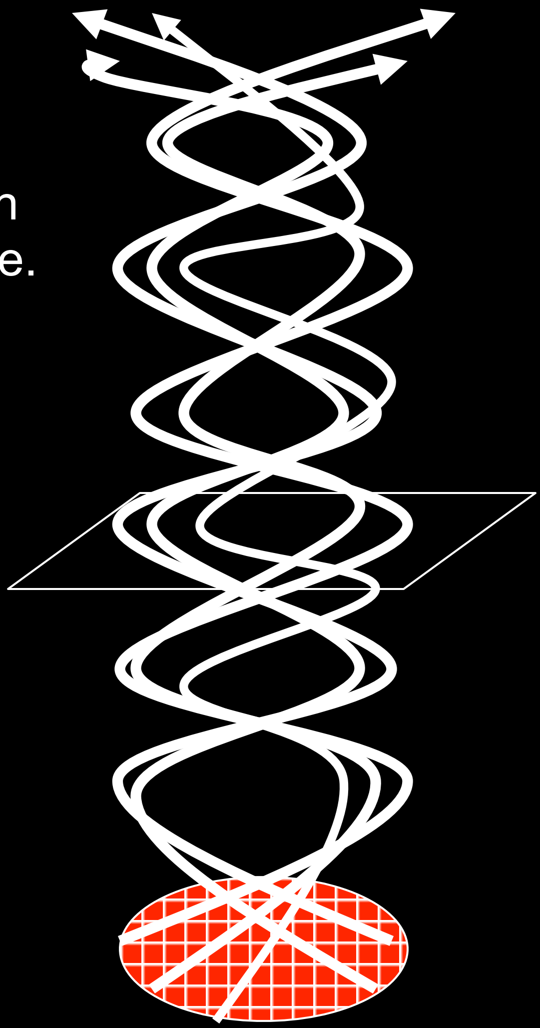




Rotation in Minkowski space-time: comparing the worldlines of rotating and non-rotating systems



A ***congruence*** of timelike curves: A set of non-intersecting timelike curves that fills a region of space-time. The worldlines of the particles of a rigid body form such a congruence. In the non-rotating case, the congruence can be “cut” by an orthogonal hypersurface. In the rotating case, no hypersurface can be orthogonal to the entire congruence of curves.



Mach's question:

How do we know that the earth's centrifugal effects arise from its own rotation, rather than from its rotation relative to the fixed stars?

There is no decision about relative and absolute which we can hit upon, to which we are forced, or from which we can obtain any intellectual or other advantage. When even modern authors let themselves be misled by the Newtonian arguments based on the bucket of water, to make a distinction between relative and absolute motion, they do not reflect that the system of the world is only given to us *once*; the Ptolemaic or Copernican view is *our* interpretation, but both are equally real. Try to hold Newton's bucket fixed, and then rotate the heaven of fixed stars around it, and then prove the absence of centrifugal forces…

The motion of a body *K* can only be estimated by reference to other bodies *A, B, C* .... When we reflect that we cannot abolish the isolated bodies *A, B, C*..., and therefore cannot decide by experiment whether the part they play is fundamental or incidental; that hitherto they have been the sole sufficient means of the orientation of motions and of the description of mechanical facts; it will be advisable to regard all motions provisionally as determined by these bodies.

(Mach, *Die Mechanik,* 1883)

“The universe is not given to us *twice*, once with an earth at rest and once with an earth in motion; but only *once*, with its relative motions, which alone are determinable. Therefore we cannot say how it would be if the earth did not rotate. We can interpret the one case that is given to us, in different ways. If, however, we interpret it in such a way that we come into conflict with experience, our interpretation is just wrong. The principles of mechanics can be so conceived, that centrifugal forces arise even for relative rotations.”

Mach’s stricture: Laws of physics should only express dependencies among observable phenomena. The laws of motion can only describe motions relative to the fixed stars.

E.g.: Every acceleration relative to the fixed stars depends on an interaction, involving equal and opposite reactions relative to the fixed stars.

A body not subject to local forces will move uniformly relative to the fixed stars.

According to Mach, this version of the laws of physics expresses only what experience justifies. Because we can't vary the circumstances of the universe as we vary the factors in the bucket experiment, the evidence we have can't decide between:

—Newton was right: the laws of motion are simply true with respect to space. In an otherwise empty space, a lone body would move uniformly in a straight line, and a rotating body would have centrifugal forces.

—Newton was wrong: the patterns of motion relative to the stars somehow physically depend on the stars. Perhaps the rotation of the stars around the earth could induce centrifugal forces in the earth.

Relative to what reference-frame do bodies obey the the laws of motion? Three ways to answer this:

The right way: The laws describe motion relative to the fixed stars. Bodies not subject to local forces move in straight lines relative to the fixed stars, and local forces cause bodies to accelerate relative to the fixed stars.

The wrong way: The laws describe motion relative to space itself. Forces cause bodies to accelerate relative to absolute space and time, and a force-free body describes a straight line in space, uniformly in time.

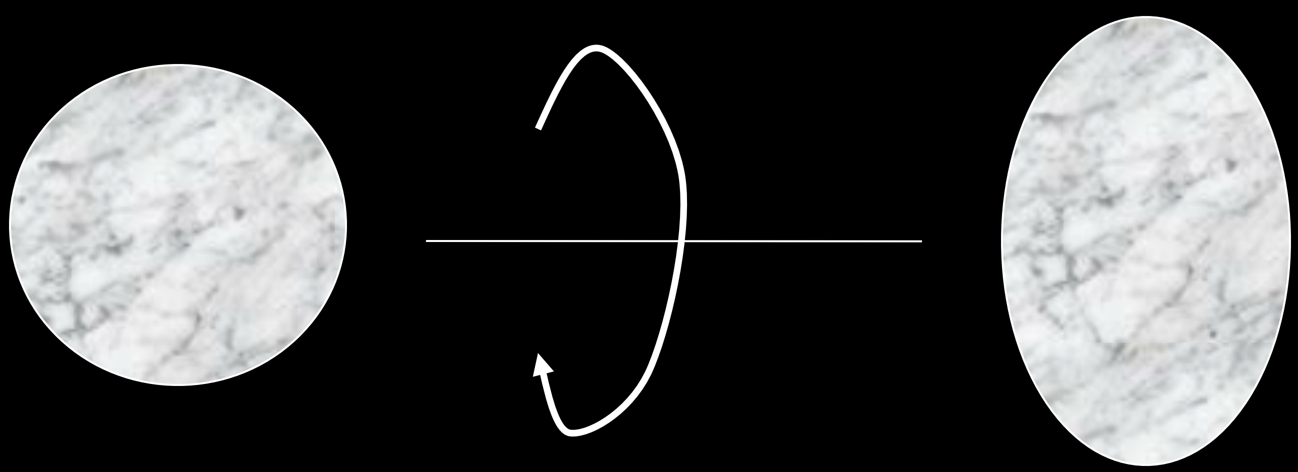
The Navy way: Don’t specify any reference-frame. Just use the laws of motion as a recipe for finding an observable reference-frame that is approximately inertial.

The right way: This is the only option that is satisfactory for empirical science, because it expresses the true empirical content of Newton's laws.

The wrong way: This is just empty metaphysics that doesn't tell us how the laws apply to the world. Space is unobservable, and no one has any idea how bodies move with respect to it.

The Navy way: This way is not strictly empiricist, but at least it makes no metaphysical assumptions. In our local universe, it has the same practical consequences as the right way. Newton used the laws of motion to determine that the fixed stars form an approximately inertial frame.

Einstein revives Mach's argument (1916): Two spheres S_1 and S_2, rotate relative to one another, and S_2 bulges at its equator; how do we explain this difference?



No answer can be admitted as epistemologically satisfactory, unless the reason given is an observable fact of experience....Newtonian mechanics does not give a satisfactory answer to this question. It pronounces as follows: The laws of mechanics apply to the space R_1, in respect to which the body S_1 is at rest, but not to the space R_2, in respect to which the body S_2 is at rest. But the privileged space R_1... is a merely factitious cause, and not a thing that can be observed.

Einstein, “The foundation of the general theory of relativity,” 1916

Note: Einstein acknowledges, as Mach did, that the set of privileged reference frames— which entails the absolute distinction between rotation and non-rotation— is a necessary feature of Newton’s laws and of special relativity.

He infers that this “epistemological defect” can only be overcome in a radically new theory.

What does Nature care about our coordinate systems?

Einstein, *The Meaning of Relativity*, 1922

What makes the situation appear particularly unpleasant is the fact that there should be infinitely many inertial systems, moving uniformly and without rotation with respect to one another, that are distinguished from all other rigid systems.

Einstein, “Autobiographical Note,” 1949

General covariance: ***The relativity principle must be extended, from inertial systems to coordinate systems in any state of motion whatsoever.***

Einstein’s arguments concerning causality:

1. The cause of a physical distinction cannot be a merely arbitrary mathematical distinction, such as the distinction between coordinate systems (the “factitious” cause).

2. The cause of an observable effect must be something observable.

3. The cause of any physical effect must itself be part of a closed causal chain, i.e., it must itself be reacted upon.

“Mach’s Principle”: The local inertial behavior of a body depends on its relation to the distribution of mass in the universe.

e.g.: the centrifugal effects exhibited by the earth (or,the inertial effects in any accelerating body) are caused by its rotation (acceleration) relative to the distant masses.

Interpreting the “general relativity” of general relativity:

What is the answer to Einstein’s question?

“The only satisfactory answer must be that the physical system consisting of S_1 and S_2 reveals within itself no imaginable cause to which the differing behaviour of S_1 and S_2 can be referred. The cause must therefore lie outside this system. We have to take it that the general laws of motion, which in particular determine the shapes of S_1 and S_2 , must be such that the mechanical behaviour of S_1 and S_2 is partly conditioned in quite essential respects, by distant masses which we have not included in the system under consideration.

...These distant masses and their motions relative to S_1 and S_2 must then be regarded as the seat of the causes (which must be susceptible to observation) of the different behaviour of our two bodies S_1 and S_2. They take over the rôle of the fictitious cause R_1. Of all imaginable spaces R_1, R_2, etc., in any kind of motion relatively to one another there is none which we may look upon as privileged a priori without reviving the above-mentioned epistemological objection. The laws of physics must be of such a nature that they apply to systems reference in any kind of motion. Along this road we arrive at an extension at the postulate of relativity.”

“Mach’s Principle”: The local inertial behavior of a body depends on its relation to the distribution of mass in the universe.

e.g.: the centrifugal effects exhibited by the earth (or,the inertial effects in any accelerating body) are caused by its rotation (acceleration) relative to the distant masses.

In other words: Einstein takes Mach’s empiricist stricture, and makes it a causal theory. The distant masses provide not just a reference frame, but also a *causal explanation* for local inertial effects.

An analogy between general relativity and quantum mechanics, from the point of view of their epistemic bases:

GR incorporates the insight, according to Einstein, that coordinates have no physical meaning, and that what is objective in our knowledge of space-time is our knowledge of space-time coincidences, or “the meetings of the material points of our measuring instruments with other material points.”

This justifies Einstein’s use and interpretation of general covariance.

But the empirical content of general relativity, in practice, is not based on point-coincidences, but on something more like a “classical mode of description.”

“The assumption of the complete physical equivalence of the systems of co-ordinates, K and K’ we call the "principle of equivalence;" this principle is evidently intimately connected with the theorem of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other….”

“Are there, in general, any inertial systems for very extended portions of the space-time continuum, or, indeed, for the whole universe? …. [T]here are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of the special theory of relativity, which have been developed above, hold with remarkable accuracy. Such regions we shall call "Galilean regions." We shall proceed from the consideration of such regions as a special case of known properties….”

“The principle of equivalence demands that in dealing with Galilean regions we may equally well make use of non-inertial systems, that is, such co-ordinate systems as, relatively to inertial systems, are not free from acceleration and rotation.” (Einstein 1921)

A freely moving body not acted on by external forces moves, according to the special relativity theory, along a straight line and uniformly. This also holds for the generalised relativity theory for any part of the four-dimensional region, in which the co-ordinates K_0 can be, and are, so chosen that the $g_{\mu\nu}$ have special constant values of the expression (4).

Let us discuss this motion from the stand-point of any arbitrary co-ordinate-system K_1; it moves with reference to (as explained in § 2) in a gravitational field. The laws of motion with reference to K_1, follow easily from the following consideration. With reference to K_0, the law of motion is a four-dimensional straight line and thus a geodesic. As a geodetic-line is defined independently of the system of co-ordinates, it would also be the law of motion for the motion of the material-point with reference to K_1; If we put

$$\Gamma^{\tau}_{\mu\nu} = \{^{\mu\nu}{}_{\tau}\}$$

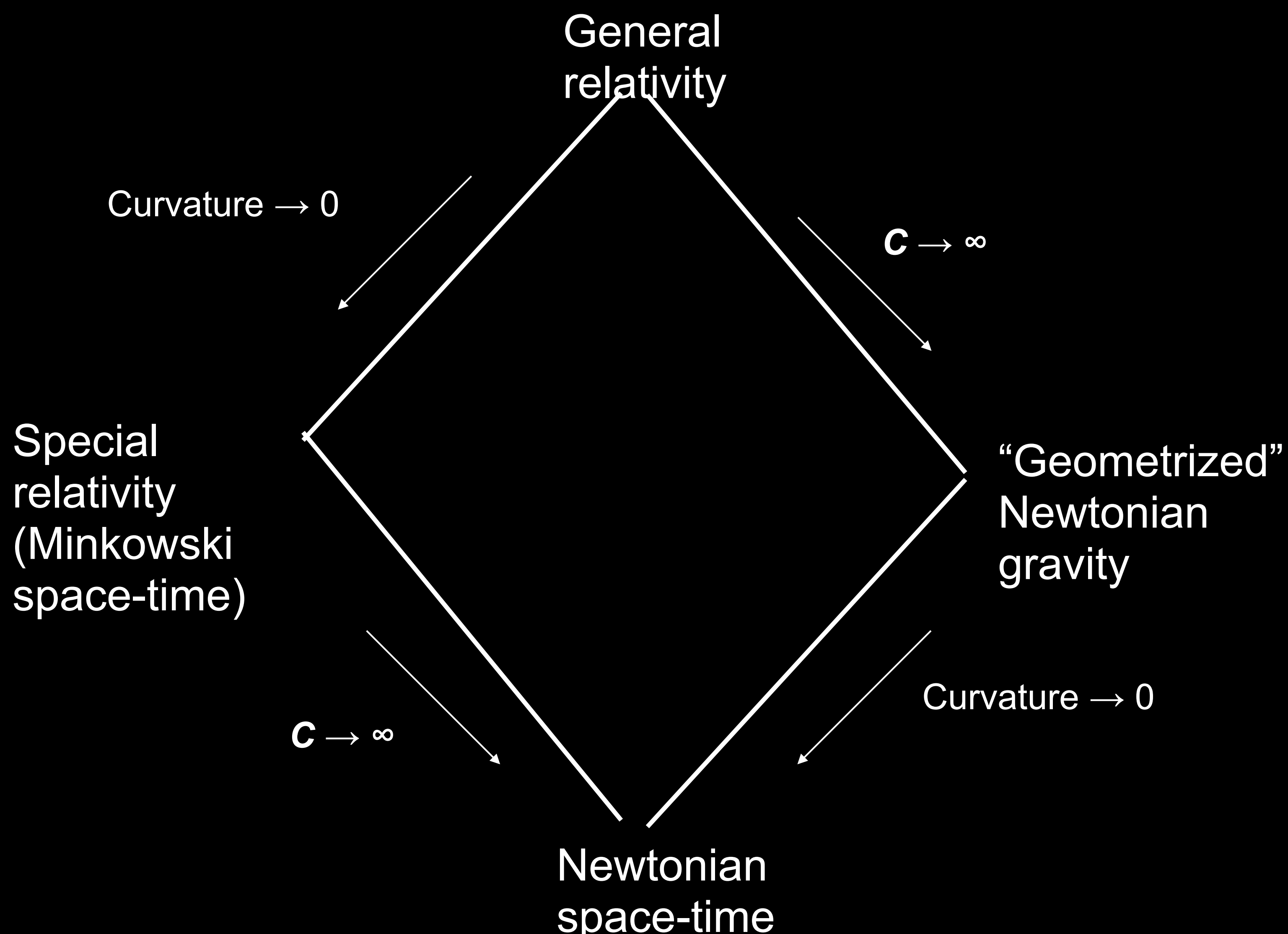
we get the motion of the point with reference to K_1 given by

$$\frac{d^2x_\tau}{ds_2} = \Gamma^{\tau}_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds}$$

We now make the very simple assumption that this general covariant system of equations defines also the motion of the point in the gravitational field, when there exists no reference-system $\boldsymbol{K_0}$, with reference to which the special relativity theory holds throughout a finite region. The assumption seems to us to be all the more legitimate, as (46) contains only the *first* differentials of $\boldsymbol{g_{\mu\nu}}$, among which there is no relation in the special case when $\boldsymbol{K_0}$ exists.

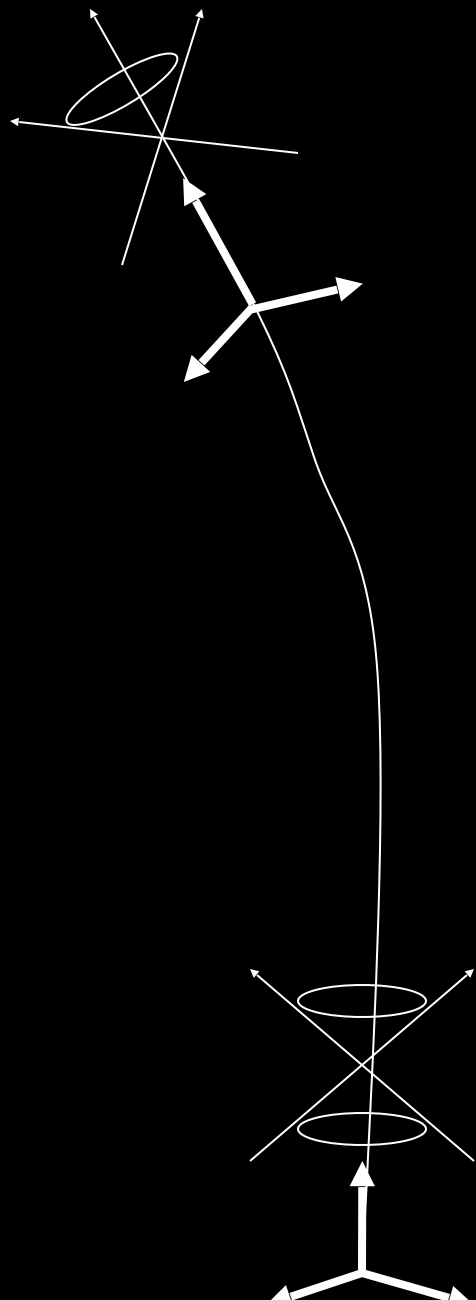
If $\boldsymbol{\Gamma^{\tau}_{\mu\nu}}$ vanish, the point moves uniformly and in a straight line; these magnitudes therefore determine the deviation from uniformity. They are the components of the gravitational field.

Limiting-case relations among classical space-time theories

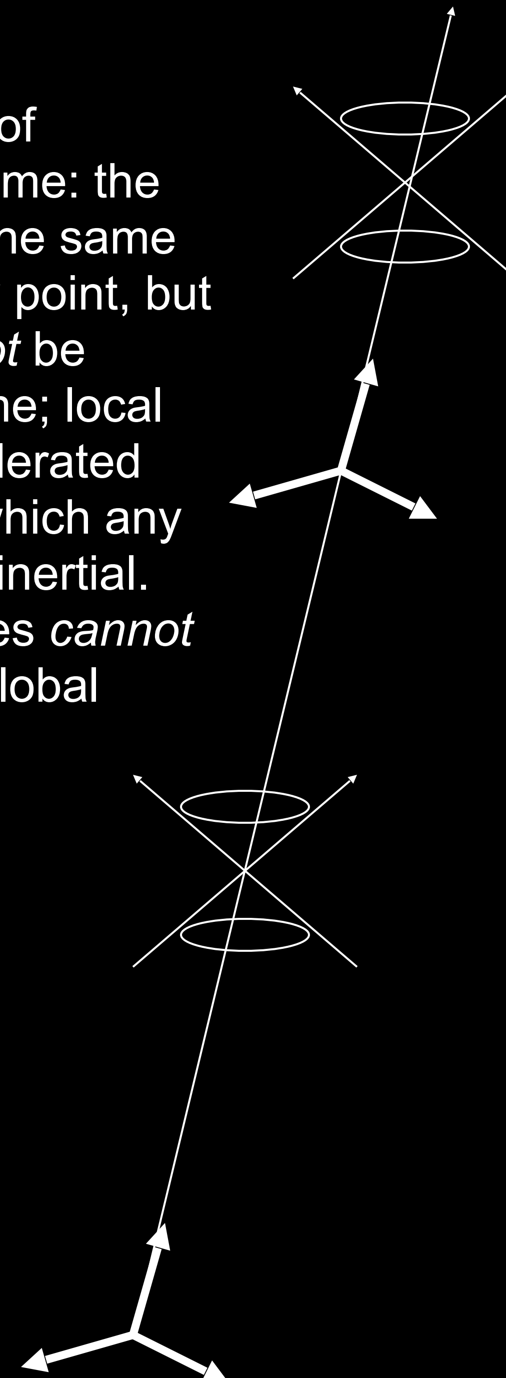


Eddington on the empirical foundations of general relativity:

The reader may not unnaturally suspect that there is an admixture of metaphysics in a theory which thus reduces the gravitational field to a modification of the metrical properties of space and time….There is nothing metaphysical in the statement that under certain circumstances the measured circumference of a circle is less than π times the measured diameter; it is purely a matter for experiment. We have simply been studying the way in which physical measures of length and time fit together– just as Maxwell's equations describe how electrical and magnetic forces fit together. The trouble is that we have inherited a preconceived idea of the way in which measures, if 'true,' ought to fit. (Eddington, 1918).

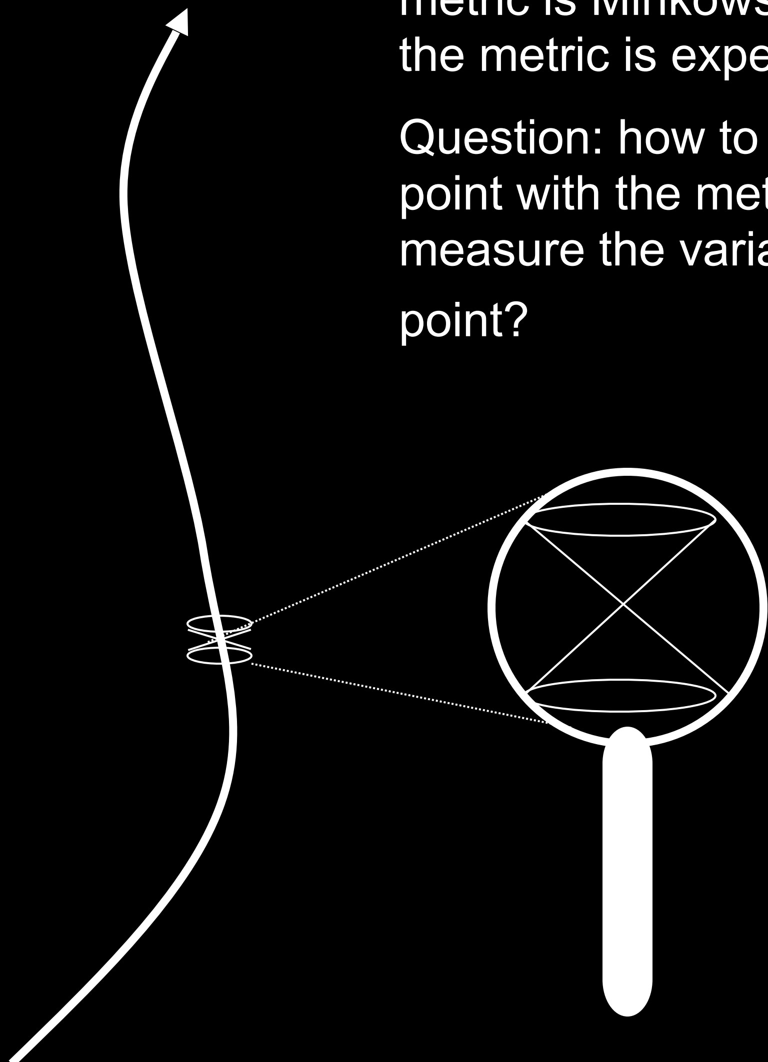


Local and global structure of general-relativistic space-time: the light-cone structure looks the same in the neighborhood of any point, but a local inertial frame *cannot* be extended into a global frame; local inertial frames will be accelerated relative to each other. in which any other inertial frame is also inertial. (So local inertial coordinates *cannot* generally be extended to global inertial coordinates.)

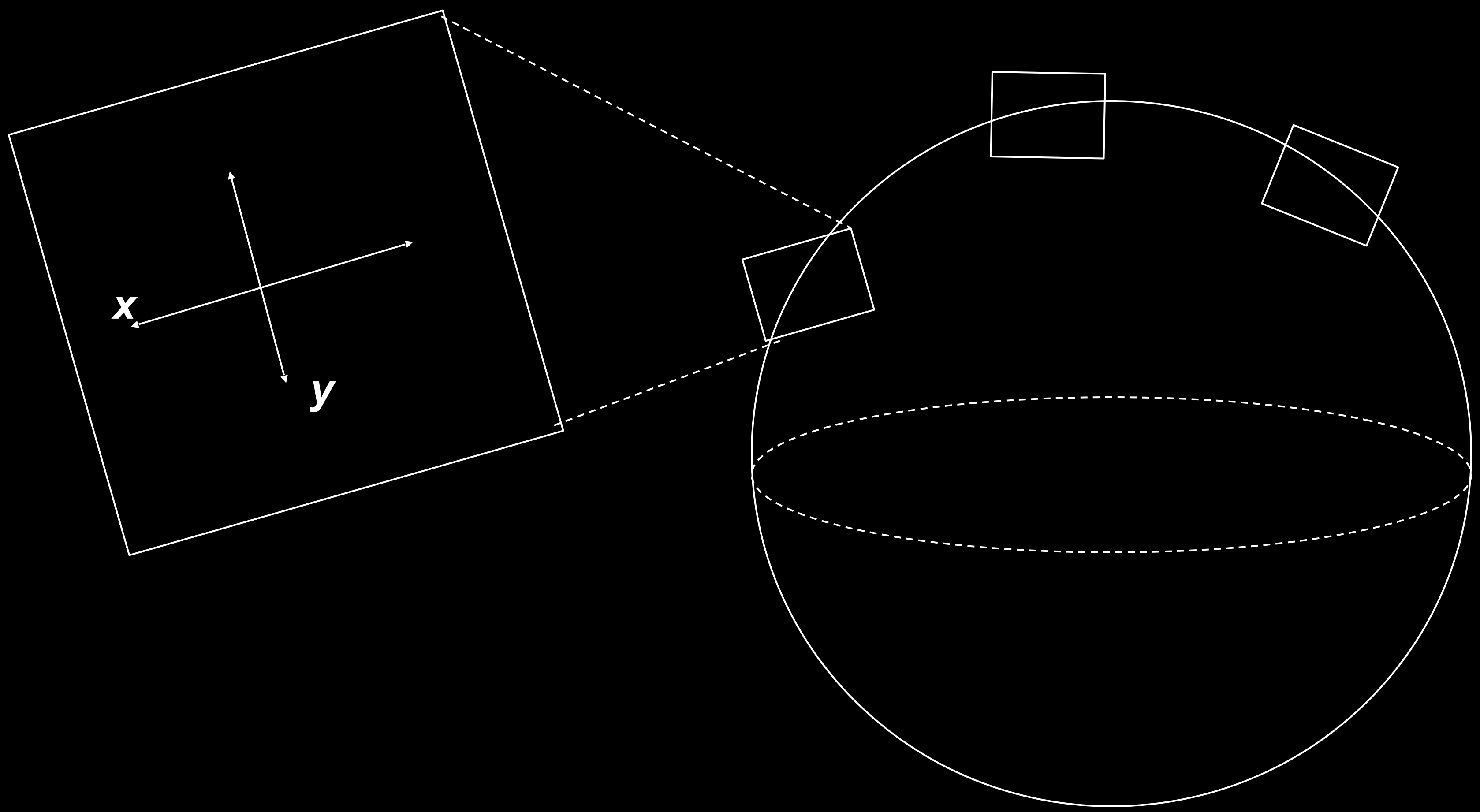


Local structure of inertial frames: in the "infinitesimal neighborhood" of any point, the metric is Minkowski's, but over any finite scale, the metric is expected to vary.

Question: how to connect the local metric at one point with the metric at any other point? How to measure the variation of the metric from point to point?



Comparison: On the surface of the sphere, the structure is “locally” that of the Euclidean plane. But the local Euclidean planes can’t be extended to include one another. The tangent space to any point is the Euclidean plane.



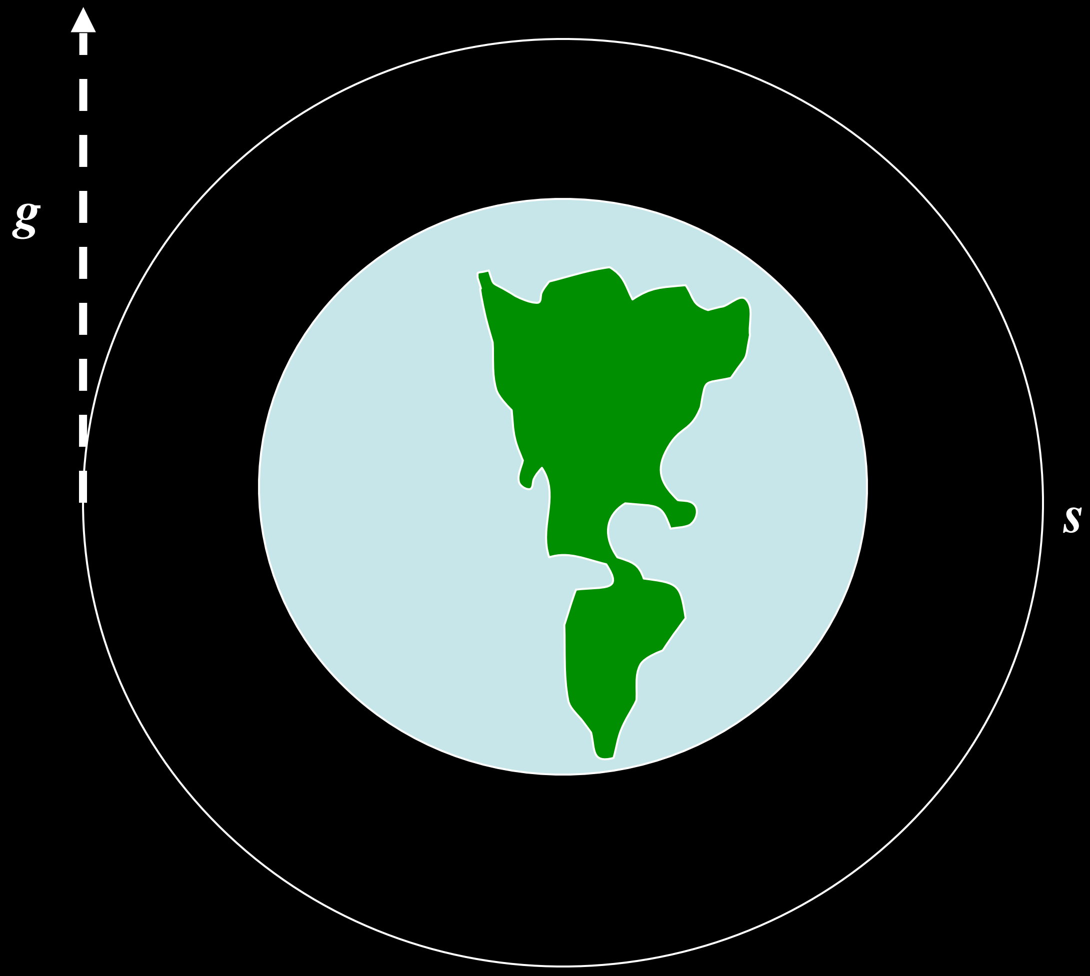
The geodesic principle: The path of a freely-falling particle is a geodesic of space-time.

Motivation: Arguments from the equivalence of gravity and inertia suggest that the path of a freely-falling particle is locally indistinguishable from the path of a Newtonian "particle not subject to forces."

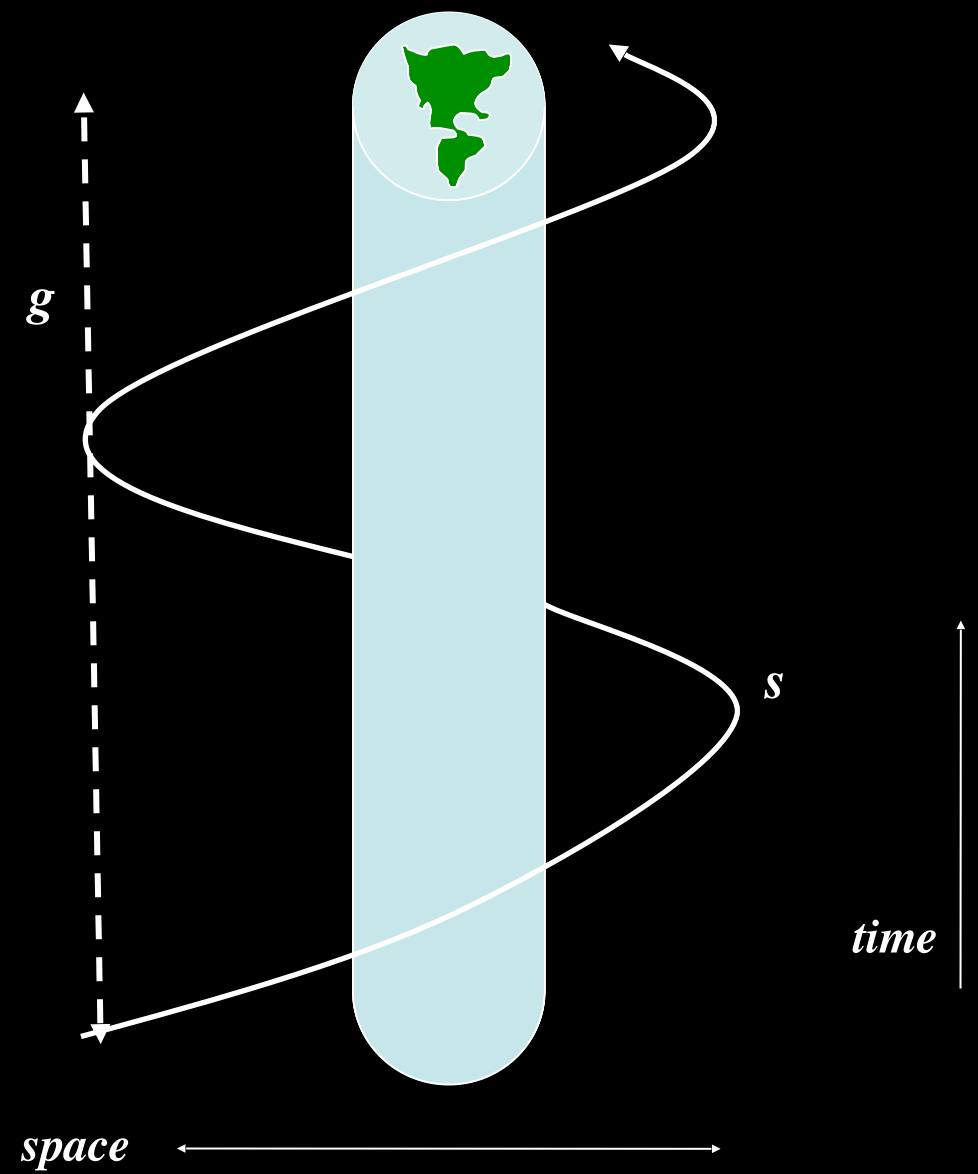
Interpretive principles of space-time geometry:

1. The path of a light-ray is a null geodesic of space-time.
2. The path of a freely-falling particle is a timelike geodesic of space-time.

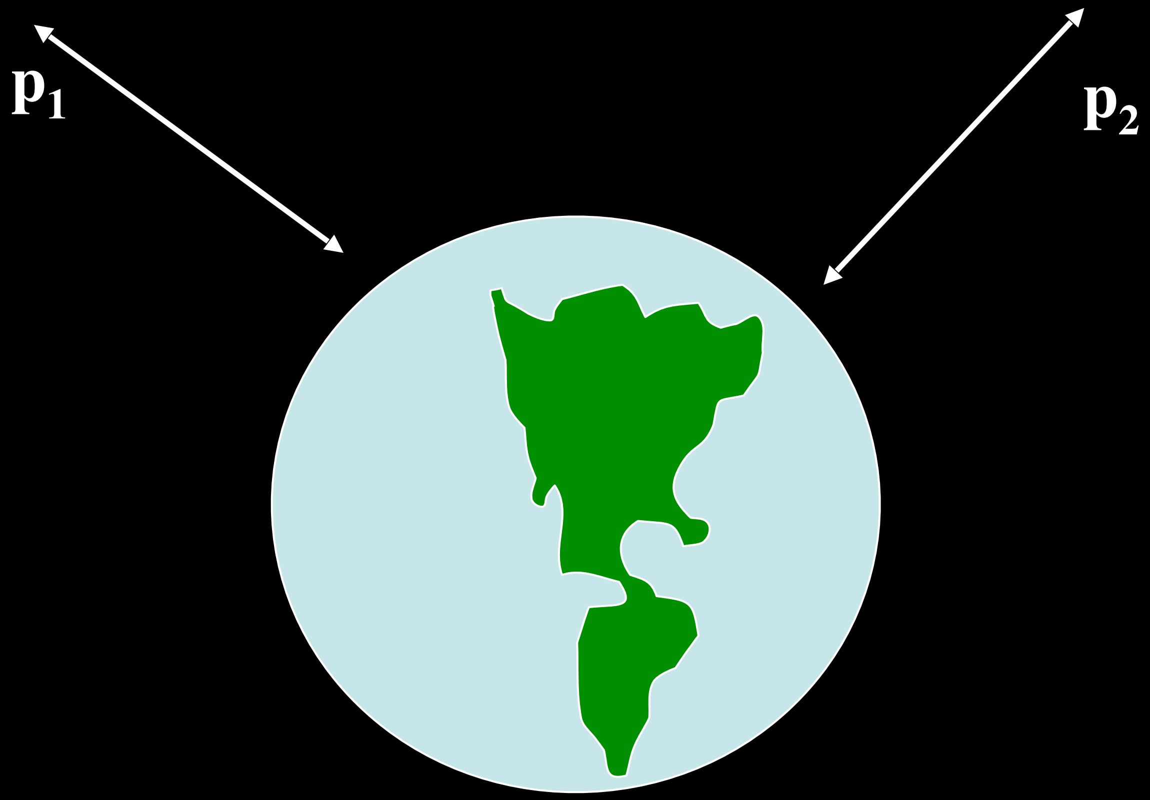
In space, a particle (e.g. a satellite) that would move uniformly in the straight line *g* is, instead, pulled by the earth's gravitational field into a closed orbit *s*.



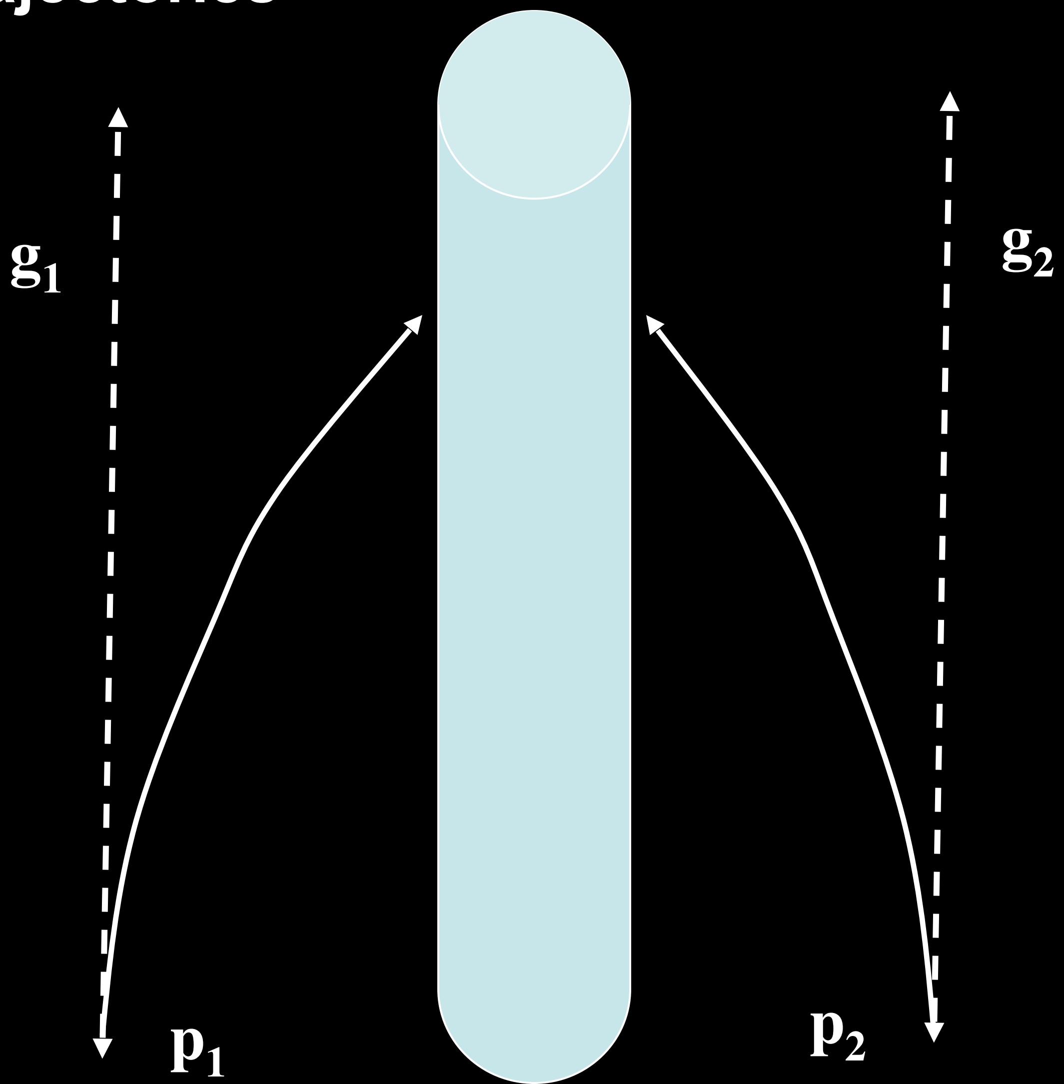
In Newtonian space-time, the particle would follow the space-time geodesic *g*, but instead is bound by gravity into the curved trajectory *s*.



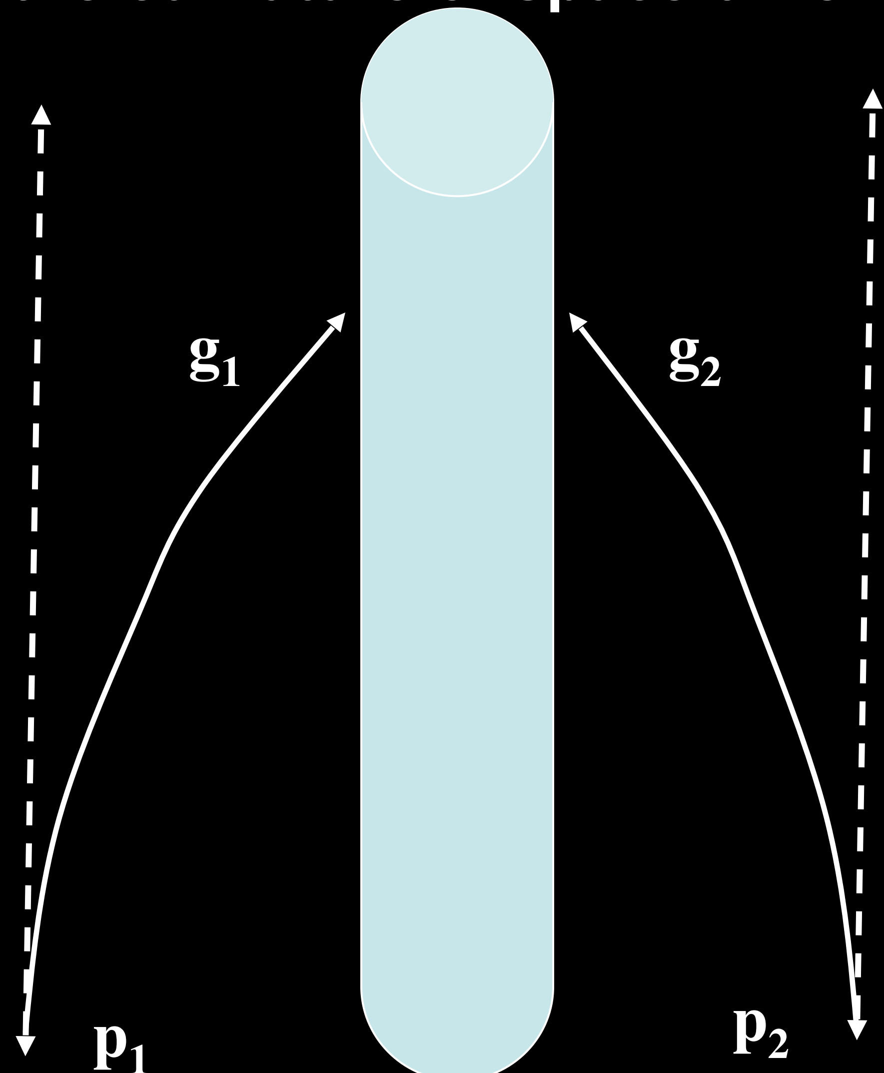
Particles p_1 and p_2 fall in the earth's gravitational field; how is this fact to be interpreted?



Newton: The particles would follow space-time geodesics g_1 and g_2, but are forced into curved space-time trajectories

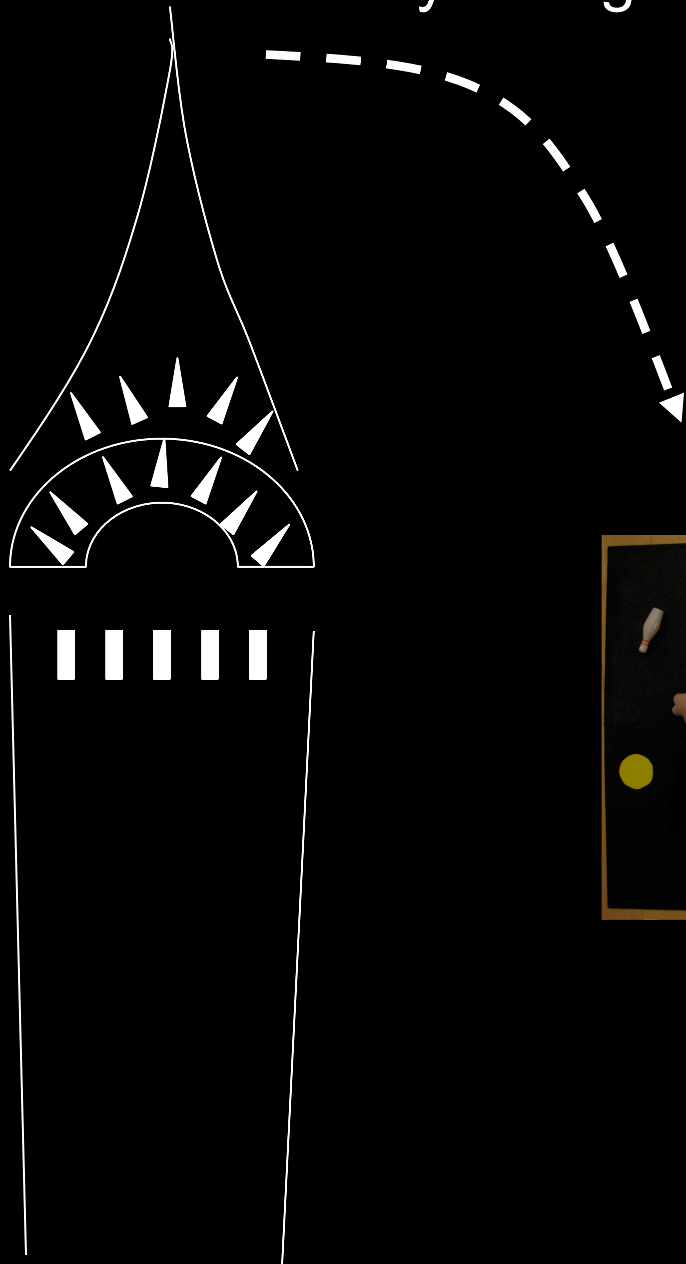


Einstein: The free-fall trajectories g_1 and g_2 are geodesics of space-time, and their convergence measures the curvature of space-time.



A force-free inertial observer alone in empty space

A freely-falling observer in a gravitational field



In a (nearly) static gravitational field such as the Earth’s, with acceleration ***g***, the man in the box will feel the weight of himself against the floor and the objects he holds in his hands.

An object that is dropped will fall to the floor with acceleration ***g***.

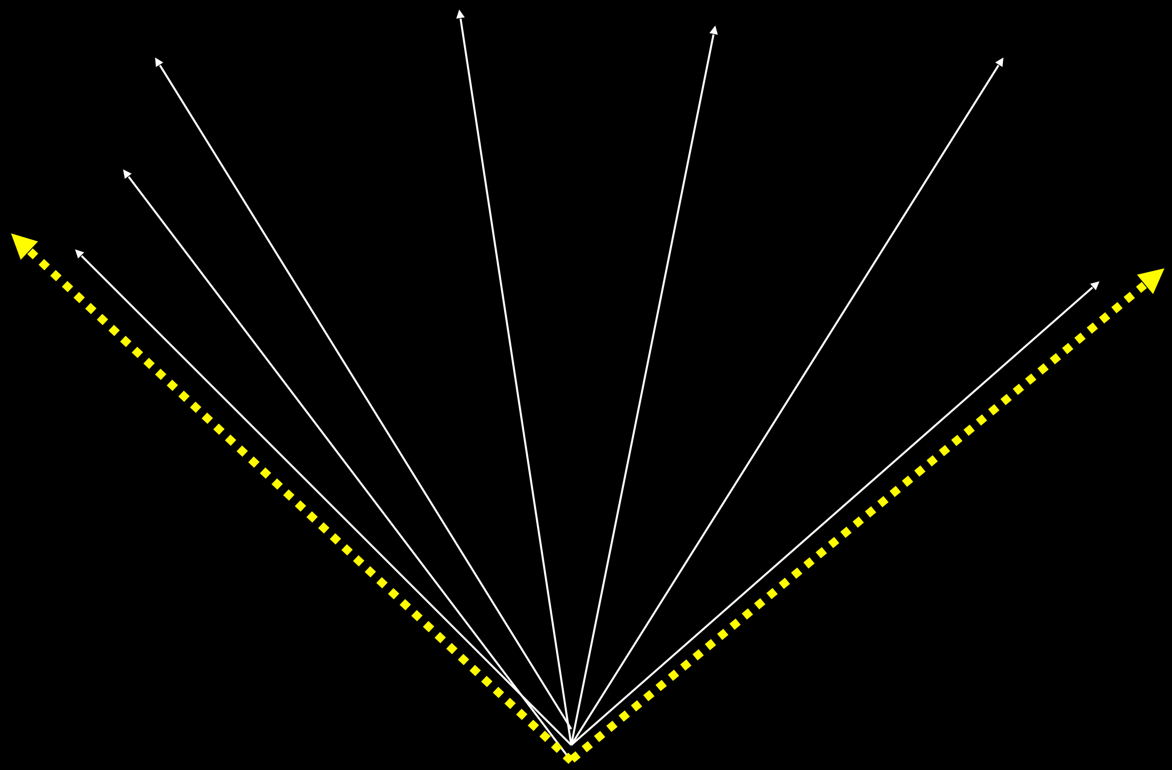
If an identical box is isolated in empty space, but is gently accelerated “upward” with acceleration -***g***, the man in this box will feel the weight of himself against the floor and the objects he holds in his hands.

In general, things will be have merely in virtue of their inertia just as they would in a gravitational field.

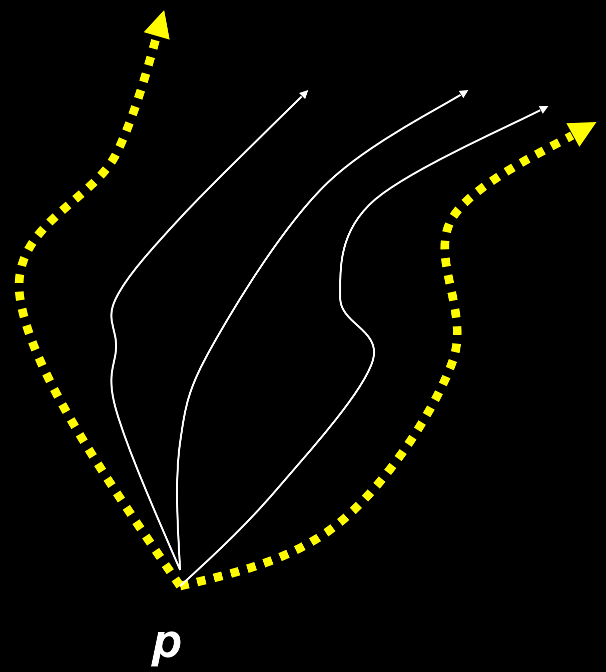
To the observer in the box, there is nothing to distinguish the two situations.

Gravity is indistinguishable from inertia.

The gravitational field is “transformed away” in the accelerated coordinate system.

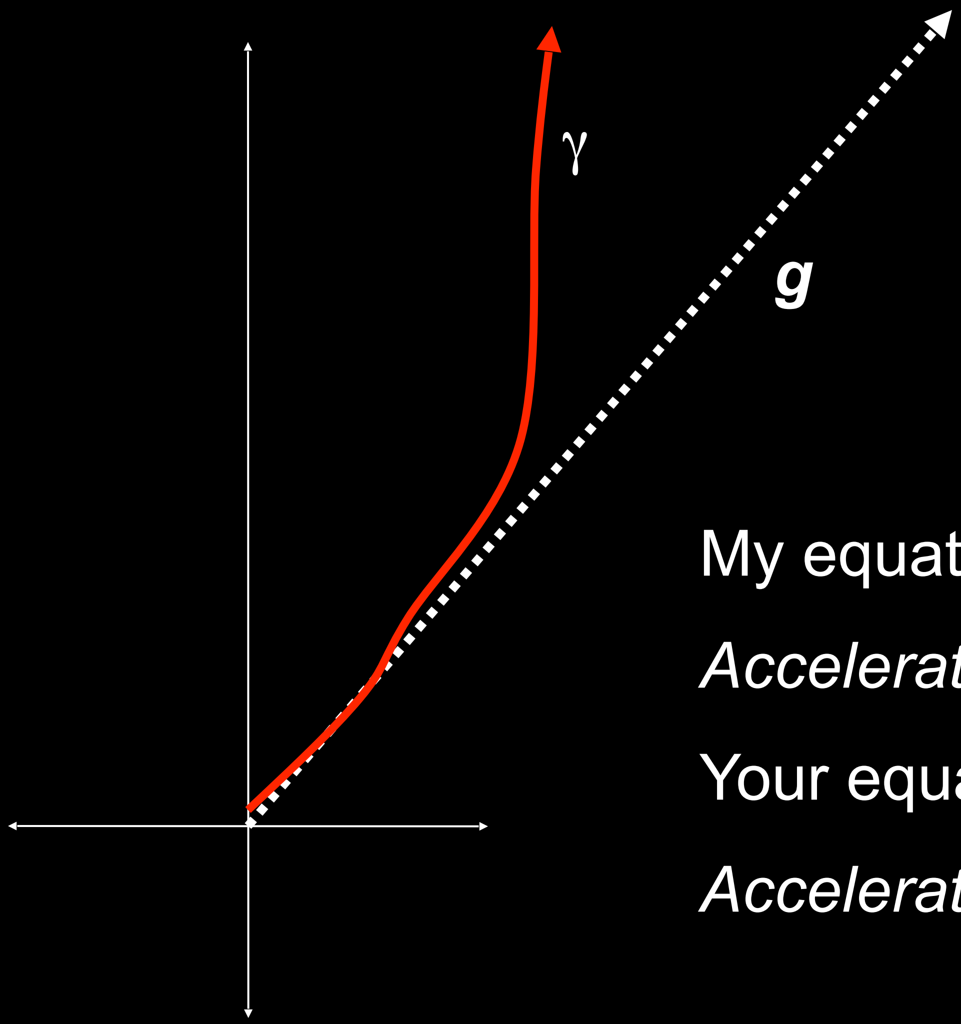


Experimental determination of space-time geometry: From a point ***p***, project particles (including photons) at all possible speeds in all possible directions.



In a curved space-time, one would expect the same experiment to yield different results.

Coordinate perspective on free-fall: In my coordinate system, you deviate from the geodesic ***g*** because the gravitational field pulls you into the trajectory γ.



My equation of motion is:

Acceleration = 0

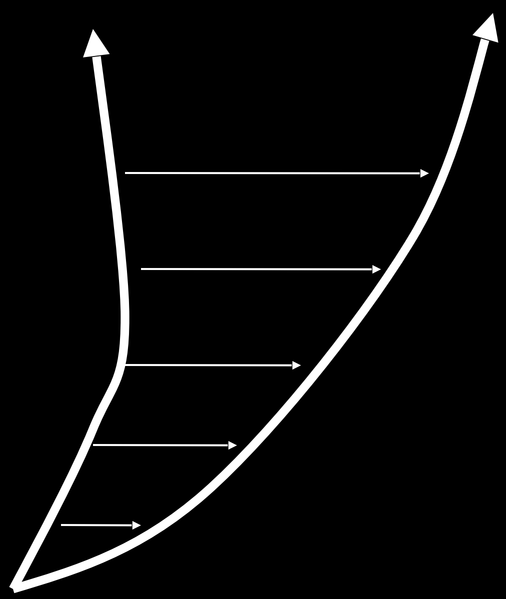
Your equation of motion is:

Acceleration = gravitational potential

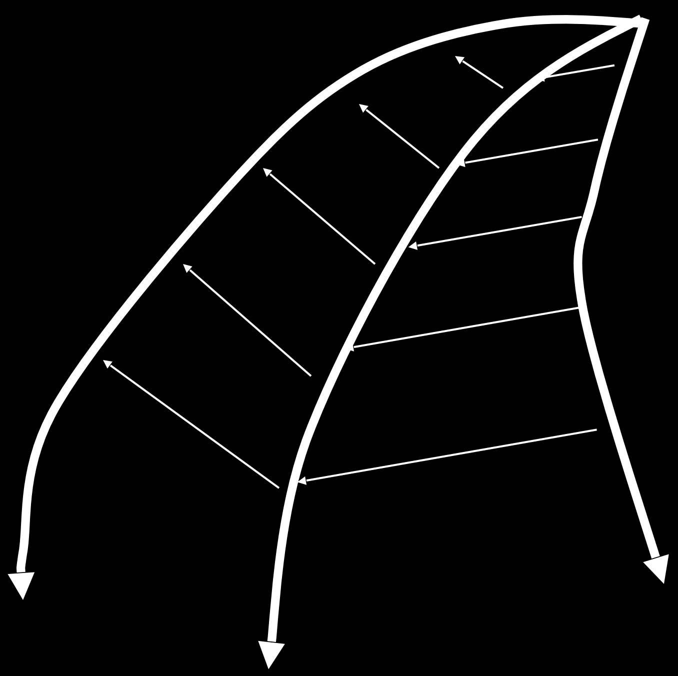
But in your coordinate system, my acceleration reveals the gravitational potential.

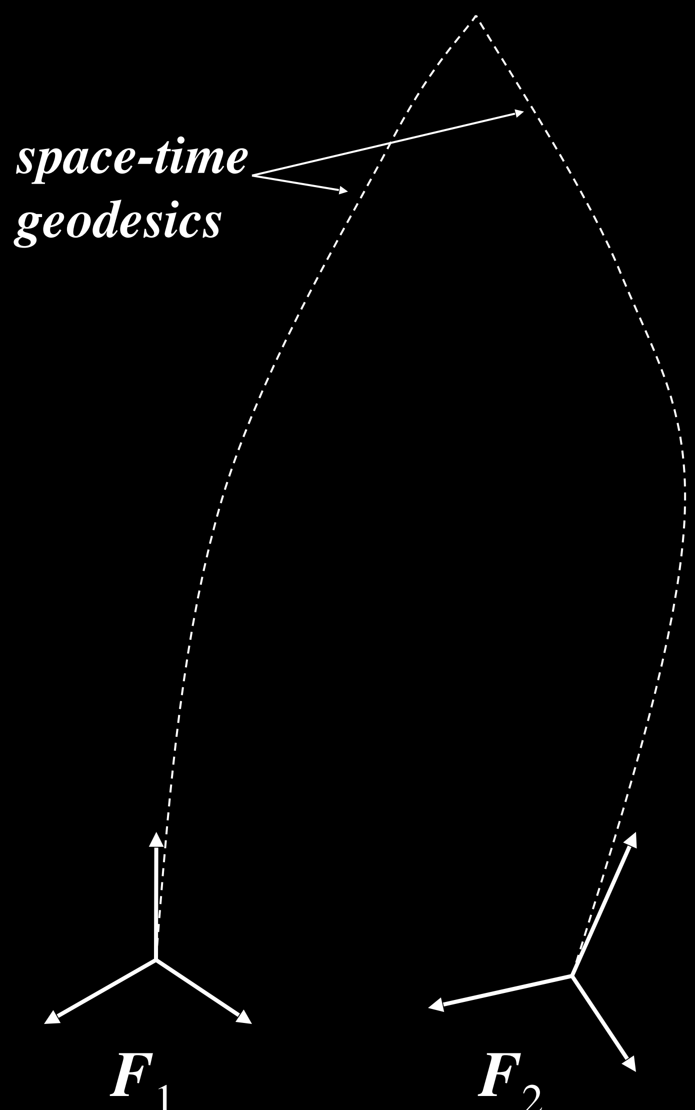
Invariant view: Both observers are following inertial trajectories, or *space-time geodesics*.

What they are measuring is the *geodesic deviation.*



What does the Newtonian observe when a handful of particles is scattered in a gravitational field?

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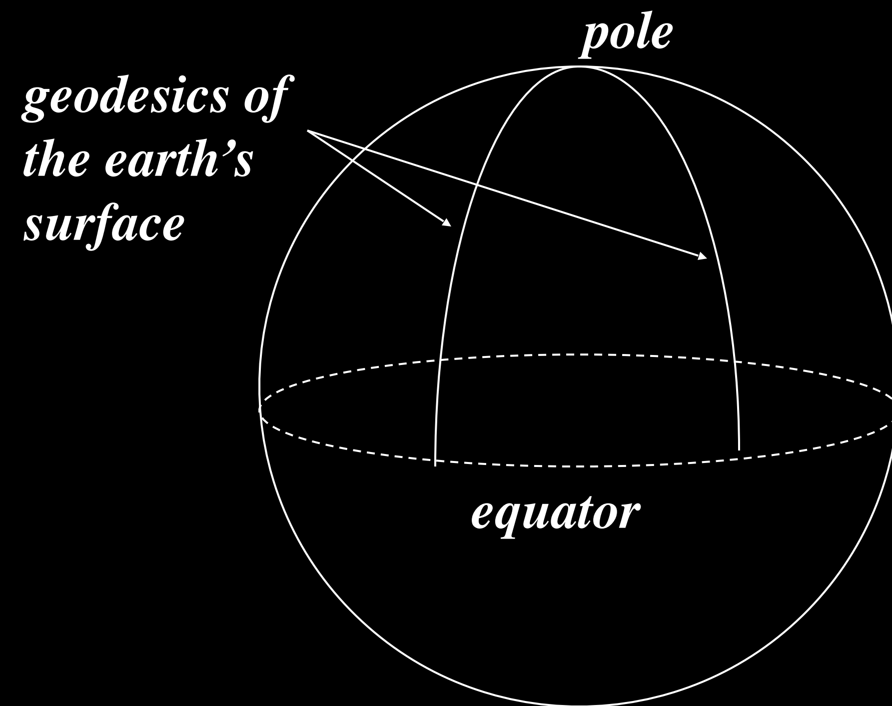
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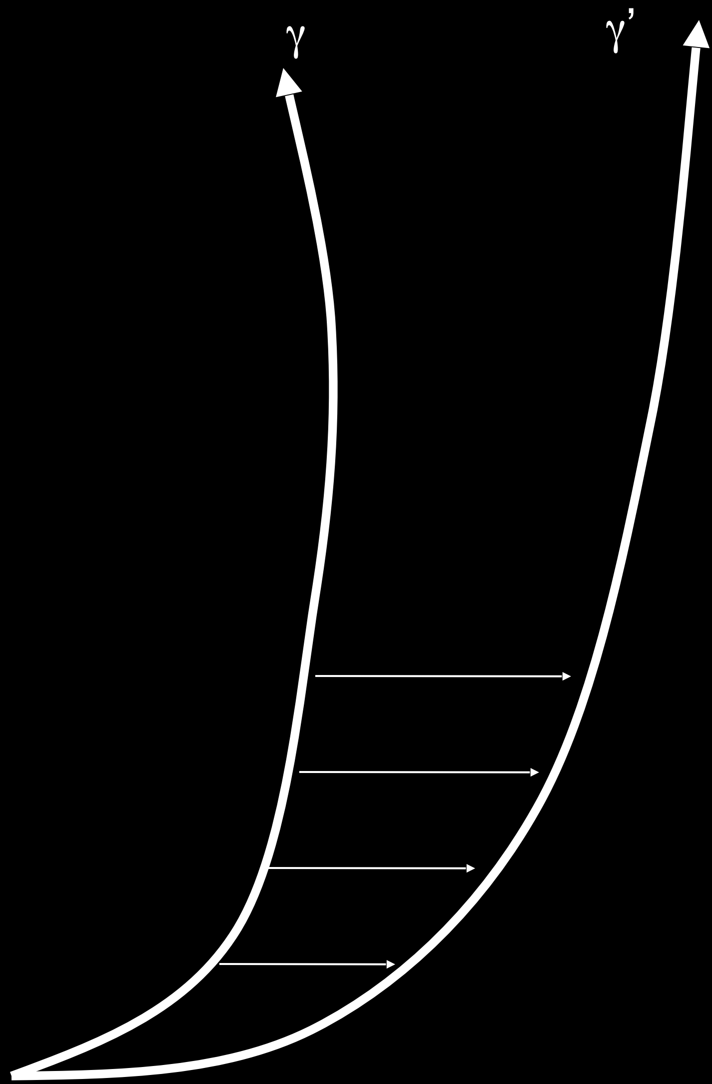
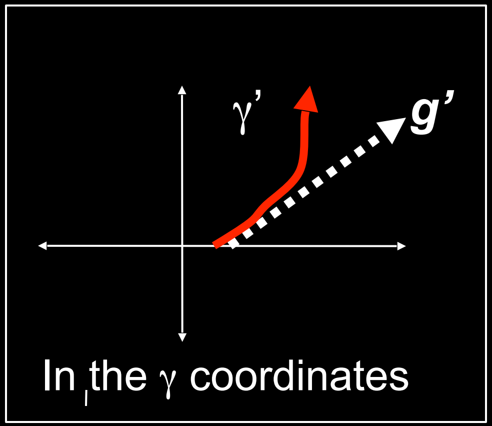
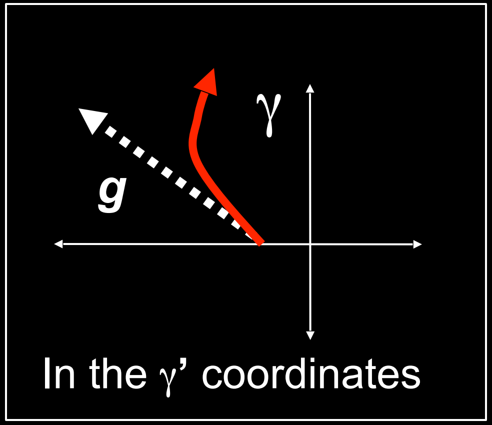
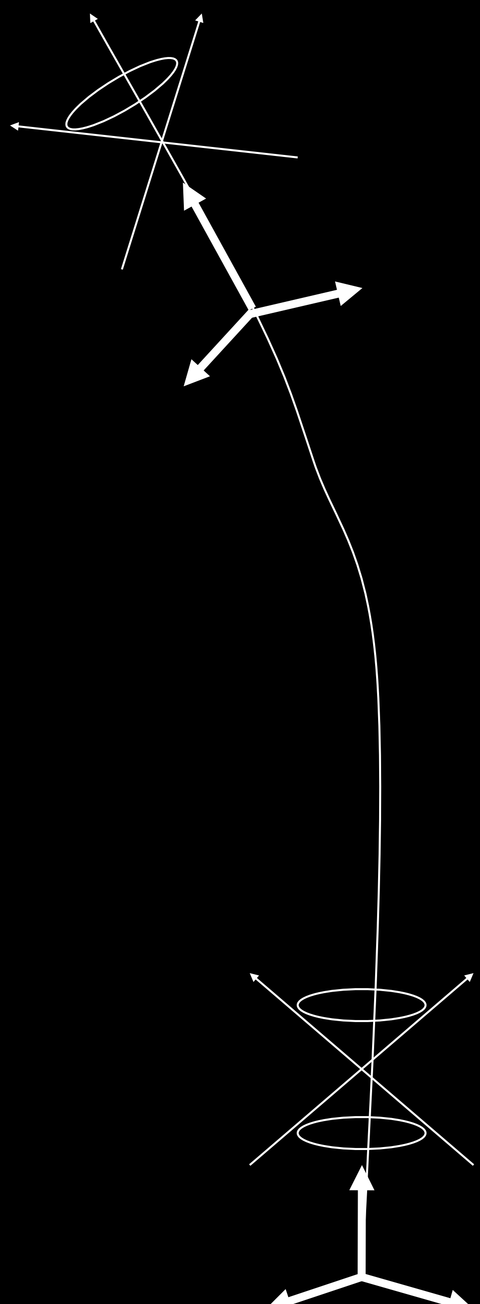
Geodesic deviation: invariant and coordinate-dependent views

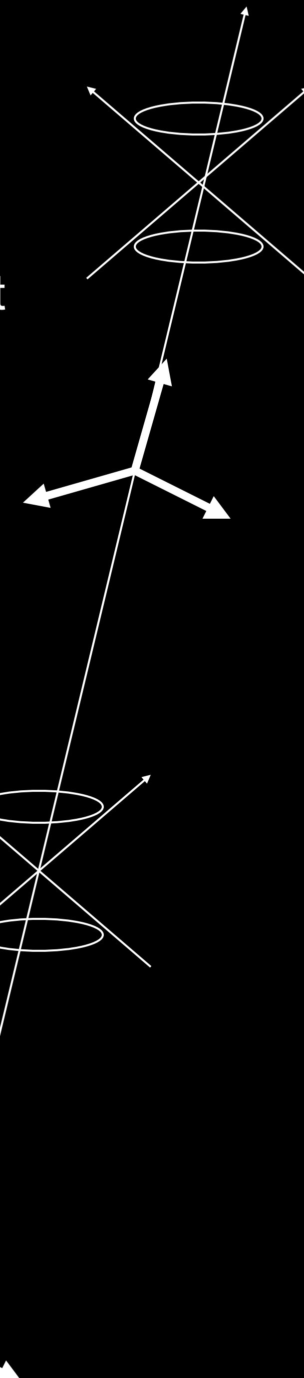








Local and global structure of general-relativistic space-time: the light-cone structure looks the same in the neighborhood of any point, but a local inertial frame *cannot* be extended into a global frame; local inertial frames will be accelerated relative to each other. in which any other inertial frame is also inertial. (So local inertial coordinates *cannot* generally be extended to global inertial coordinates.)



Einstein, Podolsky, Rosen,1935: “Can the Quantum-Mechanical Description of Physical Reality Be Considered Complete?”

“Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.”

Niels Bohr, 1935: “Can the Quantum-Mechanical Description of Physical Reality Be Considered Complete?”

EPR argument does not affect the soundness of quantum mechanics, “which is based on a coherent mathematical formalism covering automatically any procedure of measurement like that indicated. The apparent contradiction in fact only discloses an essential inadequacy of the customary viewpoint of natural philosophy for a rational account of physical phenomena of the type with which we are concerned in quantum mechanics.”

From our point of new we now see that the wording of the above-mentioned criterion of physical reality proposed by Einstein, Podolsky, and Rosen contains an ambiguity as regards the meaning of the expression ' without in any way disturbing a system.' Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system*.

Bohr: We thus see that the impossibility of carrying through a causal representation of quantum phenomena is directly connected with the assumptions underlying the use of the most elementary concepts which come into consideration for the description of experience. In this connection the view has been expressed from various sides that some future more radical departure in our mode of description from the concepts adapted to our daily experience would perhaps make it possible to preserve the ideal of causality also in the field of atomic physics. Such an opinion would, however, seem to be due to a misapprehension of the situation. For the requirement of communicability of the circumstances and results of experiments implies that we can speak of well defined experiences only within the framework of ordinary concepts.

Einstein on the presuppositions of physics:

What, “independently of quantum mechanics, is characteristic of the world of ideas of physics”:

the concepts of physics relate to a real outside world, that is, ideas are established relating to things such as bodies, fields, etc., which claim a 'real existence' that is independent of the perceiving subject - ideas which, on the other hand, have been brought into as secure a relationship as possible with sense-impressions;

these physical objects are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided that the objects ‘are situated in different parts of space’.

Unless one makes this kind of assumption about the independence of the existence (the ‘being-thus') of spatially separated objects, which stems in the first place from everyday thinking, physical thinking in the familiar sense would not be possible.

It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind.

The following idea characterizes the relative independence of objects far apart in space (A and B): external influence from A has no direct influence on B; this is known as the ‘principle of local action,’ which is applied consistently only in field theory. The complete abolition of this principle would make the idea of the existence of (quasi-) enclosed systems, and thereby the postulation of laws which can be tested empirically in the familiar sense, impossible. (Einstein 1948)

[The quantum mechanical] description, as appears from the preceding discussion, may be characterized as a rational utilisation of all possibilities of unambiguous interpretation of measurements, compatible with the finite and uncontrollable interaction between the objects and the measuring instruments in the field of quantum theory. In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which ***provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science.*** It is just this entirely new situation as regards the description of physical phenomena that the notion of complementarity aims at characterising. (Bohr 1948) [emphasis added]

It is reasonable of Einstein to ask whether, without the founding assumption that we can characterize the state of a local system, it would even be possible to undertake theoretical physics.

It is also reasonable to expect that measurements on different local systems will be integrable into a coherent classical picture.

Perhaps the former is a condition of the possibility of mathematical physics.

The latter, however, is merely a reasonable expectation that fails to be fulfilled.

As in the case of general relativity, the most reasonable extension of the local framework for measurement fails to capture the reality of a larger context.

A “ minimal”conception of realism:

Physical theories can extend our theoretical knowledge beyond what is immediately observable

Theoretical claims about the unobservable can be meaningful, in the sense of having definite truth conditions

The world can reject our theoretical pictures of it, because it has real physical features that cannot be captured within a given picture,

The replacement of one physical theory by another is, at least sometimes, an enlargement of our understanding of features of the world that are not immediately observable.

This conception of realism is not necessarily tied to the idea that any given theory, even “our best” theory at a given moment, is “true”.

The “pessimistic induction”: All those theories regarded as “someone’s best” theory, at some time, turned out to be false. Their approximate empirical successes evidently did not guarantee that they provided a “true” picture of the world. Therefore, the success of our current “best theory,” or any theory, is no reason to regard its way of representing the world as the right one.

Text

But this argument does not affect the circumstance that any successful theory— including a superseded theory—may identify some *systematic feature of reality* that survives the transition to the new theory.

A theory is an instrument—not merely for prediction and control, but for understanding.

Sometimes, a theory serves this instrumental function without being true.