BRILL'S COMPANION TO

THE RECEPTION OF PYTHAGORAS AND PYTHAGOREANISM IN THE MIDDLE AGES AND THE RENAISSANCE

Edited by

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CHAPTER 4

The Tribulations of the *Introduction to Arithmetic* from Greek to Hebrew Via Syriac and Arabic. Nicomachus of Gerasa, Ḥabib Ibn Bahrīz, al-Kindī, and Qalonymos ben Qalonymos

Gad Freudenthal

For Mauro Zonta,
in memoriam.

Nicomachus of Gerasa wrote his Neo-Pythagorean *Introduction to Arithmetic* toward the turn of the first and second century CE.[1] More than a millennium later, in 1317, the Jewish Provençal industrious translator, scientist, and man of letters translated (a version of) that work from Arabic into Hebrew.[2] During the long process of transmission through four cultures and four languages the text underwent profound changes: (i) in the Hebrew text many interpolations can be identified, some explicitly ascribed to "Abū Youssef," i.e., al-Kindī (801–866), the so-called "first philosopher of the Arabs";[3] (ii) throughout, the text is a paraphrase rather than a literal translation; and (iii) while the manuscripts of the Hebrew text reproduce many of the tables and drawings of the Greek original, they also carry drawings and tables to which nothing corresponds in the Greek text. A century ago, the great scholar Moritz Steinschneider, to whom we owe most of what we know about medieval Hebrew translations (and much more), examined these differences and made the following resigned observation: "Einiges ist auch nach Vergleichung mehrerer mss. nicht ganz klar."[4] Three

1 Greek text: Nicomachus of Gerasa 1866 (ed. Hoche). Translations: D'Ooge 1926 and Nicomachus of Gerasa 1978.
2 Steinschneider 1893a, § 320, 517.
3 Steinschneider 1893b and 1960, 227–228.
4 Steinschneider 1893a, 517.

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recent studies, which I had the pleasure to co-author, have shed some new light on the history of the text.[5] The present publication is based on these studies.

1

Nicomachus' *Introduction to Arithmetic* reached the Arabic world twice: a first version was made from a Syriac one; this version will be discussed below. Subsequently, the work was translated into Arabic a second time, now directly from the Greek, by the noted mathematician Thābit Ibn Qurra (d. 901); this version will not concern us here.[6] The Hebrew work is entitled *Sefer ha-'aritmatiqa'* ("The book of arithmetic") and the name of the author is given, following the Arabic, as Nīqūmākhūs al-gaharshīnī. The name of the translator, Qalonymos b. Qalonymos of Arles, appears in the colophons of two manuscripts (out of eight) of the text. In these colophons, the well-known translator indicates that he completed his work on 5 Nissan [50]77, i.e., March 19, 1317, when he was 30 years old.

Sefer ha-'aritmatiqa', the Hebrew version of the *Introduction to Arithmetic*, opens with a *Prologue*, which is not part of the Greek original of the work and which sheds important light on the history of the text. Its anonymous author addresses an unnamed personality, apparently of high rank; I will call that person the *Addressee*. From the Prologue we understand that the Addressee had already studied in part the *Introduction to Arithmetic* (the "famous work"), in a version that the author of the Prologue had "corrected" or "revised" "under the authority of our master, the noble Yaʿqūb ibn 'Isḥāq aṣ-Ṣabbāḥ al-Kindī" (515.3–5).[7] I will therefore call the author of the Prologue the *Revisor*. Al-Kindī was much interested in mathematics,[8] and so it is not surprising that he was interested in Nicomachus' work, to the point of "reading" it with his students while also "revising" the text. Certain works of al-Kindī indeed contain identifiable traces of his study of the *Introduction to Arithmetic*.[9] It should be noted,

5 Freudenthal and Lévy 2004. (This publication includes a critical edition of the Hebrew text of the first part of the work, accompanied by an annotated French translation.) Freudenthal and Zonta 2007; Zonta and Freudenthal 2009. Freudenthal 2005 essentially summarizes Freudenthal and Lévy 2004.

6 Kutsch 1958.

7 The references given in brackets in the text refer to the pages and lines of the published Hebrew text and its facing French translation (Freudenthal and Lévy 2004).

8 See, e.g., Rashed 1993, 7–12.

9 Brentjes 1987, 227–229. See also Endress 1997, 55; Langermann 2003.

however, that there are only few points of convergence between Nicomachus' work and al-Kindī's metaphysics.[10]

The Prologue states that al-Kindī made the revision in order to eliminate from the Arabic text that had reached him the numerous errors introduced into it by "Ḥabib Ibn Bahrīz the Nestorian," who had translated the work from Syriac into Arabic, at the request of Ṭāhir b. al-Ḥusain, "the ambidextrous" (515.5–7). Whereas translators from Syriac into Arabic were often criticized for an excess of literalism, Ibn Bahrīz is apparently taken to task for having introduced his personal philosophical ideas into the Arabic version of Nicomachus' work; we will come back to this below.

The two persons mentioned by the Revisor are well known. Ḥabib Ibn Bahrīz was a jurist, theologian and scholar, the Nestorian metropolitan of Harrān, then Mosul and Hazza; when he was consecrated bishop, he took the name of 'Abdīshū'.[11] He translated from Syriac into Arabic medical and philosophical works, including the *Introduction to Arithmetic*. He also composed original works, of which two, one legal, the other logical, have reached us and to which we shall return. Ibn Bahrīz played a significant role in the development of Arabic logic before Ḥunain ibn Isḥāq's translations were made (873). It is striking that Ibn Bahrīz and al-Kindī wrote epitomes of the same two logical works of Aristotle (the *Categoriae* and the *De interpretatione*): in the section devoted to books of logic, Ibn al-Nadim quotes the name of Ibn Bahrīz twice, and both times his name is associated with that of al-Kindī.[12] There was thus apparently a convergence of interests between the two scholars. Moreover, al-Kindī dedicated his epistle on the causes of rain to a pupil named "Ḥabib," most likely none other than Ḥabib Ibn Bahrīz.[13] A direct connection between the two scholars is all the more likely since al-Kindī attached great importance to his contacts with different translators and, moreover, both benefited from the support of al-Ma'mūn (the other patron of Ibn Bahrīz, Ṭāhir b. al-Ḥusain, was a general of al-Ma'mūn). It thus seems that Ibn Bahrīz belonged to the circle of translators around al-Kindī and it stands to reason that the latter may

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10 See al-Kindī 1974 (transl. Ivry), 20–21. Ivry concludes that "a comparison of our text and Nicomachus' yields little by way of specific comparisons" (at page 20); however, he compared al-Kindī's metaphysics with the Greek version of the *Introduction*, not with the Hebrew text, which reflects the Arabic text "corrected" by al-Kindī.

11 Troupeau 1997, and the bibliography given therein.

12 Flügel 1871, 248.27; 249.4. See further Rescher 1963, 14, and 1964, 28–29, 100.

13 This hypothesis was suggested by Steinschneider 1893a, 518, 564. The text has been edited in Bos and Burnett 2000; the dedication is on 97, 139 (transl. 161.); see also 325.

have been involved in selecting for translation this work, which responded to his theoretical interests.[14]

Ṭāhir b. al-Ḥusain is also a well-known personality, albeit of a very different profile.[15] As a general of al-Ma'mūn, he won the decisive battle that the latter fought against his brother al-Amīn, at the end of which Bagdad fell into his hands and al-Amīn was killed (813). Ṭāhir then settled for a time in the capital, where he accumulated a considerable fortune, and was later appointed governor of the territories of the caliphate to the east of Iraq (821), thus becoming the founder of the Ṭāhirid dynasty in Khorāsān. He was clearly endowed with exceptional warrior qualities – he was named "Dhul-Yamīnayn" (the ambidextrous) because in the course of a battle he cut a man into two with his left hand – but also had a penchant for the letters. Although Persian was his mother tongue, he was raised in the Arabic language, of which he had an exceptional mastery (the epistle to his son, dated 821–822, became a model of Arabic eloquence). It is thus not surprising that this close friend of al-Ma'mūn was a patron of learning too: he is known to have commissioned from scholars (including Ibn Bahrīz) at least five original works or translations.[16] Ṭāhir b. al-Ḥusain died (prematurely, possibly poisoned) in 822; this date constitutes the terminus *ante quem* for the translation of the *Introduction to Arithmetic* by Ibn Bahrīz.

2

In his Prologue, the Revisor refers to a letter he had received from the Addressee. In this letter, the latter complained that the revised text of the *Introduction to Arithmetic* was available to him only from the discussion of numbers onward. The Addressee suspected that the preceding part, to which he refers as the *Proemium* of the work, was "of great use and contains valuable information" (515.12–13) and he asked his correspondent to send it to him. The latter, the Revisor, confirms the Addressee's surmise: al-Kindī himself, he writes, stressed that the proemia of scientific and philosophical works constitute a literary genre of great significance and even commented that the proemia of the works of Nicomachus and of Ptolemy are the parts of these works

14 It is usually assumed that al-Kindī lived between 801 and 866. Given that Ibn Bahrīz completed his translation before 822 (see *infra*), the collaboration between the two scholars would have taken place when al-Kindī was only 20 years old.

15 Bosworth 1975, 90–95; Bosworth 2000.

16 See Endress 1987, 424 n. 60; Gutas 1998, 129–130.

in which the authors best explained their philosophies (517.4–8). In fact, since late Antiquity, the proemia of philosophical books constituted a codified literary genre whose purpose was to facilitate access to the works themselves.[17] The Addressee's request is therefore understandable: aware of the potential importance of the Proemium of Nicomachus' work, he wished to avail himself of it.

The Addressee's request was fulfilled. The Revisor sent the requested Proemium to the Addressee, accompanying it with a personal letter that subsequently became the Prologue as we have it in the Hebrew version (it is preserved only in that version). (Note that in what follows "Proemium" refers to the opening sections of *Introduction to Arithmetic*, construed as a proemium by the participants; "Prologue" refers to the Revisor's letter to the Addressee, inserted by the former before the Proemium.) From that Prologue we learn that at the time when the request was made to the Revisor, the revised version of the Proemium of the *Introduction to Arithmetic* did not yet exist: apparently it was only the Addressee's request that prompted the Revisor to draft it. In fact, at the end of the Proemium, just before the passage on the nature of numbers, the Revisor interpolates a remark of his own: "Here, my brother, is the whole of the Proemium of this book, up to [the passage] on the number, as you had requested. Let [the study of] this book […] be successful, and let Him, by His grace, direct you according to His will. Amen" (543.7–8). The text that the Revisor sent to the Addressee ends at this point; the bulk of the work was already in the hands of the latter. At some point in time, the Addressee combined the text that he received from the Revisor (the latter's Prologue followed by the first part of Nicomachus' *Introduction to Arithmetic*, considered as its Proemium) with the rest of the work and this reunified Arabic text (as modified by al-Kindi) became the *Vorlage* of the Hebrew version that has come down to us.

3

As already mentioned, a comparison of the Hebrew version of the *Introduction to Arithmetic* with the Greek original reveals that the two differ considerably. Thus, the Hebrew version carries numerous glosses: several of them are expressly attributed to al-Kindī, while the majority is unattributed. Some of these have a philosophical significance, indicating that they are deliberate interpolations and not the results of accidents of manuscript transmission. Moreover, throughout the text we repeatedly find the introductory formula

17 See, e.g., Westernik 1990; Mansfeld 1994; Quain 1945; Robinson 2000, 83–85; Klein-Braslavy 2002 and 2005.

"Nicomachus said" or "the author of the book said": these references to Nicomachus in the third person are para-textual elements that clearly go back to an "editor." In addition, the Hebrew text is paraphrastic throughout. The Hebrew version of *Introduction to Arithmetic* clearly resulted from heavy editing. The question arises: who is/are the editor(s) responsible for the various differences between the Greek and the Hebrew versions?

Many scholars were involved in creating, transmitting, translating and revising the text of *Introduction to Arithmetic*, and each could have played a role in its editing:

(i) Nicomachus of Gerasa, the author of the original Greek text;
(ii) The Greek commentators on the work;
(iii) The unknown translator from Greek into Syriac;
(iv) Ḥabib Ibn Bahrīz, the translator from Syriac into Arabic;
(v) Al-Kindī, who eliminated "errors" introduced by Ḥabib Ibn Bahrīz, adding at the same time his own glosses;
(vi) The anonymous Revisor, al-Kindī's student, who put in writing the corrections and glosses of his Master and wrote the Prologue;
(vii) The Addressee, who re-combined the Prologue and the Proemium he received with the bulk of *Introduction to Arithmetic*, thus creating a single continuous work;
(viii) An Andalusian scholar who (as will be seen) authored two colophons interpolated into the text;
(ix) Qalonymos ben Qalonymos, the translator from Arabic into Hebrew in 1317.

In this archaeological site, where no less than nine different layers of text are stacked on top of each other, can the different strata be identified and their paternity determined? To answer this question, let us try to appreciate the possible contribution of each of the participants, and specifically try to determine to whom the text owes its paraphrastic character and who interpolated into it the various unattributed glosses found in the Hebrew version.

4

We begin with the last link in the chain of transmission: the Hebrew translator, Qalonymos ben Qalonymos. It can be affirmed with certainty that he in no way interfered with the text: he was a prolific translator and we know his translations to have been strictly literal; nor did he have any ambition to "improve" a text he translated by interpolating additional material. This was also the general style of Hebrew translations made in Provence in the thirteenth and fourteenth centuries.[18] The paraphrastic character and the glosses thus originated in previous stages of the transmission – they may be due to the Greek-into-Syriac translator, and/or to Ibn Bahrīz (the Syriac-into-Arabic translator), and/or to al-Kindī and his student, the Revisor.

Consider now the passage from Greek into Syriac (Stage *iii*). We do not know the identity of the translator and so the date of the translation is uncertain. Philosophical translations from Greek into Syriac evolved according to the following scheme: they began in the middle of the fifth century, continued during the sixth and seventh centuries, and came to a halt during the eighth century.[19] In the ninth century there was a new wave of translations, concomitant with the great movement of translations into Arabic. Now the translation of a specialized work such as Nicomachus', which requires an elaborate philosophical and mathematical vocabulary, cannot have been very early. Given that the work was translated into Syriac before 822 (when it was already translated into Arabic), it seems reasonable to assume that the Syriac translation dates from the end of the eighth century or the very beginning of the ninth. It is not impossible that it was Ibn Bahrīz himself who translated the text first into Syriac, then from Syriac into Arabic (Ḥunayn ibn Isḥāq did such double translations a few decades later).[20]

We next ask whether the Syriac translation was paraphrastic or literal. The Syriac version is not extant, but we have textual witnesses allowing us to determine that it was a literal translation from Greek. Remains of Ibn Bahrīz's Arabic translation from Syriac into Arabic *before* it was revised by al-Kindī are preserved in the *Ta'riḥ* by Aḥmad Ibn Abū Yaʿqūb Ibn Wadiḥ, known as al-Yaʿqūbī (d. 897), which contains a short account of Nicomachus' *Introduction to Arithmetic*.[21] The comparison of the sentences borrowed from Ibn Bahrīz with the Greek version establishes that Ibn Bahrīz's original version corresponds literally to the Greek original, implying that the Greek-into-Syriac, as also the Syriac-into-Arabic, versions (Stages *iii* and *iv*), were both *literal translations*.[22] However, we will shortly see that while al-Bahrīz translated literally, he nonetheless introduced into the work significant interpolations.

Yaʿqūbī's text has further significant information in store. A close look reveals that Yaʿqūbī was under the impression that the body of Nicomachus'

18 See, e.g., Zonta 1992, XXXI, XXXVI.
19 For what follows, see Hugonnard-Roche 1990, 132–134; Brock 1977, esp. 6–10; Brock 1983.
20 On the rationale for this translation technique, see Brock 1977, 2–3.
21 Houtsma 1883, 140–143. This section is translated in Klamroth 1888, 9–16; see also Sezgin 1974, 164–166.
22 Freudenthal and Zonta 2007.

work began only at I.6 and that the foregoing text (I.1–5) was its Proemium:[23] the entrenched notion of "proemium" was projected unto Nicomachus' work, leading its ninth-century (and possibly earlier) readers to view the work's first five chapters as its proemium, the sequel as the bulk of the work.[24] In the ninth century, we realize, the *Introduction to Arithmetic* circulated in a version in which chapters I.1–5 were somehow set apart from the rest and often circulated independently. This ties in neatly with what we saw above: at some point, the Addressee received the body of the work, with the exception of what he and the Revisor considered as the work's Proemium, which – we now realize – consisted of I.1–5. The Revisor, we saw, prepared the Proemium (I.1–5) only after the Addressee had requested it; this means that he "edited" the Proemium and the bulk of the book at two different times.

The Greek version is divided into two books, consisting of 23 and 29 chapters, respectively. The Hebrew version is divided into two books (*ma'amarim*), in conformity with the Greek division. In the Hebrew version the Proemium (corresponding to I.1–5) is not subdivided, except by short phrases of the kind "Abū Yūsuf said." The sequel, from I.6 onward, is divided into a series of unnumbered sections, each identified as a "discourse" (*dibbur*, the exact equivalent of the Arabic *qaul*) and bearing a title indicating its subject-matter: e.g., *ha-dibbur be-geder ha-mispar wa-ḥaluqato* (the discourse on the definition of number and its division), *ha-dibbur be-to'ar ha-kammah ha-ṣerufi* (the discourse on relative quantity), etc. The "discourses" are units smaller than the chapters of the Greek version, of which there is no trace here anymore. Now Yaʿqūbī knows of only three "discourses," showing that the division into the numerous short "discourses" is posterior to al-Bahrīz. We infer that it was introduced by al-Kindī's student, the Revisor, according to the Master's instructions.

The Revisor (following al-Kindī's directives) interfered in the work in two ways. First, he corrected the text, eliminating from it the "false ideas" introduced by Ibn Bahrīz. These interferences can obviously not be identified in the text that has reached us. Second, the Revisor interpolated into the text glosses, introduced by the phrase "Abū Yūsuf said."[25] Some of these are veritable quotations from al-Kindī, as we shall now see. Nicomachus opens his work by a brief discussion of the definition of philosophy that he ends by endorsing the

23 I refer to the division in the Greek version.

24 See *supra*, n. 17. Indeed, medieval Jewish authors looked for – and identified – "proemia" even in the biblical books.

25 Only once does the text of a gloss attributed to al-Kindī indicate where it ends ("end of the words of Abū Yūsuf"), so that at times it is difficult to detect where a given Kindian gloss ends.

definition given by Pythagoras.[26] This discussion is found, abbreviated and modified, in the Hebrew version (519.5–8). Then follows a long gloss introduced by the sentence "Abū Yūsuf said: The Ancients have given to philosophy several definitions" (519.11). Five definitions of philosophy are then presented, followed by a sixth, which al-Kindī presents as his own (523.4–6). Now this long gloss is found with minimal textual variants in the *Book of Definitions* attributed to al-Kindī.[27] The fact that the same passage appears in both works affords us an insight into the method of the Revisor: some of the glosses attributed to the Master he borrowed from other works of his. At the same time, it confirms the authenticity of the passages attributed to al-Kindī in the *Introduction to Arithmetic*, just as it validates the (disputed) authenticity of the *Book of Definitions*. Obviously, the Proemium may include also interpolations not identified by the phrase "Abū Yūsuf said."

The Revisor assures us that his aim has always been to summarize in the most concise way what he had heard from his Master, al-Kindī: "I will refrain from embellishing the book and adding to it," he writes (517.2–3). He further states that he avoided writing longwinded discussions, preferring "short discussion[s] that I have heard from our teacher Abū Yūsuf [al-Kindī] explaining what you wished to be explained. […] I will abandon what I [myself] wished to divulge at length, [replacing it] by his concise statement" (515.14–517.2). But the Revisor does not claim to have completely abstained from any personal comments. On the contrary, when he writes, "I have explained all that can and needs to be explained, omitting all repetition and redundancy. […] In doing so, I made it more accessible to you than it was in the text of the translator, without [however] changing the ideas" (517.12–15), he clearly asserts that he gave a personal stamp to the presentation of the ideas of both Nicomachus and al-Kindī.

In the Prologue the Revisor says nothing more about his inclination for brevity and concision. But later on, following an interpolation by al-Kindī concerning the relative distances of the earth to the celestial bodies, the Revisor intrudes with the following interjection, directed to the Addressee:

> I did not go into long [details] in this discussion, although I suppose that what the author states on this subject is not known to you,

---

26 Greek text in Nicomachus of Gerasa 1866, 1.1–3.

27 Text in Abū Rīda 1950/1369 H., 172–173. A revised version with an introduction and notes has been published in Al-Kindī 1976, 7–69; for our passage, see 22–23 (text), 35 (translation), 56–60 (commentary). An English translation of this passage is included in Altmann and Stern 1958, 28.

> notwithstanding your sharp mind, your perseverance in the study of the author's words, and your love for this art, and although you count among those who possess his books in their own home. Therefore, I wanted to remind you of this matter [just discussed].
>
> [However,] I have no doubt that this book of mine will eventually fall into the hands of someone who will be unfamiliar with the views of the Master, as you know them. Now, since this [foregoing] discussion [alone] cannot lead [someone] to the truth concerning the *quaesitum*, the ideas [of the unprepared reader] will be confused, the imaginings will vanish, truths will disappear, and knowledge will be lost. May God guide you right in the light of his [al-Kindī's] commentary and allow you to apprehend the splendor of His Glory.[28]

The brevity thus seems to follow the intention to make the text impenetrable to those unworthy of it.

The above reconstruction implies that the version produced under al-Kindī's authority has to be qualified as a *recension* of the *Introduction to Arithmetic*, distinct from that of Ibn Bahrīz (to be presented below). Al-Kindī, it seems, did not himself interfere with the text: he apparently left this task to his student, the Revisor, who prepared the final version of the text according to the directions he had received from al-Kindī and according to his own good judgment in the spirit of his Master. It therefore seems appropriate to refer to this recension as that of *al-Kindī/the Revisor*.

5

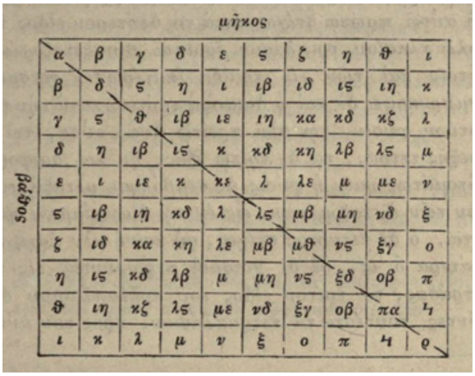
With these insights into stages *v–vi* of the transmission, we can now consider stage *iv*, the Syriac-into-Arabic translation. Let us first recall the important finding that Ibn Bahrīz's Syriac-into-Arabic translation was literal, not paraphrastic. At the same time, as we will now see, Ibn Bahrīz interfered with the text substantially. This is not surprising: we already noticed that al-Kindī observed that Ibn Bahrīz had introduced into the Arabic version his own ideas ("false ideas" in al-Kindī's judgment), precisely those that the Revisor set out to

28 MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt Yb 4° 5, fol. 19a, collated with MSS Paris, BnF, héb. 1095, fols 195a–195b and héb. 1029, fol. 11a.

eliminate following al-Kindī's instructions. As mentioned, Ibn Bahrīz authored "epitomes" of two of Aristotle's logical works, thereby evincing that his profile was not that of a "mere translator."[29] Felicitously, we are able to identify in the Hebrew version significant interpolations that can be ascribed to Ibn Bahrīz with a near-certainty.

The Hebrew version of *Introduction to Arithmetic* contains many tables and diagrams. They can be divided into two groups according to their proximity to the Greek text:

1. *Numerical tables and diagrams*. Most of the numerical tables in the Hebrew version reflect their Greek models faithfully, where the Greek letters indicating numbers are replaced by Hebrew letters also functioning as numerals. (Compare Figures 4.1 and 4.2.)


μῆκος

βάθος

α	β	γ	δ	ε	ϛ	ζ	η	ϑ	ι
β	δ	ϛ	η	ι	ιβ	ιδ	ιϛ	ιη	κ
γ	ϛ	ϑ	ιβ	ιε	ιη	κα	κδ	κζ	λ
δ	η	ιβ	ιϛ	κ	κδ	κη	λβ	λϛ	μ
ε	ι	ιε	κ	κε	λ	λε	μ	με	ν
ϛ	ιβ	ιη	κδ	λ	λϛ	μβ	μη	νδ	ξ
ζ	ιδ	κα	κη	λε	μβ	μϑ	νϛ	ξγ	ο
η	ιϛ	κδ	λβ	μ	μη	νϛ	ξδ	οβ	π
ϑ	ιη	κζ	λϛ	με	νδ	ξγ	οβ	πα	ϟ
ι	κ	λ	μ	ν	ξ	ο	π	ϟ	ϱ

FIGURE 4.1
Numerical Table
This table of the multiples (from × 2 to × 10) of the first ten integers is meant as an aid for generating an epimoric number or superparticular ratio, which is a number that contains a smaller number to which it is compared, plus an integral fraction (½, ⅓, ¼ …) of the latter.
REPR. FROM *INTRODUCTIONIS ARITHMETICAE LIBRI*, NICOMACHUS OF GERASA 1866 (ED. HOCHE), 51

29 See Gutas 1993, 35–36.

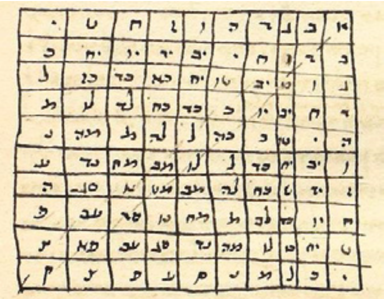


FIGURE 4.2
Numerical Table
This table, from the Ibn Bahrīz-al-Kindī (Qalonymos) version, is the faithful reproduction and translation of the corresponding Greek table, where the Greek numerals have been replaced by Hebrew numerals.
REPRODUCED FROM: HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 21R. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

Other numerical tables are more complete and detailed than their Greek models but follow the same general model (see Figures 4.3 and 4.4).

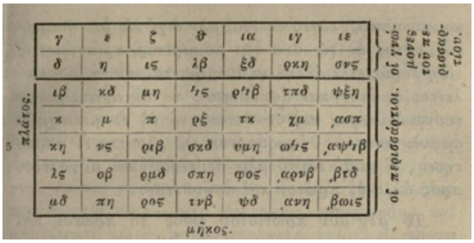


FIGURE 4.3 Numerical Table

FIGURE 4.4 Numerical Table

Figure 4.4, from the Ibn Bahrīz-al-Kindī (Qalonymos) version, is the expanded form of its Greek model in Figure 4.3. It presents the rule for generating singly even numbers, which are the product of a doubly even integer (i.e., a power of 2) multiplied by an odd integer (*Introduction to Arithmetic* l, 10). In the Greek table, the doubly even integers 4, 8, 16, …, 256 are arranged in a row below the top row of odd integers 3, 5, 7, …, 15. Each column in the third line contains the number produced by multiplying the corresponding doubly even integer in the second row by the first odd integer in the first row (3); the fourth row, the number produced by multiplying the corresponding doubly even integer in the second row by the second odd integer in the first row (5); etc. The Hebrew table is set up differently and is more complete. The two generating series are arrayed in the top row (4, 8, 16, …, 128) and the rightmost column (3, 5, 7, …, 15). The product of one term by another (a singly even number) appears where the respective row and column intersect. The Hebrew table adds something else as well: the rightmost column displays the first "doubly odd" numbers (integers divisible into two equal odd integers: 6, 10, 14, …, 30). In the Ibn Bahrīz-al-Kindī version, Nicomachus' characteristic property of a singly even number is treated at greater length than in the Greek work, both in the category of doubly even numbers and in that of doubly odd numbers.
REPR. FROM *INTRODUCTIONIS ARITHMETICAE LIBRI*, NICOMACHUS OF GERASA 1866 (ED. HOCHE), 25; HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 12R. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

Much the same holds true of the diagrams (graphical representations of mathematical statements; see Figure 4.5): the published Greek text of *Introduction to Arithmetic* carries many diagrams, but the Hebrew version has a many more, some of which correspond to a Greek model and translate it faithfully (Figure 4.6), while others have no Greek model and while they follow the same tradition, their form is more elaborate (Figure 4.7).

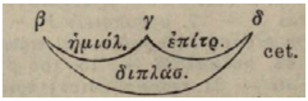


FIGURE 4.5
This diagram, found in some Greek manuscripts of *Introductionis arithmeticae Libri*, illustrates the relationship between three ratios: double, hemiolic or sesquialteral (× 1½), and epitritic (× 1⅓). Five other diagrams of the same type, found in the manuscripts but not reproduced in Hoche's edition, are reproduced in D'Ooge 1926, 235, n. 2; this diagram is also there, 234, n. 2.
FROM: *INTRODUCTIONIS ARITHMETICAE LIBRI*, NICOMACHUS OF GERASA 1866 (ED. HOCHE), 82 (APPARATUS)

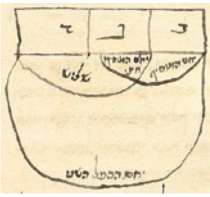


FIGURE 4.6
This diagram is the accurate reproduction and translation of the corresponding Greek diagram (Fig. 4.5), where the Greek numerals have been replaced by Hebrew numerals.
MS PARIS, BIBLIOTHÈQUE NATIONALE DE FRANCE, HÉB. 1095, FOL. 208R. REPRODUCED BY KIND PERMISSION OF THE BIBLIOTHÈQUE NATIONALE DE FRANCE

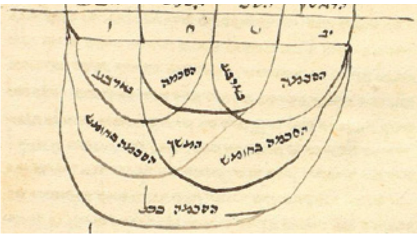


FIGURE 4.7
This diagram, absent from the Greek text, follows the Greek model (Figs. 4.5 and 4.6), but develops it further. It is inserted at the end of the book and is the most elaborated diagram of this form in the Hebrew version. It represents the numerical proportions corresponding to musical consonances: the fourth (8 to 6 or 12 to 9), the fifth (9 to 6 or 12 to 8), the octave (12 to 6), the tone (9 to 8).
MS HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 54R. REPRODUCED BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

At present, little can be said about the origin of these differences. Richard Hoche (1834–1906), the editor of the Greek text (as also its two translators), placed at their center of his concerns Nicomachus' *text*, paying almost no attention to variations in tables or diagrams in different manuscripts. Moreover, Hoche did not study all the extant manuscripts.[30] This disallows drawing any conclusions from a comparison of the Greek version with its Hebrew offspring: when faced with a table or diagram in the Hebrew version not found in the printed Greek text edition, we cannot know at what stage that item entered the text tradition. In any event, as far as the numerical tables and the diagrams are concerned, they are a continuation of an existing Greek tradition, not a radical innovation.

II. *Verbal tables*. But a second group of tables in the Hebrew version have no equivalent or model at all in the Greek text: these are rectangular tables representing *verbally* a schematic synthesis of analyses offered in Nicomachus' text; they contain no numbers at all. Their purpose is to make a theoretical statement with a certain level of generalization more easily accessible and easier to retain (see Figures 4.8, 4.9, and 4.10).

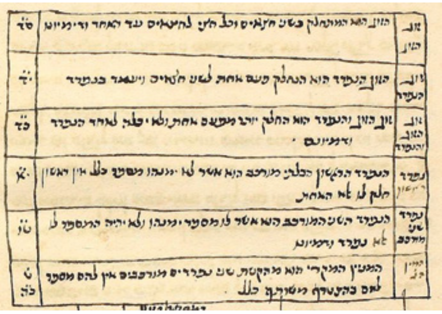


FIGURE 4.8
Textual Table
This textual table summarizes the discussions of the six species of integers: doubly even (example given: 64); doubly odd (14), singly even (24), primes (11), nonprime odd integers (15), and pairs of integers without a common divisor (9 and 25). Like the other textual tables, it does not have a Greek model.
REPRODUCED FROM: HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 16V. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT

30 Hoche used nine manuscripts (Nicomachus of Gerasa 1866 [ed. Hoche], VI–VII), whereas D'Ooge lists 44 (D'Ooge 1926, 147–151).

FIGURE 4.9 Textual Table
This textual table, inserted at the end of Treatise One, summarizes the categories of numerical relationships: equality/inequality, ratios of the greater to the smaller number (five species), ratios of the smaller to the larger number (five species paralleling the previous ones).
REPRODUCED FROM: HALLE, UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT, MS YB 4° 5, FOL. 31R. BY KIND PERMISSION OF THE UNIVERSITÄTS- UND LANDESBIBLIOTHEK SACHSEN-ANHALT.

The verbal tables constitute a true innovation, for no Greek model is known. Fortunately, they bear the unmistakable stamp of their author: Habīb Ibn Bahrīz. In fact, the latter's *Kitāb Ḥudūd al-manṭiq* (Definitions of Logic), his only philosophical work to have come down to us,[31] contains many tables whose structure and presentation are exactly identical to those of the verbal tables in the Hebrew version of the *Introduction to Arithmetic* (compare Figures 4.8, 4.9 and 4.10 with Figures 4.11 and 4.12).

Ibn Bahrīz presents these tables as one of the highlights of his *Ḥudūd al-manṭiq*. In order to expose the logical ideas of Aristotle and his commentators, he writes, he "chose the [method] of representation [*tamthīl*] and of presentation by tables [*taṣwīr*], so that [these ideas] be easier to understand

31 Published in Danišpazuh 2002, 97–126.

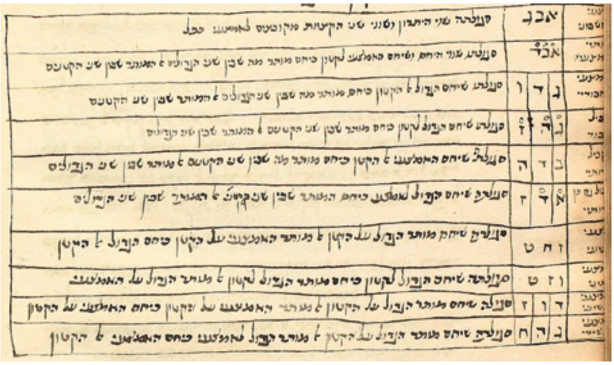
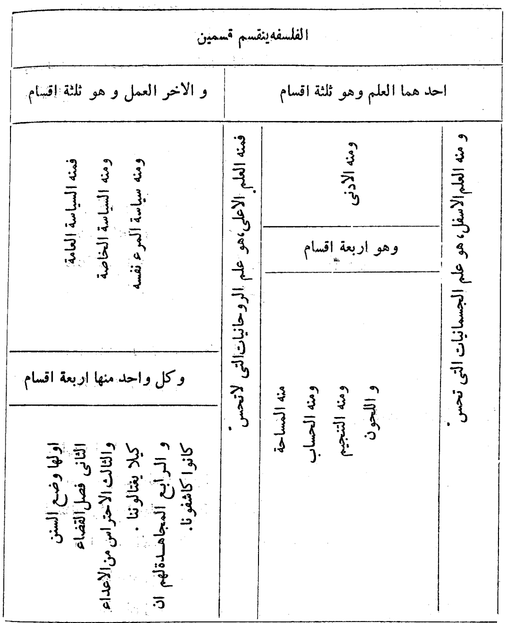


FIGURE 4.10 Numerical and Textual Table
This table is unique in that it combines the presentation of numerical and textual information. The numerical data are already in the Greek version of the *Introduction to Arithmetic* (Nicomachus of Gerasa 1866 [ed. Hoche], 144) and here they are supplemented by a summary of the ten "means".
FROM: MS NEW YORK, JEWISH THEOLOGICAL SEMINARY, 2449, F. 151V. REPRODUCED BY KIND PERMISSION OF THE JEWISH THEOLOGICAL SEMINARY, NEW YORK



الفلسفه ينقسم قسمين

احد هما العلم وهو ثلثة اقسام

و الآخر العمل و هو ثلثة اقسام

و منه العلم الاسفل وهو علم الجسمانيات التى تحس .

ومنه الادنى

وهو اربعة اقسام

منه المساحة
ومنه الحساب
ومنه التنجيم
و اللحون

فمنه العلم الاعلى وهو علم الروحانيات التى لا تحس .

فمنه السياسة العامة
ومنه السياسة الخاصة
ومنه سياسة المرء نفسه

وكل واحد منها اربعة اقسام

اولها وضع السنن
الثانى فصل القضاء
والثالث الاحتراس من الاعداء كيلا يقتلونا .
و الرابع المجاهدة لهم ان كانوا كاشفونا.

FIGURE 4.11 Textual Table

<table>
<tr><td colspan="3">وانما ينقسم العلم قسمان لأنّ الأشياء شيئان</td></tr>
<tr><td>والاخر غير محسوس</td><td colspan="2">احدهما محسوس وهو قسمان</td></tr>
<tr><td>وعلمه يسمّى العلم الأعلى بمنزلة الأمهات الاربع ومانشأ منها وعلمها يسمّى العلم الأسفل.</td><td>فمنه مالايفارق العدّة التي هو فيها ولا بالتوهّم.</td><td>و منها ما يفارق العدّة التي هو فيها بالتوهّم فقط، كالدائرة و الشكل المثلّث و المربّع، وعلمها يسمّى علم الأدب.</td></tr>
</table>

FIGURE 4.12 Textual Table

These two tables are taken from Ḥabīb ibn Bahrīz's *Kitāb Ḥudūd al-manṭiq* and are typical of this work. Their similarity to the textual tables in the Hebrew version of the *Introduction to Arithmetic* is unmistakable. This indicates that the textual tables in the *Introduction to Arithmetic* go back to ibn Bahrīz and attests to the Nestorian scholar's active contribution to this version.

REPR. FROM *AL-MANṬIQ* (*LOGIC*) *BY IBN MUQAFFA*ʿ [*AND*] *ḤUDŪD AL-MANṬIQ* (*DEFINITIONS OF LOGIC*) *BY IBN BIHRIZ*, ED. DĀNESHPAZHŪH, 112, 114

and easier to retain."[32] In order to make the definitions of logic easier to understand, he further emphasizes, he endeavored to "discover, abstract, summarize, transcribe them, and represent their divisions and their definitions in tables [*taṣwīr*], so that their representation [*tamthīl*] be in front of the reader's eyes, facilitating their understanding and favoring their memorization."[33] These sentences perfectly characterize the function of the verbal-type tables appearing both in *Ḥudūd al-manṭiq* and in the Hebrew version of the *Introduction to Arithmetic*. Ibn Bahrīz, it should be noted, did not invent the tabular

32 Danišpazuh (ed.) 1978/1398 H, 97.8.

33 Danišpazuh (ed.) 1978/1398 H, 100.19–20.

representation of "divisions": as Henri Hugonnard-Roche and Gérard Troupeau have pointed out, it was inspired by Syriac scholastic models that go back to the Alexandrian tradition.[34] This incidentally explains why "primitive" forms of certain tables can be found, e.g., in Boethius' *De institutione arithmetica*.[35]

The fact that the verbal-type tables are found in *Kitāb Ḥudūd al-manṭiq* attests that Ibn Bahrīz had the necessary competence to prepare those that we find in the Hebrew version of the *Introduction to Arithmetic*. If we consider the Nestorian's educational aims as set forth in his work on logic, we can suppose that he had the motivation to "improve" also the *Introduction to Arithmetic* by adding verbal tables. Ibn Bahrīz was similarly fond of visual representations of scientific knowledge, for in his two known works he also used graphic arborescent diagrams.[36] By contrast, we do not know of any table of the same type in the works of al-Kindī: this makes it little likely that the verbal tables were added later on, by al-Kindī himself or by the Revisor, his faithful student. In general, this didactic means of presenting knowledge was infrequent.[37] Thus, assuming (as we did) that the Syriac version of the text did not contain interpolations, it seems clear that the most likely author of the verbal-type tables is Ibn Bahrīz.

It is important to note that the verbal tables are embedded in the text in a seamless way that disallows the reader to realize that they are not by

34 Troupeau 1997, 141–142; Hugonnard-Roche 1994. Verbal tables of the same kind had been used by Ibn al-Muqaffa' half a century earlier (see Danišpazuh 2002, 1–93); this work has also other similarities with that of Ibn Bahrīz (analyzed in detail in Troupeau 1997). See, as well, Endress 1992, 48 and the bibliography; Gutas 1993, 44.

35 A "primitive" version of Figure 4.10 (which combines a verbal table with a numerical table) is found in Boethius' *De institutione arithmetica* (2, 53; Boethius 1995, 174). (I am grateful to Irene Caiazzo for having drawn my attention to this fact.) It is, however, nearly identical to one found already in the *Introduction to Arithmetic* (cf. Nicomachus of Gerasa 1866 [ed. Hoche], 144; transl. D'Ooge 1926, 284). The much more developed verbal component in the table as found in the Hebrew recension seems to go back to Ibn Bahrīz. To anticipate possible misunderstandings: it is excluded that the "additional" verbal elements entered the Hebrew text directly via Boethius. Qalonymos' translations are all faithful "replicas" of the source texts, without any interpolations by the translator; moreover, in 1317, when he translated the *Introduction to Arithmetic*, Qalonymos (then aged 30) had not been in Italy and did not yet read Latin (see *supra* n. 18). The development and transmission across cultures of the extra-textual elements in the *Introduction to Arithmetic* (tables, diagrams, etc.) deserve study.

36 In *Kitāb Ḥudūd al-manṭiq*, he uses one arborescent scheme (p. 125). In his juristic work he uses a similar device to represent the law of heritages; see Selb 1970, Schema I, II and the explanations on pp. 142–143. This way of graphic representation also has its origins in the Syriac tradition; see Hugonnard-Roche 2001, esp. the folios reproduced on p. 17.

37 Endress 1987, 471.

Nicomachus himself. The first verbal-type table, for example (Figure 4.8), is introduced by the formula: "Here is the table [*ṣurah*] summarizing the divisions [of the species of number] according to what we have discussed from the beginning of *our book* up to this place";[38] the same is true of the other tables.

Ibn Bahrīz thus "intervenes" freely in the text that he translates, although his translation itself was literal, not paraphrastic. Now if Ibn Bahrīz adhered to an ideal of translation that encourages the translator to interpolate glosses in the interest of the reader, then it can reasonably be supposed that he did not limit his interpolations to tables alone. If so, he may be the author of at least a part of the passages in the Hebrew version that have no source in the Greek original and are not explicitly attributed to al-Kindī. By the same token, it seems likely that he authored the formulas "the author of the book says" or "Nicomachus says": probably these formulas were originally placed after an interpolation, to indicate that from that point on the translation of Nicomachus' text resumes. (Presumably, many of these interpolations are no longer there, having been eliminated by al-Kindī/the Revisor.) A tiny philological detail in the Hebrew text seems to confirm that these formulae have their origin in Ibn Bahrīz: the Hebrew text refers to the author of the *Introduction to Arithmetic* with the relatively rare term *maniaḥ ha-sefer* (lit. the one who "established" or "instituted" or "put down" the book), a noun derived from the root, *n.w.ḥ* (= to place, to put, to rest), whose Arabic equivalent is *w.ḍ.*ʿ. This verb is found in the same precise meaning also in Ibn Bahrīz's *Kitāb Ḥudūd al-manṭiq*.[39]

Let us now consider one specific interpolation identified in our text: we will see that it, too, was most likely introduced by Ibn Bahrīz. This interpolation is a gloss that discusses a method "called *diallelos*," a term that does not occur in the Greek text of the *Introduction to Arithmetic*. (The Greek term appears in our Hebrew text in transliteration.) As it happens, the interpolated passage derives from the Commentary on the *Introduction to Arithmetic* by Iamblichus' (*ca* 250–330).[40] Here is the passage as it appears in the Hebrew text; the phrases in italics go back to Nicomachus' Greek text (1.7), the text in roman being the interpolation:

38 MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt Yb 4° 5, fol. 16b.

39 E.g., Danišpazuh 2002, 97.3.

40 Iamblichus 1894, 12.22–25; Iamblichus 2014, 78.17–18. The passage has been identified with the help of M. Bernard Vitrac. This is one of several Neoplatonic commentaries on Nicomachus' work; see Robbins and Karpinski 1926, 124–137; Tarán 1969.

> *The odd number is one whose parts are not equal, however you divide it; and it is impossible that its parts not be* [*one*] *odd and* [*the other*] *even.* I.e., when one of its parts is odd, the other is even. It is therefore manifest that the parts of the odd number come closest to being equal when its two parts differ by a unit, by which one exceeds the other. Indeed, in the method [lit. definition; "*geder*"] called "*diallelos*", which consists in determining one of two unknowns through the other – *for the odd number is that which differs from the even number by one unit at both its ends, be it by excess or by deficiency, and the even number is that which differs from the odd number by one unit at both its ends, either by excess or by deficiency* – well, this method does not allow one to determine the unknown odd number unless the unknown even number is known, and the even unknown number cannot be known unless the unknown odd number is known.[41]

How did this interpolation find its way from Iamblichus' Commentary into our text? It seems that no Greek manuscript of the *Introduction to Arithmetic* carries it. Nor was it the Greek-into-Syriac translator who interpolated it: for, as already mentioned, the translators into Syriac as a rule did not make interpolations. The absence of the term *diallelos* in the Syriac literature seems to confirm this assumption.[42] Since the Greek commentaries on the *Introduction to Arithmetic* were apparently unknown to Arabic writers, it seems improbable that the passage was interpolated by al-Kindī/the Revisor. These considerations leave us with Ibn Bahrīz as the most likely possibility. His intellectual "profile" as outlined above indeed makes him into an ideal suspect. But did Ibn Bahrīz have access to Greek sources? This seems possible. First, it is not excluded that he knew Greek. Moreover, we have evidence that in Ibn Bahrīz's time, Syriac translators who were confronted with difficult texts occasionally consulted Greek colleagues: for example, in a letter to Sergius, Metropolitan of Elam, dating from 799, Patriarch Timothy reports that when he translated the *Topics* from Syriac into Arabic he sought advice from Greek scholars.[43] Ibn Bahrīz may have done the same and so learned of the passage discussing the method called *diallelos*. This finding confirms the surmise reached above

41 MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt Yb 4° 5, fols. 8b.16–9a.3, checked against MSS Paris, Bibliothèque nationale de France, héb. 1095, fols. 185b.11–20, and héb. 1029, fols. 5a.15–22. The Greek text is in Treatise I, chap. 7 (Nicomachus of Gerasa 1866 [ed. Hoche], 14.4–12; transl. D'Ooge 1926, 190–191).

42 The term does not appear in Smith 1879–1901. Thanks go to Henri Hugonnard-Roche (CNRS, Paris) for his help on this point.

43 Brock 1999, 239 (§ 8).

that many silent (unidentified) extensive interpolations may go back to Ibn Bahrīz.

Ibn Bahrīz clearly invested much thought and labor to improve the Arabic text of the *Introduction to Arithmetic* and bring it closer to the reader. Given the substantial nature of his intervention, we should henceforth think of his version of the text (= the non-extant Arabic version of the text that reached al-Kindî) as a *recension*, rather than a *translation* of the Syriac text. Ibn Bahrīz's recension of *Introduction to Arithmetic* shares the characteristics of the translations carried out in the "al-Kindī circle": first the translators tinkered and glossed their translations according to their own philosophical preferences, and, in a second move, their *spiritus rector* revised them.[44] Given that al-Kindī and Ibn Bahrīz were probably in personal contact and that their respective patrons were connected, one is tempted to think that Ibn Bahrīz prepared the Arabic version of the *Introduction to Arithmetic* at the request of al-Kindī or with his encouragement. This hypothesis allows us to understand why Ibn Bahrīz at all engaged in this enterprise: while as a lawyer he had good reasons to take an interest in logic (its study was situated at the beginning of the curriculum) and to write his epitomes, there is no apparent reason why he should have undertaken to translate such a difficult and long philosophic-mathematical work as Nicomachus'. Al-Kindī, for his part, was certainly interested in Nicomachus' work (*supra*, near n. 8), so it seems plausible to think that he engaged Ibn Bahrīz to translate it for him. (It should yet be borne in mind that al-Kindī was still young when Ibn Bahrīz translated Nicomachus' work; see *supra*, n. 14.)

6

At some point in time, already in the tenth century, the Arabic recension of Nicomachus' *Introduction to Arithmetic* reached al-Andalus. Historians have found that it was already known to mathematicians gathered around Abū 'l-Qāsim Maslama b. Aḥmad al-Majrīṭī, as well as to Ibn Sayyid in the second half of the tenth century.[45] It was also known to two Arabophone Jewish scholars – R. Abraham bar Ḥiyya of Barcelona (d. 1136) and R. Abraham Ibn Ezra

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44 Gerhard Endress was the first to have identified the "circle" of translators whose *spiritus rector* was al-Kindi (Endress 1973, 192) and in this context he referred to the *Introduction to Arithmetic* (Endress 1997, 55). On other translations made in this circle and their characteristics see also: Zimmermann 1986; Fazzo and Wiesner 1993, 126ff.; Gutas 1998, 145–146.

45 Samsó 1992, 953–954.

of Tudela (1089–1164)[46] and to others.[47] But which version of the work reached them – that of Ibn Bahrīz, that of Ibn Bahrīz as revised by al-Kindī/the Revisor, or that of Thābit Ibn Qurra? The question has been answered with respect to Abraham bar Ḥiyya, who (as the late Mauro Zonta and I have shown) used the work in Ibn Bahrīz's original, unrevised Arabic version.[48] But eventually also Ibn Bahrīz's text as revised by al-Kindī/the Revisor came to al-Andalus, from where it reached Qalonymos ben Qalonymos, its Hebrew translator. This brings us to the last stage (*viii*) in the evolution of the text prior to Qalonymos ben Qalonymos, one that allegedly took place in al-Andalus.

The Hebrew text (in all eight manuscripts) carries two colophons, one at the end of Treatise I, the other at the end of the entire work. The text of the first colophon is as follows:

> This, may God direct you on the right path, will suffice up to the end of the First Treatise of the *Introduction to Arithmetic*, composed by Nicomachus of Gerasa, the Pythagorean. And it was revised [*t.q.n.*, corresponding to the Arabic *s.l.ḥ.*] in Andalus by Abu Suleiman Rabiʿ ben Yaḥyā, bishop [*usquf*] of Elvira. Help will come by studying and meditating it. God, in His mercy, will direct you, so that you may understand [it] and find in it what you seek and which is useful to your salvation. Amen. (544)

The second colophon is at the end of the book and it is shorter:

> The *Introduction to Arithmetic* is completed, [namely,] the composition of Nicomachus of Gerasa, the Pythagorean, in the revision of Rabiʿ ben Yaḥyā, bishop [*usquf*] of Elvira, the Andalusian. Praise be to God. (544).

The two colophons thus agree that *Introduction to Arithmetic* was revised by a scholar named Rabiʿ ben Yaḥyā; the first colophon adds the *kunya*, Abu Suleiman, and states that the revision was made "in Andalus," a statement which the second colophon confirms by appending the epithet "Andalusian" to name of Rabiʿ. The colophons describe Rabiʿ ben Yaḥyā as *usquf alvirah* (the Hebrew translator merely transcribed the two Arabic words). It is noteworthy that both colophons use the verb (*t.q.n.*, translating the Arabic *s.l.h.*), which is also used by the Revisor to describe his work. This confirms the Revisor's

46 Langermann 2001, 223–224.
47 Steinschneider 1893a, 519.
48 Zonta and Freudenthal 2009.

statement that he revised both the Proemium and the bulk of the book, albeit in two installments.

The explicit statements of the two colophons lead us to try to identify our Revisor as Rabiʿ ben Yaḥyā, bishop of Elvira in al-Andalus. Unfortunately, no person by this name can be found. Steinschneider already perceived the difficulty and suggested that Rabiʿ ben Yaḥyā may be a well-known scholar, Recemund, whose Arabic name could be, according to some scholars, Rabiʿ ben Zayd.[49] This person – to whom some scholars have attributed a role in drafting the Cordoba calendar – was in fact appointed bishop of Elvira by Caliph Abd al-Rahman III.[50] However, this hypothesis is invalidated by two considerations: first, the bishop in question was called "Rabiʿ ben Zayd," whereas the colophons in our text twice name Rabiʿ ben Yaḥyā; second and more significantly, the *floruit* of Rabiʿ ben Zayd is around 950–960, that is, a century after the revision of the Arabic version of *Introduction to Arithmetic* by the direct pupil of al-Kindī. Since Steinschneider, the solution of the problem has not progressed by one iota.[51]

Indeed, it seems that no Abu Sulaimān Rabiʿ ben Yaḥyā can be identified – neither in the east, around al-Kindī (the reference to al-Andalus could be erroneous), nor in the west, where a list of the bishops of Elvira since the creation of this Mozarabic episcopal See does not include this name throughout the ninth century.[52] Further, no Rabiʿ having the *kunya* "Abū Sulaimān" appears in the large computerized databases that were checked.[53] Moreover, in the present state of our knowledge on the transfer of philosophy and science from the Arabic East to the West, the very presence of a student of al-Kindī in al-Andalus in the middle of the ninth century seems unlikely.[54]

How then can we reconcile the precision of the indications of the two colophons with the impossibility to identify the person named therein? The answer suggested here is that the information according to which the Revisor's

49 Pellat 1995.

50 Steinschneider 1874, 4–6.

51 The hypothesis that Rabiʿ ben Yaḥyā could be Rabiʿ ben Zayd, alias Recemund, has become even more unlikely than it was in Steinschneider's time. Ann Christys has shown that the identification of Recemund with Rabiʿ ben Zayd, and their link with the Cordoba calendar, is an artificial construct of historians which is not sufficiently substantiated by the facts. Thus, nothing allows us to identify our Revisor either with Rabiʿ ben Zayd, or with Recemund, or with any other Rabiʿ known to historians. See Christys 2002, 108–134.

52 See Flórez 1754, 167–171.

53 The *Onomasticon arabicum* (Paris: Institut de recherche et d'histoire des textes [IRHT] du CNRS); and the website *al-warraq.com*, through which numerous Arabic works can be searched.

54 See Samsó 1992, 953–956; Van Koningsveld 1994.

name was “Rabiʿ ben Yaḥyā” is not reliable. A close look at the colophons themselves clearly shows this. For whereas colophons are usually written in the first person, here, on the contrary, the two colophons are in the *third person*: obviously, they were not written by the Revisor himself. The two colophons thus seem to be interpolations, or reworkings of the original colophons. The following reconstruction seems plausible: if (as seems likely) the original text had colophons, they were written in the first person, without mentioning the name of the author; a later scholar who was involved in the transmission of the text, perhaps a scribe, believed that he would do well if he identified him, and he replaced the first person by the name he believed to be that of the Revisor, perhaps also adding at the same time the indications relating to al-Andalus, where the colophon shuffling was presumably done. Alternatively (although less likely), the work had no colophons at all, and both colophons were interpolated by a scribe. In any event, the identification of the Revisor as Rabiʿ ben Yaḥyā the bishop of Elvira seems to be a late fabrication and we must conclude that the information transmitted by the two colophons is probably incorrect. We should remember that the text is known to us only through the Hebrew translation made by Qalonymos b. Qalonymos, from a single Arabic *Vorlage*, so that it needed only one interpolator in a single manuscript to produce the false identification in all the manuscript witnesses of the text. In any event, the replacement of the original colophons or the interpolation must have occurred after the transfer of the sciences to the Iberian Peninsula, and thus well after the Prologue was written by the Revisor, who, as a student of al-Kindī, must have lived in the East in the second half of the ninth century.

7

The developments that culminated in the Hebrew *Introduction to Arithmetic* can now be reconstructed as follows. At an unknown date, probably at the end of the eighth century, an unidentified scholar translated Nicomachus’ work from Greek into Syriac. Shortly before 822, Ibn Bahrīz prepared an Arabic translation of this translation under the patronage of Ṭahir b. al-Ḥusain. The translation was literal, but Ibn Bahrīz introduced many interpolations, some borrowed from Greek sources (like Iamblichus’ Commentary). Ibn Bahrīz was connected to al-Kindī, and his work resembles other translations prepared in the latter’s “circle.” Ibn Bahrīz probably introduced the formulas “Nicomachus [or: the author] said” when, following an interpolation, he switched back to translating. When the work left his desk, Nicomachus’ translated text and the interpolations were still clearly distinguished (this contention will be corroborated shortly). Ibn Bahrīz's most characteristic interpolations are the verbal-type tables, a didactic device which Ibn Bahrīz used in other works. This is why his version should be described as a distinct recension of the *Introduction to Arithmetic*. Remnants of it are found in al-Yaʿqūbī's *Taʾriḥ* as well as in Abraham bar Ḥiyya's encyclopedia *Yesodey ha-tevunah u-migdal ha-emunah*. The active role played by Ibn Bahrīz in the history of the transmission of the *Introduction to Arithmetic* is one of the most important findings of the research presented here.

A few years later, the text reached al-Kindī. He made up his mind that Ibn Bahrīz had introduced into the text many errors. He manifestly was confident that he could distinguish between the authentic Nicomachean text and the "false ideas" introduced by Ibn Bahrīz: at this stage, we again conclude, the latter's interpolations were still clearly identifiable as such. (Al-Kindī did not know Syriac and thus could not compare Ibn Bahrīz's version with his model; as noted, the interpolations seem to have been marked by certain para-textual elements, such as the phrase "the author said.") Al-Kindī apparently did not himself "edit" the text that had reached him, but left the task to one of his students, the Revisor, who, however, assures us that he interfered in the text only following ideas of al-Kindī. By this time, the book was already separated into two parts: I.1–5 was considered as the Proemium of the *Introduction to Arithmetic*, the sequel was considered the book itself. At first, the Revisor corrected only the body of the work, leaving its Proemium for later. His interpolations were of two kinds. First, seven passages (of which only one in the second treatise) were interpolated and clearly identified by the introductory formula "Abū Yūsuf said," which probably goes back to the Revisor. At least some of these interpolations attributed to al-Kindī consist of passages taken from other works of his. Second, the Revisor informs us that he meddled with the text also in other ways – shortening and summarizing, embellishing and simplifying – although (he assures us) always in the spirit of al-Kindī.[55] Like Ibn Bahrīz, al-Kindī/the Revisor created a new recension of Nicomachus' *Introduction to*

55 What, then, distinguishes the interpolations that the Revisor expressly ascribed to al-Kindī from the others? If we recall that one of the interpolations that is expressly attributed to al-Kindī is borrowed *verbatim* from a known treatise by him then the following hypothesis suggests itself: when the Revisor had at his disposal a text from the very pen of al-Kindī, he inserted it in the body of the *Introduction to Arithmetic* preceded by the formula "Abū Yūsuf said"; it is a genuine citation. When he did not have such a text at his disposal and relied on his memory or on notes, then the Revisor interfered with the text without signaling it, introducing into the text an interpolation in which he summarized al-Kindī's thought in his own words. This hypothesis needs to be checked systematically.

Arithmetic. This new recension did not do away systematically with *all* of Ibn Bahrīz's interpolations, for the verbal tables were left in place and reached the Hebrew version; apparently Al-Kindī/the Revisor approved of these tables and did not consider them as "false ideas."

At this stage, the revised book (*Treatises* I.6 *sqq*. and II, without the Proemium) was made available to users. One of them – we called him the "Addressee" – wrote to the Revisor requesting the "Proemium" to the work. He already had the bulk of the book and wished to have the opening part too. The Revisor complied: he edited the Proemium as he had previously done with the rest of the book, affixed a personal letter to the Addressee (now: the Prologue), and sent both to the addressee. The latter combined all three texts and this is what became the *Vorlage* that Qalonymos was to translate. It is important to retain that the entire work was revised under the authority of al-Kindī by one and the same person –the Revisor.

With this reconstruction in mind, we realize that the numerous unattributed interpolated passages in the Hebrew version of the *Introduction to Arithmetic* must go back to either Ibn Bahrīz or to al-Kindī/the Revisor. Future research (based on a scientific edition of the entire text) will have to try to assign each interpolation to its author. This task is not necessarily beyond reach. For example, interpolations that reflect a high mathematical level are in all probability *not* by Ibn Bahrīz, for it seems unlikely that this jurist had the mathematical skill to write such passages.[56] They presumably are by al-Kindī/the Revisor.

In 1317 the recension produced by al-Kindī/the Revisor finally reached the prolific Jewish translator, Qalonymos ben Qalonymos of Arles. He prepared a faithful rendition of the Arabic text, a sort of a photograph in Hebrew of the Arabic model that had reached him. Since Qalonymos added nothing and omitted nothing, he retained all the traces left by the previous scholars involved in the long transmission process, allowing us to disentangle some threads in its complicated and long history. The Hebrew text enjoyed some popularity, as evidenced by the eight preserved manuscripts, as well as the commentaries written on it.[57]

56 E.g., at the end of Treatise I there is a passage with an explicit reference to Euclid and a mention of the Euclidean definition of proportion, which are not in the Greek original (MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt, Yb 4° 5, fol. 28b.20–29b.17).

57 Langermann 2001.

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