

# ΣΤΑΣΙΜΟ ΚΥΜΑ σε ΚΟΙΛΟΤΗΤΑ

Θα κατασκευάσουμε Χαμιλιτονιανή τού ΗΜ πεδίου,  
 ή θηλα να μπορέ να μετασχηματιστεί  
 από τη γλώσσα  $\vec{E}, \vec{B}$   
 στη γλώσσα τού αριθμού τών φωτονίων

Τρέχοντα κύμα

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t + \delta)}$$

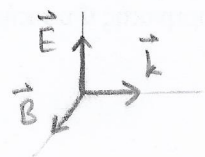
$$\vec{E} \times \vec{B} \parallel \vec{k}$$

3Δ

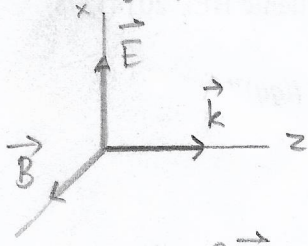
3Δ  
KE

$$\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{B}(\vec{r}, t) = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t + \delta)}$$



διαλέγουμε



$$\nabla^2 \vec{E}_x = \frac{1}{c^2} \frac{\partial^2 \vec{E}_x}{\partial t^2}$$

$$\vec{E}(\vec{r}, t) = \vec{E}_{x0} e^{i(k_z z - \omega t + \delta)} = \vec{E}_x(z, t)$$

$$\nabla^2 \vec{B}_y = \frac{1}{c^2} \frac{\partial^2 \vec{B}_y}{\partial t^2}$$

$$\vec{B}(\vec{r}, t) = \vec{B}_{y0} e^{i(k_z z - \omega t + \delta)} = \vec{B}_y(z, t)$$

δηότε

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}$$

KE1ΔE

$$\frac{\partial^2 B_y}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 B_y}{\partial t^2}$$

KE1ΔB

1Δ  
KE

1<sup>η</sup> Εξ.Μ  $\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (E_x, E_y, E_z) = 0 \Rightarrow$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \Rightarrow \frac{\partial E_x}{\partial x} = 0 \quad \text{ἀναμενόμενο}$$

2<sup>η</sup> Εξ.Μ  $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (B_x, B_y, B_z) = 0 \Rightarrow$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \Rightarrow \frac{\partial B_y}{\partial y} = 0 \quad \text{ἀναμενόμενο}$$

3<sup>η</sup> Εξ.Μ  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\frac{\partial}{\partial t} (0, B_y, 0) \Rightarrow$

$$+\hat{j} \frac{\partial E_x}{\partial z} = -\hat{j} \frac{\partial B_y}{\partial t} \Rightarrow$$

$$\boxed{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} \quad \underline{E_x B_y}$$

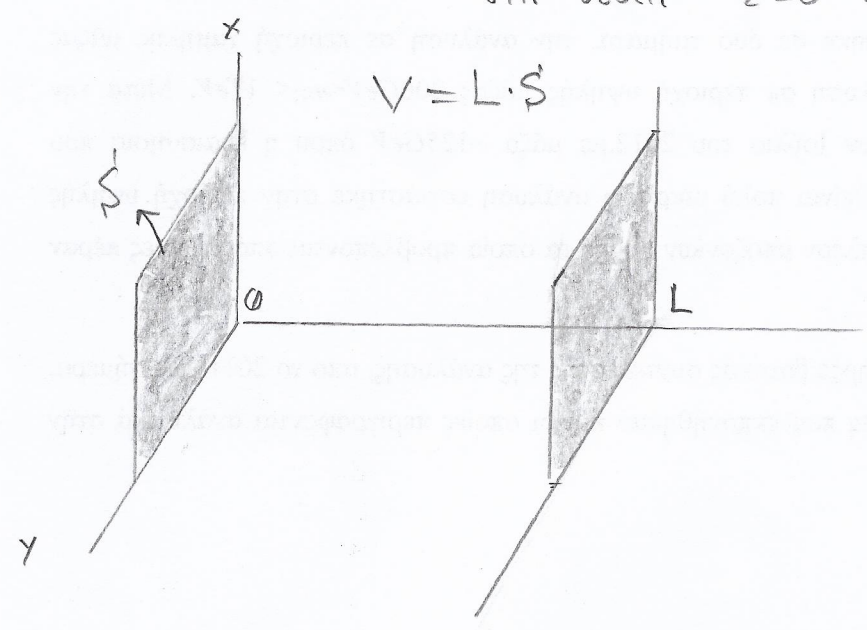
4<sup>η</sup> Εξ.Μ  $\vec{\nabla} \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_y & 0 \end{vmatrix} = \epsilon_0 \mu_0 \frac{\partial}{\partial t} (E_x, 0, 0) \Rightarrow$

$$-\hat{i} \frac{\partial B_y}{\partial z} = \epsilon_0 \mu_0 \frac{\partial E_x}{\partial t} \hat{i} \Rightarrow$$

$$\boxed{\frac{\partial B_y}{\partial z} = -\frac{1}{c^2} \frac{\partial E_x}{\partial t}} \quad \underline{B_y E_x}$$

Τώρα βάλουμε ιδανικό αγωγό κάθετα

στις θέσεις  $z=0$  και  $z=L$



θα δημιουργηθούν  
στάσιμα κύματα  
από το συνδυασμό  
προσπίπτουσών και  
ἀνακλωμένων κυμάτων

οι ΚΕΙΔΕ και ΚΕΙΔΒ  
 $E_x B_y$  και  $B_y E_x$

έφακογονόσι να ίσχυουν  
για το γραμμικό συνδυασμό  
των προσπίπτουσών και ἀνακλωμένων κυμάτων

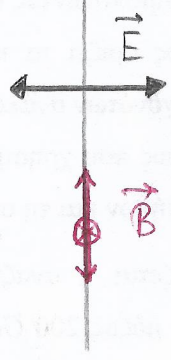
Αναζητούμε λύση με τη μέθοδο των χωριζόμενων μεταβλητών  
δηλαδή για

$$E_x(z, t) = \mathcal{N} Z(z) T(t)$$

ΕΣΣ\*  $B_{z\perp} = 0$   
 $E_{z\parallel} = 0$

ΙΔΑΝΙΚΟΣ  
ΑΓΩΓΟΣ  
①

ΚΕΝΟ ή ΑΕΡΑΣ  
②



ξοπηρεπτοί  
προσφατο λισχοί  
ση διεπιφάνεια

ΑΡΑ

$$E_x(0, t) = 0 = E_x(L, t) \quad \forall t$$

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}$$

$$E_x(z, t) = \mathcal{N} Z(z) T(t)$$

$$\Rightarrow \cancel{\mathcal{N}} T(t) \frac{d^2 Z(z)}{dz^2} = \frac{1}{c^2} \cancel{\mathcal{N}} Z(z) \frac{d^2 T(t)}{dt^2}$$

$$\Rightarrow \underbrace{\frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2}}_{f(z)} = \underbrace{\frac{1}{c^2} \frac{1}{T(t)} \frac{d^2 T(t)}{dt^2}}_{g(t)} \quad \text{APA σταθερά: } = -k^2$$

$$\underbrace{\hspace{10em}}_{=}$$

$$\forall z, \forall t$$

Da πρέπει  $Z(z) \neq 0$  και  $T(t) \neq 0$  για να γράψουμε  $\frac{1}{Z(z)}$  ή  $\frac{1}{T(t)}$   
 αν  $Z(z) = 0$  ή  $T(t) = 0$  τότε η } ικανοποιείται

"Αρα

$\frac{d^2 Z(z)}{dz^2} + k^2 Z(z) = 0$	(Z)
$\frac{d^2 T(t)}{dt^2} + k^2 c^2 T(t) = 0$	(T)

Αν δώσουμε αρχικές για την (Z) ΔΛΜ  $e^{\tilde{\lambda} z}$

$$\tilde{\lambda}^2 + k^2 = 0 \Rightarrow \tilde{\lambda} = \pm ik \quad \text{ο.κ. αν διαλέξουμε } \underline{\underline{k \in \mathbb{R}^+}}$$

"Αρα η λύση της (Z) θα είναι της μορφής

$$Z(z) = A e^{ikz} + B e^{-ikz}$$

ΣΣ1  $Z(0) = A + B = 0 \Rightarrow B = -A$

ΣΣ2  $Z(L) = A e^{ikL} + B e^{-ikL} = 0 \Rightarrow A e^{ikL} - A e^{-ikL} = 0$

$$\Rightarrow e^{ikL} = e^{-ikL} \Rightarrow \cos(kL) + i \sin(kL) = \cos(kL) - i \sin(kL)$$

$$\Rightarrow \sin(kL) = 0$$

$$\sin(kL) = 0 \Rightarrow kL = m\pi, m \in \mathbb{Z} \quad \left. \vphantom{\sin(kL) = 0} \right\} \Rightarrow$$

άλλα σνοδείσασε  $k \in \mathbb{R}_+$

$$kL = m\pi, m \in \mathbb{N}$$

άλλε άρ  $m=0 \Rightarrow k=0 \Rightarrow Z(z) = A + B = 0$

Θάσετ γα γη γυδερνίη λυσή

$$k_m = \frac{m\pi}{L}, m \in \mathbb{N}^* \quad (KM)$$

ΑΡΑ  $Z(z) = A e^{ikt} - A e^{-ikt} = A \cos kz + A i \sin kz \Rightarrow$   
 $-A \cos kz + A i \sin kz$

$$Z(z) = 2iA \sin(kz)$$

ΑΡΑ

$$Z_m(z) = 2iA \sin\left(\frac{m\pi}{L} z\right) \quad m \in \mathbb{N}^*$$

κι άρ άναμίσουσε of  $Z_m(z)$  να είναι ορθοκανονική

$$\int_0^L Z_m^*(z) Z_l(z) dz = \delta_{ml}$$

$$\int_0^L dz \ 2(-i)A^* \sin\left(\frac{m\pi z}{L}\right) 2iA \sin\left(\frac{l\pi z}{L}\right) = \delta_{ml} \Rightarrow$$

$$4|A|^2 \int_0^L dz \ \sin\left(\frac{m\pi z}{L}\right) \sin\left(\frac{l\pi z}{L}\right) = \delta_{ml}$$

$$\psi := \frac{\pi z}{L}$$

$$d\psi = \frac{\pi}{L} dz \Rightarrow dz = \frac{L}{\pi} d\psi$$

$$4|A|^2 \frac{L}{\pi} \int_0^{\pi} d\psi \sin(m\psi) \sin(l\psi) = \delta_{ml}$$

$$\int_0^{\pi} d\psi \sin(m\psi) \sin(l\psi) = \frac{\pi}{2} \delta_{ml}$$

$$\int_0^{\pi} d\psi \cos(m\psi) \cos(l\psi) = \frac{\pi}{2} \delta_{ml}$$

$$4|A|^2 \frac{L}{\pi} \frac{\pi}{2} \delta_{ml} = \delta_{ml} \Rightarrow 2L|A|^2 = 1 \Rightarrow |A|^2 = \frac{1}{2L}$$

ως διαλέγουμε κάτι βολικό π.χ.  $A = \frac{1}{\sqrt{2L}} (-i)$

Συνένωση

$$\sum_m(z) = 2i \frac{1}{\sqrt{2L}} (-i) \sin\left(\frac{m\pi z}{L}\right) \quad m \in \mathbb{N}^*$$

$$\boxed{\sum_m(z) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{m\pi z}{L}\right)} \quad , m \in \mathbb{N}^*$$

• Αν δεχόμαστε τώρα γι' την  $\textcircled{T}$ , "Αν δρίσουμε

$$\Delta \Lambda M \quad e^{\tilde{\lambda} t}$$

$$\frac{d^2 T(t)}{dt^2} + k^2 c^2 T(t) = 0$$

$$\tilde{\lambda}^2 + \omega^2 = 0 \Rightarrow \lambda = \pm i\omega$$

$$\lambda = \pm i\omega_m$$

$$\omega := kc > 0$$

$$\Rightarrow \omega_m = kmc$$

$$\omega_m = \frac{m\pi c}{L}$$

$$m \in \mathbb{N}^*$$

$$\omega_m$$

"Αρα η λύση της  $\textcircled{T}$  θα είναι της μορφής

$$T(t) = \Gamma e^{i\omega_m t} + \Delta e^{-i\omega_m t}$$

Ki. ar dioroute tuv arxiu sunthimi  $T(0) = 0$

$$\Rightarrow \Gamma + \Delta = 0 \Rightarrow \Delta = -\Gamma$$

$$T(t) = \Gamma e^{i\omega_m t} - \Gamma e^{-i\omega_m t} = \Gamma \cos \omega_m t + \Gamma i \sin \omega_m t - \Gamma \cos \omega_m t + \Gamma i \sin \omega_m t$$

$$\Rightarrow T(t) = 2i\Gamma \sin \omega_m t \Rightarrow$$

$$T_m(t) = 2i\Gamma \sin\left(\frac{m\pi c}{L}t\right) \quad m \in \mathbb{N}^*$$

Ki av anamirovete oi  $T_m(t)$  va einai orthogonoties st ein xronio tlaiois anō to xrono 0 tōte to xrono  $t_k$

$$\int_0^{t_k} dt T_m^*(t) T_l(t) = \delta_{ml} \Rightarrow$$

$$\int_0^{t_k} dt 2(-i)\Gamma^* \sin\left(\frac{m\pi c}{L}t\right) 2i\Gamma \sin\left(\frac{l\pi c}{L}t\right) = \delta_{ml} \Rightarrow$$

$$4|\Gamma|^2 \int_0^{t_k} dt \sin\left(\frac{m\pi c}{L}t\right) \sin\left(\frac{l\pi c}{L}t\right) = \delta_{ml}$$

$$x := \frac{\pi c t}{L} \quad dx = \frac{\pi c}{L} dt \Rightarrow dt = \frac{L}{\pi c} dx$$

$$4|\Gamma|^2 \frac{L}{\pi c} \int_0^{\frac{\pi c t_k}{L}} dx \sin(mx) \sin(lx) = \delta_{ml}$$

Χρόνος ηχητικού  
Τοῦ φωτός διαφύλαξης  
στὴν κοιλότητα

$$\text{Ἄρα einai βολὴς va διοορυεῖται} \quad \frac{\pi c}{L} t_k = \pi \Rightarrow t_k = \frac{L}{c} := \tau$$

$\tau$  time of photon flight through cavity

ΛΟΓΙΚΗ  
ΕΚΛΟΓΗ...

$$4|\Gamma|^2 \frac{L}{\pi c} \int_0^{\pi} dx \sin(mx) \sin(lx) = \delta_{nl}$$

$$4|\Gamma|^2 \frac{L}{\pi c} \cdot \frac{\pi}{2} \delta_{nl} = \delta_{nl} \Rightarrow |\Gamma|^2 = \frac{c}{2L}$$

ki óv staðjafngætt hönn þolius  $\Gamma = \sqrt{\frac{c}{2L}} (-i)$

$$T_m(t) = 2i \sqrt{\frac{c}{2L}} (-i) \sin\left(\frac{m\pi c}{L} t\right)$$

$$T_m(t) = \sqrt{\frac{2c}{L}} \cdot \sin\left(\frac{m\pi c}{L} t\right)$$

Þá er  $E_x^m(z, t) = \mathcal{N} \sqrt{\frac{2}{L}} \sin\left(\frac{m\pi z}{L}\right) \sqrt{\frac{2c}{L}} \cdot \sin\left(\frac{m\pi c}{L} t\right)$

$$E_x^m(z, t) = \frac{2\mathcal{N}}{L} \sqrt{c} \sin\left(\frac{m\pi z}{L}\right) \sin\left(\frac{m\pi c}{L} t\right) \quad \text{Exm}$$

$$[\mathcal{N}] = \frac{V}{m} \frac{m \sqrt{s}}{\sqrt{m}} = \frac{V \sqrt{s}}{\sqrt{m}} = \frac{V}{\sqrt{m/s}}$$



$$\frac{\partial B_y}{\partial z} = -\frac{1}{c^2} \frac{\partial E_x}{\partial t} \quad \underline{B_y E_x} \quad \left. \vphantom{\frac{\partial B_y}{\partial z}} \right\} \Rightarrow$$

$$E_x^m(z, t) = \frac{2N\sqrt{c}}{L} \sin\left(\frac{m\pi z}{L}\right) \sin\left(\frac{m\pi ct}{L}\right)$$

$$\frac{\partial B_y^m}{\partial z} = -\frac{1}{c^2} \frac{2N\sqrt{c}}{L} \sin\left(\frac{m\pi z}{L}\right) \frac{m\pi c}{L} \cos\left(\frac{m\pi ct}{L}\right) \Rightarrow$$

$$\frac{\partial B_y^m}{\partial z} = -\frac{2N\pi m}{L^2\sqrt{c}} \sin\left(\frac{m\pi z}{L}\right) \cos\left(\frac{m\pi ct}{L}\right)$$

$$\int_0^{z'} \frac{\partial B_y^m}{\partial z} dz = -\frac{2N\pi m}{L^2\sqrt{c}} \int_0^{z'} dz \sin\left(\frac{m\pi z}{L}\right) \cdot \cos\left(\frac{m\pi ct}{L}\right)$$

$$B_y^m(z', t) - B_y^m(0, t) = +\frac{2N\pi m}{L^2\sqrt{c}} \cdot \cos\left(\frac{m\pi ct}{L}\right) \cdot \left(\frac{L}{m\pi}\right) \left[ +\cos\left(\frac{m\pi z}{L}\right) \right]_0^{z'}$$

$$B_y^m(z', t) - B_y^m(0, t) = \frac{2N\pi m}{L^2\sqrt{c}} \frac{L}{m\pi} \cos\left(\frac{m\pi ct}{L}\right) \cdot \left\{ \cos\left(\frac{m\pi z'}{L}\right) - 1 \right\}$$

$$B_y^m(z', t) - B_y^m(0, t) = \frac{2N}{L\sqrt{c}} \cos\left(\frac{m\pi ct}{L}\right) \left\{ \cos\left(\frac{m\pi z'}{L}\right) - 1 \right\}$$

οριζόντιες καταλλήλως το  $B_y^m(z)$   $\Rightarrow$   
 και κάθετες το υπερεπίπεδο  $z' \rightarrow z$

$B_y^m(z, t) = \frac{2N}{L\sqrt{c}} \cos\left(\frac{m\pi ct}{L}\right) \cos\left(\frac{m\pi z}{L}\right)$	<div style="border: 1px solid black; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin: 0 auto;"> <math>B_{ym}</math> </div>
---	---

Πυκνότητα  
Ενέργειας

$$U = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 = \frac{\epsilon_0}{2} [E^2 + c^2 B^2] \quad [U] = \frac{J}{m^3}$$

Πυκνότητα  
Ενέργειας  
in τμήνου

$$U_m = \frac{\epsilon_0}{2} \left[ \frac{4\mathcal{W}^2}{L^2} c \sin^2\left(\frac{m\pi z}{L}\right) \sin^2\left(\frac{m\pi ct}{L}\right) + \frac{c^2 4\mathcal{W}^2}{L^2} \cos^2\left(\frac{m\pi z}{L}\right) \cdot \cos^2\left(\frac{m\pi ct}{L}\right) \right]$$

$$\Rightarrow U_m = \frac{\epsilon_0}{2} \frac{4\mathcal{W}^2 c}{L^2} \left[ \sin^2\left(\frac{m\pi z}{L}\right) \sin^2\left(\frac{m\pi ct}{L}\right) + \cos^2\left(\frac{m\pi z}{L}\right) \cos^2\left(\frac{m\pi ct}{L}\right) \right]$$

Άρα η ενέργεια του m τμήνου

$$E_m = \int_{V=LS} d^3r U_m = \left( \frac{\epsilon_0 4\mathcal{W}^2 c}{2 L^2} \right) \left[ \sin^2\left(\frac{m\pi ct}{L}\right) S \int_0^L dz \sin^2\left(\frac{m\pi z}{L}\right) + \cos^2\left(\frac{m\pi ct}{L}\right) \cdot S \int_0^L dz \cos^2\left(\frac{m\pi z}{L}\right) \right]$$

$$d^3r = dV = S dz$$

$$E_m = \frac{2\epsilon_0 \mathcal{W}^2 c S}{L^2} \left[ \sin^2\left(\frac{m\pi ct}{L}\right) \frac{L}{2} + \cos^2\left(\frac{m\pi ct}{L}\right) \cdot \frac{L}{2} \right]$$

$$\psi := \frac{\pi z}{L}$$
$$d\psi = \frac{\pi dz}{L}$$

$$\int_0^L dz \sin^2\left(\frac{m\pi z}{L}\right) = \frac{L}{\pi} \int_0^\pi d\psi \sin^2(m\psi) = \frac{L}{\pi} \cdot \frac{\pi}{2} = \frac{L}{2}$$

$$\int_0^L dz \cos^2\left(\frac{m\pi z}{L}\right) = \frac{L}{\pi} \int_0^\pi d\psi \cos^2(m\psi) = \frac{L}{\pi} \cdot \frac{\pi}{2} = \frac{L}{2}$$

$$E_m = \frac{\epsilon_0 \omega^2 c^2 S}{L} \left[ \sin^2\left(\frac{m\pi c t}{L}\right) + \cos^2\left(\frac{m\pi c t}{L}\right) \right] = \frac{\epsilon_0 \omega^2 c^2 S}{L}$$

? Ar σοδύτ άγο άδιδώζινα τιν  $E_m$  ...

$$E_m = \frac{\epsilon_0 c \omega^2 S}{L^3} \left[ L^2 \sin^2\left(\frac{m\pi c t}{L}\right) + L^2 \cos^2\left(\frac{m\pi c t}{L}\right) \right]$$

$$q_m(t) := L \sin\left(\frac{m\pi c t}{L}\right)$$

γενικευμένη δύση  
 $[q_m(t)] = m$

$$\dot{q}_m(t) := L \frac{m\pi c}{L} \cos\left(\frac{m\pi c t}{L}\right)$$

$$\dot{q}_m(t) = m\pi c \cos\left(\frac{m\pi c t}{L}\right)$$

γενικευμένη ταχύτητα  
 $[\dot{q}_m(t)] = \frac{m}{s}$

$$E_m = \frac{\epsilon_0 c \omega^2 S}{L^3} \left[ (q_m(t))^2 + \frac{(q_m(t))^2 L^2}{(m\pi c)^2} \right]$$

αναλογία με AAT



$$E = \frac{K}{2} x^2 + \frac{M}{2} v^2 = \frac{K}{2} \left[ x^2 + \frac{M}{K} v^2 \right]$$

$$\frac{K}{2} = \frac{\epsilon_0 c \omega^2 S}{L^3}$$

$$\frac{L^2}{m^2 \pi^2 c^2} = \frac{M}{K}$$

Σημειώστε την ποσότητα ή σταθερά έλαμψεως

$$K = \frac{2\epsilon_0 c N^2 S}{L^3}$$

και η γω'ε

$$M = K \frac{L^2}{m^2 n^2 c^2} = \frac{2\epsilon_0 c N^2 S}{L^3} \frac{L^2}{m^2 n^2 c^2}$$

$$M_m = \frac{2\epsilon_0 N^2 S}{L c m^2 n^2}$$

$$[M_m] = kg$$

$$K = M_m \omega_m^2$$

$$[M_m] = \frac{F}{m} \cdot \frac{V^2}{\frac{m}{s}} \frac{m^2}{\frac{m}{s}} = \frac{F V^2}{(\frac{m}{s})^2} = \frac{C V^2 s^2}{m^2} = \frac{J s^2}{m^2}$$

$$= \frac{kg \cdot m \cdot m \cdot s^2}{s^2 \cdot m^2} = kg$$

$$M_m \omega_m^2 = \frac{2\epsilon_0 N^2 S}{L c m^2 n^2} \frac{m^2 n^2 c^2}{L^2} = \frac{2\epsilon_0 N^2 S c}{L^3} = K$$

$$\frac{K}{M} = \frac{2\epsilon_0 c N^2 S}{L^3 2\epsilon_0 N^2 S} \frac{L c m^2 n^2}{m^2 n^2 c^2} = \frac{m^2 n^2 c^2}{L^2} \Rightarrow K = M_m \cdot \left(\frac{m n c}{L}\right)^2$$

$$K = M_m \cdot \omega_m^2$$

$$[K] = kg \cdot \frac{m^2}{s^2} \frac{1}{m^2} = \frac{kg}{s^2} = \frac{kg \cdot m}{s^2 \cdot m} = \frac{N}{m}$$

$$E_m = \frac{M_m \omega_m^2}{2} (q_m(t))^2 + \frac{M_m}{2} (\dot{q}_m(t))^2$$

$$E_m = \frac{M_m \omega_m^2}{2} q_m^2 + \frac{M_m}{2} \dot{q}_m^2$$

↓ κβαντικός διαχωρισμός

$$\hat{H}_{HM,m} = \frac{M_m \omega_m^2}{2} \hat{q}_m^2 + \frac{M_m}{2} \hat{\dot{q}}_m^2$$

χαμiltoniana  
in terms of HM modes

$$E_{m,n_m} = \hbar \omega_m \left( n_m + \frac{1}{2} \right)$$

$$n_m \in \mathbb{N}$$

$$m \in \mathbb{N}^*$$

ιδιοτιμήτες ενέργειας  
in terms of HM modes

$$\hat{H}_{HM} = \sum_m \hat{H}_{HM,m}$$

χαμiltoniana  
HM modes

$$E_x^m(z, t) = \frac{2\sqrt{c}}{L^2} \mathcal{N} \sin\left(\frac{m\pi z}{L}\right) q_m(t)$$

$$\hat{E}_x^m(z, t) = \frac{2\sqrt{c}}{L^2} \mathcal{N} \sin\left(\frac{m\pi z}{L}\right) \hat{q}_m(t)$$

$$B_y^m(z, t) = \frac{2W}{L\sqrt{c}} \frac{1}{m\pi c} \cos\left(\frac{m\pi z}{L}\right) \dot{q}_m(t)$$

$$\hat{B}_y^m(z, t) = \frac{2W}{L\sqrt{c}} \frac{1}{m\pi c} \cos\left(\frac{m\pi z}{L}\right) \dot{\hat{q}}_m(t)$$

$$E_x^m(z, t) = \left(\frac{2M_m \omega_m^2}{\epsilon_0 V}\right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) q_m(t)$$

$$\hat{E}_x^m(z, t) = \left(\frac{2M_m \omega_m^2}{\epsilon_0 V}\right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) \hat{q}_m(t)$$

$$B_y^m(z, t) = \frac{1}{c} \left(\frac{2M_m}{\epsilon_0 V}\right)^{1/2} \cos\left(\frac{m\pi z}{L}\right) \dot{q}_m(t)$$

$$\hat{B}_y^m(z, t) = \frac{1}{c} \left(\frac{2M_m}{\epsilon_0 V}\right)^{1/2} \cos\left(\frac{m\pi z}{L}\right) \dot{\hat{q}}_m(t)$$

$$\begin{bmatrix} E_x^m \\ B_y^m \end{bmatrix} = [c] \begin{bmatrix} \left(\frac{2M_m \omega_m^2}{\epsilon_0 V}\right)^{1/2} \\ \frac{1}{c} \left(\frac{2M_m}{\epsilon_0 V}\right)^{1/2} \end{bmatrix} \frac{m}{s}$$

$$= [c] [\omega_m] s = \frac{m}{s} \cdot \frac{1}{s} s = \frac{m}{s}$$

$$= [c]$$