Είχαμε δείξει πώς για την κατάσταση

$$|\psi_A(t)\rangle = c_1(t)|\downarrow n\rangle + c_2(t)|\uparrow n-1\rangle = |A\rangle$$

$$\left.\begin{array}{l}\langle \hat{a}^{+}\hat{a}\rangle = n - |c_2(t)|^2 \\ \langle \hat{a}\hat{a}^{+}\rangle = n + |c_1(t)|^2\end{array}\right\} \Rightarrow \langle \hat{a}\hat{a}^{+}\rangle - \langle \hat{a}^{+}\hat{a}\rangle = 1$$

$$\left.\begin{array}{l}\langle \hat{S}_+\hat{S}_-\rangle = |c_2(t)|^2 \\ \langle \hat{S}_-\hat{S}_+\rangle = |c_1(t)|^2\end{array}\right\} \Rightarrow \langle \hat{S}_+\hat{S}_-\rangle + \langle \hat{S}_-\hat{S}_+\rangle = 1$$

$$\langle \hat{a}^{+}\hat{a}\rangle + \langle \hat{S}_+\hat{S}_-\rangle = n$$

$$\langle \hat{S}_+\hat{a}\rangle = c_2^{*}(t)\, c_1(t)\sqrt{n} \qquad \langle \hat{S}_+\hat{a}^{+}\rangle = 0$$

$$\langle \hat{S}_-\hat{a}^{+}\rangle = c_1^{*}(t)\, c_2(t)\sqrt{n} \qquad \langle \hat{S}_-\hat{a}\rangle = 0$$

β'

$$|\psi_A(t)\rangle = c_1(t)\,|\downarrow n\rangle + c_2(t)\,|\uparrow n-1\rangle$$

$$i\hbar\frac{\partial}{\partial t}|\psi_A(t)\rangle = \hat H\,|\psi(t)\rangle$$

$$\hat H = \hat H_{JCM} = \hbar\omega_m \hat a_m^{\dagger}\hat a_m + \hbar\Omega\hat S_+\hat S_- + \hbar g_m\left(\hat S_+\hat a_m + \hat S_-\hat a_m^{\dagger}\right)$$

Α.Σ. $c_1(0)=1, \quad c_2(0)=0$

$$A' = i\hbar\frac{\partial}{\partial t}|\psi_A(t)\rangle = \left(\dot c_1(t)\,|\downarrow n\rangle + \dot c_2(t)\,|\uparrow n-1\rangle\right) i\hbar$$

$$\Delta' = \left(\hbar\omega_m \hat a_m^{\dagger}\hat a_m + \hbar\Omega\hat S_+\hat S_- + \hbar g_m\hat S_+\hat a + \hbar g_m\hat S_-\hat a^{\dagger}\right)\left(c_1(t)\,|\downarrow n\rangle + c_2(t)\,|\uparrow n-1\rangle\right) =$$

$$= \hbar\omega_m c_1(t)\,n\,|\downarrow n\rangle + \hbar\omega_m c_2(t)\,(n-1)\,|\uparrow n-1\rangle +$$

$$\hbar\Omega\, c_1(t)\,|0\, n\rangle + \hbar\Omega\, c_2(t)\,|\uparrow n-1\rangle +$$

$$\hbar g_m c_1(t)\sqrt{n}\,|\uparrow n-1\rangle + \hbar g_m c_2(t)\sqrt{n-1}\,|0\, n-2\rangle +$$

$$\hbar g_m c_1(t)\sqrt{n+1}\,|0\, n+1\rangle + \hbar g_m c_2(t)\sqrt{n}\,|\downarrow n\rangle =$$

Επί $\langle\downarrow n|$

$$\left.\begin{aligned} A' &= i\hbar\,\dot c_1(t)\\ \Delta' &= \hbar\omega_m c_1(t)\,n + \hbar g_m c_2(t)\sqrt{n}\end{aligned}\right\}\ i\cancel{\hbar}\,\dot c_1(t) = \cancel{\hbar}\omega n\, c_1(t) + \cancel{\hbar} g_m\sqrt{n}\, c_2(t)$$

Επί $\langle\uparrow n-1|$

$$\left.\begin{aligned} A' &= i\hbar\,\dot c_2(t)\\ \Delta' &= \hbar\omega_m c_2(t)(n-1) + \hbar\Omega\, c_2(t) + \hbar g_m\sqrt{n}\, c_1(t)\end{aligned}\right\} \Rightarrow$$

$$i\cancel{\hbar}\,\dot c_2(t) = \cancel{\hbar} g_m\sqrt{n}\, c_1(t) + \left[\cancel{\hbar}\omega_m(n-1) + \cancel{\hbar}\Omega\right] c_2(t)$$

$$i\begin{bmatrix}\dot c_1(t)\\ \dot c_2(t)\end{bmatrix} = \begin{bmatrix} n_m\omega_m & \sqrt{n_m}\,g_m\\ \sqrt{n_m}\,g_m & \Omega+(n_m-1)\omega_m\end{bmatrix}\begin{bmatrix}c_1(t)\\ c_2(t)\end{bmatrix}$$

υπενθυμίζεται πως για απλότητα "ξεχνάμε" το δείκτη m

$\omega = \omega_m$

$g = g_m$

$n = n_m$

Ορίζουμε

$$\Omega_n = \left[\left(\frac{\omega-\Omega}{2}\right)^2 + g^2 n\right]^{1/2} = \sqrt{\left(\frac{\Delta}{2}\right)^2 + g^2 n}$$

και παραλείποντας τις πράξεις προκύπτει

$$c_1(t) = \exp\left[-i\left(n\omega + \frac{\Omega-\omega}{2}\right)t\right]\left\{\cos(\Omega_n t) + i\,\frac{\Omega-\omega}{2\Omega_n}\sin(\Omega_n t)\right\}$$

$$c_2(t) = \exp\left[-i\left(n\omega + \frac{\Omega-\omega}{2}\right)t\right]\left\{-i\,\frac{g\sqrt{n}}{\Omega_n}\sin(\Omega_n t)\right\}$$

Ἄρα $\boxed{|c_2(t)|^2 = \dfrac{ng^2}{\Omega_n^2}\sin^2(\Omega_n t)}$ $\qquad |c_1(t)|^2 = 1 - |c_2(t)|^2 = \dots$

$$\boxed{\langle \hat{a}^{+}\hat{a}\rangle_{Ⓐ} = n - \frac{ng^2}{\Omega_n^2}\sin^2(\Omega_n t)} \qquad \boxed{\langle \hat{S}_{+}\hat{S}_{-}\rangle_{Ⓐ} = \frac{ng^2}{\Omega_n^2}\cdot\sin^2(\Omega_n t)}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\langle \hat{a}^{+}\hat{a}\rangle_{Ⓐ} = n - \frac{ng^2}{2\Omega_n^2} + \frac{ng^2}{2\Omega_n^2}\cos(2\Omega_n t)$$

$$\langle \hat{S}_{+}\hat{S}_{-}\rangle_{Ⓐ} = \frac{ng^2}{2\Omega_n^2} - \frac{ng^2}{2\Omega_n^2}\cos(2\Omega_n t)$$

maximum transfer percentage
μέγιστο ποσοστό μεταβιβάσεως $\qquad \alpha_R = \dfrac{ng^2}{\Omega_n^2} = \dfrac{ng^2}{\frac{\Delta^2}{4} + ng^2} = \dfrac{4ng^2}{4ng^2 + \Delta^2}$

περίοδος ταλαντώσεων $\qquad T_R = \dfrac{2\pi}{2\Omega_n} = \dfrac{\pi}{\Omega_n} = \dfrac{2\pi}{2\sqrt{\frac{\Delta^2}{4} + g^2 n}} = \dfrac{2\pi}{\sqrt{\Delta^2 + 4ng^2}}$

$\Omega_n = \sqrt{\dfrac{\Delta^2}{4} + g^2 n}$ $\qquad \Omega_R = 2\sqrt{n}\,g$ (κυκλική) συχνότητα Rabi

$2\Omega_n = \sqrt{\Delta^2 + 4ng^2}$ $\qquad \alpha_R = \dfrac{\Omega_R^2}{\Omega_R^2 + \Delta^2}$

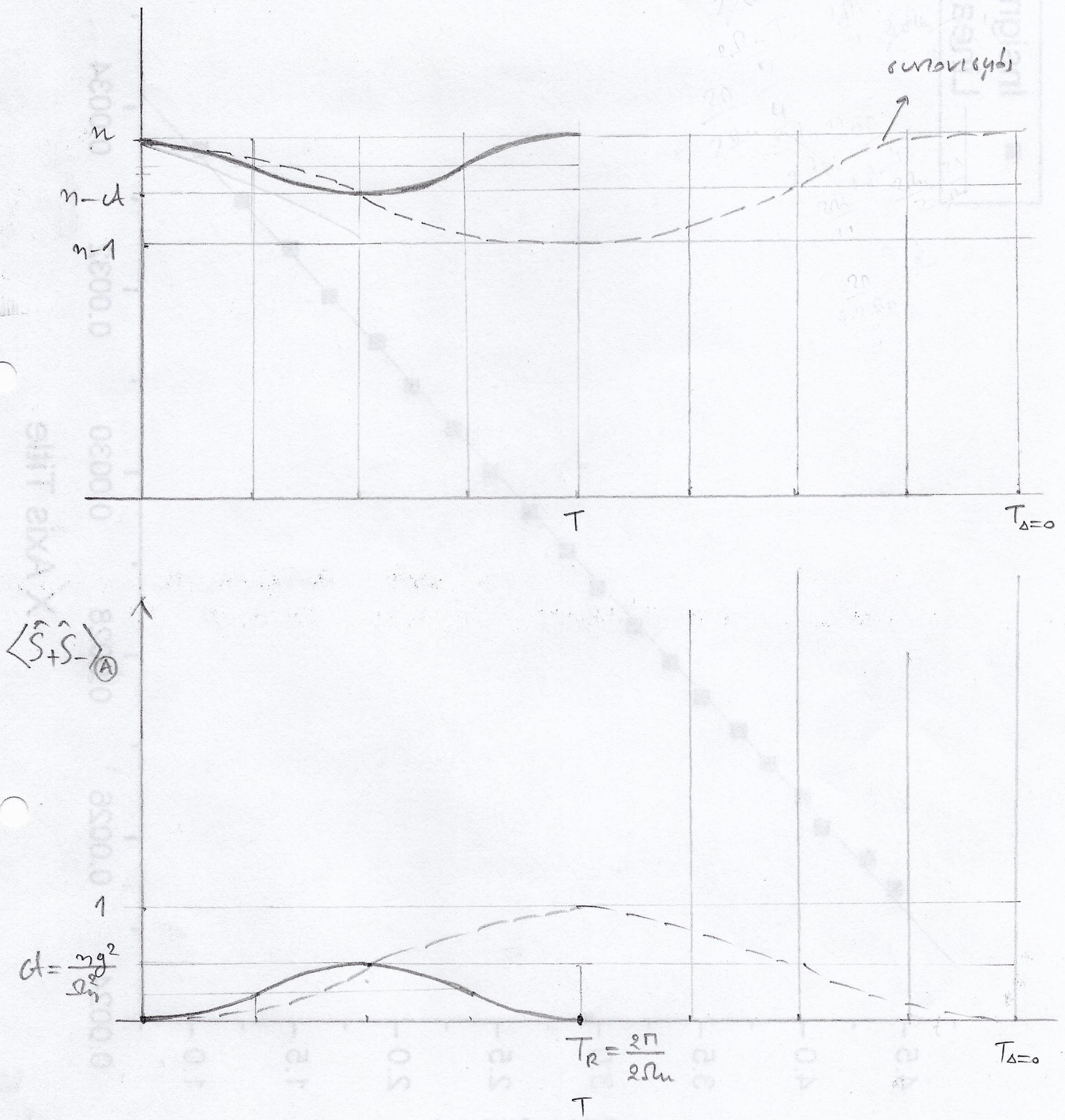
$2\Omega_n = \sqrt{\Delta^2 + \Omega_R^2}$

$$T_R = \frac{2\pi}{\sqrt{\Delta^2 + \Omega_R^2}} = \frac{1}{f_R}$$

Δηλαδή η (κυκλική) συχνότητα Rabi Ω_R
και ο αποσυντονισμός Δ καθορίζουν
την περίοδο και το μέγιστο ποσοστό μεταβιβάσεως

Ἀν $\Delta = 0 \Rightarrow \alpha_R = 1$

$$T_R = \frac{2\pi}{\Omega_R}$$


συντονισμός
n
$n-A$
$n-1$
T
$T_{\Delta=0}$
$\langle \hat{S}_+ \hat{S}_- \rangle_A$
1
$A = \frac{n g^2}{\Omega_n^2}$
$T_R = \frac{2\pi}{2\Omega_n}$
T
$T_{\Delta=0}$

$$i\begin{pmatrix}\dot c_1\\ \dot c_2\end{pmatrix} = \underbrace{\begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega\end{pmatrix}}_{A}\begin{pmatrix}c_1\\ c_2\end{pmatrix} \quad \text{θέσω} \quad \vec x(t) = \begin{pmatrix}c_1\\ c_2\end{pmatrix} \quad \dot{\vec x}(t) = \begin{pmatrix}\dot c_1\\ \dot c_2\end{pmatrix}$$ 5

$$i\dot{\vec x}(t) = A\vec x(t) \Rightarrow \boxed{\dot{\vec x}(t) = -iA\vec x(t)} \Rightarrow \vec u(-i\lambda)e^{-i\lambda t} = -iA\vec u e^{-i\lambda t}$$

ΔΛΜ $\vec x(t) = \vec u e^{-i\lambda t} \Rightarrow \dot{\vec x}(t) = \vec u(-i\lambda)e^{-i\lambda t}$

$$\Rightarrow \boxed{A\vec u = \lambda\vec u} \; (*) \Rightarrow A\vec u = \lambda I\vec u \Rightarrow (A-\lambda I)\vec u = 0$$

πρόβλημα ιδιοτιμών

πρέπει $\det(A-\lambda I) = 0$

$$\begin{vmatrix} n\omega-\lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega-\lambda \end{vmatrix} = 0 \Rightarrow (n\omega-\lambda)[\Omega+(n-1)\omega-\lambda] - ng^2 = 0$$

$$\lambda^2 - [\Omega+(n-1)\omega+n\omega]\lambda + n\omega[\Omega+(n-1)\omega] - ng^2 = 0$$

Έστω $n=1$

ένα φωτόνιο στην κοιλότητα

$$A = \begin{pmatrix}\omega & g\\ g & \Omega\end{pmatrix} \quad \text{και} \quad \det(A-\lambda I) = 0 \Rightarrow$$

$$\begin{vmatrix}\omega-\lambda & g\\ g & \Omega-\lambda\end{vmatrix} = 0$$

$$(\omega-\lambda)(\Omega-\lambda) - g^2 = 0$$

$$(*) \begin{bmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \lambda\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\lambda^2 - (\omega+\Omega)\lambda + \omega\Omega - g^2 = 0$$

$$\begin{bmatrix} n\omega-\lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega-\lambda\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Delta' = (\omega+\Omega)^2 - 4(\omega\Omega - g^2)$$

$$= \omega^2+\Omega^2+2\omega\Omega-4\omega\Omega+4g^2$$

$$= (\omega-\Omega)^2+4g^2 \Rightarrow$$

$$\Delta' = \Delta^2+4g^2$$

$$\lambda_{2,1} = \frac{(\omega+\Omega) \pm\sqrt{\Delta^2+4g^2}}{2}$$

$$\lambda_{2,1} = \frac{\omega+\Omega}{2} \pm\sqrt{\left(\frac{\Delta}{2}\right)^2+g^2}$$

$$\boxed{\lambda_{2,1} = H_1 \pm \Omega_1} \qquad H_1 = \frac{\omega+\Omega}{2}$$

$$\Omega_1 = \sqrt{\left(\frac{\Delta}{2}\right)^2+g^2}$$

$\boxed{\lambda_1 = H_1 - \Omega_1}$ $\qquad A\vec{u}_1 = \lambda_1 \vec{u}_1 \Rightarrow \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix}\begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = (H_1 - \Omega_1)\begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix}$

$$\Rightarrow \omega u_{11} + g u_{21} = (H_1 - \Omega_1) u_{11} \Rightarrow g u_{21} = (H_1 - \Omega_1 - \omega) u_{11}$$

$$g u_{11} + \Omega u_{21} = (H_1 - \Omega_1) u_{21} \Rightarrow g u_{11} = (H_1 - \Omega_1 - \Omega) u_{21}$$

$$g u_{21} = \frac{(H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) u_{21}}{g}$$

$$g^2 u_{21} = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) u_{21}$$

Άρα $u_{21} = 0 \Rightarrow u_{11} = 0$

ή $g^2 = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega)$

$$g^2 = \left(\frac{\Omega - \omega}{2} - \Omega_1\right)\left(\frac{\omega - \Omega}{2} - \Omega_1\right) \Rightarrow$$

$$g^2 = -\left(\frac{\Delta}{2} + \Omega_1\right)\left(\frac{\Delta}{2} - \Omega_1\right) \Rightarrow g^2 = -\left[\frac{\Delta^2}{4} - \Omega_1^2\right] \Rightarrow$$

$g^2 + \frac{\Delta^2}{4} = \Omega_1^2$ που ισχύει εξ ορισμού του Ω_1

$\boxed{\text{δηλαδή το } u_{21} \text{ μπορεί να είναι οτιδήποτε πχ } u_{21} = 1} \Rightarrow$

$$g u_{11} = (H_1 - \Omega_1 - \Omega) \cdot 1 = \frac{\Delta}{2} - \Omega_1 \Rightarrow u_{11} = \frac{\frac{\Delta}{2} - \Omega_1}{g} \Rightarrow$$

$$\boxed{u_{11} = \frac{\Delta - 2\Omega_1}{2g}}$$

$$\vec{u}_1 = \begin{bmatrix} \frac{\Delta - 2\Omega_1}{2g} \\ 1 \end{bmatrix}$$

$\boxed{\lambda_2 = H_1 + \Omega_1}$ $\Rightarrow$ πράξεις $\Rightarrow \vec{u}_2 = \begin{bmatrix} \frac{\Delta + 2\Omega_1}{2g} \\ 1 \end{bmatrix}$

$\boxed{\lambda_2 = H_1+\Omega_1}$ $\quad A\vec{u}_2=\lambda_2 \vec{u}_2 \Rightarrow \begin{pmatrix}\omega & g\\ g & \Omega\end{pmatrix}\begin{pmatrix}u_{12}\\ u_{22}\end{pmatrix} = (H_1+\Omega_1)\begin{pmatrix}u_{12}\\ u_{22}\end{pmatrix}$ 7'

$$\Rightarrow \omega u_{12} + g u_{22} = (H_1+\Omega_1)u_{12} \Rightarrow g u_{22} = (H_1+\Omega_1-\omega)u_{12}$$

$$g u_{12} + \Omega u_{22} = (H_1+\Omega_1)u_{22} \Rightarrow g u_{12} = (H_1+\Omega_1-\Omega)u_{22}$$

$$g u_{22} = \frac{(H_1+\Omega_1-\omega)(H_1+\Omega_1-\Omega)u_{22}}{g}$$

Άρα $u_{22}=0$ $\quad (\Rightarrow u_{12}=0)$

ή $\quad g^2 = (H_1+\Omega_1-\omega)(H_1+\Omega_1-\Omega)$

$$g^2 = \left(\frac{\omega+\Omega-2\omega}{2}+\Omega_1\right)\left(\frac{\omega+\Omega-2\Omega}{2}+\Omega_1\right)$$

$$g^2=\left(\frac{\Omega-\omega}{2}+\Omega_1\right)\left(\frac{\omega-\Omega}{2}+\Omega_1\right)$$

$$g^2 = \left(\Omega_1 - \frac{\omega-\Omega}{2}\right)\left(\Omega_1+\frac{\omega-\Omega}{2}\right) = \left(\Omega_1-\frac{\Delta}{2}\right)\left(\Omega_1+\frac{\Delta}{2}\right)$$

$$g^2 = \Omega_1^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow \Omega_1^2 = \left(\frac{\Delta}{2}\right)^2+g^2$$

το οποίον ισχύει εξ ορισμού του Ω_1

$\boxed{\text{δηλαδή το } u_{22} \text{ μπορεί να είναι οτιδήποτε π.χ. } u_{22}=1}$

$$g u_{12} = (H_1+\Omega_1-\Omega)\cdot 1 = \frac{\omega+\Omega-2\Omega}{2}+\Omega_1 = \frac{\omega-\Omega}{2}+\Omega_1$$

$$\boxed{u_{12} = \frac{\Delta+2\Omega_1}{2g}}$$

$$\vec{u}_2 = \begin{bmatrix}\frac{\Delta+2\Omega_1}{2g}\\ 1\end{bmatrix}$$

Η γενική λύση είναι $\vec{x}(t) = \sigma_1 \vec{u}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{u}_2 e^{-i\lambda_2 t}$

$$\vec{x}(t)=\begin{bmatrix}c_1(t)\\ c_2(t)\end{bmatrix} = \begin{bmatrix} \sigma_1 \frac{\Delta-2\Omega_1}{2g} e^{-i(H_1-\Omega_1)t} + \sigma_2 \frac{\Delta+2\Omega_1}{2g} e^{-i(H_1+\Omega_1)t} \\ \sigma_1\cdot 1\cdot e^{-i(H_1-\Omega_1)t} + \sigma_2 \cdot 1 \cdot e^{-i(H_1+\Omega_1)t}\end{bmatrix}$$

ΑΡΧΙΚΕΣ ΣΥΝΘΗΚΕΣ $c_1(0)=1, \ c_2(0)=0$

$$\left.\begin{aligned} 1 &= \sigma_1\frac{\Delta-2\Omega_1}{2g} + \sigma_2\frac{\Delta+2\Omega_1}{2g} \\ 0 &= \sigma_1+\sigma_2 \Rightarrow \sigma_2=-\sigma_1 \end{aligned}\right\} \Rightarrow 2g = \sigma_1(\Delta-2\Omega_1)-\sigma_1(\Delta+2\Omega_1)$$

$$2g = -2\Omega_1\sigma_1 - 2\Omega_1\sigma_1$$

$$g = -2\Omega_1\sigma_1 \Rightarrow \sigma_1 = -\frac{g}{2\Omega_1} = -\sigma_2$$

Άρα $c_2(t) = -\frac{g}{2\Omega_1} e^{-i(H_1-\Omega_1)t} + \frac{g}{2\Omega_1} e^{-i(H_1+\Omega_1)t} \Rightarrow$

$$c_2(t) = -\frac{g}{2\Omega_1} e^{-iH_1 t} e^{i\Omega_1 t} + \frac{g}{2\Omega_1} e^{-iH_1 t} e^{-i\Omega_1 t}$$

$$c_2(t) = \frac{g}{2\Omega_1} e^{-iH_1 t}\left\{-\cos(\Omega_1 t) - i\sin(\Omega_1 t) + \cos(\Omega_1 t) - i\sin(\Omega_1 t)\right\}$$

$$c_2(t) = \frac{g}{2\Omega_1} e^{-iH_1 t}(-2i)\sin(\Omega_1 t) = e^{-i\frac{\omega+\Omega}{2}t}\left\{-i\frac{g}{\Omega_1}\sin(\Omega_1 t)\right\}$$

$$\boxed{|c_2(t)|^2 = \frac{g^2}{\Omega_1^2}\sin^2(\Omega_1 t)} \qquad |c_1(t)|^2 = 1-|c_2(t)|^2 = 1-\frac{g^2}{\Omega_1^2}\left(1-\cos^2(\Omega_1 t)\right)$$

$$= \left(1-\frac{g^2}{\Omega_1^2}\right) + \frac{g^2}{\Omega_1^2}\cos^2(\Omega_1 t)$$

$$\Omega_1^2 = \frac{\Delta^2}{4} + g^2 \Rightarrow \Omega_1^2 - g^2 = \frac{\Delta^2}{4}$$

$$\boxed{|c_1(t)|^2 = \frac{\left(\frac{\Delta^2}{4}\right)}{\Omega_1^2} + \frac{g^2}{\Omega_1^2}\cos^2(\Omega_1 t)}$$

Γ' Έστω η φωτόνια στην κοιλότητα

$$A=\begin{bmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega \end{bmatrix} \quad \text{και} \quad \det(A-\lambda I)=0 \Rightarrow$$

$$\begin{vmatrix} n\omega-\lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega-\lambda \end{vmatrix}=0$$

$$\Rightarrow (n\omega-\lambda)\left[\Omega+(n-1)\omega-\lambda\right]-ng^2=0$$

$$\lambda^2-\lambda\left[\Omega+(n-1)\omega+n\omega\right]+n\omega\left[\Omega+(n-1)\omega\right]-ng^2=0$$

$$\Delta'=\left[\Omega+(n-1)\omega+n\omega\right]^2-4\left(n\omega\left[\Omega+(n-1)\omega\right]-ng^2\right)$$

$$\Delta'=\underbrace{\left[\Omega+(n-1)\omega+n\omega\right]^2-4n\omega\left[\Omega+(n-1)\omega\right]}+4ng^2$$

$$\Delta'=\underbrace{\left[\Omega+(n-1)\omega-n\omega\right]^2}+4ng^2>0$$

$$\lambda_{2,1}=\frac{\left[\Omega+(n-1)\omega+n\omega\right]\pm\sqrt{\left[\Omega+(n-1)\omega-n\omega\right]^2+4ng^2}}{2}$$

$$\lambda_{2,1}=\frac{\Omega+(n-1)\omega+n\omega}{2}\pm\sqrt{\left(\frac{\Omega+(n-1)\omega-n\omega}{2}\right)^2+ng^2}$$

$$\lambda_{2,1}=\frac{\Omega+(n-1)\omega+n\omega}{2}\pm\sqrt{\left(\frac{\omega-\Omega}{2}\right)^2+ng^2}$$

$$\boxed{\lambda_{2,1}=H_n\pm\Omega_n} \qquad H_n=\frac{\Omega+(n-1)\omega+n\omega}{2}$$

$$\Omega_n=\sqrt{\left(\frac{\Delta}{2}\right)^2+ng^2}$$

$\lambda_1 = H_n - \Omega_n$ $\quad A\vec{U}_1 = \lambda_1 \vec{U}_1 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega \end{pmatrix} \begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix} = (H_n - \Omega_n) \begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix}$ (6')

$$n\omega U_{11} + g\sqrt{n} U_{21} = (H_n - \Omega_n) U_{11}$$

$$g\sqrt{n} U_{11} + [\Omega + (n-1)\omega] U_{21} = (H_n - \Omega_n) U_{21}$$

$$g\sqrt{n} U_{21} = (H_n - \Omega_n - n\omega) U_{11}$$

$$g\sqrt{n} U_{11} = \left(H_n - \Omega_n - [\Omega + (n-1)\omega]\right) U_{21}$$

$$g\sqrt{n} U_{21} = \frac{(H_n - \Omega_n - n\omega)\left(H_n - \Omega_n - [\Omega + (n-1)\omega]\right) U_{21}}{g\sqrt{n}}$$

"Αν $U_{21} = 0$ ή $g^2 n = (H_n - \Omega_n - n\omega)\left(H_n - \Omega_n - [\Omega + (n-1)\omega]\right)$

$\left(\begin{matrix} \Downarrow \\ U_{11} = 0 \end{matrix} \right)$

$$H_n - \Omega_n - n\omega = \frac{\Omega + (n-1)\omega + n\omega - 2n\omega}{2} - \Omega_n$$

$$= \frac{\Omega - \omega}{2} - \Omega_n = -\frac{\Delta}{2} - \Omega_n$$

$$H_n - \Omega_n - [\Omega + (n-1)\omega] = \frac{\Omega + (n-1)\omega + n\omega - 2\Omega - 2(n-1)\omega}{2} - \Omega_n$$

$$= \frac{\omega - \Omega}{2} - \Omega_n = \frac{\Delta}{2} - \Omega_n$$

$$\Rightarrow g^2 n = -\left(\frac{\Delta}{2} + \Omega_n\right) \cdot \left(\frac{\Delta}{2} - \Omega_n\right) = -\left(\frac{\Delta}{2}\right)^2 + \Omega_n^2$$

$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + n g^2}$ το οποίον ισχύει εξ ορισμού του Ω_n

$\boxed{\text{δηλαδή το } U_{21} \text{ μπορεί να είναι οτιδήποτε π.χ. } U_{21} = 1}$

$$g\sqrt{n} U_{11} = H_n - \Omega_n - [\Omega + (n-1)\omega] = \frac{\Delta}{2} - \Omega_n$$

$$\boxed{U_{11} = \frac{\Delta - 2\Omega_n}{2g\sqrt{n}}}$$

$$\vec{U}_1 = \begin{bmatrix} \dfrac{\Delta - 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix}$$

$$\boxed{\lambda_2 = H_n + \Omega_n} \qquad A\vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix} \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_n + \Omega_n) \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix}$$

$$n\omega U_{12} + g\sqrt{n}\, U_{22} = (H_n + \Omega_n) U_{12}$$

$$g\sqrt{n}\, U_{12} + [\Omega + (n-1)\omega] U_{22} = (H_n + \Omega_n) U_{22}$$

$$g\sqrt{n}\, U_{22} = (H_n + \Omega_n - n\omega) U_{12}$$

$$g\sqrt{n}\, U_{12} = \big(H_n + \Omega_n - [\Omega + (n-1)\omega]\big) U_{22}$$

$$g\sqrt{n}\, U_{22} = \frac{(H_n + \Omega_n - n\omega)\big(H_n + \Omega_n - [\Omega + (n-1)\omega]\big)\, U_{22}}{g\sqrt{n}}$$

Άρα $U_{22} = 0$ ή $g^2 n = (H_n + \Omega_n - n\omega)\big(H_n + \Omega_n - [\Omega + (n-1)\omega]\big)$

$$\left(\begin{matrix} U_{22} = 0 \\ \Downarrow \\ U_{12} = 0 \end{matrix} \right)$$

$$H_n + \Omega_n - n\omega = \frac{\Omega + (n-1)\omega + n\omega - 2n\omega}{2} + \Omega_n$$

$$= \frac{\Omega - \omega}{2} + \Omega_n = -\frac{\Delta}{2} + \Omega_n$$

$$H_n + \Omega_n - [\Omega + (n-1)\omega] = \frac{\Omega + (n-1)\omega + n\omega - 2\Omega - 2(n-1)\omega}{2} + \Omega_n$$

$$= \frac{\omega - \Omega}{2} + \Omega_n = \frac{\Delta}{2} + \Omega_n$$

$$g^2 n = \left(\Omega_n + \frac{\Delta}{2}\right)\left(\Omega_n - \frac{\Delta}{2}\right) = \Omega_n^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow$$

$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + n g^2}$ το οποίον ισχύει εξ ορισμού του Ω_n

$\boxed{\text{δηλαδή το } U_{22} \text{ μπορεί να είναι ό,τιδήποτε π.χ. } U_{22} = 1}$

$$g\sqrt{n}\, U_{12} = H_n + \Omega_n - [\Omega + (n-1)\omega] = \frac{\Delta}{2} + \Omega_n \Rightarrow$$

$$\boxed{U_{12} = \frac{\Delta + 2\Omega_n}{2g\sqrt{n}}}$$

$$\vec{U}_2 = \begin{bmatrix} \dfrac{\Delta + 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix}$$

Η γενική λύση είναι $\vec{x}(t) = \sigma_1 \vec{u}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{u}_2 e^{-i\lambda_2 t}$

$$\vec{x}(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \sigma_1 \begin{bmatrix} \frac{\Delta - 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix} e^{-i(H_n - \Omega_n)t} + \sigma_2 \begin{bmatrix} \frac{\Delta + 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix} e^{-i(H_n + \Omega_n)t}$$

Αρχικές Συνθήκες $c_1(0) = 1 \qquad c_2(0) = 0$

$$\left. \begin{array}{l} \sigma_1 \frac{\Delta - 2\Omega_n}{2g\sqrt{n}} + \sigma_2 \frac{\Delta + 2\Omega_n}{2g\sqrt{n}} = 1 \\ \sigma_1 + \sigma_2 = 0 \Rightarrow \sigma_2 = -\sigma_1 \end{array} \right\} \Rightarrow \sigma_1 \frac{\Delta - 2\Omega_n - \Delta - 2\Omega_n}{2g\sqrt{n}} = 1$$

$$\Rightarrow \sigma_1 \frac{-4\Omega_n}{2g\sqrt{n}} = 1 \Rightarrow \sigma_1 = \frac{-g\sqrt{n}}{2\Omega_n} = -\sigma_2$$

$$c_2(t) = \frac{-g\sqrt{n}}{2\Omega_n} e^{-i(H_n - \Omega_n)t} + \frac{g\sqrt{n}}{2\Omega_n} e^{-i(H_n + \Omega_n)t}$$

$$c_2(t) = \frac{g\sqrt{n}}{2\Omega_n} e^{-iH_n t} \left[e^{-i\Omega_n t} - e^{i\Omega_n t} \right] \Rightarrow c_2(t) = \frac{g\sqrt{n}}{2\Omega_n} e^{-iH_n t} (-2i) \sin(\Omega_n t)$$

$$c_2(t) = -i \frac{g\sqrt{n}}{\Omega_n} e^{-iH_n t} \cdot \sin(\Omega_n t)$$

$$|c_2(t)|^2 = \frac{n g^2}{\Omega_n^2} \sin^2(\Omega_n t)$$