Είχαμε βρεί για $\Delta = 0$

ΑΣΚΗΣΗ 1

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} \frac{\sigma_1}{\sqrt{2}} e^{i\frac{\Omega_R t}{2}} + \frac{\sigma_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \\ \frac{\sigma_1}{\sqrt{2}} e^{i\frac{\Omega_R t}{2}} - \frac{\sigma_2}{\sqrt{2}} e^{-i\frac{\Omega_R t}{2}} \end{bmatrix}$$

ας βάλουμε αρχικές συνθήκες $C_1(0) = \frac{1}{\sqrt{2}} e^{i\theta}$ κ $C_2(0) = \frac{1}{\sqrt{2}} e^{i\varphi} \Rightarrow$

$$|C_1(0)|^2 = \frac{1}{2} = |C_2(0)|^2$$

- $\frac{1}{\sqrt{2}} e^{i\theta} = \frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}} \Rightarrow \sigma_1 + \sigma_2 = e^{i\theta}$
- $\frac{1}{\sqrt{2}} e^{i\varphi} = \frac{\sigma_1}{\sqrt{2}} - \frac{\sigma_2}{\sqrt{2}} \Rightarrow \sigma_1 - \sigma_2 = e^{i\varphi}$

$\oplus$ $2\sigma_1 = e^{i\theta} + e^{i\varphi}$

$\ominus$ $2\sigma_2 = e^{i\theta} - e^{i\varphi}$

$$\sigma_1 = \frac{e^{i\theta} + e^{i\varphi}}{2} \qquad \sigma_2 = \frac{e^{i\theta} - e^{i\varphi}}{2}$$

- $C_1(t) = \frac{e^{i\theta} + e^{i\varphi}}{2\sqrt{2}} e^{i\frac{\Omega_R t}{2}} + \frac{e^{i\theta} - e^{i\varphi}}{2\sqrt{2}} e^{-i\frac{\Omega_R t}{2}}$
- $C_2(t) = \frac{e^{i\theta} + e^{i\varphi}}{2\sqrt{2}} e^{i\frac{\Omega_R t}{2}} - \frac{e^{i\theta} - e^{i\varphi}}{2\sqrt{2}} e^{-i\frac{\Omega_R t}{2}}$

$\Rightarrow$

- $2\sqrt{2}\, C_1(t) = \underline{e^{i\theta} e^{i\frac{\Omega_R t}{2}}} + \underline{e^{i\varphi} e^{i\frac{\Omega_R t}{2}}} + \underline{e^{i\theta} e^{-i\frac{\Omega_R t}{2}}} - \underline{e^{i\varphi} e^{-i\frac{\Omega_R t}{2}}}$
- $2\sqrt{2}\, C_2(t) = \underline{e^{i\theta} e^{i\frac{\Omega_R t}{2}}} + \underline{e^{i\varphi} e^{i\frac{\Omega_R t}{2}}} - \underline{e^{i\theta} e^{-i\frac{\Omega_R t}{2}}} + \underline{e^{i\varphi} e^{-i\frac{\Omega_R t}{2}}}$

$\Rightarrow$

- $2\sqrt{2}\, C_1(t) = e^{i\theta}\, 2\cos\left(\frac{\Omega_R t}{2}\right) + e^{i\varphi}\, 2i\sin\left(\frac{\Omega_R t}{2}\right)$
- $2\sqrt{2}\, C_2(t) = e^{i\theta}\, 2i\sin\left(\frac{\Omega_R t}{2}\right) + e^{i\varphi}\, 2\cos\left(\frac{\Omega_R t}{2}\right)$

$\Rightarrow$

- $8|C_1(t)|^2 = 4\cos^2\left(\frac{\Omega_R t}{2}\right) + 4\sin^2\left(\frac{\Omega_R t}{2}\right) + e^{i\theta}\, 2\cos\left(\frac{\Omega_R t}{2}\right) e^{-i\varphi}\, 2(-i)\sin\left(\frac{\Omega_R t}{2}\right)$
$\quad e^{i\varphi}\, 2i\sin\left(\frac{\Omega_R t}{2}\right) 2e^{-i\theta}\cos\left(\frac{\Omega_R t}{2}\right) \Rightarrow$

$$2|C_1(t)|^2 = \cos^2\left(\frac{\Omega_R t}{2}\right) + \sin^2\left(\frac{\Omega_R t}{2}\right) - i e^{i\theta} e^{-i\varphi} \cos\left(\frac{\Omega_R t}{2}\right) \cdot \sin\left(\frac{\Omega_R t}{2}\right) + i e^{i\varphi} e^{-i\theta} \cos\left(\frac{\Omega_R t}{2}\right) \cdot \sin\left(\frac{\Omega_R t}{2}\right)$$

$$\frac{1}{2}\sin(\Omega_R t)\, i \left\{ e^{i(\varphi-\theta)} - e^{-i(\varphi-\theta)} \right\} = \frac{i}{2}\sin(\Omega_R t)\, 2i\sin\psi = -\sin(\Omega_R t)\sin\psi$$

$$= \sin(\Omega_R t) \cdot \sin(\theta - \varphi)$$

$\psi := \varphi - \theta$

$\cos\psi \quad i\sin\psi$

$-\cos\psi + i\sin\psi$

$$\boxed{|C_1(t)|^2 = \frac{1}{2} + \frac{1}{2}\sin(\Omega_R t)\sin(\theta-\varphi)}$$ γενικώς, ∃ ταλάντωση

$$\boxed{\text{αν όμως } \theta=\varphi \Rightarrow |C_1(t)|^2 = \frac{1}{2} \text{ κ } \nexists \text{ ταλάντωση}}$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Αν θέλω $\frac{1}{2} + \frac{1}{2}\sin(\Omega_R t)\sin(\theta-\varphi) = \frac{1}{2} + \frac{1}{2}\cos\left(\Omega_R t + \frac{\pi}{2}\right) \Rightarrow$

$$\sin(\Omega_R t)\sin(\theta-\varphi) = -\sin(\Omega_R t) \Rightarrow$$

$$\sin(\theta-\varphi) = -1 \Rightarrow \theta-\varphi = -\frac{\pi}{2} \Rightarrow \theta = \varphi - \frac{\pi}{2}$$

- $8|C_2(t)|^2 = 4\sin^2\left(\frac{\Omega_R t}{2}\right) + 4\cos^2\left(\frac{\Omega_R t}{2}\right) + e^{i\theta}2i\sin\left(\frac{\Omega_R t}{2}\right)\cdot e^{-i\varphi}2\cos\left(\frac{\Omega_R t}{2}\right)$

$$e^{i\varphi}2\cos\left(\frac{\Omega_R t}{2}\right)\cdot e^{-i\theta}2(-i)\sin\left(\frac{\Omega_R t}{2}\right) \Rightarrow$$

$$2|C_2(t)|^2 = 1 + \frac{1}{2}\sin(\Omega_R t)\, i\left\{e^{i\theta}e^{-i\varphi} - e^{i\varphi}e^{-i\theta}\right\}$$

$$e^{i(\theta-\varphi)} - e^{-i(\theta-\varphi)}$$

$$\psi' = \theta-\varphi \qquad \psi' = -\psi$$

$$e^{i\psi'} - e^{-i\psi'}$$

$$\cos\psi' \quad i\sin\psi'$$

$$-\cos\psi' + i\sin\psi'$$

$$2|C_2(t)|^2 = 1 + \frac{1}{\cancel{2}}\sin(\Omega_R t)\, i\,\cancel{2}\, i\sin\psi'$$

$$\boxed{|C_2(t)|^2 = \frac{1}{2} - \frac{1}{2}\sin(\Omega_R t)\sin(\theta-\varphi)}$$ γενικώς, ∃ ταλάντωση

$$\boxed{\text{αν όμως } \theta=\varphi \Rightarrow |C_2(t)|^2 = \frac{1}{2} \text{ κ } \nexists \text{ ταλάντωση}}$$

Αν θέλω $\frac{1}{2} - \frac{1}{2}\sin(\Omega_R t)\sin(\theta-\varphi) = \frac{1}{2} - \frac{1}{2}\cos\left(\Omega_R t + \frac{\pi}{2}\right) \Rightarrow$

$$\sin(\Omega_R t)\sin(\theta-\varphi) = -\sin(\Omega_R t) \Rightarrow$$

$$\sin(\theta-\varphi) = -1 \Rightarrow \theta-\varphi = -\frac{\pi}{2} \Rightarrow \theta = \varphi - \frac{\pi}{2}$$

ΔΣ $\Delta = 0$ ἀρχικὲς συνθῆκες $C_1(0)=1$, $C_2(0)=0$ δηλ. ⫨

ἀποσυντονισμός

$\Delta := \omega - \Omega$

$$P_1(t) = |C_1(t)|^2 = \cos^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} + \frac{1}{2}\cos(\Omega_R t)$$

$$P_2(t) = |C_2(t)|^2 = \sin^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t)$$

$\Omega_R := \dfrac{\wp E_0}{\hbar}$ $(\wp > 0)$

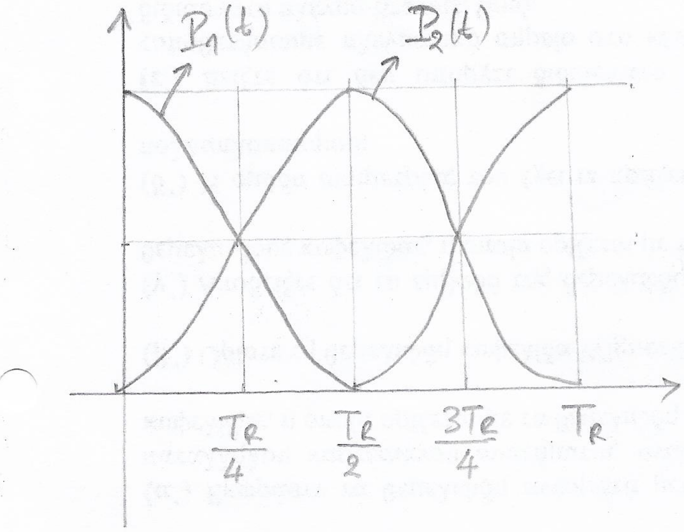
συχνότητα Rabi

βρίσκεται θετική

$\Omega_R := \dfrac{-\wp E_0}{\hbar}$ $(\wp < 0)$

Ω_R: ἐκφράζει την ταχύτητα διαταραχής

Δ: ἐκφράζει την ἀπόσταση τῶν ω (ΗΜ πεδίο) κ Ω (ΔΣ)



περίοδος (period)

$$T_R = \frac{2\pi}{\Omega_R}$$

$A_R = 1$

μέγιστο ποσοστό μεταβιβάσεως (maximum transfer percentage)

$$\langle P_1(t)\rangle = \langle |C_1(t)|^2\rangle = \frac{1}{2}$$ μέση πιθανότητα παρουσίας στη στάθμη 1

$$\langle P_2(t)\rangle = \langle |C_2(t)|^2\rangle = \frac{1}{2}$$ μέση πιθανότητα παρουσίας στη στάθμη 2

μέγιστος ρυθμός μεταβιβάσεως (maximum transfer rate) $\boxed{\dfrac{A_R}{T_R} = \dfrac{1}{\frac{2\pi}{\Omega_R}} = \dfrac{\Omega_R}{2\pi}}$

t_{2mean} := ὁ χρόνος, ὁ ὁποῖος ἀπαιτεῖται ὥστε ἡ $P_2(t)$ να πιάσει 1η φορά την $\langle P_2(t)\rangle$

$$\Rightarrow \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t_{2mean}) = \frac{1}{2} \Rightarrow \cos(\Omega_R t_{2mean}) = 0 \Rightarrow$$

$$\Omega_R t_{2mean} = \frac{\pi}{2} \Rightarrow \boxed{t_{2mean} = \frac{\pi}{2\Omega_R}}$$

μέσος ρυθμός μεταβιβάσεως (mean transfer rate)

$$k := \frac{\langle |C_2(t)|^2\rangle}{t_{2mean}} = \frac{\frac{1}{2}}{\frac{\pi}{2\Omega_R}} = \frac{\Omega_R}{\pi} \Rightarrow \boxed{k = 2\frac{A_R}{T_R}}$$

• ΛΥΣΗ για $\Delta \neq 0$ μηδε για $\xi<0$

A

$$\begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \lambda \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\lambda_{2,1} = \pm \frac{\sqrt{\Omega_R^2+\Delta^2}}{2} = \pm\lambda$$

$\lambda > 0$

$$\vec{U}_1 = \begin{bmatrix} \frac{1}{\sqrt{1+a^2}} \\ \frac{a}{\sqrt{1+a^2}} \end{bmatrix} \qquad a = \frac{\frac{\Delta}{2} + \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}}{\frac{\Omega_R}{2}}$$

οι πράξεις υπάρχουν στο βιβλίο

$$\vec{U}_2 = \begin{bmatrix} \frac{1}{\sqrt{1+a'^2}} \\ \frac{a'}{\sqrt{1+a'^2}} \end{bmatrix} \qquad a' = \frac{\frac{\Delta}{2} - \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}}{\frac{\Omega_R}{2}}$$

Γενική λύση

$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} C_1(t)e^{-i\frac{\Delta}{2}t} \\ C_2(t)e^{i\frac{\Delta}{2}t} \end{bmatrix} = \sum_{k=1}^{2} c_k \vec{U}_k e^{-i\lambda_k t} = c_1\vec{U}_1 e^{-i\lambda_1 t} + c_2 \vec{U}_2 e^{-i\lambda_2 t}$$

$$= c_1 \begin{bmatrix} \frac{1}{\sqrt{1+a^2}} \\ \frac{a}{\sqrt{1+a^2}} \end{bmatrix} e^{-i\lambda_1 t} + c_2 \begin{bmatrix} \frac{1}{\sqrt{1+a'^2}} \\ \frac{a'}{\sqrt{1+a'^2}} \end{bmatrix} e^{-i\lambda_2 t}$$

Έστωσαν Αρχικές Συνθήκες $C_1(0)=1 \quad C_2(0)=0$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{c_1}{\sqrt{1+a^2}} + \frac{c_2}{\sqrt{1+a'^2}} \\ \frac{c_1 a}{\sqrt{1+a^2}} + \frac{c_2 a'}{\sqrt{1+a'^2}} \end{bmatrix} \Rightarrow \ldots \quad c_1 = \frac{a'\sqrt{1+a^2}}{a'-a} \quad c_2 = -\frac{a\sqrt{1+a'^2}}{a'-a}$$

Άρα...

$$\begin{bmatrix} C_1(t)e^{-i\frac{\Delta}{2}t} \\ C_2(t)e^{i\frac{\Delta}{2}t} \end{bmatrix} = \frac{a'\sqrt{1+a^2}}{a'-a}\begin{bmatrix} \frac{1}{\sqrt{1+a^2}} \\ \frac{a}{\sqrt{1+a^2}} \end{bmatrix} e^{-i\lambda_1 t} - \frac{a\sqrt{1+a'^2}}{a'-a}\begin{bmatrix} \frac{1}{\sqrt{1+a'^2}} \\ \frac{a'}{\sqrt{1+a'^2}} \end{bmatrix} e^{-i\lambda_2 t}$$

B

$$C_1(t)\, e^{-i\frac{\Delta}{2}t} = \frac{a'}{a'-a} e^{-i\lambda_1 t} - \frac{a}{a'-a} e^{-i\lambda_2 t}$$

$$C_2(t)\, e^{i\frac{\Delta}{2}t} = \frac{aa'}{a'-a} e^{-i\lambda_1 t} - \frac{aa'}{a'-a} e^{-i\lambda_2 t}$$

$$\frac{a'}{a'-a} = \frac{\sqrt{\Omega_R^2+\Delta^2}-\Delta}{2\sqrt{\Omega_R^2+\Delta^2}} = \gamma_1 \qquad \frac{aa'}{a'-a} = \frac{\Omega_R}{2\sqrt{\Omega_R^2+\Delta^2}} = \gamma_3$$

$$\frac{a}{a'-a} = -\frac{\sqrt{\Omega_R^2+\Delta^2}+\Delta}{2\sqrt{\Omega_R^2+\Delta^2}} = -\gamma_2$$

$$C_1(t)\, e^{-i\frac{\Delta}{2}t} = \gamma_1 e^{-i\lambda_1 t} + \gamma_2 e^{-i\lambda_2 t} \Rightarrow C_1(t) = \left(\gamma_1 e^{-i\lambda_1 t} + \gamma_2 e^{-i\lambda_2 t}\right) e^{i\frac{\Delta}{2}t}$$

$$C_2(t)\, e^{i\frac{\Delta}{2}t} = \gamma_3\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right) \Rightarrow C_2(t) = \gamma_3\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right) e^{-i\frac{\Delta}{2}t}$$

$$|C_1(t)|^2 = \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2\, e^{i(\lambda_1-\lambda_2)t} + \gamma_1\gamma_2\, e^{i(\lambda_2-\lambda_1)t}$$

$$|C_2(t)|^2 = \gamma_3^2\left[1 + 1 - e^{i(\lambda_1-\lambda_2)t} - e^{i(\lambda_2-\lambda_1)t}\right]$$

$$\lambda_1-\lambda_2 = -\lambda-\lambda = -2\lambda \qquad \lambda_2-\lambda_1 = 2\lambda$$

$$|C_1(t)|^2 = \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2\, e^{-i2\lambda t} + \gamma_1\gamma_2\, e^{i2\lambda t} = \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2\, 2\cos(2\lambda t)$$

$$|C_2(t)|^2 = \gamma_3^2\left[2 - e^{-i2\lambda t} - e^{i2\lambda t}\right] = \gamma_3^2\left[2 - 2\cos(2\lambda t)\right]$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{4(\Omega_R^2+\Delta^2)} \cdot 2\left(1-\cos(2\lambda t)\right) = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\, 2\sin^2(\lambda t) = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \cdot \sin^2(\lambda t)$$

$$\lambda = \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}$$

$$\gamma_1^2+\gamma_2^2 = \frac{\Omega_R^2+2\Delta^2}{2(\Omega_R^2+\Delta^2)}, \quad 2\gamma_1\gamma_2 = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \Rightarrow \gamma_1^2+\gamma_2^2+2\gamma_1\gamma_2 = 1 \qquad 2\gamma_3^2 = 2\gamma_1\gamma_2$$

$$\gamma_3^2 = \gamma_1\gamma_2$$

$$|C_1(t)|^2 = 1 - 2\gamma_1\gamma_2 + 2\gamma_1\gamma_2\cos(2\lambda t) = 1 + 2\gamma_1\gamma_2\left[\cos(2\lambda t) - 1\right]$$

$$|c_1(t)|^2 = 1 + \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \cdot (-2)\sin^2(\lambda t)$$

$$|c_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \cdot \sin^2(\lambda t)$$ όπως αναμενόταν

διότι $|c_1(t)|^2 + |c_2(t)|^2 = 1$

<u>Συνοπτικώς:</u>

$$\boxed{\begin{aligned} |c_1(t)|^2 &= 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \cdot \sin^2(\lambda t) \\ |c_2(t)|^2 &= \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \cdot \sin^2(\lambda t) \end{aligned}}$$

$$\boxed{\lambda = \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}}$$

ή

$$|c_1(t)|^2 = 1 - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \cdot \cos(2\lambda t)$$

$$\boxed{\begin{aligned} |c_1(t)|^2 &= \frac{\Omega_R^2+2\Delta^2}{2(\Omega_R^2+\Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \cdot \cos(2\lambda t) = P_1(t) \\ |c_2(t)|^2 &= \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \cdot \cos(2\lambda t) = P_2(t) \end{aligned}}$$

περίοδος ταλαντώσεων

$$\boxed{T_R = \frac{2\pi}{2\lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Delta^2}} = \frac{1}{\nu_R}}$$

ΝΑ ΔΙΟΡΘΩΘΕΙ κ ΣΤΟ ΒΙΒΛΙΟ

~~πλάτος~~

Ωογ ~~ταλαντώσεων~~

$$\boxed{\alpha_R = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}}$$

μέγιστο ποσοστό μεταβιβάσεως (maximum transfer percentage)

$\Delta \uparrow \Rightarrow \alpha_R \downarrow$ και $\nu_R \uparrow$ $(T_R \downarrow)$

$\Delta = 0 \Rightarrow \alpha_R = 1$ και $T_R = \frac{2\pi}{\Omega_R}$

$$\langle P_1(t)\rangle = \langle |C_1(t)|^2\rangle = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2+\Delta^2)}$$ μέση πιθανότητα παρουσίας στη στάθμη 1

$$\langle P_2(t)\rangle = \langle |C_2(t)|^2\rangle = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}$$ μέση πιθανότητα παρουσίας στη στάθμη 2

μέγιστος ρυθμός μεταβιβάσεως
(maximum transfer rate)

$$\boxed{\frac{A_R}{T_R} = \frac{\Omega_R^2\sqrt{\Omega_R^2+\Delta^2}}{(\Omega_R^2+\Delta^2)\,2\pi} = \frac{\Omega_R^2}{2\pi\sqrt{\Omega_R^2+\Delta^2}}}$$

t_{2mean} := ο χρόνος, ο οποίος απαιτείται ώστε η $P_2(t)$ να πιάσει 1η φορά την $\langle P_2(t)\rangle$

$$\Rightarrow \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\cdot\cos(2\lambda t_{2mean}) = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}$$

$$\Rightarrow \cos(2\lambda t_{2mean}) = 0 \Rightarrow 2\lambda t_{2mean} = \frac{\pi}{2} \Rightarrow \boxed{t_{2mean} = \frac{\pi}{4\lambda}}$$

μέσος ρυθμός μεταβιβάσεως
(mean transfer rate)

$$\boxed{k := \frac{\langle |C_2(t)|^2\rangle}{t_{2mean}} = \frac{\Omega_R^2\, 4\sqrt{\Omega_R^2+\Delta^2}}{2(\Omega_R^2+\Delta^2)\,\pi\, 2} = \frac{\Omega_R^2}{\pi\sqrt{\Omega_R^2+\Delta^2}}}$$

$$\Rightarrow \boxed{k = 2\frac{A_R}{T_R}}$$

E

* όταν $|\Delta|\uparrow$ (δηλ απομακρυνόμαστε από το συντονισμό) $\Rightarrow$ $\mathcal{A}_R \searrow$

$T_R \searrow$

δηλ το φαινόμενο γίνεται <u>πιο μικρό</u> και πιο <u>γρήγορο</u>

* όταν $\Omega_R << |\Delta|$ (μικρή διαταραχή σε σχέση με την απόλυτη τιμή του αποσυντονισμού)

$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\sin^2\left(\frac{\sqrt{\Omega_R^2+\Delta^2}}{2}t\right)$$

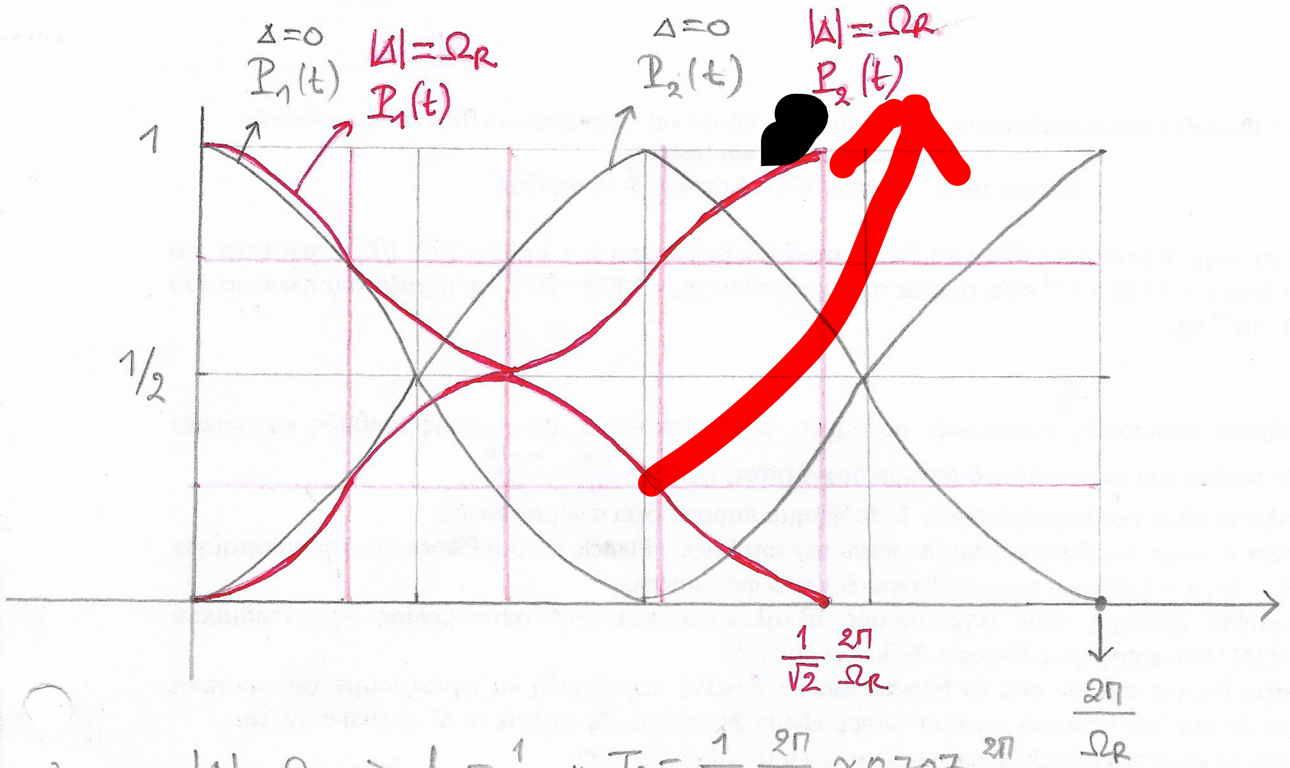
$$\approx \frac{\Omega_R^2}{\Delta^2}\sin^2\left(\frac{|\Delta|}{2}t\right)$$

ή
$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\cdot\cos\left(\cancel{2}\frac{\sqrt{\Omega_R^2+\Delta^2}}{\cancel{2}}t\right)$$

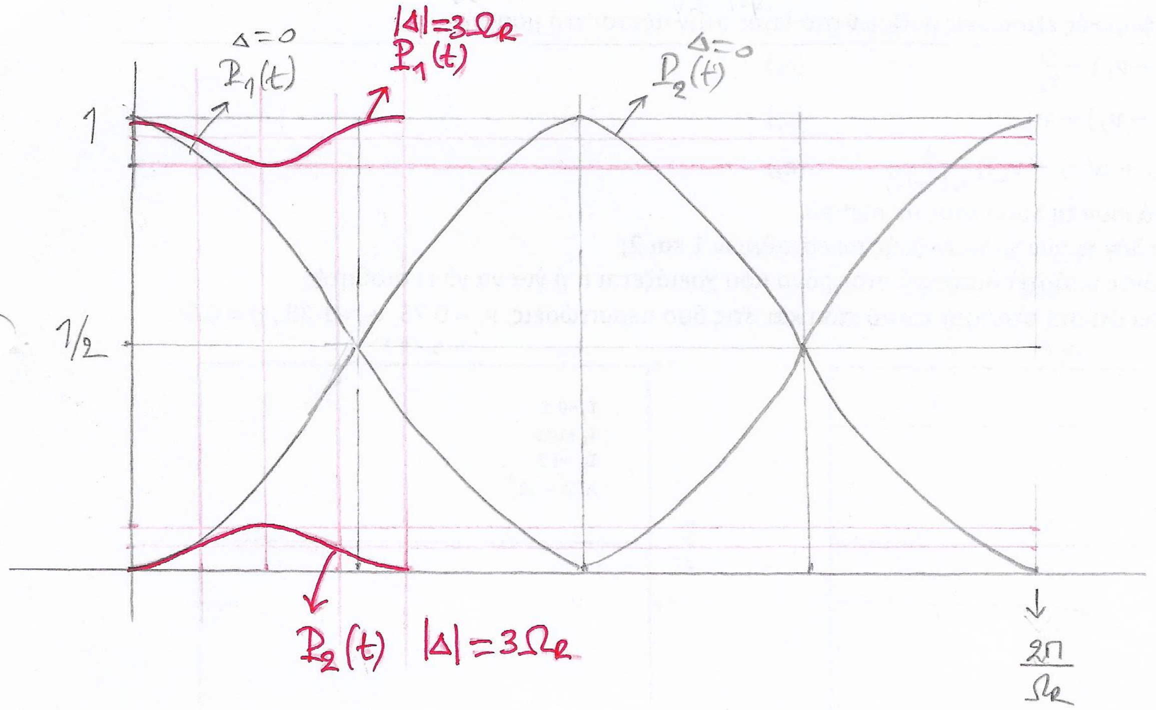
$$\approx \frac{\Omega_R^2}{2\Delta^2} - \frac{\Omega_R^2}{2\Delta^2}\cos(|\Delta|\cdot t)$$

$$\Rightarrow T_R \approx \frac{2\pi}{|\Delta|} \qquad \mathcal{A}_R \approx \frac{\Omega_R^2}{\Delta^2}$$

$$\lim_{\Omega_R\to 0} T_R = \frac{2\pi}{|\Delta|} \qquad \lim_{\Omega_R\to 0}\mathcal{A}_R = 0$$



αν π.χ. $|\Delta| = \Omega_R \Rightarrow A_R = \frac{1}{2}$ ϗ $T_R = \frac{1}{\sqrt{2}}\frac{2\pi}{\Omega_R} \simeq 0.707\,\frac{2\pi}{\Omega_R}$



αν π.χ. $|\Delta| = 3\Omega_R \Rightarrow A_R = \frac{1}{10}$ ϗ $T_R = \frac{2\pi}{\sqrt{10}\,\Omega_R} \simeq 0.316\,\frac{2\pi}{\Omega_R}$

Z

ΑΣΚΗΣΗ

Να λυθεί το πρόβλημα $\Delta=0$ και αρχική συνθήκη

$$C_1(0)=\frac{1}{\sqrt{2}}=C_2(0)$$

$$\Rightarrow |C_1(0)|^2=\frac{1}{2}=|C_2(0)|^2$$

Δηλ. το ηλεκτρόνιο βρίσκεται εξ ίσου στις δύο στάθμες τη χρονική στιγμή 0.

ΛΥΣΗ

Είχαμε βρεί για $\Delta=0$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} \frac{c_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} + \frac{c_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \\ \frac{c_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} - \frac{c_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \end{bmatrix}$$

με αρχική συνθήκη $C_1(0)=\frac{1}{\sqrt{2}}=C_2(0) \Rightarrow$

$$\frac{1}{\sqrt{2}}=\frac{c_1}{\sqrt{2}}+\frac{c_2}{\sqrt{2}} \Rightarrow 1=c_1+c_2$$

$$\frac{1}{\sqrt{2}}=\frac{c_1}{\sqrt{2}}-\frac{c_2}{\sqrt{2}} \Rightarrow 1=c_1-c_2$$

$$2=2c_1 \Rightarrow c_1=1$$

$$c_2=0$$

Άρα

$$C_1(t)=\frac{1}{\sqrt{2}}e^{i\frac{\Omega_R}{2}t} \Rightarrow |C_1(t)|^2=\frac{1}{2}=\text{σταθερό}$$

$$C_2(t)=\frac{1}{\sqrt{2}}e^{i\frac{\Omega_R}{2}t} \Rightarrow |C_2(t)|^2=\frac{1}{2}=\text{σταθερό}$$

Δηλαδή δεν υπάρχει ταλάντωση φορτίου...

H

ΑΣΚΗΣΗ

Να λυθεί το πρόβλημα $\Delta=0$ με αρχική συνθήκη $C_1(0)=0$, $C_2(0)=1$

δηλ. το ηλεκτρόνιο βρίσκεται αρχικώς στην ΑΝΩ ΣΤΑΘΜΗ

ΛΥΣΗ

Είχαμε βρεί για $\Delta=0$

$$C_1(t) = \frac{c_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} + \frac{c_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t}$$

$$C_2(t) = \frac{c_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} - \frac{c_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t}$$

με αρχική συνθήκη $C_1(0)=0$, $C_2(0)=1 \Rightarrow$

$$0 = \frac{c_1+c_2}{\sqrt{2}} \Rightarrow c_2 = -c_1 := -c$$

$$1 = \frac{c_1-c_2}{\sqrt{2}} \Rightarrow \sqrt{2} = c+c \Rightarrow 2c=\sqrt{2} \Rightarrow c=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$

Άρα $C_1(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} - \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} = i\sin\left(\frac{\Omega_R}{2}t\right)$

$$C_2(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} + \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} = \frac{1}{2}2\cos\left(\frac{\Omega_R}{2}t\right) = \cos\left(\frac{\Omega_R}{2}t\right)$$

$$\hat{H} = \hat{H}_0 + U_\mathcal{E}(\vec{r},t)$$

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = \hat{H}\,\Psi(\vec{r},t)$$

αρχ. συν. $\Psi(\vec{r},0) = \Phi(\vec{r})$ = γνωστή

$$\Phi(\vec{r}) = \sum_k f_k \Phi_k(\vec{r})$$

$$\Psi(\vec{r},t) = \sum_k C_k e^{-i\Omega_k t} \Phi_k(\vec{r})$$

$$\hat{H}_0 \Phi_k(\vec{r}) = E_k \Phi_k(\vec{r})$$

$$E_k = \hbar \Omega_k$$

$$\Downarrow$$

$$\dot{C}_{k'}(t) = -\frac{i}{\hbar}\sum_k C_k(t)\, e^{i(\Omega_{k'}-\Omega_k)t}\, U_{\mathcal{E}k'k}(t)$$

όπου $U_{\mathcal{E}k'k}(t) = \int d^3r\, \Phi_{k'}^*(\vec{r})\, U_\mathcal{E}(\vec{r},t)\, \Phi_k(\vec{r}) = \langle \Phi_{k'} | U_\mathcal{E}(\vec{r},t) | \Phi_k \rangle$

$$U_{\mathcal{E}k'k}(t) = \begin{cases} \wp \mathcal{E}_0 \cos\omega t, & k \neq k' \\ 0, & k = k' \end{cases}$$

χρονικώς εξαρτώμενοι συντελεστές

$\wp = \wp_{z12} = -e z_{12} = -e z_{21} = \wp_{z21}$ για $\Phi_k(\vec{r})$ πραγματικές

$\vec{\mathcal{E}}(t) = \mathcal{E}_0 \hat{z} \cos\omega t$ μετά από προσέγγιση δίπολου $|\vec{r}| \sim a_0 << \lambda$ για οπτικό μήκος κύματος

συχνοτική απόσταση ΗΜ πεδίου − ΔΣ

$$\Delta := \omega - \Omega$$

detuning

αποσυντονισμός

@ΔΣ

$$\Omega := \Omega_2 - \Omega_1 = \frac{E_2 - E_1}{\hbar}$$

RWA

γρήγοροι διώξαμε όρους $e^{\pm i(\omega+\Omega)t}$

αργοί κρατήσαμε όρους $e^{\pm i(\omega-\Omega)t}$

$\Omega_R = \frac{\wp \mathcal{E}_0}{\hbar}$ συχνότητα Rabi

ισχύς διαταραχής

$$\Downarrow$$

$$\dot{C}_1(t) = \frac{i}{2}\Omega_R e^{i\Delta t} C_2(t)$$

$$\dot{C}_2(t) = \frac{i}{2}\Omega_R e^{-i\Delta t} C_1(t)$$

(M) $\begin{cases} C_1(t) = \mathcal{C}_1(t)\, e^{i\frac{\Delta}{2}t} \\ C_2(t) = \mathcal{C}_2(t)\, e^{-i\frac{\Delta}{2}t} \end{cases} \rightarrow \begin{bmatrix} \dot{\mathcal{C}}_1(t) \\ \dot{\mathcal{C}}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{i\Delta}{2} & \frac{i\Omega_R}{2} \\ \frac{i\Omega_R}{2} & \frac{i\Delta}{2} \end{bmatrix} \begin{bmatrix} \mathcal{C}_1(t) \\ \mathcal{C}_2(t) \end{bmatrix}$

χρονικώς ανεξάρτητοι συντελεστές

$$\dot{\vec{x}}(t) = \tilde{A}\, \vec{x}(t)$$

ΔΛΜ $\vec{x}(t) = \vec{u}\, e^{\tilde{\lambda} t}$

$$\tilde{A}\vec{u} = \tilde{\lambda}\vec{u}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & -\frac{\Omega_R}{2} \\ -\frac{\Omega_R}{2} & -\frac{\Delta}{2} \end{bmatrix}$$

ή $A\vec{u} = \lambda\vec{u}$ $\quad \tilde{A} = -iA$, $\tilde{\lambda} = -i\lambda$

γενική λύση $\vec{x}(t) = \sum_k c_k \vec{u}_k e^{-i\lambda_k t}$

$A\vec{u} = \lambda \vec{u}$

$$\begin{bmatrix} \frac{\Delta}{2} & -\frac{\Omega_R}{2} \\ -\frac{\Omega_R}{2} & -\frac{\Delta}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \lambda \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \frac{\Delta}{2} - \lambda & -\frac{\Omega_R}{2} \\ -\frac{\Omega_R}{2} & -\frac{\Delta}{2} - \lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow \lambda_{2,1} = \pm \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}$$

$\lambda_{2,1} = \pm \frac{\Omega_R}{2}$ στην περίπτωση συντονισμού $(\Delta = 0)$

Θα χρησιμοποιήσουμε αρχικές συνθήκες $C_1(0) = 1, \; C_2(0) = 0 \; \left(\frac{t=0}{\bullet}\right)$

$$\Downarrow$$

$$C_1(0) = 1, \; C_2(0) = 0$$

ΛΥΣΗ για $\Delta = 0$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \lambda_1 = -\frac{\Omega_R}{2}$$

$$\vec{V}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \lambda_2 = \frac{\Omega_R}{2}$$

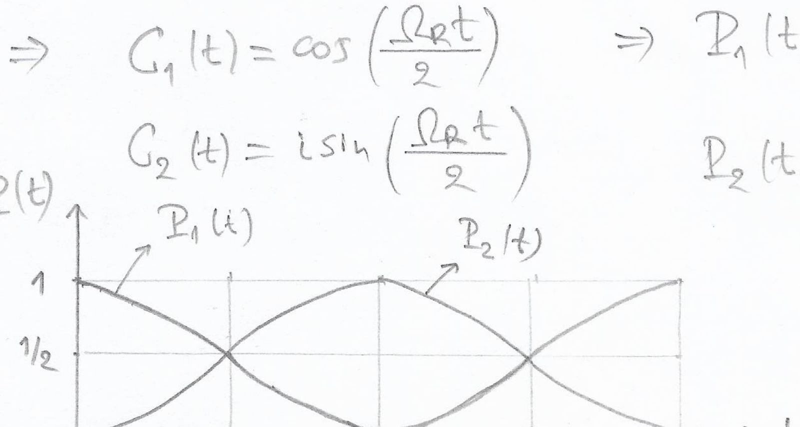
$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sum_{k=1}^{2} c_k \vec{v}_k e^{-i\lambda_k t} = \frac{c_1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{+i\frac{\Omega_R}{2}t} + \frac{c_2}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-i\frac{\Omega_R}{2}t}$$

αρχ. συν.

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{c_1 + c_2}{\sqrt{2}} \\ \frac{c_1 - c_2}{\sqrt{2}} \end{bmatrix} \Rightarrow c_1 = c_2 = \frac{\sqrt{2}}{2}$$

$$\Rightarrow C_1(t) = \cos\left(\frac{\Omega_R t}{2}\right) \qquad \Rightarrow P_1(t) = \cos^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} + \frac{1}{2}\cos(\Omega_R t)$$

$$C_2(t) = i\sin\left(\frac{\Omega_R t}{2}\right) \qquad P_2(t) = \sin^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t)$$



$T = \frac{2\pi}{\Omega_R}$ περίοδος

$A = 1$ πλάτος

● ΛΥΣΗ για $\Delta \neq 0$

$$\vec{V}_1 = \begin{bmatrix} \frac{1}{\sqrt{1+a^2}} \\ \frac{a}{\sqrt{1+a^2}} \end{bmatrix} \qquad \lambda_1 = -\frac{\sqrt{\Omega_R^2+\Delta^2}}{2} := -\lambda < 0$$

$$a = \frac{\frac{\Delta}{2} + \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}}{\frac{\Omega_R}{2}}$$

$$\vec{V}_2 = \begin{bmatrix} \frac{1}{\sqrt{1+a'^2}} \\ \frac{a'}{\sqrt{1+a'^2}} \end{bmatrix} \qquad \lambda_2 = +\frac{\sqrt{\Omega_R^2+\Delta^2}}{2} := \lambda > 0$$

$$a' = \frac{\frac{\Delta}{2} - \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}}{\frac{\Omega_R}{2}}$$

© ΒΙΒΛΙΟ

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2 \sin x \cos x$

$\cos^2 x + \sin^2 x = 1$

$\cos 2x = 2\cos^2 x - 1$

$$P_1(t) = |C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \sin^2(\lambda t)$$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\sin^2 x = 1 - \cos^2 x$

$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \sin^2(\lambda t)$$

$\sin^2 x = 1 - \frac{\cos 2x + 1}{2}$

$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$

$$P_1(t) = 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\left(\frac{1}{2} - \frac{\cos(2\lambda t)}{2}\right) = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2+\Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \cos(2\lambda t)$$

$$P_2(t) = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\left(\frac{1}{2} - \frac{\cos(2\lambda t)}{2}\right) = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \cdot \cos(2\lambda t)$$

ΝΑ ΔΙΟΡΘΩΘΕΙ ~~πλάτος~~ Α
ΣΤΟ ΒΙΒΛΙΟ
SOS

$$A = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}$$

μέγιστο ποσοστό μεταφοράς
maximum transfer percentage

για $\Delta = 0 \Rightarrow A = 1$

περίοδος

$$T = \frac{2\pi}{2\lambda} = \frac{2\pi}{2\frac{\sqrt{\Omega_R^2+\Delta^2}}{2}} = \frac{2\pi}{\sqrt{\Omega_R^2+\Delta^2}}$$

$T = \frac{2\pi}{\Omega_R}$

όπως αναμενόταν

maximum transfer rate

$$:= \frac{\mathcal{A}}{T} = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\,\frac{\sqrt{\Omega_R^2+\Delta^2}}{2\pi} = \frac{\Omega_R^2}{2\pi\sqrt{\Omega_R^2+\Delta^2}}$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\sin^2(\lambda t) = $$

$$= \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\cdot\cos(2\lambda t) \Rightarrow$$

$$\langle |C_2(t)|^2\rangle = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}$$

ορισμός

t_{2mean} —

ο απαιτούμενος χρόνος

για να πιάσει τη μέση τιμή

$\downarrow$

$$\frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\cdot\cos(2\lambda t_{2mean})$$

$$\Rightarrow \cos(2\lambda t_{2mean}) = 0 \Rightarrow 2\lambda t_{2mean} = \frac{\pi}{2}$$

$$\Rightarrow t_{2mean} = \frac{\pi}{4\lambda} = \frac{2\pi}{4\sqrt{\Omega_R^2+\Delta^2}}$$

mean transfer rate

$$k := \frac{\langle |C_2(t)|^2\rangle}{t_{2mean}} = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\,\frac{4\sqrt{\Omega_R^2+\Delta^2}}{2\pi} = \frac{\Omega_R^2}{\sqrt{\Omega_R^2+\Delta^2}\cdot\pi}$$

$$\frac{k}{\frac{\mathcal{A}}{T}} = 2 \Rightarrow k = 2\,\frac{\mathcal{A}}{T}$$

$$P_1(t) = |C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\sin^2(\lambda t) \qquad \lambda = \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}$$

ίδιο για 3'

$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\sin^2(\lambda t)$$

$$\sin^2(\lambda t) = \frac{1}{2} - \frac{\cos(2\lambda t)}{2} \qquad A = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}$$

$$\langle |C_1(t)|^2 \rangle = 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\,\frac{1}{2} \qquad T = \frac{2\pi}{2\lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Delta^2}}$$

$$\langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\cdot\frac{1}{2}$$

$$\langle |C_1(t)|^2 \rangle + \langle |C_2(t)|^2 \rangle = 1$$

αν $\Delta = 0 \Rightarrow \langle |C_1(t)|^2 \rangle = \frac{1}{2} = \langle |C_2(t)|^2 \rangle$

αλλά για $\Delta \neq 0$ οι πιθανότητες δεν είναι ίσες

π.χ. με $\Delta = \sqrt{3}\,\Omega_R$

$$\langle |C_1(t)|^2 \rangle = 1 - \frac{\cancel{\Omega_R^2}}{4\cancel{\Omega_R^2}}\cdot\frac{1}{2} = \frac{7}{8}$$

$$\langle |C_2(t)|^2 \rangle = \frac{\cancel{\Omega_R^2}}{4\cancel{\Omega_R^2}}\,\frac{1}{2} = \frac{1}{8}$$

pure maximum transfer rate $:= \frac{A}{T} = \frac{\Omega_R^2}{(\Omega_R^2+\Delta^2)}\,\frac{\sqrt{\Omega_R^2+\Delta^2}}{2\pi} = \frac{\Omega_R^2}{2\pi}\cdot\frac{1}{\sqrt{\Omega_R^2+\Delta^2}}$

t_{2mean} $\overset{\text{ορισμός}}{\Rightarrow}$ $\cancel{\frac{\Omega_R^2}{\Omega_R^2+\Delta^2}}\,\frac{1}{2} = \cancel{\frac{\Omega_R^2}{\Omega_R^2+\Delta^2}}\cdot\sin^2(\lambda t_{2mean}) \Rightarrow \sin^2(\lambda t_{2mean}) = \frac{1}{2}$

$$\sin(\lambda t_{2mean}) = \pm\frac{\sqrt{2}}{2} = \pm\sin\left(\frac{\pi}{4}\right)$$

$$\lambda t_{2mean} = \frac{\pi}{4} \Rightarrow \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}\cdot t_{2mean} = \frac{\pi}{4} \Rightarrow t_{2mean} = \frac{\pi}{2}\,\frac{1}{\sqrt{\Omega_R^2+\Delta^2}}$$

pure mean transfer rate

ίδιο με 3'

$$k := \frac{\langle |c_2(t)|^2 \rangle}{t_{2mean}} = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \frac{1}{\cancel{2}} \frac{\cancel{2}\sqrt{\Omega_R^2 + \Delta^2}}{\pi} = \frac{\Omega_R^2}{\pi} \frac{1}{\sqrt{\Omega_R^2 + \Delta^2}}$$

$$k = 2 \frac{A}{T}$$