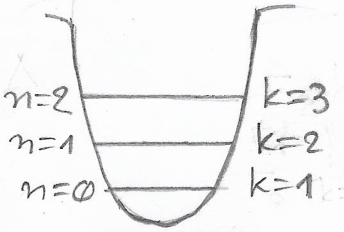
ΤΡΙΣΤΑΘΜΙΚΟ ΜΕ ΑΑΤ

$$\boxed{\dot{C}_{k'} = -\frac{i}{\hbar}\sum_k C_k(t)\, e^{i(\Omega_{k'}-\Omega_k)t}\, U_{\varepsilon k'k}(t)}$$

①



ΑΑΤ $\boxed{E_n = \hbar\Omega\left(n+\frac{1}{2}\right)}$ $\quad E_{n+1}-E_n = \hbar\Omega$

$$E_0 = \frac{\hbar\Omega}{2}, \quad E_1 = \frac{3\hbar\Omega}{2} \quad E_2 = \frac{5\hbar\Omega}{2}$$

$\boxed{k = n+1}$

$$\boxed{\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2}\hat{z}^2}$$

✱ ΔΕΙΤΕ (1') $\quad \overrightarrow{3\Delta}$

Υποτίθεται πώς

$$\Phi_n(\vec{r}) = X_1(x)\,Y_1(y)\,Z_n(z)$$

Ας θεωρήσουμε ότι μπορούμε να περιοριστούμε στις 3 κατώτερες στάθμες του ΑΑΤ

$$U_{\varepsilon k'k}(t) = e\mathcal{E}_0 \cos\omega t\; z_{k'k}$$

ιδιοσυναρτήσεις 1Δ ΑΑΤ $\quad a = \left(\frac{\hbar}{m\Omega}\right)^{1/2}$

$$Z_n(z) = u_n(z)\exp\left(-\frac{m\Omega z^2}{2\hbar}\right)$$

k		n	$u_n(z)$
1	Α	0	$(1/a\sqrt{\pi})^{1/2}$
2	Π	1	$(1/a\sqrt{\pi})^{1/2}\, 2\frac{z}{a}$
3	Α	2	$(1/8a\sqrt{\pi})^{1/2}\left[2-4\left(\frac{z}{a}\right)^2\right]$
4	Π	3	$(1/48a\sqrt{\pi})^{1/2}\left[12\left(\frac{z}{a}\right)-8\left(\frac{z}{a}\right)^3\right]$
		n	$(1/n!\,2^n a\sqrt{\pi})^{1/2} H_n\left(\frac{z}{a}\right)$

πολυώνυμα Hermite

$$z_{k'k} = \int d^3r\, \Phi^*_{k'}(\vec{r})\, z\, \Phi_k(\vec{r})$$

$$z_{kk} = \int d^3r\, \underbrace{\underbrace{|\Phi_k(\vec{r})|^2}_{A}\; \underbrace{z}_{\Pi}}_{\Pi} = 0$$

$$z_{12} = \int d^3r\, \underset{A}{\Phi^*_1(\vec{r})}\; \underset{\Pi}{z}\; \underset{\Pi}{\Phi_2(\vec{r})} \neq 0$$

$\|$

$$z_{21} = \int d^3r\, \Phi^*_2(\vec{r})\, z\, \Phi_1(\vec{r}) \neq 0$$

$$z_{13} = \int d^3r\, \underbrace{\underset{A}{\Phi^*_1(\vec{r})}\; \underset{\Pi}{z}\; \underset{A}{\Phi_3(\vec{r})}}_{\Pi} = 0$$

$\|$

$$z_{31} = \ldots = 0$$

μάλλον...

$$z_{23} = \int d^3r\, \underbrace{\underset{\Pi}{\Phi^*_2(\vec{r})}\; \underset{\Pi}{z}\; \underset{A}{\Phi_3(\vec{r})}}_{A} \neq 0$$

$\|$

$$z_{32} = z_{23}$$

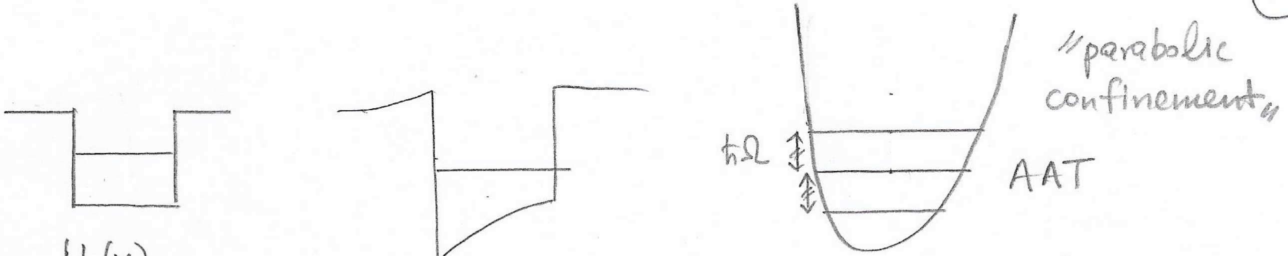
$$z_{12} = z_{21} \neq z_{23} = z_{32}$$

$$\boxed{\begin{aligned} -e z_{12} &= -e z_{21} \\ \wp_{z12} &= \wp_{z21} := \wp \end{aligned}}$$

$$U_{\varepsilon 12}(t) = e\mathcal{E}_0\cos\omega t\; z_{12} = -\wp_{z12}\cdot \mathcal{E}_0\cos\omega t$$

$$U_{\varepsilon 21}(t) = e\mathcal{E}_0\cos\omega t\; z_{21} = -\wp_{z21}\, \mathcal{E}_0\cos\omega t$$

π.χ



$U_1(x)$ $U_2(y)$ $U_3(z)=\frac{m\Omega^2}{2}z^2$

$$U(\vec{r})=U_1(x)+U_2(y)+U_3(z)$$

$U_1(x)$	$U_2(y)$	$U_3(z)$
χωρέ 1 μόνο στάθμη	χωρέ 1 μόνο στάθμη	ΑΑΤ …

$$\Phi_n(\vec{r})=X_1(x)Y_1(y)Z_n(z)$$

ΑΣΚΗΣΗ Να υπολογιστεί ο λόγος $\frac{P}{P'}=\frac{\Omega_R}{\Omega_R'}$ σε αυτό το σύστημα.

$$\boxed{\begin{aligned}\wp_{\Sigma 23} &= \wp_{\Sigma 32} := \wp' \\ &= -eZ_{23} = -eZ_{32}\end{aligned}} \longrightarrow \quad \underline{\text{ὁμοίως:..}}$$

$$\dot{C}_1(t) = -\frac{i}{\hbar} C_1(t)\, e^{i(\Omega_1-\Omega_1)t}\, \cancel{U_{\Sigma 11}(t)}^{\,0} - \frac{i}{\hbar} C_2(t)\, e^{i(\Omega_1-\Omega_2)t}\, U_{\Sigma 12}(t)$$

$$-\frac{i}{\hbar} C_3(t)\, e^{i(\Omega_1-\Omega_3)t}\, \cancel{U_{\Sigma 13}(t)}^{\,0}$$

$$\boxed{\dot{C}_1(t) = +\frac{i}{\hbar} C_2(t)\, e^{-i\Omega t}\, \wp \mathcal{E}_0 \cos\omega t} \quad (1)$$

$$\dot{C}_2(t) = -\frac{i}{\hbar} C_1(t)\, e^{i(\Omega_2-\Omega_1)t}\, U_{\Sigma 21}(t) - \frac{i}{\hbar} C_2(t)\, e^{i(\Omega_2-\Omega_2)t}\, \cancel{U_{\Sigma 22}(t)}^{\,0}$$

$$-\frac{i}{\hbar} C_3(t)\, e^{i(\Omega_2-\Omega_3)t}\, U_{\Sigma 23}(t)$$

$$\boxed{\dot{C}_2(t) = +\frac{i}{\hbar} C_1(t)\, e^{i\Omega t}\, \wp \mathcal{E}_0 \cos\omega t + \frac{i}{\hbar} C_3(t)\, e^{-i\Omega t}\, \wp' \mathcal{E}_0 \cos\omega t} \quad (2)$$

$$\dot{C}_3(t) = -\frac{i}{\hbar} C_1(t)\, e^{i(\Omega_3-\Omega_1)t}\, \cancel{U_{\Sigma 31}(t)}^{\,0} - \frac{i}{\hbar} C_2(t)\, e^{i(\Omega_3-\Omega_2)t}\, U_{\Sigma 32}(t)$$

$$-\frac{i}{\hbar} C_3(t)\, e^{i(\Omega_3-\Omega_3)t}\, \cancel{U_{\Sigma 33}(t)}^{\,0}$$

$$\boxed{\dot{C}_3(t) = +\frac{i}{\hbar} C_2(t)\, e^{i\Omega t}\, \wp' \mathcal{E}_0 \cos\omega t} \quad (3)$$

μετασχηματισμός (M)
$$\left\{\begin{aligned} C_1(t) &= \mathcal{C}_1(t)\, e^{\frac{i\Delta t}{2}} \\ C_2(t) &= \mathcal{C}_2(t)\, e^{-\frac{i\Delta t}{2}} \\ C_3(t) &= \mathcal{C}_3(t)\, e^{-\frac{3i\Delta t}{2}} \end{aligned}\right.$$

$$\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

ώστε να κάνουμε RWA

③

①⇒ $\dot{C}_1(t) = \frac{i}{\hbar}\wp\mathcal{E}_0\, C_2(t)\, e^{-i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2}$

$\boxed{\Delta := \omega - \Omega \qquad \Omega_R = \frac{\wp\mathcal{E}_0}{\hbar} > 0 \text{ για } \wp > 0 \ldots}$

$\boxed{\text{RWA}}$

$\dot{C}_1(t) = \frac{i}{2\hbar}\wp\mathcal{E}_0\, C_2(t)\, e^{i(\omega-\Omega)t} \Rightarrow \boxed{\dot{C}_1(t) = \frac{i}{2}\Omega_R\, C_2(t)\, e^{i\Delta t}}$ (1')

②⇒ $\dot{C}_2(t) = \frac{i}{\hbar}\wp\mathcal{E}_0\, C_1(t)\, e^{i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2} + \frac{i}{\hbar}\wp'\mathcal{E}_0\, C_3(t)\, e^{i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2}$

για $\wp<0$ $\quad \Omega_R = \frac{-\wp\mathcal{E}_0}{\hbar}$ μπλε $\quad \Omega_R' = \frac{-\wp'\mathcal{E}_0}{\hbar}$ κλπ...

$\boxed{\text{RWA}}$

$\dot{C}_2(t) = \frac{i\wp\mathcal{E}_0}{2\hbar}\, C_1(t)\, e^{-i(\omega-\Omega)t} + \frac{i}{2\hbar}\wp'\mathcal{E}_0\, C_3(t)\, e^{i(\omega-\Omega)t}$

$\boxed{\Omega_R' = \frac{\wp'\mathcal{E}_0}{\hbar} > 0}$ για $\wp>0$ ...

$\boxed{\dot{C}_2(t) = \frac{i}{2}\Omega_R\, C_1(t)\, e^{-i\Delta t} + \frac{i}{2}\Omega_R'\, C_3(t)\, e^{i\Delta t}}$ (2')

Η 2 είναι δ διαμεσολαβητής.

αν ορίζουμε τις Ω_R, Ω_R' θετικές

③⇒ $\dot{C}_3(t) = \frac{i\wp'\mathcal{E}_0}{\hbar}\, C_2(t)\, e^{i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2}$

$\boxed{\text{RWA}}$

$\boxed{\dot{C}_3(t) = \frac{i\wp'\mathcal{E}_0}{2\hbar}\, C_2(t)\, e^{-i\Delta t}}$ (3')

(1') (2') (3')

οι παράγωγοι των συναρτήσεων σχετίζονται με τις συναρτήσεις με χρονικώς εξαρτώμενους συντελεστές

$$\dot{c}_1(t) = \dot{C}_1(t)\, e^{\frac{i\Delta t}{2}} + C_1(t)\,\frac{i\Delta}{2}\, e^{\frac{i\Delta t}{2}}$$

$$\dot{c}_2(t) = \dot{C}_2(t)\, e^{-\frac{i\Delta t}{2}} + C_2(t)\left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}}$$

$$\dot{c}_3(t) = \dot{C}_3(t)\, e^{-\frac{3 i\Delta t}{2}} + C_3(t)\left(\frac{-3 i\Delta}{2}\right) e^{-\frac{3 i\Delta t}{2}}$$

$\Longleftarrow$ (M)

(4)

(1')(M)

$$\dot{C}_1(t)\, \cancel{e^{\frac{i\Delta t}{2}}} + C_1(t)\,\frac{i\Delta}{2}\, \cancel{e^{\frac{i\Delta t}{2}}} = \frac{i}{2}\Omega_R\, C_2(t)\, \cancel{e^{-\frac{i\Delta t}{2}}}\, \cancel{e^{i\Delta t}}$$

$$\boxed{\dot{C}_1(t) = -i\frac{\Delta}{2}\, C_1(t) + i\frac{\Omega_R}{2}\, C_2(t)}$$ (1'')

(2')(M)

$$\dot{C}_2(t)\, \cancel{e^{-\frac{i\Delta t}{2}}} + C_2(t)\left(-\frac{i\Delta}{2}\right) \cancel{e^{-\frac{i\Delta t}{2}}} = i\frac{\Omega_R}{2}\, C_1(t)\, \cancel{e^{\frac{i\Delta t}{2}}}\, \cancel{e^{-i\Delta t}} + \frac{i}{2}\Omega_R'\, C_3(t)\, \cancel{e^{-\frac{3 i\Delta t}{2}}}\, \cancel{e^{i\Delta t}}$$

$$\boxed{\dot{C}_2(t) = i\frac{\Omega_R}{2}\, C_1(t) + i\frac{\Delta}{2}\, C_2(t) + i\frac{\Omega_R'}{2}\, C_3(t)}$$ (2'')

(3')(M)

$$\dot{C}_3(t)\, \cancel{e^{-\frac{3 i\Delta t}{2}}} + C_3(t)\left(-\frac{3 i\Delta}{2}\right) \cancel{e^{-\frac{3 i\Delta t}{2}}} = \frac{i}{2}\Omega_R'\, C_2(t)\, \cancel{e^{-\frac{i\Delta t}{2}}}\, \cancel{e^{-i\Delta t}}$$

$$\boxed{\dot{C}_3(t) = i\frac{\Omega_R'}{2}\, C_2(t) + i\frac{3\Delta}{2}\, C_3(t)}$$ (3'')

(1'') (2'') (3'')

οι παράγωγοι των συναρτήσεων
σχετίζονται με τις συναρτήσεις
με χρονικώς ανεξάρτητους συντελεστές

$$\underbrace{\begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \\ \dot{C}_3(t) \end{bmatrix}}_{\dot{\vec{x}}(t)} = \underbrace{\begin{bmatrix} -i\frac{\Delta}{2} & -\frac{i\Omega_R}{2} & 0 \\ -i\frac{\Omega_R}{2} & i\frac{\Delta}{2} & -i\frac{\Omega_R'}{2} \\ 0 & -i\frac{\Omega_R'}{2} & i\frac{3\Delta}{2} \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}}_{\vec{x}(t)}$$

$$\boxed{\tilde{A} = -iA}$$

ΔΛΜ

$$\boxed{\vec{x}(t) = \vec{u}\, e^{\tilde{\lambda} t}} \qquad \dot{\vec{x}}(t) = \tilde{A}\,\vec{x}(t)$$

$$\vec{u}\,\tilde{\lambda}\,\cancel{e^{\tilde{\lambda} t}} = \tilde{A}\,\vec{u}\,\cancel{e^{\tilde{\lambda} t}}$$

$$\tilde{A}\,\vec{u} = \tilde{\lambda}\,\vec{u} \qquad \boxed{\tilde{\lambda} = -i\lambda}$$

$$\boxed{A\vec{u} = \lambda\vec{u}}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} \end{bmatrix} \qquad (A - \lambda I)\vec{u} = \vec{0}$$

$$\det \begin{bmatrix} \frac{\Delta}{2} - \lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} - \lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} - \lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ΛΥΣΗ για $\boxed{\Delta = 0}$

$$\begin{bmatrix} -\lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\lambda \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow -\lambda \begin{vmatrix} -\lambda & +\frac{\Omega_R'}{2} \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} \mp \frac{\Omega_R}{2} \begin{vmatrix} +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} = 0$$

$$-\lambda\left[\lambda^2 - \frac{\Omega_R'^2}{4}\right] + \frac{\Omega_R}{2}\frac{\Omega_R}{2}\lambda = 0$$

από εδώ η πέρα ίδιο...

$$-\lambda^3 + \lambda\frac{\Omega_R'^2}{4} + \lambda\frac{\Omega_R^2}{4} = 0 \Rightarrow \lambda\left[-\lambda^2 + \frac{\Omega_R'^2}{4} + \frac{\Omega_R^2}{4}\right] = 0$$

$\boxed{\lambda = 0}$ ή $\lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$

$$\boxed{\lambda = \pm\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}}$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} \qquad \lambda_2 = 0 \qquad \lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\Lambda < 0 \qquad \lambda_2 = 0 \qquad \lambda_3 = \Lambda > 0$$

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$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} = -\Lambda < 0$$

$$\begin{bmatrix} \Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & \Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & \Lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda u_1 - \frac{\Omega_R}{2} u_2 = 0 \Rightarrow \Lambda u_1 = \frac{\Omega_R}{2} u_2 \Rightarrow u_1 = \frac{\Omega_R}{2\Lambda} u_2$$

$$-\frac{\Omega_R}{2} u_1 + \Lambda u_2 - \frac{\Omega_R'}{2} u_3 = 0$$

$$-\frac{\Omega_R'}{2} u_2 + \Lambda u_3 = 0 \Rightarrow \Lambda u_3 = \frac{\Omega_R'}{2} u_2 \Rightarrow u_3 = \frac{\Omega_R'}{2\Lambda} u_2$$

$$-\frac{\Omega_R}{2}\frac{\Omega_R}{2\Lambda} u_2 + \Lambda u_2 - \frac{\Omega_R'}{2}\frac{\Omega_R'}{2\Lambda} u_2 = 0$$

$$\Rightarrow u_2 \left[-\frac{\Omega_R^2}{4\Lambda} + \Lambda - \frac{\Omega_R'^2}{4\Lambda} \right] = 0 \qquad \Lambda = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\Rightarrow u_2 \underbrace{\left[\frac{-\Omega_R^2 + 4\Lambda^2 - \Omega_R'^2}{4\Lambda} \right]}_{\text{μηδέν}} = 0 \qquad \Lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$$

αν $u_2 = 0$
$\left(\Rightarrow u_1 = u_2 = u_3 = 0 \right) \Rightarrow u_2$ ό,τι θέλουμε (μη μηδενικό)

π.χ. $u_2 = 1$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 = \beta \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} \qquad \vec{V}_1 \cdot \vec{V}_1 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_R'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + 4\Lambda^2 + \Omega_R'^2}{4\Lambda^2} = 1$$

$$|\beta|^2 \frac{2}{1} = 1 \Rightarrow \text{π.χ. } \beta = \frac{1}{\sqrt{2}}$$

• $\lambda_2 = 0$

$$\begin{bmatrix} 0 & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & 0 & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-\frac{\Omega_R}{2} u_2 = 0 \qquad u_2 = 0$

$-\frac{\Omega_R}{2} u_1 - \frac{\Omega_R'}{2} u_3 = 0 \qquad -\frac{\Omega_R}{2} u_1 = \frac{\Omega_R'}{2} u_3 \Rightarrow \boxed{u_3 = -\frac{\Omega_R}{\Omega_R'} u_1}$

$-\frac{\Omega_R'}{2} u_2 = 0 \qquad u_2 = 0$

$$\vec{V}_2 = \beta \begin{bmatrix} 1 \\ 0 \\ -\frac{\Omega_R}{\Omega_R'} \end{bmatrix} \qquad \vec{V}_2 \cdot \vec{V}_2 = 1 \Rightarrow |\beta|^2 \left(1 + \frac{\Omega_R^2}{\Omega_R'^2}\right) = 1 \Rightarrow |\beta|^2 \frac{\Omega_R'^2 + \Omega_R^2}{\Omega_R'^2} = 1$$

$$|\beta|^2 \frac{4\Lambda^2}{\Omega_R'^2} = 1 \Rightarrow \text{π.χ. } \beta = \frac{\Omega_R' \, 2}{2\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

• $\lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} = \Lambda > 0$

$$\begin{bmatrix} -\Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & -\Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & -\Lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \quad \vec{V}_2 = \begin{bmatrix} \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \\ 0 \\ -\frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \end{bmatrix} = \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix}$$

$-\Lambda u_1 - \frac{\Omega_R}{2} u_2 = 0 \Rightarrow -\frac{\Omega_R}{2} u_2 = \Lambda u_1 \Rightarrow u_1 = -\frac{\Omega_R}{2\Lambda} u_2$

$-\frac{\Omega_R}{2} u_1 - \Lambda u_2 - \frac{\Omega_R'}{2} u_3 = 0$

$-\frac{\Omega_R'}{2} u_2 - \Lambda u_3 = 0 \Rightarrow -\frac{\Omega_R'}{2} u_2 = \Lambda u_3 \Rightarrow u_3 = -\frac{\Omega_R'}{2\Lambda} u_2$

$$+\frac{\Omega_R}{2}\frac{\Omega_R}{2\Lambda} u_2 - \Lambda u_2 + \frac{\Omega_R'}{2}\frac{\Omega_R'}{2\Lambda} u_2 = 0 \Rightarrow u_2 \left[\frac{\Omega_R^2}{4\Lambda} - \frac{4\Lambda^2}{4\Lambda} + \frac{\Omega_R'^2}{4\Lambda}\right] = 0$$

$$u_2 \left[\underbrace{\frac{\Omega_R^2 + \Omega_R'^2 - 4\Lambda^2}{4\Lambda}}_{\text{μηδέν}}\right] = 0 \Rightarrow u_2 \text{ ό,τι θέλουμε (μη μηδενικό)}$$

π.χ. $u_2 = 1$

$$\vec{V}_3 = \beta \begin{bmatrix} -\frac{\Omega_R}{2\Lambda} \\ 1 \\ -\frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_3 \cdot \vec{V}_3 = 1 \Longrightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_R'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + \Omega_R'^2 + 4\Lambda^2}{4\Lambda^2} = 1 \Longrightarrow |\beta|^2 \cdot 2 = 1 \Longrightarrow$$

$$\pi.\chi. \quad \beta = -\frac{1}{\sqrt{2}}$$

$$\vec{V}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

γενική λύση

$$\vec{x}(t) = \sum_{k=1}^{3} \sigma_k \vec{v}_k e^{-i\lambda_k t}$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2}$$

$$4\Lambda^2 = (\Omega_R^2 + \Omega_R'^2)$$

$$16\Lambda^4 = (\Omega_R^2 + \Omega_R'^2)^2$$

$c_1(0)=1 \qquad c_2(0)=0 \qquad c_3(0)=0$ έστωσαν αρχ. συνθήκες

(10)

$$\vec{x}(t)=\begin{bmatrix} c_1(t)\,e^{-i\frac{\Delta t}{2}} \\ c_2(t)\,e^{i\frac{\Delta t}{2}} \\ c_3(t)\,e^{3i\frac{\Delta t}{2}} \end{bmatrix} = \frac{\sigma_1}{\sqrt{2}}\begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_1 t} + \sigma_2 \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix} e^{-i\lambda_2 t} + \frac{\sigma_3}{\sqrt{2}}\begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_3 t}$$

$$1 = \frac{\sigma_1}{\sqrt{2}}\frac{\Omega_R}{2\Lambda} + \sigma_2\frac{\Omega_R'}{2\Lambda} + \frac{\sigma_3}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}$$

$$0 = \frac{\sigma_1}{\sqrt{2}} - \frac{\sigma_3}{\sqrt{2}} \Longrightarrow \boxed{\sigma_1=\sigma_3=\sigma}$$

$$0 = \frac{\sigma_1}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda} - \sigma_2\frac{\Omega_R}{2\Lambda} + \frac{\sigma_3}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}$$

$$0 = \frac{\sigma}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda} - \frac{\sigma_2\Omega_R}{2\Lambda} + \frac{\sigma}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda} \Longrightarrow 0 = \frac{\sigma\Omega_R'}{\sqrt{2}\Lambda} - \frac{\sigma_2\Omega_R}{2\Lambda} \Longrightarrow$$

$$\frac{\sigma_2\Omega_R}{2\Lambda} = \frac{\sigma\Omega_R'}{\sqrt{2}\Lambda} \Longrightarrow \boxed{\sigma_2 = \sigma\sqrt{2}\frac{\Omega_R'}{\Omega_R}}$$

$$1 = \frac{\sigma}{\sqrt{2}}\frac{\Omega_R}{2\Lambda} + \sigma\sqrt{2}\frac{\Omega_R'}{\Omega_R}\frac{\Omega_R'}{2\Lambda} + \frac{\sigma}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}$$

$$2\Lambda = \sigma\left(\frac{\Omega_R}{\sqrt{2}} + \sqrt{2}\frac{\Omega_R'^2}{\Omega_R} + \frac{\Omega_R}{\sqrt{2}}\right) = \sigma\frac{\Omega_R^2 + 2\Omega_R'^2 + \Omega_R^2}{\sqrt{2}\,\Omega_R} = \sigma\frac{2}{\sqrt{2}}\frac{\Omega_R^2+\Omega_R'^2}{\Omega_R}$$

$$\sigma = \frac{\sqrt{\Omega_R^2+\Omega_R'^2}\,\sqrt{2}\,\Omega_R}{2(\Omega_R^2+\Omega_R'^2)} \Longrightarrow \boxed{\sigma = \frac{\Omega_R}{\sqrt{2}\sqrt{\Omega_R^2+\Omega_R'^2}} = \frac{\Omega_R}{\sqrt{2}\,2\Lambda}}$$

$$\sigma = \frac{\Omega_R}{\sqrt{2}\,2\Lambda}$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R}{2\underbrace{\sqrt{\Omega_R^2+\Omega_R'^2}}_{2\Lambda}} \cdot \frac{\Omega_R \cancel{2}}{\cancel{2}\underbrace{\sqrt{\Omega_R^2+\Omega_R'^2}}_{2\Lambda}}\, e^{+i\Lambda t}$$

$$+ \frac{\cancel{\Omega_R}}{\sqrt{2}\underbrace{\sqrt{}}_{2\Lambda}} \cdot \frac{\sqrt{2}\,\Omega_R'}{\cancel{\Omega_R}} \cdot \frac{\Omega_R'}{\underbrace{\sqrt{}}_{2\Lambda}}\, e^{-i0}$$

$$+ \frac{\Omega_R}{\sqrt{2}\underbrace{\sqrt{}}_{2\Lambda}} \cdot \frac{\Omega_R \cancel{2}}{\cancel{2}\underbrace{\sqrt{}}_{2\Lambda}}\, e^{-i\Lambda t}$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2\underbrace{(\Omega_R^2+\Omega_R'^2)}_{4\Lambda^2}} \cdot e^{i\Lambda t} + \frac{\Omega_R'^2}{\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2}} + \frac{\Omega_R^2}{2\underbrace{(\Omega_R^2+\Omega_R'^2)}_{4\Lambda^2}} \cdot e^{-i\Lambda t} \leftarrow$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\cancel{2}\,\Omega_R^2}{\cancel{2}\underbrace{(\Omega_R^2+\Omega_R'^2)}_{4\Lambda^2}} \cos\Lambda t + \frac{\Omega_R'^2}{\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2}} \leftarrow$$

$$C_2(t)\, e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{}}\, 1 \cdot e^{+i\Lambda t} - \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{}} \cdot e^{-i\Lambda t}$$

$$= \frac{\Omega_R}{2\sqrt{}} \left(e^{+i\Lambda t} - e^{-i\Lambda t} \right)$$

$\cos + i\sin$

$- \cos + i\sin$

$$C_2(t)\, e^{i\frac{\Delta t}{2}} = \frac{\Omega_R}{\cancel{2}\sqrt{}}\, \cancel{2}\, i \sin\Lambda t$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\sqrt{\Omega_R^2+\Omega_R'^2}} \sin^2(\Lambda t) = \frac{\Omega_R^2}{\Omega_R^2+\Omega_R'^2} \cdot \left(\frac{1}{2} - \frac{\cos(2\Lambda t)}{2} \right)$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$|c_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2} \cdot \left(\frac{1}{2} - \frac{\cos(2\Lambda t)}{2} \right)$$

$$= \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_R'^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_R'^2)} \cdot \cos(2\Lambda t)$$

$$A_2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2}$$

maximum transfer percentage

μέγιστο ποσοστό μεταφοράς

$$T_2 = \frac{2\pi}{2\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

για $\Omega_R = \Omega_R'$ $\Rightarrow$ $A_2 = \frac{1}{2}$

$$T_2 = \frac{2\pi}{\sqrt{2}\,|\Omega_R|} = \frac{1}{\sqrt{2}} \frac{2\pi}{|\Omega_R|}$$

Περίοδος αντίστοιχου δισταθμικού

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2}\,\Omega_R$$

$$|c_2(t)|^2 = \frac{1}{2}\left(\frac{1}{2} - \frac{\cos(\sqrt{2}\,\Omega_R t)}{2} \right)$$

$$|c_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cdot \cos(\underbrace{\sqrt{2}\,\Omega_R}_{} t)$$

$$2\omega_1 = \omega_2$$

$$|c_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\omega_2 t)$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2\cdot 4\Lambda^2} e^{+i\Lambda t} + \frac{\Omega_R'^2}{4\Lambda^2} + \frac{\Omega_R^2}{2\cdot 4\Lambda^2} e^{-i\Lambda t}$$ (13)

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{4\Lambda^2}\cos(\Lambda t) + \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + 2\,\frac{\Omega_R^2}{4\Lambda^2}\cdot\cos(\Lambda t)\cdot\frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos(2\Lambda t)+1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \qquad T_B = \frac{2\pi}{\Lambda} \qquad \frac{T_B}{T_A} = 2 \Rightarrow$$ περιοδική κίνηση

με περίοδο $$\boxed{T_1 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Omega_R'^2}}\cdot 2 = 2T_2}$$

$$\left|C_1\left(\tfrac{2\pi}{\Lambda}\right)\right|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos\left(2\Lambda\frac{2\pi}{\Lambda}\right)+1}{2} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos\left(\Lambda\frac{2\pi}{\Lambda}\right) =$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2+\Omega_R'^2)^2}{16\Lambda^4} = 1$$

$$\left|C_1\left(\tfrac{2\pi}{2\Lambda}\right)\right|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos\left(2\Lambda\frac{2\pi}{2\Lambda}\right)+1}{2} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos\left(\Lambda\frac{2\pi}{2\Lambda}\right)$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$\frac{d}{dt}|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot 2\cos(\Lambda t)\cdot(-\Lambda)\sin(\Lambda t) + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}(-\Lambda)\sin(\Lambda t)$$

$$\frac{d}{dt}|C_1(t)|^2 = \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\cdot\sin(\Lambda t)\cdot\left[\Omega_R^2\cdot\cos(\Lambda t) + \Omega_R'^2\right]$$

$$\frac{d^2}{dt^2}|C_1(t)|^2 = \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\cdot\Lambda\cdot\cos(\Lambda t)\cdot\left[\Omega_R^2\cos(\Lambda t) + \Omega_R'^2\right] + \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\sin(\Lambda t)\cdot\Omega_R^2(-\Lambda)\cdot\sin(\Lambda t)$$

$$= \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\cdot\Lambda\cdot\left\{\Omega_R^2\cdot\cos^2(\Lambda t) + \Omega_R'^2\cos(\Lambda t) - \Omega_R^2\sin^2(\Lambda t)\right\}$$

$$\frac{d|C_1(t)|^2}{dt} = \frac{(-2\Lambda)\cdot\Omega_R^2}{16\Lambda^4}\cdot\sin(\Lambda t)\cdot\left[\Omega_R^2\cdot\cos(\Lambda t)+\Omega_R'^2\right]$$

$$\frac{d^2|C_1(t)|^2}{dt^2} = \frac{(-2\Lambda^2)\cdot\Omega_R^2}{16\Lambda^4}\left[\Omega_R^2\sin^2(\Lambda t)-\Omega_R^2\cos^2(\Lambda t)-\Omega_R'^2\cos(\Lambda t)\right]$$

$$\frac{d|C_1(t)|^2}{dt} = 0 \Rightarrow \sin(\Lambda t)=0 \quad \text{ή} \quad \cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$$

$$\Downarrow$$

$$\Lambda t = n\pi,\ n\in\mathbb{Z} \qquad \boxed{\Pi 1}$$

Προσοχή, πρέπει $\Omega_R'^2 \leq \Omega_R^2$ $\boxed{\Pi 2}$

$\boxed{\Pi 2}$

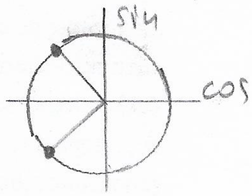
$$\frac{d^2}{dt^2}|C_1(t)|^2 = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\cdot\left[\Omega_R^2\cdot\sin^2(\Lambda t)-\Omega_R^2\cdot\frac{\Omega_R'^4}{\Omega_R^4}+\Omega_R'^2\,\frac{\Omega_R'^2}{\Omega_R^2}\right]$$

$$= \frac{2\Lambda^2}{16\Lambda^4}\left[\Omega_R^4\cdot\sin^2(\Lambda t)-\Omega_R'^4+\Omega_R'^4\right] = \frac{2\Lambda^2}{16\Lambda^4}\cdot\Omega_R^4\cdot\sin^2(\Lambda t) > 0$$

Δηλ. στην $\boxed{\Pi 2}$ έχουμε ελάχιστα, με τιμή μηδενική διότι τότε:

$$|C_1(t)|^2 = \frac{\cancel{\Omega_R^4}}{16\Lambda^4}\cdot\frac{\Omega_R'^4}{\cancel{\Omega_R^4}} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\cancel{\Omega_R^2}\,\Omega_R'^2}{16\Lambda^4}(-1)\frac{\Omega_R'^2}{\cancel{\Omega_R^2}} = \frac{2\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R'^4}{16\Lambda^4} = 0$$

Μέγιστα σε μία περίοδο $T_1 = \frac{2\pi}{\Lambda}$, η οποία είναι και η περίοδος του όρου $\cos(\Lambda t)$, $\exists$ 2 φορές όπου έχουμε $\cos(\Lambda t) = \frac{-\Omega_R'^2}{\Omega_R^2}$ δηλαδή θα έχουμε 2 μηδενισμούς σε μία περίοδο T_1.



Οπότε τότε, $G_1 = 1$ μέγιστο ποσοστό μεταβιβάσεων
maximum transfer percentage

$\boxed{\Pi 1}$

$$\frac{d^2}{dt^2}|C_1(t)|^2 = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left[-\Omega_R^2\cdot\cos^2(\Lambda t)-\Omega_R'^2\cos(\Lambda t)\right]$$

$\sin(\Lambda t)=0 \Rightarrow \cos(\Lambda t) = \pm 1$

Π1α $\quad \Lambda t = 0, 2\pi, 4\pi, \ldots \quad \cos(\Lambda t) = 1 \quad \rightarrow t = 0, T_1, 2T_1, \ldots$

Π1β $\quad \Lambda t = \pi, 3\pi, 5\pi, \ldots \quad \cos(\Lambda t) = -1 \quad \rightarrow t = \frac{T_1}{2}, \frac{3T_1}{2}, \frac{5T_1}{2}, \ldots$

Π1α) $\dfrac{d^2|C_1(t)|^2}{dt^2} = \dfrac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left(-\Omega_R^2 - \Omega_R'^2\right) < 0 \Rightarrow$ τοπικό μέγιστο

13''

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot 1 + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2+\Omega_R'^2)^2}{16\Lambda^4} = 1$$

δηλ. είναι ολικό μέγιστο

Π1β) $\dfrac{d^2|C_1(t)|^2}{dt^2} = \dfrac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left(-\Omega_R^2 + \Omega_R'^2\right) > 0 \Rightarrow$ Τοπικό ελάχιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

η οποία είναι η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$d_1 = 1 - \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2} = \frac{4\cdot\Omega_R^2\cdot\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

για $\Omega_R = \Omega_R'$ $\quad T_1 = \dfrac{2\pi}{\sqrt{2}\,\Omega_R}\cdot 2 = \sqrt{2}\,\dfrac{2\pi}{\Omega_R}$

περίοδος αντίστοιχου διστάθμικού

Π2 $\Rightarrow d_1 = 1$ $\quad \cos(\Lambda t) = -1 \Rightarrow \Lambda t = \pi, 3\pi, \ldots$ $\quad t = \dfrac{\pi}{\Lambda} = \dfrac{2\pi}{2\Lambda} =$ Το ήμισυ της περιόδου

Π1 $\Rightarrow d_1 = 1$ στο ήμισυ της περιόδου

1 φορά μηδενισμός...

δηλ. για $\Omega_R = \Omega_R'$ οι Π1, Π2 ταυτίζονται

$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2}\,\Omega_R$

$4\Lambda^2 = 2\Omega_R^2$

$16\Lambda^4 = 4\Omega_R^4$

π.χ. όπως στο GG, GGG

$T_{GG} = 20.6783$ fs

$T_{GGG} = 29.2436$ fs $= \sqrt{2}\,T_{GG}$

$T_{31} = 14.6218$ fs $= \dfrac{T_{GG}}{\sqrt{2}}$

$T_{32} = 29.2436$ fs $= T_{GGG} = \sqrt{2}\,T_{GG}$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4}\cdot\frac{\cos(\sqrt{2}\,\Omega_R t)+1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} + \frac{2\Omega_R^4}{4\Omega_R^4}\cdot\cos\left(\frac{\sqrt{2}}{2}\Omega_R t\right)$$

$13'''$

$$|c_1(t)|^2 = \frac{1}{4}\left(\frac{1}{2}\cdot\cos(\underbrace{\sqrt{2}\Omega_R}_{2w_1 = w_2} t) + \frac{1}{2}\right) + \frac{1}{4} + \frac{1}{2}\cos\left(\underbrace{\frac{\sqrt{2}}{2}\Omega_R}_{w_1} t\right)$$

$$\frac{1}{8}\cos(2w_1 t) + \frac{1}{8} + \frac{1}{4} + \frac{1}{2}\cos(w_1 t) =$$

$$\frac{1}{8}\Big(\underbrace{\cos(2w_1 t) + 1}_{2\cos^2(w_1 t)} + 2 + 4\cos(w_1 t)\Big) = \frac{1}{8}\left(2\cos^2(w_1 t) + 2 + 4\cos(w_1 t)\right)$$

$$= \frac{1}{4}\left(\cos^2(w_1 t) + 1 + 2\cos(w_1 t)\right) = \frac{1}{4}\Big(\underbrace{\cos(w_1 t) + 1}_{2\cos^2\left(\frac{w_1 t}{2}\right)}\Big)^2 = \cos^4\left(\frac{w_1 t}{2}\right)$$

$$C_3(t)\, e^{3\frac{i\Delta t}{2}} = \frac{\Omega_R}{4\Lambda}\cdot\frac{\Omega_R'}{2\Lambda}\, e^{+i\Lambda t} + \frac{\Omega_R'}{2\Lambda}\,\frac{\Omega_R'}{\cancel{\Omega_R}}(-1)\frac{\Omega_R}{2\Lambda} + \frac{\Omega_R}{4\Lambda}\cdot\frac{\Omega_R'}{2\Lambda}\, e^{-i\Lambda t}$$ (14)

$$C_3(t)\, e^{3\frac{i\Delta}{2}t} = \frac{\Omega_R\Omega_R'}{4\Lambda^2}\cos(\Lambda t) - \frac{\Omega_R\Omega_R'}{4\Lambda^2}$$

$$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos^2(\Lambda t) + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - 2\,\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,\frac{\cos(2\Lambda t)+1}{2} + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \qquad T_B = \frac{2\pi}{\lambda} \qquad \frac{T_B}{T_A} = 2 \Rightarrow$$ η κίνηση είναι περιοδική "(

με περίοδο

$$T_3 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Omega_R'^2}}\,2 = 2T_2 = T_1$$

$$\frac{d}{dt}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2\cos(\Lambda t)(-\Lambda)\sin(\Lambda t) - 2\,\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}(-\Lambda)\sin(\Lambda t)$$

$$\frac{d}{dt}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2(-\Lambda)\left[\cos(\Lambda t)\cdot\sin(\Lambda t) - \sin(\Lambda t)\right]$$

$$\frac{d}{dt}|C_3(t)|^2 = 0 \Rightarrow \sin(\Lambda t) = 0 \quad ή \quad \cos(\Lambda t) = 1$$

$$\Rightarrow t = T_3,\ 2T_3, \ldots$$

Π1 $\Lambda t = 0, 2\pi, 4\pi, \ldots \Rightarrow \sin(\Lambda t) = 0$ και $\cos(\Lambda t) = 1$

Π2 $\Lambda t = \pi, 3\pi, 5\pi, \ldots \Rightarrow \sin(\Lambda t) = 0$ και $\cos(\Lambda t) = -1$

$$\Rightarrow t = \frac{T_3}{2}, \frac{3T_3}{2}, \ldots$$

Π1, Π2 $$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot 1 + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}(\pm 1) = \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} \mp \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}$$

Π1 $\Rightarrow |C_3(t)|^2 = 0$ ελάχιστο

Π2 $\Rightarrow |C_3(t)|^2 = \frac{4\Omega_R^2\Omega_R'^2}{16\Lambda^4}$ μέγιστο δηλ...

$$\frac{d^2}{dt^2}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2(-\Lambda)\left[\frac{1}{\cancel{2}}\cancel{2}\Lambda\cos(2\Lambda t) - \Lambda\cos(\Lambda t)\right] = \underbrace{\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}2\Lambda^2}_{\vartheta}\left(\cos(\Lambda t) - \cos(2\Lambda t)\right)$$

Π1 $\Rightarrow (\ldots) = \cos(2\pi) - \cos(4\pi) = 0$

Π2 $\Rightarrow (\ldots) = \cos(\pi) - \cos(2\pi) = -1-1 = -2 \Rightarrow \frac{d^2}{dt^2}|C_3(t)|^2 < 0$ μέγιστο

Αν τα πράγματα είναι έτσι, τότε

$$\frac{d^3|C_3(t)|^2}{dt^3} = \vartheta\cdot\Big(\Lambda(-1)\cdot\sin(\Lambda t) + 2\Lambda\sin(2\Lambda t)\Big)$$

$$= \vartheta\Lambda\Big(2\sin(2\Lambda t) - \sin(\Lambda t)\Big)$$

και για $\Lambda t = 2\pi$ Π1 $= \vartheta\Lambda\Big(2\sin(4\pi) - \sin(2\pi)\Big) = 0$

$$\frac{d^4|C_3(t)|^2}{dt^4} = \vartheta\Lambda\Big(2\cdot 2\Lambda\cos(2\Lambda t) - \Lambda\cdot\cos(\Lambda t)\Big)$$

$$= \vartheta\Lambda^2\Big(4\cdot\cos(2\Lambda t) - \cos(\Lambda t)\Big)$$

και για $\Lambda t = 2\pi$ Π1 $= \vartheta\Lambda^2\Big(4\cdot\underbrace{\cos(4\pi)}_{1} - \underbrace{\cos(2\pi)}_{1}\Big) > 0$

ΞΑΝΑ $\dfrac{d^2}{dt^2}|C_3(t)|^2 = \vartheta\Big(\underbrace{\cos(\Lambda t)}_{1} - \underbrace{\cos(2\Lambda t)}_{1}\Big) = 0$

για $\Lambda t = 2\pi$ Π1

$$\frac{d^3}{dt^3}|C_3(t)|^2 = \vartheta\Big(-\Lambda\underbrace{\sin(\Lambda t)}_{0} + 2\Lambda\underbrace{\sin(2\Lambda t)}_{0}\Big) = 0$$

για $\Lambda t = 2\pi$ Π1

$$\frac{d^4}{dt^4}|C_3(t)|^2 = \vartheta\Big(-\Lambda^2\cos(\Lambda t) + 2\Lambda\cdot 2\Lambda\cos(2\Lambda t)\Big) = \vartheta\Lambda^2\Big(4\underbrace{\cos(2\Lambda t)}_{1} - \underbrace{\cos(\Lambda t)}_{1}\Big)$$

για $\Lambda t = 2\pi$ Π1 $= \vartheta\Lambda^2\cdot 3 > 0$

δηλαδή στα ελάχιστα μηδενίζονται η $|C_3(t)|^2$, καθώς και η 1η, 2η και 3η παράγωγός της!

ενώ, η 4η παράγωγός της είναι θετική...

μοιάζει κάπως με επίπεδη συνάρτηση (flat function)
[όπου μηδενίζονται σε κάποιο σημείο όλες οι παράγωγοι]

Άρα $\boxed{\lambda_3 = \dfrac{4\Omega_R^2\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}}$ μέγιστο ποσοστό μεταβιβάσεως maximum transfer percentage (14'')

για $\Omega_R' = \Omega_R$ $\quad T_3 = \dfrac{2\pi}{\sqrt{2}\,\Omega_R}\cdot 2 = \sqrt{2}\,\dfrac{2\pi}{\Omega_R}$ $\quad 2\Lambda = \sqrt{2\Omega_R^2} = \sqrt{2}\,\Omega_R$

περίοδος αντιστοίχου διστάθμικού

$$|c_3(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4}\,\frac{\cos(\sqrt{2}\Omega_R t)+1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} - \frac{2\Omega_R^4}{4\Omega_R^4}\cdot\cos\left(\frac{\sqrt{2}\,\Omega_R t}{2}\right)$$

$$|c_3(t)|^2 = \frac{1}{4}\left(\frac{1}{2}\cos(\sqrt{2}\,\Omega_R t) + \frac{1}{2} + \frac{1}{4} - \frac{1}{2}\cos\left(\frac{\sqrt{2}\,\Omega_R t}{2}\right)\right)$$

$2\omega_3 = 2\omega_1 = \omega_2 \qquad \omega_1 = \omega_3$

$$= \frac{1}{8}\cos(2\omega_3 t) + \frac{1}{8} + \frac{1}{4} - \frac{1}{2}\cos(\omega_3 t)$$

$$= \frac{1}{8}\left(\cos(2\omega_3 t) + 1 + 2 - 4\cos(\omega_3 t)\right)$$

$$= \frac{1}{8}\left(2\cos^2(\omega_3 t) + 2 - 4\cos(\omega_3 t)\right)$$

$$= \frac{1}{4}\left(\cos^2(\omega_3 t) + 1 - 2\cos(\omega_3 t)\right)$$

$$= \frac{1}{4}\left(1 - \cos(\omega_3 t)\right)^2$$

$$= \frac{1}{4}\left(2\sin^2\left(\frac{\omega_3 t}{2}\right)\right)^2 = \sin^4\left(\frac{\omega_3 t}{2}\right)$$

Άρα $$A_3 = \frac{4\Omega_R^2\,\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

μέγιστος ρυθμός μεταβιβάσεως
maximum transfer rate $1\to3$

$$:= \frac{A_3}{T_3} = \frac{A_3}{T_3} = \frac{4\Omega_R^2\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}\,\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2\pi\cdot 2}$$

Μέσες πιθανότητες παρουσίας του ηλεκτρονίου σε κάθε στάθμη

$$\langle|c_3(t)|^2\rangle = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,\frac{1}{2} + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{3\Omega_R^2\Omega_R'^2}{2\cdot16\Lambda^4}$$

$$\langle|c_1(t)|^2\rangle = \frac{\Omega_R^4}{16\Lambda^4}\,\frac{1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} = \frac{\Omega_R^4+2\Omega_R'^4}{2\cdot16\Lambda^4}$$

$$\langle|c_2(t)|^2\rangle = \frac{\Omega_R^2}{\Omega_R^2+\Omega_R'^2}\,\frac{1}{2} = \frac{\Omega_R^2}{4\Lambda^2}\,\frac{1}{2} = \frac{4\Omega_R^2\Lambda^2}{2\cdot16\Lambda^4}.$$

$$\langle|c_1(t)|^2\rangle + \langle|c_2(t)|^2\rangle + \langle|c_3(t)|^2\rangle = \frac{3\Omega_R^2\Omega_R'^2+\Omega_R^4+2\Omega_R'^4+4\Omega_R^2\Lambda^2}{2\cdot16\Lambda^4}$$

$$= \frac{3\Omega_R^2\Omega_R'^2+\Omega_R^4+2\Omega_R'^4+\Omega_R^2(\Omega_R^2+\Omega_R'^2)}{2\cdot16\Lambda^4} = \frac{4\Omega_R^2\Omega_R'^2+2\Omega_R^4+2\Omega_R'^4}{2\cdot16\Lambda^4} =$$

$$\frac{\Omega_R^4+2\Omega_R^2\Omega_R'^2+\Omega_R'^4}{(\Omega_R^2+\Omega_R'^2)^2} = 1$$

ορισμός

t_{3mean} $$\frac{3\cancel{\Omega_R^2\Omega_R'^2}}{2\cdot\cancel{16\Lambda^4}} = \frac{\cancel{\Omega_R^2\Omega_R'^2}}{\cancel{16\Lambda^4}}\cos^2(\Lambda t_{3mean}) + \frac{\cancel{\Omega_R^2\Omega_R'^2}}{\cancel{16\Lambda^4}} - \frac{2\cancel{\Omega_R^2\Omega_R'^2}}{\cancel{16\Lambda^4}}\cos(\Lambda t_{3mean})$$

ο απαιτούμενος χρόνος για να φτάσει τη μέση του τιμή

$$\frac{3}{2} = \cos^2(\Lambda t_{3mean}) + 1 - 2\cos(\Lambda t_{3mean})$$

$$\frac{1}{2} = \cos^2(\Lambda t_{3mean}) - 2\cdot\cos(\Lambda t_{3mean})$$

$$1 = 2\cos^2 x - 4\cos x$$

$$2\psi^2 - 4\psi - 1 = 0 \qquad \psi = \cos x$$

$$\delta = 16 + 4\cdot 2 = 24$$

$$\frac{4\pm\sqrt{24}}{2\cdot 2} = 1 \pm \frac{\sqrt{24}}{4} = 1 \pm \sqrt{\frac{24}{16}} = 1 \pm \sqrt{\frac{3\cdot \cancel{8}}{2\cdot \cancel{8}}}$$

$$= 1 \pm \frac{\sqrt{3}}{\sqrt{2}} \qquad 1-\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}} \qquad 1+\frac{\sqrt{3}}{\sqrt{2}} > 1$$

απορρίπτεται

$$\cos(\Lambda\, t_{3mean}) = 1-\frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos\left(\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2}\, t_{3mean}\right) = 1-\frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2}\, t_{3mean} = 1.797477\ldots$$

$$t_{3mean} = \frac{2\cdot 1.797477}{\sqrt{\Omega_R^2+\Omega_R'^2}}$$

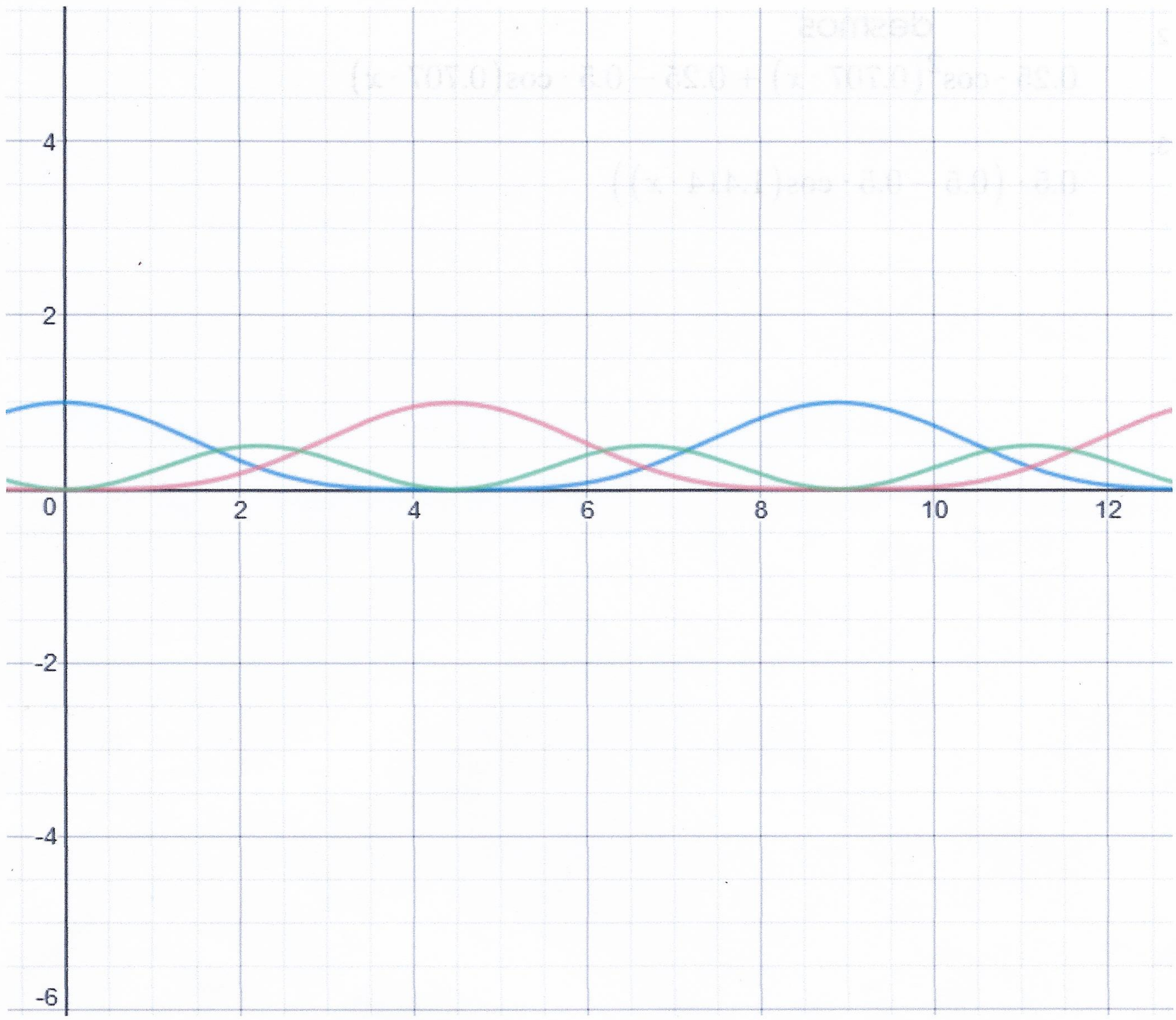
mean tranfer rate

<u>μέσος ρυθμός μεταβάσεως</u>

$$k := \frac{\langle |c_3(t)|^2\rangle}{t_{3mean}} = \frac{3\,\Omega_R^2\,\Omega_R'^2}{2\cdot 16\,\Lambda^4}\,\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2\cdot 1.797477}$$

$$\frac{k}{\frac{\Lambda_3}{\Gamma_3}} = \frac{3\,\cancel{\Omega_R^2}\,\cancel{\Omega_R'^2}}{2\,\cancel{(\Omega_R^2+\Omega_R'^2)^2}}\,\frac{\cancel{\sqrt{\Omega_R^2+\Omega_R'^2}}}{2\cdot 1.797477}\cdot\frac{\cancel{(\Omega_R^2+\Omega_R'^2)^2}\; 2\pi\; 2}{4\,\cancel{\Omega_R^2}\,\cancel{\Omega_R'^3}\,\cancel{\sqrt{\Omega_R^2+\Omega_R'^2}}}$$

$$= \frac{3\cdot\pi}{4\cdot 1.797477} \simeq 1.31083\ldots$$



— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 + 0.5 \cdot \cos(0.707 \cdot x)$

— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 - 0.5 \cdot \cos(0.707 \cdot x)$

— $0.5 \cdot (0.5 - 0.5 \cdot \cos(1.414 \cdot x))$

πράσινο

για

$\Omega_R = \Omega_R' = 1$

$2\Lambda = \sqrt{1^2 + 1^2} = \sqrt{2} \Rightarrow \Lambda = \frac{\sqrt{2}}{2}$

ΑΣΚΗΣΗ (matlab)

Χρησιμοποιώντας το πρόγραμμα Oscillations.m
να γίνει η γραφική παράσταση των ταλαντώσεων ΔΣ
με $|\Delta| = \sqrt{3}\,\Omega_R$ $\left(\Rightarrow \alpha = \frac{1}{4},\ T = \frac{1}{2}\frac{2\pi}{\Omega_R}\right)$

ΑΣΚΗΣΗ (matlab)

Να φτιαχθεί ανάλογο πρόγραμμα για Τριστάθμικό Σύστημα
και να γίνει η γραφική παράσταση για $\Omega_R = \Omega_L = 1$ $\quad \Delta = 0$