ΣΤΑΣΙΜΟ ΚΥΜΑ σε ΚΟΙΛΟΤΗΤΑ

θα κατασκευάσουμε **Χαμιλτονιανή του ΗΜ πεδίου** η οποία να μπορεί να μετασχηματίζεται από τη γλώσσα $\vec{E}, \vec{B}$ στη γλώσσα του αριθμού των φωτονίων

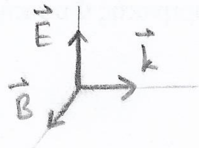
Τρέχοντα κύματα

3Δ ΚΕ

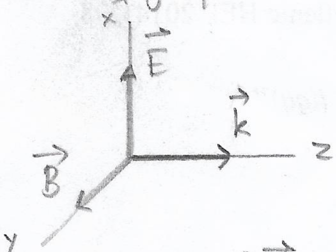
$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \vec{E}(\vec{r},t) = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t + \delta)} \qquad \vec{E} \times \vec{B} \uparrow\uparrow \vec{k}$$

3Δ

$$\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \vec{B}(\vec{r},t) = \vec{B}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t + \delta)}$$



διαλέγουμε



$$\nabla^2 \vec{E}_x = \frac{1}{c^2} \frac{\partial^2 \vec{E}_x}{\partial t^2} \qquad \vec{E}(\vec{r},t) = \vec{E}_{x0} \, e^{i(k_z z - \omega t + \delta)} = \vec{E}_x(z,t)$$

$$\nabla^2 \vec{B}_y = \frac{1}{c^2} \frac{\partial^2 \vec{B}_y}{\partial t^2} \qquad \vec{B}(\vec{r},t) = \vec{B}_{y0} \, e^{i(k_z z - \omega t + \delta)} = \vec{B}_y(z,t)$$

οπότε

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}} \qquad \boxed{\frac{\partial^2 B_y}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 B_y}{\partial t^2}}$$

1Δ ΚΕ

ΚΕ1ΔΕ ΚΕ1ΔΒ

1η Εξ.Μ $\vec{\nabla}\cdot\vec{E}=0 \Rightarrow \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\cdot(E_x,E_y,E_z)=0 \Rightarrow$

$$\frac{\partial E_x}{\partial x}+\underbrace{\cancel{\frac{\partial E_y}{\partial y}}}_{0}+\underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{0}=0 \Rightarrow \frac{\partial E_x}{\partial x}=0 \quad \text{αναμενόμενο}$$

2η Εξ.Μ $\vec{\nabla}\cdot\vec{B}=0 \Rightarrow \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\cdot(B_x,B_y,B_z)=0 \Rightarrow$

$$\underbrace{\cancel{\frac{\partial B_x}{\partial x}}}_{0}+\frac{\partial B_y}{\partial y}+\underbrace{\cancel{\frac{\partial B_z}{\partial z}}}_{0}=0 \Rightarrow \frac{\partial B_y}{\partial y}=0 \quad \text{αναμενόμενο}$$

3η Εξ.Μ $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ E_x & 0 & 0\end{vmatrix}=-\frac{\partial}{\partial t}(0,B_y,0) \Rightarrow$

$$+\hat{j}\,\frac{\partial E_x}{\partial z}=-\hat{j}\,\frac{\partial B_y}{\partial t} \Rightarrow$$

$$\boxed{\frac{\partial E_x}{\partial z}=-\frac{\partial B_y}{\partial t}} \qquad \underline{E_x B_y}$$

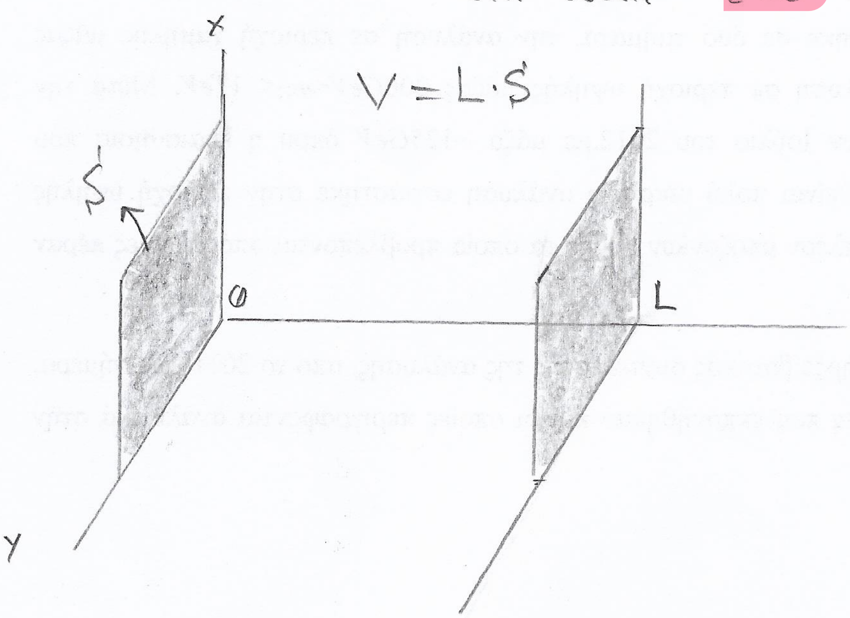
4η Εξ.Μ $\vec{\nabla}\times\vec{B}=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ 0 & B_y & 0\end{vmatrix}=\varepsilon_0\mu_0\frac{\partial}{\partial t}(E_x,0,0) \Rightarrow$

$$-\hat{i}\,\frac{\partial B_y}{\partial z}=\varepsilon_0\mu_0\frac{\partial E_x}{\partial t}\,\hat{i} \Rightarrow$$

$$\boxed{\frac{\partial B_y}{\partial z}=-\frac{1}{c^2}\frac{\partial E_x}{\partial t}} \qquad \underline{B_y E_x}$$

Τώρα βάζουμε ιδανικώς αγώγιμα κάτοπτρα

στις θέσεις $z=0$ και $z=L$



θα δημιουργηθούν στάσιμα κύματα από το συνδυασμό προσπιπτόντων και ανακλωμένων κυμάτων

οι ΚΕ1ΔΕ και ΚΕ1ΔΒ εξακολουθούν να ισχύουν για το γραμμικό συνδυασμό $E_x B_y$ και $B_y E_x$ των προσπιπτόντων και ανακλωμένων κυμάτων

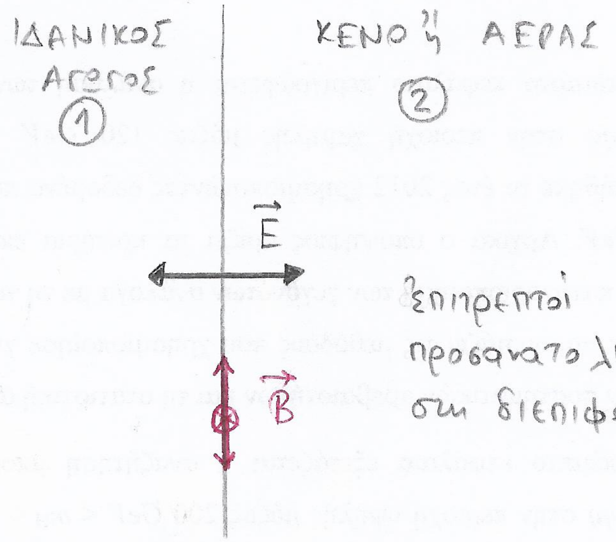
Αναζητούμε λύση με τη μέθοδο των χωριζομένων μεταβλητών υποθέτοντας ότι

$$\boxed{E_x(z,t) = N\, Z(z)\, T(t)}$$

ΕΣΣ*

$B_{2\perp} = 0$

$E_{2\parallel} = 0$



επιτρεπτοί προσανατολισμοί στη διεπιφάνεια

ΑΡΑ $E_x(0,t) = 0 = E_x(L,t) \quad \forall t$

$$\left.\begin{aligned} &\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \\ &E_x(z,t) = N Z(z) T(t) \end{aligned}\right\} \Rightarrow \cancel{N} T(t) \frac{d^2 Z(z)}{dz^2} = \frac{1}{c^2} \cancel{N} Z(z) \frac{d^2 T(t)}{dt^2}$$

$$\Rightarrow \underbrace{\underbrace{\frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2}}_{f(z)} = \underbrace{\frac{1}{c^2} \frac{1}{T(t)} \frac{d^2 T(t)}{dt^2}}_{g(t)}}_{\forall z,\ \forall t} \quad \text{ΑΡΑ σταθερά} := -k^2$$

θα πρέπει $Z(z) \neq 0$ και $T(t) \neq 0$ για να γράψουμε $\frac{1}{Z(z)}$ ή $\frac{1}{T(t)}$

αν $Z(z) = 0$ ή $T(t) = 0$ τότε η $\}$ ικανοποιείται

Άρα

$$\boxed{\frac{d^2 Z(z)}{dz^2} + k^2 Z(z) = 0} \quad (Z)$$

$$\boxed{\frac{d^2 T(t)}{dt^2} + k^2 c^2 T(t) = 0} \quad (T)$$

Ας ασχοληθούμε αρχικώς με την (Z) ΔΛΜ $e^{\tilde{\lambda} z}$

$$\tilde{\lambda}^2 + k^2 = 0 \Rightarrow \tilde{\lambda} = \pm ik \quad \text{π.χ. ας διαλέξουμε } \underline{k \in \mathbb{R}_+}$$

Άρα η λύση της (Z) θα είναι της μορφής

$$Z(z) = A e^{ikz} + B e^{-ikz}$$

ΣΣ1 $\quad Z(0) = A + B = 0 \Rightarrow B = -A$

ΣΣ2 $\quad Z(L) = A e^{ikL} + B e^{-ikL} = 0 \Rightarrow A e^{ikL} - A e^{-ikL} = 0$

$$\Rightarrow e^{ikL} = e^{-ikL} \Rightarrow \cos(kL) + i\sin(kL) = \cos(kL) - i\sin(kL)$$

$$\Rightarrow \sin(kL) = 0$$

$$\sin(kL)=0 \Rightarrow kL = m\pi,\ m\in\mathbb{Z}$$

αλλά υποθέσαμε $k\in\mathbb{R}_+$ $\Rightarrow$

$$kL = m\pi,\ m\in\mathbb{N}$$

αλλά αν $m=0 \Rightarrow k=0 \Rightarrow Z(z)=A+B=0$

Οπότε για μη μηδενική λύση

$$k_m = \frac{m\pi}{L},\ m\in\mathbb{N}^*$$

KM

ΑΡΑ $Z(z) = Ae^{ikz} - Ae^{-ikz} = A\cos kz + Ai\sin kz \Rightarrow$
$- A\cos kz + Ai\sin kz$

$$Z(z) = 2iA\sin(kz)$$

ΑΡΑ

$$\boxed{Z_m(z) = 2iA\sin\left(\frac{m\pi}{L}z\right) \quad m\in\mathbb{N}^*}$$

Κι αν απαιτήσουμε οι $Z_m(z)$ να είναι ορθοκανονικές

$$\int_0^L Z_m^*(z)\,Z_\ell(z)\,dz = \delta_{m\ell}$$

$$\int_0^L dz\ 2(-i)A^*\sin\left(\frac{m\pi z}{L}\right) 2iA\sin\left(\frac{\ell\pi z}{L}\right) = \delta_{m\ell} \Rightarrow$$

$$4|A|^2\int_0^L dz\ \sin\left(\frac{m\pi z}{L}\right)\sin\left(\frac{\ell\pi z}{L}\right) = \delta_{m\ell}$$

$$\varphi := \frac{\pi z}{L} \qquad d\varphi = \frac{\pi}{L}dz \Rightarrow dz = \frac{L}{\pi}d\varphi$$

$$4|A|^2 \frac{L}{\pi} \int_0^{\pi} d\psi \sin(m\psi) \sin(l\psi) = \delta_{ml}$$

$$\int_0^{\pi} d\psi \sin(m\psi) \sin(l\psi) = \frac{\pi}{2} \delta_{ml}$$

$$\int_0^{\pi} d\psi \cos(m\psi) \cos(l\psi) = \frac{\pi}{2} \delta_{ml}$$

$$4|A|^2 \frac{L}{\cancel{\pi}} \frac{\cancel{\pi}}{2} \cancel{\delta_{ml}} = \cancel{\delta_{ml}} \Rightarrow 2L|A|^2 = 1 \Rightarrow |A|^2 = \frac{1}{2L}$$

ας διαλέξουμε κάτι βολικό π.χ. $A = \frac{1}{\sqrt{2L}}(-i)$

Συνεπώς

$$Z_m(z) = 2i \frac{1}{\sqrt{2L}} (-i) \sin\left(\frac{m\pi z}{L}\right) \qquad m \in \mathbb{N}^*$$

$$\boxed{Z_m(z) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{m\pi z}{L}\right)} \quad , m \in \mathbb{N}^*$$

Ας ασχοληθούμε τώρα με την (T). Ας βρούμε $\omega := kc > 0$

ΔΛΜ $e^{\tilde{\lambda} t}$ $\quad \frac{d^2 T(t)}{dt^2} + k^2 c^2 T(t) = 0 \quad \Rightarrow \omega_m = k_m c$

$$\tilde{\lambda}^2 + \omega^2 = 0 \Rightarrow \lambda = \pm i\omega$$

$$\omega_m = \frac{m\pi c}{L} \qquad m \in \mathbb{N}^*$$

$$\lambda = \pm i\omega_m$$

ΩΜ

Άρα η λύση της (T) θα είναι της μορφής

$$T(t) = \Gamma e^{i\omega_m t} + \Delta e^{-i\omega_m t}$$

Κι αν θέσουμε την αρχική συνθήκη $T(0)=0$

$$\Rightarrow \Gamma + \Delta = 0 \Rightarrow \Delta = -\Gamma$$

$$T(t) = \Gamma e^{i\omega_m t} - \Gamma e^{-i\omega_m t} = \Gamma \cos\omega_m t + \Gamma i \sin\omega_m t - \Gamma\cos\omega_m t + \Gamma i \sin\omega_m t$$

$$\Rightarrow T(t) = 2i\Gamma \sin\omega_m t \Rightarrow$$

$$T_m(t) = 2i\Gamma \sin\left(\frac{m\pi c}{L} t\right) \qquad m\in\mathbb{N}^*$$

Κι αν απαιτήσουμε οι $T_m(t)$ να είναι ορθοκανονικές σε ένα χρονικό πλαίσιο από το χρόνο 0 έως το χρόνο t_k

$$\int_0^{t_k} dt\, T_m^*(t)\, T_l(t) = \delta_{ml} \Rightarrow$$

$$\int_0^{t_k} dt\, 2(-i)\Gamma^* \sin\left(\frac{m\pi c t}{L}\right) 2i\Gamma \sin\left(\frac{l\pi c t}{L}\right) = \delta_{ml} \Rightarrow$$

$$4|\Gamma|^2 \int_0^{t_k} dt\, \sin\left(\frac{m\pi c t}{L}\right) \sin\left(\frac{l\pi c t}{L}\right) = \delta_{ml}$$

$$x := \frac{\pi c t}{L} \qquad dx = \frac{\pi c}{L} dt \Rightarrow dt = \frac{L}{\pi c} dx$$

$$4|\Gamma|^2 \frac{L}{\pi c} \int_0^{\frac{\pi c t_k}{L}} dx\, \sin(mx) \sin(lx) = \delta_{ml}$$

χρόνος πτήσεως του φωτονίου διαμέσου της κοιλότητας

Άρα είναι βολικό να θέσουμε $\frac{\pi c}{L} t_k = \pi \Rightarrow t_k = \frac{L}{c} := \tau$

ΛΟΓΙΚΗ ΕΚΛΟΓΗ...

τ time of photon flight through cavity

$$4|\Gamma|^2 \frac{L}{\pi c} \int_0^\pi dx \sin(mx)\sin(lx) = \delta_{ml}$$

$$4|\Gamma|^2 \frac{L}{\cancel{\pi} c} \cdot \frac{\cancel{\pi}}{2} \cancel{\delta_{ml}} = \cancel{\delta_{ml}} \Rightarrow |\Gamma|^2 = \frac{c}{2Lc}$$

κι άν διαλέξουμε κάτι βολικό $\Gamma = \sqrt{\frac{c}{2L}}\,(-i)$

$$T_m(t) = 2i\sqrt{\frac{c}{2L}}\,(-i)\sin\left(\frac{m\pi c}{L}t\right)$$

$$\boxed{T_m(t) = \sqrt{\frac{2c}{L}} \cdot \sin\left(\frac{m\pi c}{L}t\right)}$$

Άρα $E_x^m(z,t) = N\sqrt{\frac{2}{L}}\sin\left(\frac{m\pi z}{L}\right)\sqrt{\frac{2c}{L}} \cdot \sin\left(\frac{m\pi c\,t}{L}\right)$

$$\boxed{E_x^m(z,t) = \frac{2N}{L}\sqrt{c}\,\sin\left(\frac{m\pi z}{L}\right)\sin\left(\frac{m\pi c t}{L}\right)}$$

E_{xm}

$$[N] = \frac{V}{\cancel{m}} \frac{\cancel{m}\sqrt{s}}{\sqrt{m}} = \frac{V\sqrt{s}}{\sqrt{m}} = \frac{V}{\sqrt{m/s}}$$

$$\left.\begin{aligned}&\frac{\partial B_y}{\partial z} = -\frac{1}{c^2}\frac{\partial E_x}{\partial t} \qquad \underline{B_y E_x}\\[2ex] &E_x^m(z,t) = \frac{2N\sqrt{c}}{L}\sin\left(\frac{m\pi z}{L}\right)\sin\left(\frac{m\pi c t}{L}\right)\end{aligned}\right\} \Rightarrow$$

$$\frac{\partial B_y^m}{\partial z} = -\frac{1}{c^2}\,\frac{2N\sqrt{c}}{L}\sin\left(\frac{m\pi z}{L}\right)\frac{m\pi c}{L}\cos\left(\frac{m\pi c t}{L}\right) \Rightarrow$$

$$\frac{\partial B_y^m}{\partial z} = -\frac{2N\pi m}{L^2\sqrt{c}}\sin\left(\frac{m\pi z}{L}\right)\cos\left(\frac{m\pi c t}{L}\right)$$

$$\int_0^{z'}\frac{\partial B_y^m}{\partial z}dz = -\frac{2Nm\pi}{L^2\sqrt{c}}\int_0^{z'} dz\,\sin\left(\frac{m\pi z}{L}\right)\cdot\cos\left(\frac{m\pi c t}{L}\right)$$

$$B_y^m(z',t) - B_y^m(0,t) = +\frac{2Nm\pi}{L^2\sqrt{c}}\cdot\cos\left(\frac{m\pi c t}{L}\right)\cdot\left(\frac{L}{m\pi}\right)\left[+\cos\left(\frac{m\pi z}{L}\right)\right]_0^{z'}$$

$$B_y^m(z',t) - B_y^m(0,t) = \frac{2N\cancel{m\pi}}{L^2\sqrt{c}}\frac{\cancel{L}}{\cancel{m\pi}}\cos\left(\frac{m\pi c t}{L}\right)\cdot\left\{\cos\left(\frac{m\pi z'}{L}\right) - 1\right\}$$

$$B_y^m(z',t) - B_y^m(0,t) = \frac{2N}{L\sqrt{c}}\cos\left(\frac{m\pi c t}{L}\right)\left\{\cos\left(\frac{m\pi z'}{L}\right) - 1\right\}$$

ορίζονται κατάλληλως το $B_y^m(0)$ $\Rightarrow$
και αλλάζονται τη μεταβλητή $z' \to z$

$$\boxed{B_y^m(z,t) = \frac{2N}{L\sqrt{c}}\cos\left(\frac{m\pi c t}{L}\right)\cos\left(\frac{m\pi z}{L}\right)}$$

B_{ym}

Πυκνότητα ενέργειας

$$U = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 = \frac{\varepsilon_0}{2}\left[E^2 + c^2B^2\right] \qquad [U] = \frac{J}{m^3}$$

Πυκνότητα ενέργειας m τρόπου

$$U_m = \frac{\varepsilon_0}{2}\left[\frac{4\omega^2}{L^2}c\,\sin^2\left(\frac{m\pi z}{L}\right)\sin^2\left(\frac{m\pi ct}{L}\right) + \frac{c^{\cancel{2}}4\omega^2}{L^2\cancel{c}}\cos^2\left(\frac{m\pi z}{L}\right)\cdot\cos^2\left(\frac{m\pi ct}{L}\right)\right]$$

$$\Rightarrow U_m = \frac{\varepsilon_0}{2}\frac{4\omega^2 c}{L^2}\left[\sin^2\left(\frac{m\pi z}{L}\right)\sin^2\left(\frac{m\pi ct}{L}\right) + \cos^2\left(\frac{m\pi z}{L}\right)\cos^2\left(\frac{m\pi ct}{L}\right)\right]$$

Άρα η ενέργεια του m τρόπου

$$\boxed{E_m = \int_{V=LS} d^3r\, U_m} = \frac{\varepsilon_0}{2}\frac{4\omega^2 c}{L^2}\left[\sin^2\left(\frac{m\pi ct}{L}\right)S\int_0^L dz\,\sin^2\left(\frac{m\pi z}{L}\right) + \cos^2\left(\frac{m\pi ct}{L}\right)\cdot S\int_0^L dz\,\cos^2\left(\frac{m\pi z}{L}\right)\right]$$

$d^3r = dV = S\,dz$

$$E_m = \frac{2\varepsilon_0\omega^2 cS}{L^2}\left[\sin^2\left(\frac{m\pi ct}{L}\right)\frac{L}{2} + \cos^2\left(\frac{m\pi ct}{L}\right)\cdot\frac{L}{2}\right]$$

$\psi := \frac{\pi z}{L}$

$d\psi = \frac{\pi\,dz}{L}$

$$\int_0^L dz\,\sin^2\left(\frac{m\pi z}{L}\right) = \frac{L}{\pi}\int_0^\pi d\psi\,\sin^2(m\psi) = \frac{L}{\pi}\cdot\frac{\pi}{2} = \frac{L}{2}$$

$$\int_0^L dz\,\cos^2\left(\frac{m\pi z}{L}\right) = \frac{L}{\pi}\int_0^\pi d\psi\,\cos^2(m\psi) = \frac{L}{\pi}\cdot\frac{\pi}{2} = \frac{L}{2}$$

$$E_m = \frac{\varepsilon_0 N^2 c S}{L}\left[\sin^2\left(\frac{m\pi c t}{L}\right) + \cos^2\left(\frac{m\pi c t}{L}\right)\right] = \frac{\varepsilon_0 N^2 c S}{L}$$

Ας δόσουμε λίγο αλλιώτικα την E_m ...

$$E_m = \frac{\varepsilon_0 c N^2 S}{L^3}\left[L^2 \sin^2\left(\frac{m\pi c t}{L}\right) + L^2 \cos^2\left(\frac{m\pi c t}{L}\right)\right]$$

$$q_m(t) := L \sin\left(\frac{m\pi c t}{L}\right)$$

γενικευμένη θέση

$$[q_m(t)] = m$$

$$\dot{q}_m(t) := L \frac{m\pi c}{L} \cos\left(\frac{m\pi c t}{L}\right)$$

$$\dot{q}_m(t) = m\pi c \cos\left(\frac{m\pi c t}{L}\right)$$

γενικευμένη ταχύτητα

$$[\dot{q}_m(t)] = \frac{m}{s}$$

$$E_m = \frac{\varepsilon_0 c N^2 S}{L^3}\left[(q_m(t))^2 + \frac{(\dot{q}_m(t))^2}{(m\pi c)^2} L^2 \right]$$

αναλογία με ΑΑΤ

$$E = \frac{K}{2} x^2 + \frac{M}{2} \upsilon^2 = \frac{K}{2}\left[x^2 + \frac{M}{K}\upsilon^2 \right]$$

$$\frac{K}{2} = \frac{\varepsilon_0 c N^2 S}{L^3} \qquad \frac{L^2}{m^2\pi^2c^2} = \frac{M}{K}$$

δηλαδή προκύπτει ή σταθερά ελατηρίου

$$K = \frac{2\varepsilon_0 c N^2 S}{L^3}$$

και ή μάζα

$$M = K\frac{L^2}{m^2\pi^2c^2} = \frac{2\varepsilon_0 c N^2 S}{L^3}\,\frac{L^2}{m^2\pi^2c^2}$$

$$M_m = \frac{2\varepsilon_0 N^2 S}{L c\, m^2 \pi^2}$$

$$\boxed{[M_m] = kg}$$

$$\boxed{K = M_m \omega_m^2}$$

$$[M_m] = \frac{F}{\cancel{m}}\cdot\frac{V^2}{\frac{m}{s}}\,\frac{\cancel{m^2}}{\cancel{m}\frac{m}{s}} = \frac{FV^2}{\left(\frac{m}{s}\right)^2} = \frac{CV^{\cancel{2}}s^2}{\cancel{V}\,m^2} = \frac{Js^2}{m^2}$$

$$= \frac{\cancel{kg}\,m\,\cancel{m}\,\cancel{s^2}}{\cancel{s^2}\,\cancel{m^2}} = kg$$

$$M_m\omega_m^2 = \frac{2\varepsilon_0 N^2 S}{L\,c\,\cancel{m^2\pi^2}}\,\frac{\cancel{m^2\pi^2}c^{\cancel{2}}}{L^2} = \frac{2\varepsilon_0 N^2 S c}{L^3} = K$$

ή $$\frac{K}{M} = \frac{\cancel{2\varepsilon_0 c N^2 S}\,L c\, m^2\pi^2}{L^3\,\cancel{2\varepsilon_0 c N^2 S}} = \frac{m^2\pi^2c^2}{L^2} \Rightarrow K = M_m\cdot\left(\frac{m\pi c}{L}\right)^2$$

$$K = M_m\cdot\omega_m^2$$

$$[K] = kg\cdot\frac{m^2}{s^2}\,\frac{1}{m^2} = \frac{kg}{s^2} = \frac{kg\,m}{s^2\cdot m} = \frac{N}{m}$$

$$E_m = \frac{M_m \omega_m^2}{2} \left(q_m(t)\right)^2 + \frac{M_m}{2} \left(\dot{q}_m(t)\right)^2$$

$$E_m = \frac{M_m \omega_m^2}{2} q_m^2 + \frac{M_m}{2} \dot{q}_m^2$$

↓ κβαντικό αντίστοιχο

$$\hat{H}_{HM,m} = \frac{M_m \omega_m^2}{2} \hat{q}_m^2 + \frac{M_m}{2} \hat{\dot{q}}_m^2$$

χαμιλτονιανή
m τρόπου ΗΜ πεδίου

$$E_{m,n_m} = \hbar \omega_m \left(n_m + \frac{1}{2}\right)$$

$$n_m \in \mathbb{N}$$

$$m \in \mathbb{N}^*$$

ιδιοτιμές ενέργειας
m τρόπου ΗΜ πεδίου

$$\hat{H}_{HM} = \sum_m \hat{H}_{HM,m}$$

χαμιλτονιανή
ΗΜ πεδίου

$$E_x^m(z,t) = \frac{2\sqrt{c}}{L^2} \mathcal{N} \sin\left(\frac{m\pi z}{L}\right) q_m(t)$$

$$\hat{E}_x^m(z,t) = \frac{2\sqrt{c}}{L^2} \mathcal{N} \sin\left(\frac{m\pi z}{L}\right) \hat{q}_m(t)$$

$$B_y^m(z,t) = \frac{2\mathcal{N}}{L\sqrt{c}} \frac{1}{m\pi c} \cos\left(\frac{m\pi z}{L}\right) \dot{q}_m(t)$$

$$\hat{B}_y^m(z,t) = \frac{2\mathcal{N}}{L\sqrt{c}} \frac{1}{m\pi c} \cdot \cos\left(\frac{m\pi z}{L}\right) \hat{\dot{q}}_m(t)$$

$$E_x^m(z,t) = \left(\frac{2M_m\omega_m^2}{\varepsilon_0 V}\right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) q_m(t)$$

$$\hat{E}_x^m(z,t) = \left(\frac{2M_m\omega_m^2}{\varepsilon_0 V}\right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) \hat{q}_m(t)$$

$$B_y^m(z,t) = \frac{1}{c}\left(\frac{2M_m}{\varepsilon_0 V}\right)^{1/2} \cos\left(\frac{m\pi z}{L}\right) \dot{q}_m(t)$$

$$\hat{B}_y^m(z,t) = \frac{1}{c}\left(\frac{2M_m}{\varepsilon_0 V}\right)^{1/2} \cos\left(\frac{m\pi z}{L}\right) \hat{\dot{q}}_m(t)$$

$$\left[\frac{E_x^m}{B_y^m}\right] = [c]\left[\left(\frac{\cancel{2M_m}\omega_m^2 \cancel{\varepsilon_0 V}}{\cancel{\varepsilon_0 V}\,\cancel{2M_m}}\right)^{1/2}\right] \frac{m}{\frac{m}{s}}$$

$$= [c]\,[\omega_m]\, s = \frac{m}{s} \cdot \frac{1}{\cancel{s}} \cancel{s} = \frac{m}{s}$$

$$= [c]$$