Είχαμε δείξει πώς για την κατάσταση

$$|\psi_A(t)\rangle = c_1(t)|\downarrow n\rangle + c_2(t)|\uparrow n-1\rangle = |A\rangle$$

$$\langle \hat{a}^\dagger \hat{a}\rangle = n - |c_2(t)|^2 \quad \Big\}$$
$$\langle \hat{a}\hat{a}^\dagger\rangle = n + |c_1(t)|^2 \quad \Big\} \Rightarrow \langle \hat{a}\hat{a}^\dagger\rangle - \langle \hat{a}^\dagger \hat{a}\rangle = 1$$

$$\langle \hat{S}_+ \hat{S}_-\rangle = |c_2(t)|^2 \quad \Big\}$$
$$\langle \hat{S}_- \hat{S}_+\rangle = |c_1(t)|^2 \quad \Big\} \Rightarrow \langle \hat{S}_+ \hat{S}_-\rangle + \langle \hat{S}_- \hat{S}_+\rangle = 1$$

$$\langle \hat{a}^\dagger \hat{a}\rangle + \langle \hat{S}_+ \hat{S}_-\rangle = n$$

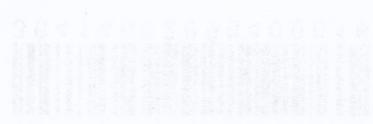$$\langle \hat{S}_+ \hat{a}\rangle = c_2^*(t)\, c_1(t)\sqrt{n} \qquad\qquad \langle \hat{S}_+ \hat{a}^\dagger\rangle = \varnothing$$

$$\langle \hat{S}_- \hat{a}^\dagger\rangle = c_1^*(t)\, c_2(t)\sqrt{n} \qquad\qquad \langle \hat{S}_- \hat{a}\rangle = 0$$

$$|\Psi_A(t)\rangle = c_1(t)\,|\downarrow n\rangle + c_2(t)\,|\uparrow n-1\rangle$$

$$i\hbar\frac{\partial}{\partial t}|\Psi_A(t)\rangle = \hat{H}|\Psi(t)\rangle$$

$$\hat{H} = \hat{H}_{JCM} = \hbar\omega_m\,\hat{a}_m^\dagger\hat{a}_m + \hbar\Omega\,\hat{S}_+\hat{S}_- + \hbar g_m(\hat{S}_+\hat{a}_m + \hat{S}_-\hat{a}_m^\dagger)$$

A.Σ. $c_1(0)=1$, $c_2(0)=0$

$$A' = i\hbar\frac{\partial}{\partial t}|\Psi_A(t)\rangle = \left(\dot{c_1}(t)\,|\downarrow n\rangle + \dot{c_2}(t)\,|\uparrow n-1\rangle\right)i\hbar$$

$$\Delta' = \left(\hbar\omega_m\,\hat{a}_m^\dagger\hat{a}_m + \hbar\Omega\,\hat{S}_+\hat{S}_- + \hbar g_m\,\hat{S}_+\hat{a} + \hbar g_m\,\hat{S}_-\hat{a}^\dagger\right)\left(c_1(t)\,|\downarrow n\rangle + c_2(t)\,|\uparrow n-1\rangle\right) =$$

$$= \hbar\omega_m\,c_1(t)\,n\,|\downarrow n\rangle + \hbar\omega_m\,c_2(t)\,(n-1)\,|\uparrow n-1\rangle +$$

$$\hbar\Omega\,c_1(t)\,|0\,n\rangle + \hbar\Omega\,c_2(t)\,|\uparrow n-1\rangle +$$

$$\hbar g_m\,c_1(t)\,\sqrt{n}\,|\uparrow n-1\rangle + \hbar g_m\,c_2(t)\,\sqrt{n-1}\,|0\,n-2\rangle +$$

$$\hbar g_m\,c_1(t)\,\sqrt{n+1}\,|0\,n+1\rangle + \hbar g_m\,c_2(t)\,\sqrt{n}\,|\downarrow n\rangle =$$

επί $\langle\downarrow n|$ $A' = i\hbar\,\dot{c_1}(t)$

$$\Delta' = \hbar\omega_m\,c_1(t)\,n + \hbar g_m\,c_2(t)\,\sqrt{n}$$

$$\Big\{\;i\hbar\,\dot{c_1}(t) = \hbar\omega_m\,c_1(t) + \hbar g_m\sqrt{n}\,c_2(t)\;\Big\}$$

επί $\langle\uparrow n-1|$ $A' = i\hbar\,\dot{c_2}(t)$

$$\Delta' = \hbar\omega_m\,c_2(t)\,(n-1) + \hbar\Omega\,c_2(t) + \hbar g_m\sqrt{n}\,c_1(t)\;\Big\} \Rightarrow$$

$$\Big\{\;i\hbar\,\dot{c_2}(t) = \hbar g_m\sqrt{n}\,c_1(t) + \big[\hbar\omega_m(n-1) + \hbar\Omega\big]\,c_2(t)\;\Big\}$$

$$i\begin{bmatrix}\dot{c_1}(t)\\[4pt]\dot{c_2}(t)\end{bmatrix} = \begin{bmatrix} n\omega_m & \sqrt{n}\,g_m \\[4pt] \sqrt{n}\,g_m & \Omega+(n-1)\omega_m \end{bmatrix}\begin{bmatrix}c_1(t)\\[4pt]c_2(t)\end{bmatrix}$$

υπενθυμίζεται πως για απλότητα
$\omega = \omega_m$ "ξεχνάμε"
$g = g_m$ το δείκτη m
$n = n_m$

Ορίζουμε $\Omega_n := \left[\left(\dfrac{\omega-\Omega}{2}\right)^2 + g^2 n\right]^{1/2} = \sqrt{\left(\dfrac{\Delta}{2}\right)^2 + g^2 n}$

και παραλείποντας τις πράξεις προκύπτει

$$c_1(t) = \exp\left[-i\left(n\omega + \frac{\Omega-\omega}{2}\right)t\right]\left\{\cos(\Omega_n t) + i\,\frac{\Omega-\omega}{2\Omega_n}\cdot\sin(\Omega_n t)\right\}$$

$$c_2(t) = \exp\left[-i\left(n\omega + \frac{\Omega-\omega}{2}\right)t\right]\left\{-i\,\frac{g\sqrt{n}}{\Omega_n}\cdot\sin(\Omega_n t)\right\}$$

"Άρα $\boxed{|C_2(t)|^2 = \dfrac{ng^2}{\Omega_n^2}\sin^2(\Omega_n t)}$ $\qquad$ $|C_1(t)|^2 = 1 - |C_2(t)|^2 = \cdots$

$\boxed{\langle \hat{a}^+\hat{a}\rangle_A = n - \dfrac{ng^2}{\Omega_n^2}\sin^2(\Omega_n t)}$ $\qquad$ $\boxed{\langle \hat{S}_+\hat{S}_-\rangle_A = \dfrac{ng^2}{\Omega_n^2}\cdot\sin^2(\Omega_n t)}$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \;\Rightarrow\; \sin^2 x = \dfrac{1-\cos 2x}{2}$$

$\langle \hat{a}^+\hat{a}\rangle_A = n - \dfrac{ng^2}{2\Omega_n^2} + \dfrac{ng^2}{2\Omega_n^2}\cos(2\Omega_n t)$

$\langle \hat{S}_+\hat{S}_-\rangle_A = \dfrac{ng^2}{2\Omega_n^2} - \dfrac{ng^2}{2\Omega_n^2}\cos(2\Omega_n t)$

maximum transfer percentage $\qquad$ $d_R = \dfrac{ng^2}{\Omega_n^2} = \dfrac{ng^2}{\dfrac{\Delta^2}{4}+ng^2} = \dfrac{4ng^2}{4ng^2+\Delta^2}$

μέγιστο ποσοστό μεταβιβάσεως

περίοδος ταλαντώσεων $\qquad$ $T_R = \dfrac{2\pi}{2\Omega_n} = \dfrac{\pi}{\Omega_n} = \dfrac{2\pi}{2\sqrt{\dfrac{\Delta^2}{4}+g^2 n}} = \dfrac{2\pi}{\sqrt{\Delta^2+4ng^2}}$

$\Omega_n = \sqrt{\dfrac{\Delta^2}{4}+g^2 n}$ $\qquad$ $\Omega_R := 2\sqrt{n}\,g$ $\quad$ (κυκλική) συχνότητα Rabi

$2\Omega_n = \sqrt{\Delta^2+4ng^2}$ $\qquad$ $d_R = \dfrac{\Omega_R^2}{\Omega_R^2+\Delta^2}$

$2\Omega_n = \sqrt{\Delta^2+\Omega_R^2}$ $\qquad$ $T_R = \dfrac{2\pi}{\sqrt{\Delta^2+\Omega_R^2}} = \dfrac{1}{f_R}$

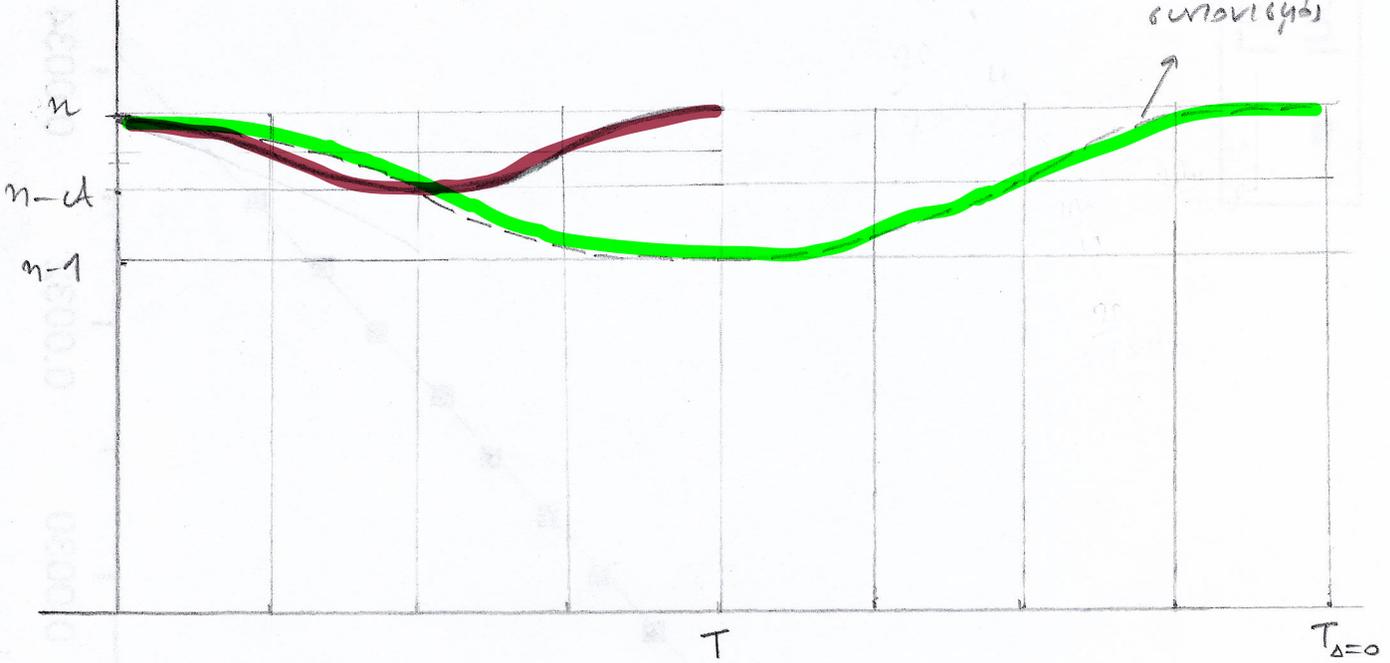Δηλαδή η (κυκλική) συχνότητα Rabi Ω_R και ο αποσυντονισμός Δ καθορίζουν την περίοδο και το μέγιστο ποσοστό μεταβιβάσεως

Άν $\Delta = 0 \Rightarrow d_R = 1$

$$T_R = \dfrac{2\pi}{\Omega_R}$$

$\langle \hat{a}^+ a \rangle$ Ⓐ

n

$n - \alpha$

$n - 1$

one cycle

T

$T_{\Delta = 0}$

$\langle \hat{S}_+ \hat{S}_- \rangle$ Ⓐ

1

$\alpha = \dfrac{n g^2}{2 \Omega_n^2}$

$T_R = \dfrac{2\pi}{2\Omega_n}$

T

$T_{\Delta = 0}$

⑧'
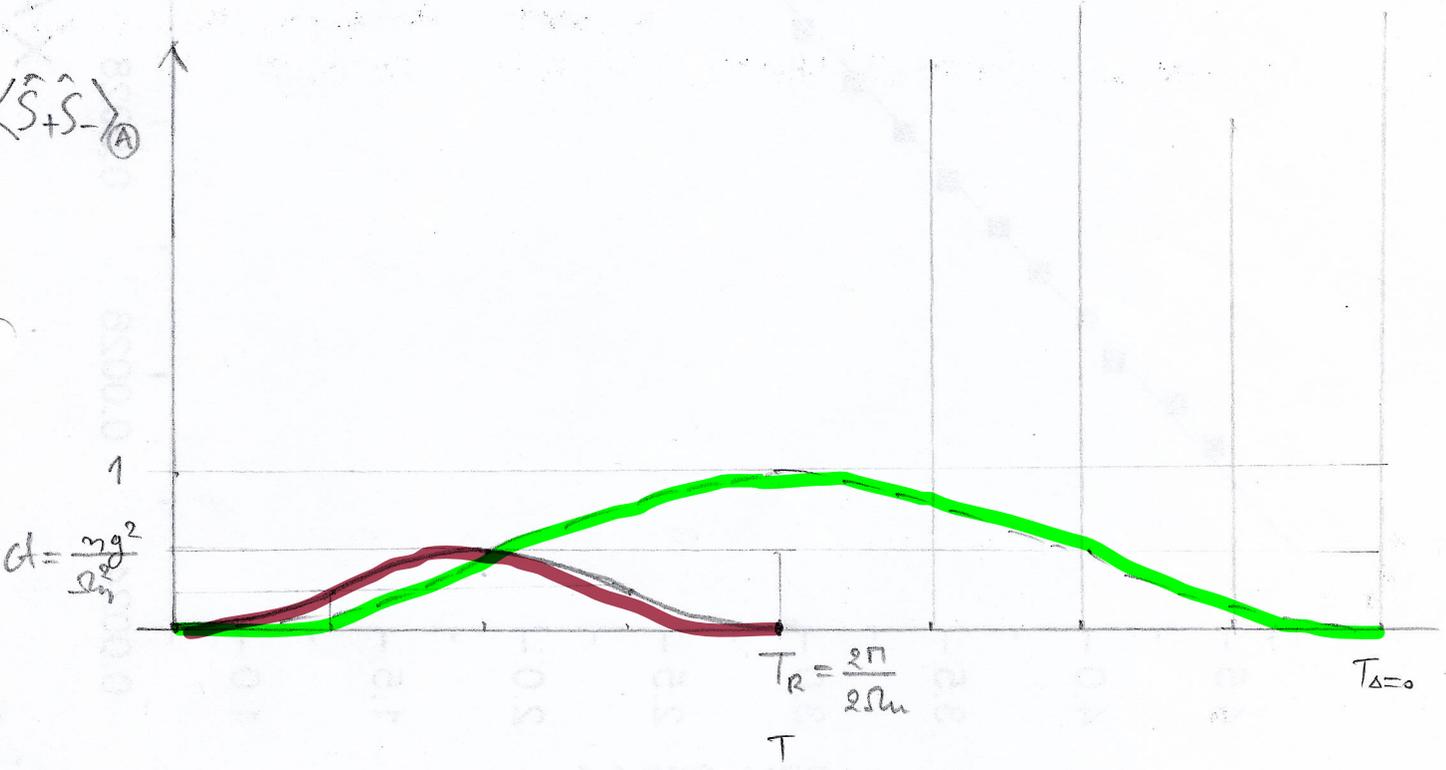
$$i \begin{pmatrix} \dot{c_1} \\ \dot{c_2} \end{pmatrix} = \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \vec{x}(t) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \dot{\vec{x}}(t) = \begin{pmatrix} \dot{c_1} \\ \dot{c_2} \end{pmatrix} \qquad \text{⑤}$$

$$\underbrace{\qquad\qquad}_{A}$$

$$i \dot{\vec{x}}(t) = A \vec{x}(t) \implies \boxed{\dot{\vec{x}}(t) = -i A \vec{x}(t)}$$

$$\Delta\Lambda M \quad \vec{x}(t) = \vec{u}\, e^{-i\lambda t} \implies \dot{\vec{x}}(t) = \vec{u}(-i\lambda) e^{-i\lambda t} \Biggr\} \implies \vec{u}(-i\lambda)\, \overline{e^{-i\lambda t}} = -i A \vec{u}\, \overline{e^{-i\lambda t}}$$

$$\implies \boxed{A\vec{u} = \lambda \vec{u}} \quad \text{⊛} \quad \Longleftrightarrow \quad A\vec{u} = \lambda I \vec{u} \Longleftrightarrow (A - \lambda I)\vec{u} = 0 \qquad 0 = \vec{u}(I\lambda - A)$$

πρόβλημα ιδιοτιμών

πρέπει $\det(A - \lambda I) = 0$

$$\begin{vmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{vmatrix} = 0 \implies (n\omega - \lambda)\left[\Omega + (n-1)\omega - \lambda\right] - n g^2 = 0$$

$$\lambda^2 - \left[\Omega + (n-1)\omega + n\omega\right]\lambda + n\omega\left[\Omega + (n-1)\omega\right] - n g^2 = 0$$

Έστω $n = 1$
ένα φωτόνιο στην κοιλότητα

$$A = \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix} \qquad \text{και} \qquad \det(A - \lambda I) = 0 \implies$$

$$\begin{vmatrix} \omega - \lambda & g \\ g & \Omega - \lambda \end{vmatrix} = 0 \implies$$

$$\text{⊛} \begin{bmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \lambda \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$(\omega - \lambda)(\Omega - \lambda) - g^2 = 0$$

$$\lambda^2 - (\omega + \Omega)\lambda + \omega\Omega - g^2 = 0$$

$$\begin{bmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Delta' = (\omega + \Omega)^2 - 4(\omega\Omega - g^2) =$$
$$= \omega^2 + \Omega^2 + 2\omega\Omega - 4\omega\Omega + 4g^2$$
$$= (\omega - \Omega)^2 + 4g^2 \implies$$

$$\Delta' = \Delta^2 + 4g^2$$

$$\lambda_{2,1} = \frac{(\omega + \Omega) \pm \sqrt{\Delta^2 + 4g^2}}{2}$$

$$\lambda_{2,1} = \frac{\omega + \Omega}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + g^2}$$

$$\boxed{\lambda_{2,1} = H_1 \pm \Omega_1}$$

$$H_1 = \frac{\omega + \Omega}{2}$$

$$\Omega_1 = \sqrt{\left(\frac{\Delta}{2}\right)^2 + g^2}$$

$\boxed{\lambda_1 = H_1 - \Omega_1}$ $\qquad A\vec{U}_1 = \lambda_1 \vec{U}_1 \Rightarrow \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix}\begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix} = (H_1 - \Omega_1)\begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix}$

$\Rightarrow \omega U_{11} + g U_{21} = (H_1 - \Omega_1) U_{11} \Rightarrow g U_{21} = (H_1 - \Omega_1 - \omega) U_{11}$

$g U_{11} + \Omega U_{21} = (H_1 - \Omega_1) U_{21} \Rightarrow g U_{11} = (H_1 - \Omega_1 - \Omega) U_{21}$

$g U_{21} = \dfrac{(H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) U_{21}}{g}$

$g^2 U_{21} = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) U_{21}$

Άρα $U_{21} = 0 \Rightarrow U_{11} = 0$

ή $g^2 = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) \Rightarrow$

$g^2 = \left(\dfrac{\Omega - \omega}{2} - \Omega_1\right)\left(\dfrac{\omega - \Omega}{2} - \Omega_1\right) \Rightarrow$

$g^2 = -\left(\dfrac{\Delta}{2} + \Omega_1\right)\left(\dfrac{\Delta}{2} - \Omega_1\right) \Rightarrow g^2 = -\left[\dfrac{\Delta^2}{4} - \Omega_1^2\right] \Rightarrow$

$g^2 + \dfrac{\Delta^2}{4} = \Omega_1^2$ που ισχύει εξ ορισμού της Ω_1

$\boxed{\text{δηλαδή το } U_{21} \text{ μπορεί να είναι οτιδήποτε π.χ. } U_{21} = 1} \Rightarrow$

$g U_{11} = (H_1 - \Omega_1 - \Omega) \cdot 1 = \dfrac{\Delta}{2} - \Omega_1 \Rightarrow U_{11} = \dfrac{\frac{\Delta}{2} - \Omega_1}{g} \Rightarrow$

$\boxed{U_{11} = \dfrac{\Delta - 2\Omega_1}{2g}}$

$\vec{U}_1 = \begin{bmatrix} \dfrac{\Delta - 2\Omega_1}{2g} \\ 1 \end{bmatrix}$

$\boxed{\lambda_2 = H_1 + \Omega_1} \Rightarrow \text{πράξεις} \Rightarrow \vec{U}_2 = \begin{bmatrix} \dfrac{\Delta + 2\Omega_1}{2g} \\ 1 \end{bmatrix}$

$$\boxed{\lambda_2 = H_1 + \Omega_1}$$

$$A\vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix}\begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_1 + \Omega_1)\begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix}$$

$$\Rightarrow \omega U_{12} + g U_{22} = (H_1 + \Omega_1)U_{12} \Rightarrow g U_{22} = (H_1 + \Omega_1 - \omega)U_{12}$$

$$g U_{12} + \Omega U_{22} = (H_1 + \Omega_1)U_{22} \Rightarrow g U_{12} = (H_1 + \Omega_1 - \Omega)U_{22}$$

$$g U_{22} = \frac{(H_1 + \Omega_1 - \omega)(H_1 + \Omega_1 - \Omega)}{g} U_{22}$$

"Άρα $U_{22} = \emptyset$ ($\Rightarrow U_{12} = \emptyset$)

"ή

$$g^2 = (H_1 + \Omega_1 - \omega)(H_1 + \Omega_1 - \Omega)$$

$$g^2 = \left(\frac{\omega + \Omega - 2\omega}{2} + \Omega_1\right)\left(\frac{\omega + \Omega - 2\Omega}{2} + \Omega_1\right)$$

$$g^2 = \left(\frac{\Omega - \omega}{2} + \Omega_1\right)\left(\frac{\omega - \Omega}{2} + \Omega_1\right)$$

$$g^2 = \left(\Omega_1 - \frac{\omega - \Omega}{2}\right)\left(\Omega_1 + \frac{\omega - \Omega}{2}\right) = \left(\Omega_1 - \frac{\Delta}{2}\right)\left(\Omega_1 + \frac{\Delta}{2}\right)$$

$$g^2 = \Omega_1^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow \Omega_1^2 = \left(\frac{\Delta}{2}\right)^2 + g^2$$

Το οποίον ?ισχύει 'ξ ορισμός τω Ω_1

$$\boxed{\delta\eta\lambda\alpha\delta\acute{\eta} \text{ το } U_{22} \text{ μπορεί να είναι οτιδήποτε π.χ. } U_{22} = 1}$$

$$g U_{12} = (H_1 + \Omega_1 - \Omega)\cdot 1 = \frac{\omega + \Omega - 2\Omega}{2} + \Omega_1 = \frac{\omega - \Omega}{2} + \Omega_1$$

$$\boxed{U_{12} = \frac{\Delta + 2\Omega_1}{2g}}$$

$$\vec{U}_2 = \boxed{\begin{array}{c} \frac{\Delta + 2\Omega_1}{2g} \\ 1 \end{array}}$$

Η γενική λύση είναι $\vec{X}(t) = \sigma_1 \vec{u}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{u}_2 e^{-i\lambda_2 t}$

$$\vec{X}(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} \sigma_1 \dfrac{\Delta - 2\Omega_1}{2g} e^{-i(H_1 - \Omega_1)t} + \sigma_2 \cdot \dfrac{\Delta + 2\Omega_1}{2g} e^{-i(H_1 + \Omega_1)t} \\[4mm] \sigma_1 \cdot 1 \cdot e^{-i(H_1 - \Omega_1)t} + \sigma_2 \cdot 1 \cdot e^{-i(H_1 + \Omega_1)t} \end{bmatrix}$$

ΑΡΧΙΚΕΣ ΣΥΝΘΗΚΕΣ $\quad c_1(0) = 1 \quad c_2(0) = 0 \quad \Rightarrow$

$1 = \sigma_1 \dfrac{\Delta - 2\Omega_1}{2g} + \sigma_2 \dfrac{\Delta + 2\Omega_1}{2g}$ $\left. \begin{array}{l} \\ \\ \end{array} \right\}$ $\Rightarrow 2g = \sigma_1(\Delta - 2\Omega_1) - \sigma_1(\Delta + 2\Omega_1)$

$0 = \sigma_1 + \sigma_2 \Rightarrow \sigma_2 = -\sigma_1$ $\qquad 2g = -2\Omega_1\sigma_1 - 2\Omega_1\sigma_1$

$g = -2\Omega_1\sigma_1 \Rightarrow \boxed{\sigma_1 = -\dfrac{g}{2\Omega_1} = -\sigma_2}$

Άρα $c_2(t) = -\dfrac{g}{2\Omega_1} e^{-i(H_1 - \Omega_1)t} + \dfrac{g}{2\Omega_1} e^{-i(H_1 + \Omega_1)t} \Rightarrow$

$c_2(t) = -\dfrac{g}{2\Omega_1} e^{-iH_1 t} e^{i\Omega_1 t} + \dfrac{g}{2\Omega_1} e^{-iH_1 t} e^{-i\Omega_1 t}$

$c_2(t) = \dfrac{g}{2\Omega_1} e^{-iH_1 t} \left\{ -\cos(\Omega_1 t) - i\sin(\Omega_1 t) + \cos(\Omega_1 t) - i\sin(\Omega_1 t) \right\}$

$c_2(t) = \dfrac{g}{2\Omega_1} e^{-iH_1 t} (-2i) \sin(\Omega_1 t) = e^{-i\frac{\omega + \Omega}{2} t} \left\{ -i \dfrac{g}{\Omega_1} \sin(\Omega_1 t) \right\}$

$$\boxed{|c_2(t)|^2 = \dfrac{g^2}{\Omega_1^2} \sin^2(\Omega_1 t)}$$

$|c_1(t)|^2 = 1 - |c_2(t)|^2 = 1 - \dfrac{g^2}{\Omega_1^2}\left(1 - \cos^2(\Omega_1 t)\right)$

$= \left(1 - \dfrac{g^2}{\Omega_1^2}\right) + \dfrac{g^2}{\Omega_1^2}\cos^2(\Omega_1 t)$

$\Omega_1^2 = \dfrac{\Delta^2}{4} + g^2 \qquad \Omega_1^2 - g^2 = \dfrac{\Delta^2}{4}$

$$\boxed{|c_1(t)|^2 = \dfrac{\left(\dfrac{\Delta^2}{4}\right)}{\Omega_1^2} + \dfrac{g^2}{\Omega_1^2}\cos^2(\Omega_1 t)}$$

$$A = \begin{bmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{bmatrix} \quad \text{και} \quad \det(A - \lambda I) = 0 \Rightarrow$$

$$\begin{vmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (n\omega - \lambda)\left[\Omega + (n-1)\omega - \lambda\right] - ng^2 = 0$$

$$\lambda^2 - \lambda\left[\Omega + (n-1)\omega + n\omega\right] + n\omega\left[\Omega + (n-1)\omega\right] - ng^2 = 0$$

$$\Delta' = \left[\Omega + (n-1)\omega + n\omega\right]^2 - 4\left(n\omega\left[\Omega + (n-1)\omega\right] - ng^2\right)$$

$$\Delta' = \left[\Omega + (n-1)\omega + n\omega\right]^2 - 4n\omega\left[\Omega + (n-1)\omega\right] + 4ng^2$$

$$\Delta' = \left[\Omega + (n-1)\omega - n\omega\right]^2 + 4ng^2 > 0$$

$$\lambda_{2,1} = \frac{\left[\Omega + (n-1)\omega + n\omega\right] \pm \sqrt{\left[\Omega + (n-1)\omega - n\omega\right]^2 + 4ng^2}}{2}$$

$$\lambda_{2,1} = \frac{\Omega + (n-1)\omega + n\omega}{2} \pm \sqrt{\left(\frac{\Omega + (n-1)\omega - n\omega}{2}\right)^2 + ng^2}$$

$$\lambda_{2,1} = \frac{\Omega + (n-1)\omega + n\omega}{2} \pm \sqrt{\left(\frac{\omega - \Omega}{2}\right)^2 + ng^2}$$

$$\boxed{\lambda_{2,1} = H_n \pm \Omega_n} \qquad H_n = \frac{\Omega + (n-1)\omega + n\omega}{2}$$

$$\Omega_n = \sqrt{\left(\frac{\Delta}{2}\right)^2 + ng^2}$$

$$\boxed{\lambda_1 = H_n - \Omega_n} \qquad A\vec{u}_1 = \lambda_1 \vec{u}_1 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega \end{pmatrix}\begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix} = (H_n-\Omega_n)\begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix} \; ⓘ$$

$$n\omega\, U_{11} + g\sqrt{n}\, U_{21} = (H_n-\Omega_n)U_{11}$$

$$g\sqrt{n}\, U_{11} + [\Omega+(n-1)\omega]\, U_{21} = (H_n-\Omega_n)U_{21}$$

$$g\sqrt{n}\, U_{21} = (H_n-\Omega_n - n\omega)\, U_{11}$$

$$g\sqrt{n}\, U_{11} = \Big(H_n-\Omega_n - [\Omega+(n-1)\omega]\Big)\, U_{21}$$

$$g\sqrt{n}\, U_{21} = \frac{(H_n-\Omega_n-n\omega)\Big(H_n-\Omega_n-[\Omega+(n-1)\omega]\Big)\, U_{21}}{g\sqrt{n}}$$

$$\text{Άρα}\quad U_{21} = 0 \;\overset{\text{ή}}{\underset{}{\quad}}\; g^2 n = (H_n-\Omega_n-n\omega)\Big(H_n-\Omega_n-[\Omega+(n-1)\omega]\Big)$$

$$\left(\;\overset{\Downarrow}{U_{11}=0}\;\right)$$

$$H_n-\Omega_n-n\omega = \frac{\Omega+(n-1)\omega+n\omega-2n\omega}{2} - \Omega_n$$

$$= \frac{\Omega-\omega}{2} - \Omega_n = -\frac{\Delta}{2} - \Omega_n$$

$$H_n-\Omega_n-[\Omega+(n-1)\omega] = \frac{\Omega+(n-1)\omega+n\omega-2\Omega-2(n-1)\omega}{2} - \Omega_n$$

$$= \frac{\omega-\Omega}{2} - \Omega_n = \frac{\Delta}{2} - \Omega_n$$

$$\Rightarrow g^2 n = -\left(\frac{\Delta}{2}+\Omega_n\right)\cdot\left(\frac{\Delta}{2}-\Omega_n\right) = -\left(\frac{\Delta}{2}\right)^2 + \Omega_n^2$$

$$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + ng^2}\quad \text{το οποίον ισχύει εξ ορισμού του } \Omega_n$$

$$\boxed{\text{δηλαδή το } U_{21} \text{ μπορεί να είναι ὁτιδήποτε πχ. } U_{21} = 1}$$

$$g\sqrt{n}\, U_{11} = H_n-\Omega_n-[\Omega+(n-1)\omega] = \frac{\Delta}{2} - \Omega_n$$

$$\boxed{U_{11} = \frac{\Delta-2\Omega_n}{2g\sqrt{n}}}$$

$$\boxed{\vec{u}_1 = \begin{bmatrix} \dfrac{\Delta-2\Omega_n}{2g\sqrt{n}} \\[2mm] 1 \end{bmatrix}}$$

$$\boxed{\lambda_2 = H_n + \Omega_n} \qquad A\vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix} \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_n + \Omega_n) \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix}$$

$$n\omega U_{12} + g\sqrt{n}\, U_{22} = (H_n + \Omega_n) U_{12}$$

$$g\sqrt{n}\, U_{12} + \left[\Omega + (n-1)\omega\right] U_{22} = (H_n + \Omega_n) U_{22}$$

$$g\sqrt{n}\, U_{22} = (H_n + \Omega_n - n\omega) U_{12}$$

$$g\sqrt{n}\, U_{12} = \left(H_n + \Omega_n - \left[\Omega + (n-1)\omega\right]\right) U_{22}$$

$$g\sqrt{n}\, U_{22} = \frac{(H_n + \Omega_n - n\omega)\left(H_n + \Omega_n - \left[\Omega + (n-1)\omega\right]\right)}{g\sqrt{n}} U_{22}$$

$$\overset{U_{22}=0}{_{\displaystyle n_?}} \quad \overset{\text{ή}}{} \quad g^2 n = (H_n + \Omega_n - n\omega)\left(H_n + \Omega_n - \left[\Omega + (n-1)\omega\right]\right)$$

$$\left(\underset{U_{12}=0}{\Downarrow}\right)$$

$$H_n + \Omega_n - n\omega = \frac{\Omega + (n-1)\omega + n\omega - 2n\omega}{2} + \Omega_n$$

$$= \frac{\Omega - \omega}{2} + \Omega_n = -\frac{\Delta}{2} + \Omega_n$$

$$H_n + \Omega_n - \left[\Omega + (n-1)\omega\right] = \frac{\Omega + (n-1)\omega + n\omega - 2\Omega - 2(n-1)\omega}{2} + \Omega_n$$

$$= \frac{\omega - \Omega}{2} + \Omega_n = \frac{\Delta}{2} + \Omega_n$$

$$g^2 n = \left(\Omega_n + \frac{\Delta}{2}\right)\left(\Omega_n - \frac{\Delta}{2}\right) = \Omega_n^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow$$

$$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + n g^2} \quad \text{Το οποίον ισχύει εξ ορισμού του } \Omega_n$$

$$\boxed{\text{δηλαδή το } U_{22} \text{ μπορεί να είναι δ,τιδήποτε ο.χ. } U_{22} = 1}$$

$$g\sqrt{n}\, U_{12} = H_n + \Omega_n - \left[\Omega + (n-1)\omega\right] = \frac{\Delta}{2} + \Omega_n \Rightarrow$$

$$\boxed{U_{12} = \frac{\Delta + 2\Omega_n}{2g\sqrt{n}}}$$

$$\boxed{\vec{U}_2 = \begin{bmatrix} \dfrac{\Delta + 2\Omega_n}{2g\sqrt{n}} \\[4mm] 1 \end{bmatrix}}$$

Η γενική λύση είναι $\vec{x}(t) = \sigma_1 \vec{v}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{v}_2 e^{-i\lambda_2 t}$

$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sigma_1 \begin{bmatrix} \dfrac{\Delta - 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix} e^{-i(H_n - \Omega_n)t} + \sigma_2 \begin{bmatrix} \dfrac{\Delta + 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix} e^{-i(H_n + \Omega_n)t}$$

Αρχικές Συνθήκες $C_1(0) = 1$ $C_2(0) = 0$

$$\sigma_1 \frac{\Delta - 2\Omega_n}{2g\sqrt{n}} + \sigma_2 \frac{\Delta + 2\Omega_n}{2g\sqrt{n}} = 1 \left.\right\} \Rightarrow \sigma_1 \frac{\Delta - 2\Omega_n - \Delta - 2\Omega_n}{2g\sqrt{n}} = 1$$

$$\sigma_1 + \sigma_2 = 0 \Rightarrow \sigma_2 = -\sigma_1$$

$$\Rightarrow \sigma_1 \frac{-4\Omega_n}{2g\sqrt{n}} = 1 \Rightarrow \boxed{\sigma_1 = \frac{-g\sqrt{n}}{2\Omega_n} = -\sigma_2}$$

$$C_2(t) = \frac{-g\sqrt{n}}{2\Omega_n} e^{-i(H_n - \Omega_n)t} + \frac{g\sqrt{n}}{2\Omega_n} e^{-i(H_n + \Omega_n)t}$$

$$C_2(t) = \frac{g\sqrt{n}}{2\Omega_n} e^{-iH_n t} \left[e^{-i\Omega_n t} - e^{i\Omega_n t} \right] \Rightarrow C_2(t) = \frac{g\sqrt{n}}{2\Omega_n} e^{-iH_n t} (-2i) \sin(\Omega_n t)$$

$$C_2(t) = -i \frac{g\sqrt{n}}{\Omega_n} e^{-iH_n t} \cdot \sin(\Omega_n t)$$

$$\boxed{|C_2(t)|^2 = \frac{n g^2}{\Omega_n^2} \sin^2(\Omega_n t)}$$