

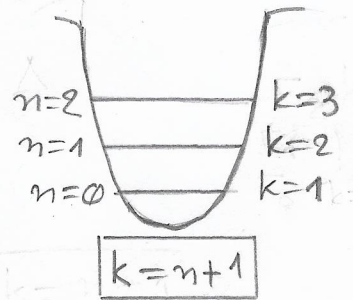
ΤΡΙΣΤΑΘΜΙΚΟ ΜΕ ΑΑΤ

$$\dot{C}_k = -\frac{i}{\hbar} \sum_k C_k(t) e^{i(\Omega_k - \Omega_k)t} U_{\epsilon k'k}(t) \quad (1)$$

AAT

$$E_n = \hbar \Omega \left(n + \frac{1}{2} \right) \quad E_{n+1} - E_n = \hbar \Omega$$

$$E_0 = \frac{\hbar \Omega}{2}, \quad E_1 = \frac{3\hbar \Omega}{2}, \quad E_2 = \frac{5\hbar \Omega}{2}$$



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2} \hat{z}^2$$

* ΔΕΙΤΕ (1)
Υποτίθεται πως $\vec{\Phi}_n(\vec{r}) = X_1(x) Y_1(y) Z_n(z)$

As θεωρούμε ότι γράφουμε να περιοριστούμε στις 3 κενώστερες στάθμες ως ΑΑΤ

$$U_{\epsilon k'k}(t) = e E_0 \cos \omega t z_{k'k}$$

$$z_{kk} = \int d^3r \Phi_k^*(\vec{r}) z \Phi_k(\vec{r})$$

$$z_{kk} = \int d^3r |\Phi_k(\vec{r})|^2 z = 0$$

$$z_{12} = \int d^3r \underbrace{\Phi_1^*(\vec{r})}_A z \underbrace{\Phi_2(\vec{r})}_\Pi \neq 0$$

$$\parallel z_{21} = \int d^3r \Phi_2^*(\vec{r}) z \Phi_1(\vec{r}) \neq 0$$

$$z_{13} = \int d^3r \underbrace{\Phi_1^*(\vec{r})}_A z \underbrace{\Phi_3(\vec{r})}_\Pi = 0$$

$$\parallel z_{31} = \dots = 0$$

$$z_{23} = \int d^3r \underbrace{\Phi_2^*(\vec{r})}_\Pi z \underbrace{\Phi_3(\vec{r})}_\Pi \neq 0$$

$$\parallel z_{32} = z_{23}$$

$$-e z_{12} = -e z_{21} \Rightarrow \mathcal{P}_{z_{12}} = \mathcal{P}_{z_{21}} := \mathcal{P}$$

2 διακυματισμοί 1Δ ΑΑΤ $a = \left(\frac{\hbar}{m\Omega}\right)^{1/2}$
 $Z_n(z) = u_n(z) \exp\left(-\frac{m\Omega z^2}{2\hbar}\right)$

k	n	$u_n(z)$
1 (A)	0	$(1/a\sqrt{\pi})^{1/2}$
2 (Π)	1	$(1/2a\sqrt{\pi})^{1/2} z/a$
3 (A)	2	$(1/8a\sqrt{\pi})^{1/2} [2 - 4(\frac{z}{a})^2]$
4 (Π)	3	$(1/48a\sqrt{\pi})^{1/2} [12(\frac{z}{a}) - 8(\frac{z}{a})^3]$
n		$(1/n! 2^n a\sqrt{\pi})^{1/2} H_n(\frac{z}{a})$

↑
πολυώνυμα Hermite

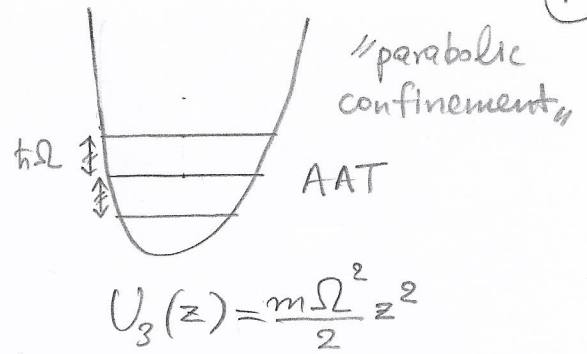
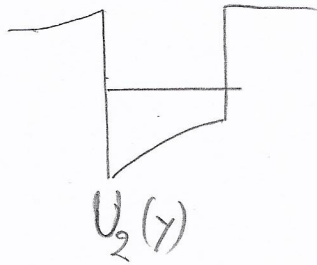
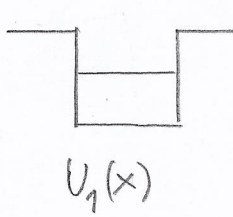
μάλιστα...

$$z_{12} = z_{21} \neq z_{23} = z_{32}$$

$$U_{\epsilon 12}(t) = e E_0 \cos \omega t z_{12} = -\mathcal{P}_{z_{12}} E_0 \cos \omega t$$

$$U_{\epsilon 21}(t) = e E_0 \cos \omega t z_{21} = -\mathcal{P}_{z_{21}} E_0 \cos \omega t$$

7. X



7'

$$U(\vec{r}) = U_1(x) + U_2(y) + U_3(z)$$

x $\omega \rho \bar{e}$	x $\omega \rho \bar{e}$	AAT
1 μ νο	1 μ νο	...
σείδυν	σείδυν	

$$\Phi_m(\vec{r}) = X_1(x) Y_1(y) Z_n(z)$$

ΑΣΚΗΣΗ Να υπολογιστεί ο λόγος $\frac{\mathcal{P}}{\mathcal{P}'}$ = $\frac{\Omega_R}{\Omega_k}$ σε αντίστοιχα σύστημα.

$$\boxed{\mathcal{F}_{223} = \mathcal{F}_{232} := \mathcal{F}'}$$

$$= -e z_{22} = -e z_{32}$$

→ οπότες... (2)

$$\dot{C}_1(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_1 - \Omega_1)t} U_{\mathcal{F}11}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_1 - \Omega_2)t} U_{\mathcal{F}12}(t)$$

$$- \frac{i}{\hbar} C_3(t) e^{i(\Omega_1 - \Omega_3)t} U_{\mathcal{F}13}(t)$$

$$\boxed{\dot{C}_1(t) = +\frac{i}{\hbar} C_2(t) e^{-i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t} \quad (1)$$

$$\dot{C}_2(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_2 - \Omega_1)t} U_{\mathcal{F}21}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_2 - \Omega_2)t} U_{\mathcal{F}22}(t)$$

$$- \frac{i}{\hbar} C_3(t) e^{i(\Omega_2 - \Omega_3)t} U_{\mathcal{F}23}(t)$$

$$\boxed{\dot{C}_2(t) = +\frac{i}{\hbar} C_1(t) e^{i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t + \frac{i}{\hbar} C_3(t) e^{-i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t} \quad (2)$$

$$\dot{C}_3(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_3 - \Omega_1)t} U_{\mathcal{F}31}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_3 - \Omega_2)t} U_{\mathcal{F}32}(t)$$

$$- \frac{i}{\hbar} C_3(t) e^{i(\Omega_3 - \Omega_3)t} U_{\mathcal{F}33}(t)$$

$$\boxed{\dot{C}_3(t) = +\frac{i}{\hbar} C_2(t) e^{i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t} \quad (3)$$

μετασχηματισμός (M)

$$\left\{ \begin{aligned} C_1(t) &= \tilde{C}_1(t) e^{\frac{i\Delta t}{2}} \\ C_2(t) &= \tilde{C}_2(t) e^{-\frac{i\Delta t}{2}} \\ C_3(t) &= \tilde{C}_3(t) e^{\frac{3i\Delta t}{2}} \end{aligned} \right.$$

$$\cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

ώστε να κέρνουμε RWA

$$\textcircled{1} \Rightarrow C_1(t) = \frac{i}{\hbar} \mathcal{P} \Sigma_0 C_2(t) e^{-i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2} \quad \text{RWA} \quad \Delta := \omega - \Omega \quad \textcircled{3}$$

$$\Omega_R = \frac{\mathcal{P} \Sigma_0}{\hbar} > 0 \text{ για } \mathcal{P} > 0 \dots$$

$$\dot{C}_1(t) = \left(\frac{i}{2\hbar} \mathcal{P} \Sigma_0 C_2(t) \right) e^{-i(\omega - \Omega)t} \Rightarrow \dot{C}_1(t) = \frac{i}{2} \Omega_R C_2(t) e^{i\Delta t} \quad \textcircled{1'}$$

$$\textcircled{2} \Rightarrow C_2(t) = \frac{i}{\hbar} \mathcal{P} \Sigma_0 C_1(t) e^{-i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2} + \frac{i}{\hbar} \mathcal{P}' \Sigma_0 C_3(t) e^{-i\Omega t} \frac{e^{i\omega t} - e^{-i\omega t}}{2}$$

$$\text{RWA} \quad \text{για } \mathcal{P} < 0 \quad \Omega_R = \frac{-\mathcal{P} \Sigma_0}{\hbar} \text{ μηδενικό} \quad \Omega_R' = \frac{\mathcal{P}' \Sigma_0}{\hbar} \text{ κλπ...}$$

$$\dot{C}_2(t) = \frac{i \mathcal{P} \Sigma_0}{2\hbar} C_1(t) e^{-i(\omega - \Omega)t} + \frac{i}{2\hbar} \mathcal{P}' \Sigma_0 C_3(t) e^{-i(\omega - \Omega)t}$$

$$\dot{C}_2(t) = \frac{i}{2} \Omega_R C_1(t) e^{-i\Delta t} + \frac{i}{2} \Omega_R' C_3(t) e^{i\Delta t} \quad \textcircled{2'}$$

Η 2 είναι ο διαμεσογερμίν.

από όριζουμε τις Ω_R, Ω_R' θετικά

$$\textcircled{3} \Rightarrow C_3(t) = \frac{i \mathcal{P}' \Sigma_0}{\hbar} C_2(t) e^{i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2} \quad \text{RWA}$$

$$\dot{C}_3(t) = \frac{i \mathcal{P}' \Sigma_0}{2\hbar} C_2(t) e^{-i\Delta t} \quad \textcircled{3'}$$

$\textcircled{1'} \textcircled{2'} \textcircled{3'}$

οι παράγωγοι των συναρτήσεων σχετίζονται με τις συναρτήσεις με χρονικά έφευρισμένους συντελεστές

$$\left. \begin{aligned} \dot{C}_1(t) &= C_1(t) e^{\frac{i\Delta t}{2}} + C_1(t) \frac{i\Delta}{2} e^{\frac{i\Delta t}{2}} \\ \dot{C}_2(t) &= C_2(t) e^{-\frac{i\Delta t}{2}} + C_2(t) \left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}} \\ \dot{C}_3(t) &= C_3(t) e^{\frac{3i\Delta t}{2}} + C_3(t) \left(\frac{3i\Delta}{2}\right) e^{\frac{3i\Delta t}{2}} \end{aligned} \right\} \leftarrow \textcircled{M}$$

$$\textcircled{1'} \textcircled{M} \quad \cancel{C_1(t) e^{\frac{i\Delta t}{2}} + C_1(t) \frac{i\Delta}{2} e^{\frac{i\Delta t}{2}}} = \frac{i}{2} \Omega_R C_2(t) e^{-\frac{i\Delta t}{2}} \quad \cancel{e^{\frac{i\Delta t}{2}}}$$

$$\boxed{\dot{C}_1(t) = -i \frac{\Delta}{2} C_1(t) + i \frac{\Omega_R}{2} C_2(t)} \quad \textcircled{1''}$$

$$\textcircled{2'} \textcircled{M} \quad \cancel{C_2(t) e^{-\frac{i\Delta t}{2}} + C_2(t) \left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}}} = i \frac{\Omega_R}{2} C_1(t) e^{\frac{i\Delta t}{2}} \quad \cancel{e^{-i\Delta t}} \\ + i \frac{\Omega_R}{2} C_3(t) e^{\frac{3i\Delta t}{2}} \quad \cancel{e^{i\Delta t}}$$

$$\boxed{\dot{C}_2(t) = -i \frac{\Omega_R}{2} C_1(t) + i \frac{\Delta}{2} C_2(t) + i \frac{\Omega_R}{2} C_3(t)} \quad \textcircled{2''}$$

$$\textcircled{3'} \textcircled{M} \quad \cancel{C_3(t) e^{\frac{3i\Delta t}{2}} + C_3(t) \left(-\frac{3i\Delta}{2}\right) e^{\frac{3i\Delta t}{2}}} = i \frac{\Omega_R}{2} C_2(t) e^{-\frac{i\Delta t}{2}} \quad \cancel{e^{-i\Delta t}}$$

$$\boxed{\dot{C}_3(t) = i \frac{\Omega_R}{2} C_2(t) + i \frac{3\Delta}{2} C_3(t)} \quad \textcircled{3''}$$

$\textcircled{1''} \textcircled{2''} \textcircled{3''}$

οι παράγωγοι των συναρτήσεων
 οξειδώνονται με τις συναρτήσεις
 με χρονικούς αντισταθμισμούς σωστά

$$\begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \\ \dot{C}_3(t) \end{bmatrix} = \begin{bmatrix} -i\frac{\Delta}{2} & -i\frac{\Omega_R}{2} & 0 \\ +i\frac{\Omega_R}{2} & i\frac{\Delta}{2} & -i\frac{\Omega_R'}{2} \\ 0 & -i\frac{\Omega_R'}{2} & i\frac{3\Delta}{2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

$\vec{x}(t) \qquad \tilde{A} \qquad \vec{x}(t)$

$$\tilde{A} = -iA$$

Assume

$$\vec{x}(t) = \vec{u} e^{\tilde{\lambda}t}$$

$$\vec{x}(t) = \tilde{A} \vec{x}(t)$$

$$\vec{u} \tilde{\lambda} e^{\tilde{\lambda}t} = \tilde{A} \vec{u} e^{\tilde{\lambda}t}$$

$$\tilde{A} \vec{u} = \tilde{\lambda} \vec{u}$$

$$\tilde{\lambda} = -i\lambda$$

$$A \vec{u} = \lambda \vec{u}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} \end{bmatrix}$$

$$(A - \lambda I) \vec{u} = \vec{0}$$

$$\det \begin{bmatrix} \frac{\Delta}{2} - \lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} - \lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} - \lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{3\Delta}{8} - \frac{2\Delta\Omega_R^2}{8} + \frac{3\Delta - \lambda}{8} = 0 \Rightarrow 3\Delta^3 - 2\Delta\Omega_R^2 + 3\Delta - \lambda = 0$$

λ λ λ H
για

$$\Delta = 0$$

(6)

$$\begin{bmatrix} -\lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow -\lambda \begin{vmatrix} -\lambda & +\frac{\Omega_R'}{2} \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} + \frac{\Omega_R}{2} \begin{vmatrix} +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} = 0$$

$$-\lambda \left[\lambda^2 - \frac{\Omega_R'^2}{4} \right] + \frac{\Omega_R}{2} \frac{\Omega_R}{2} \lambda = 0$$

δύο 0 και ένα 160...

$$-\lambda^3 + \lambda \frac{\Omega_R'^2}{4} + \lambda \frac{\Omega_R^2}{4} = 0 \Rightarrow \lambda \left[-\lambda^2 + \frac{\Omega_R'^2}{4} + \frac{\Omega_R^2}{4} \right] = 0$$

$$\lambda = 0$$

(2)
3)

$$\lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$$

$$\lambda = \pm \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} \quad \lambda_2 = 0 \quad \lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\Lambda < 0$$

$$\lambda_2 = 0$$

$$\lambda_3 = \Lambda > 0$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_a'^2}}{2} = -\Lambda < 0$$

(7)

$$\begin{bmatrix} \Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & \Lambda & -\frac{\Omega_a'}{2} \\ 0 & -\frac{\Omega_a'}{2} & \Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow \Lambda U_1 = \frac{\Omega_R}{2} U_2 \Rightarrow U_1 = \frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 + \Lambda U_2 - \frac{\Omega_a'}{2} U_3 = 0$$

$$-\frac{\Omega_a'}{2} U_2 + \Lambda U_3 = 0 \Rightarrow \Lambda U_3 = \frac{\Omega_a'}{2} U_2 \Rightarrow U_3 = \frac{\Omega_a'}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} U_2 + \Lambda U_2 - \frac{\Omega_a'}{2} \frac{\Omega_a'}{2\Lambda} U_2 = 0$$

$$\Rightarrow U_2 \left[-\frac{\Omega_R^2}{4\Lambda} + \Lambda - \frac{\Omega_a'^2}{4\Lambda} \right] = 0$$

$$\Lambda = \frac{\sqrt{\Omega_R^2 + \Omega_a'^2}}{2}$$

$$\Rightarrow U_2 \left[\frac{-\Omega_R^2 + 4\Lambda^2 - \Omega_a'^2}{4\Lambda} \right] = 0$$

$$\Lambda^2 = \frac{\Omega_R^2 + \Omega_a'^2}{4}$$

or $U_2 = 0$
 $\Rightarrow U_1 = U_2 = U_3 = 0$ μηδέν
 $\Rightarrow U_2$ οτι δέχουμε (μη κενετικό)
 ο.κ. $U_2 = 1$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_a'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 = \beta \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_a'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 \cdot \vec{V}_1 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_a'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + 4\Lambda^2 + \Omega_a'^2}{4\Lambda^2} = 1$$

$$|\beta|^2 \frac{2}{1} = 1 \Rightarrow \text{o.k. } \beta = \frac{1}{\sqrt{2}}$$

• $\lambda_2 = 0$

(8)

$$\begin{bmatrix} 0 & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & 0 & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\frac{\Omega_R}{2} U_2 = 0 \quad (U_2 = 0)$$

$$-\frac{\Omega_R}{2} U_1 - \frac{\Omega_R'}{2} U_3 = 0$$

$$-\frac{\Omega_R}{2} U_1 = \frac{\Omega_R'}{2} U_3 \Rightarrow U_3 = -\frac{\Omega_R}{\Omega_R'} U_1$$

$$-\frac{\Omega_R'}{2} U_2 = 0 \quad (U_2 = 0)$$

$$\vec{V}_2 = \beta \begin{bmatrix} 1 \\ 0 \\ -\frac{\Omega_R}{\Omega_R'} \end{bmatrix}$$

$$\vec{V}_2 \cdot \vec{V}_2 = 1 \Rightarrow |\beta|^2 \left(1 + \frac{\Omega_R^2}{\Omega_R'^2} \right) = 1 \Rightarrow |\beta|^2 \frac{\Omega_R'^2 + \Omega_R^2}{\Omega_R'^2} = 1$$

$$|\beta|^2 \frac{4\Lambda^2}{\Omega_R'^2} = 1 \Rightarrow \eta \times \beta = \frac{\Omega_R'}{2\sqrt{\Omega_R'^2 + \Omega_R^2}} = \frac{\Omega_R'}{\sqrt{\Omega_R'^2 + \Omega_R^2}}$$

• $\lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} = \Lambda > 0$

$$\begin{bmatrix} -\Lambda & -\frac{\Omega_R}{2} & 0 \\ \frac{\Omega_R}{2} & -\Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & -\Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\vec{V}_2 = \begin{bmatrix} \frac{\Omega_R'}{\sqrt{\Omega_R'^2 + \Omega_R^2}} \\ 0 \\ -\frac{\Omega_R}{\sqrt{\Omega_R'^2 + \Omega_R^2}} \end{bmatrix} = \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix}$$

$$-\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow -\frac{\Omega_R}{2} U_2 = \Lambda U_1 \Rightarrow U_1 = -\frac{\Omega_R}{2\Lambda} U_2$$

$$\frac{\Omega_R}{2} U_1 - \Lambda U_2 - \frac{\Omega_R'}{2} U_3 = 0$$

$$-\frac{\Omega_R'}{2} U_2 - \Lambda U_3 = 0 \Rightarrow -\frac{\Omega_R'}{2} U_2 = \Lambda U_3 \Rightarrow U_3 = -\frac{\Omega_R'}{2\Lambda} U_2$$

$$+\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} U_2 - \Lambda U_2 + \frac{\Omega_R'}{2} \frac{\Omega_R'}{2\Lambda} U_2 = 0 \Rightarrow U_2 \left[\frac{\Omega_R^2}{4\Lambda} - \frac{4\Lambda^2}{4\Lambda} + \frac{\Omega_R'^2}{4\Lambda} \right] = 0$$

$$U_2 \left[\frac{\Omega_R^2 + \Omega_R'^2 - 4\Lambda^2}{4\Lambda} \right] = 0 \Rightarrow U_2 \stackrel{\text{δηλωση (μη μηδενικη)}}{\eta \times} U_2 = 1$$

$$\vec{V}_3 = \beta \begin{bmatrix} -\frac{\Omega_R}{2\Lambda} \\ 1 \\ -\frac{\Omega'_R}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_3 \cdot \vec{V}_3 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega'^2_R}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + \Omega'^2_R + 4\Lambda^2}{4\Lambda^2} = 1 \Rightarrow |\beta|^2 \cdot 2 = 1 \Rightarrow \beta = -\frac{1}{\sqrt{2}}$$

$$\vec{V}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega'_R}{2\Lambda} \end{bmatrix}$$

γενική λύση

$$\vec{x}(t) = \sum_{k=1}^3 \sigma_k \vec{V}_k e^{-i\lambda_k t}$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega'^2_R}$$

$$4\Lambda^2 = (\Omega_R^2 + \Omega'^2_R)$$

$$16\Lambda^4 = (\Omega_R^2 + \Omega'^2_R)^2$$

$$C_1(0) = 1$$

$$C_2(0) = 0$$

$$C_3(0) = 0$$

ΣΕΤΩΘΕΝ
αρχ. συνθήκες \bullet (10)

$$\vec{x}(t) = \begin{bmatrix} C_1(t) e^{-\frac{i\Delta t}{2}} \\ C_2(t) e^{\frac{i\Delta t}{2}} \\ C_3(t) e^{\frac{3i\Delta t}{2}} \end{bmatrix} = \frac{\sigma_1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_1 t} + \sigma_2 \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ \frac{\Omega_R}{2\Lambda} \end{bmatrix} e^{-i\lambda_2 t} + \frac{\sigma_3}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_3 t}$$

$$1 = \frac{\sigma_1}{\sqrt{2}} \frac{\Omega_R}{2\Lambda} + \sigma_2 \frac{\Omega_R'}{2\Lambda} + \frac{\sigma_3}{\sqrt{2}} \frac{\Omega_R}{2\Lambda}$$

$$0 = \frac{\sigma_1}{\sqrt{2}} - \frac{\sigma_3}{\sqrt{2}} \Rightarrow \boxed{\sigma_1 = \sigma_3 = \sigma}$$

$$0 = \frac{\sigma}{\sqrt{2}} \frac{\Omega_R'}{2\Lambda} - \sigma_2 \frac{\Omega_R}{2\Lambda} + \frac{\sigma}{\sqrt{2}} \frac{\Omega_R'}{2\Lambda}$$

$$0 = \left(\frac{\sigma}{\sqrt{2}}\right) \frac{\Omega_R'}{2\Lambda} - \frac{\sigma_2 \Omega_R}{2\Lambda} + \left(\frac{\sigma}{\sqrt{2}}\right) \frac{\Omega_R'}{2\Lambda} \Rightarrow 0 = \frac{\sigma \Omega_R'}{\sqrt{2} \Lambda} - \frac{\sigma_2 \Omega_R}{2\Lambda} \Rightarrow$$

$$\frac{\sigma_2 \Omega_R}{2\Lambda} = \frac{\sigma \Omega_R'}{\sqrt{2} \Lambda} \Rightarrow \boxed{\sigma_2 = \sigma \sqrt{2} \frac{\Omega_R'}{\Omega_R}}$$

$$1 = \left(\frac{\sigma}{\sqrt{2}}\right) \frac{\Omega_R}{2\Lambda} + \left(\sigma \sqrt{2} \frac{\Omega_R'}{\Omega_R}\right) \frac{\Omega_R'}{2\Lambda} + \left(\frac{\sigma}{\sqrt{2}}\right) \frac{\Omega_R}{2\Lambda}$$

$$2\Lambda = \sigma \left(\frac{\Omega_R}{\sqrt{2}} + \sqrt{2} \frac{\Omega_R'^2}{\Omega_R} + \frac{\Omega_R}{\sqrt{2}} \right) = \sigma \frac{\Omega_R^2 + 2\Omega_R'^2 + \Omega_R^2}{\sqrt{2} \Omega_R} = \sigma \frac{2}{\sqrt{2}} \frac{\Omega_R^2 + \Omega_R'^2}{\Omega_R}$$

$$\sigma = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2} \cdot \sqrt{2} \Omega_R}{2(\Omega_R^2 + \Omega_R'^2)} \Rightarrow \boxed{\sigma = \frac{\Omega_R}{\sqrt{2} \sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{\Omega_R}{\sqrt{2} 2\Lambda}}$$

$$\sigma = \frac{\Omega_R}{\sqrt{2} 2\Lambda}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} e^{+i\Delta t} + \frac{\Omega_k'}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} \frac{\Omega_k'}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} e^{-i0} + \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} e^{-i\Delta t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_k'^2)} e^{i\Delta t} + \frac{\Omega_k'^2}{2(\Omega_R^2 + \Omega_k'^2)} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_k'^2)} e^{-i\Delta t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{2\Omega_R^2}{2(\Omega_R^2 + \Omega_k'^2)} \cos \Delta t + \frac{\Omega_k'^2}{\Omega_R^2 + \Omega_k'^2}$$

$$C_2(t) e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{\Omega_R^2 + \Omega_k'^2}} e^{-i\Delta t + i\Delta t} - \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{\Omega_R^2 + \Omega_k'^2}} e^{-i\Delta t}$$

$$= \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k'^2}} (e^{+i\Delta t} - e^{-i\Delta t})$$

$\cos + i\sin$
 $-\cos + i\sin$

$$C_2(t) e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_k'^2}} i \sin \Delta t$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_k'^2} \sin^2(\Delta t) = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_k'^2} \left(\frac{1}{2} - \frac{\cos(2\Delta t)}{2} \right)$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$|C_2(t)|^2 = \frac{\Omega_r^2}{\Omega_r^2 + \Omega_r'^2} \cdot \left(\frac{1}{2} - \frac{\cos(2\Omega t)}{2} \right)$$

$$= \frac{\Omega_r^2}{2(\Omega_r^2 + \Omega_r'^2)} - \frac{\Omega_r^2}{2(\Omega_r^2 + \Omega_r'^2)} \cos(2\Omega t)$$

$$d_2 = \frac{\Omega_r^2}{\Omega_r^2 + \Omega_r'^2}$$

maximum transfer percentage
μέγιστο ποσοστό μεταβίβασης

$$T_2 = \frac{2\pi}{2\Omega} = \frac{2\pi}{2\sqrt{\Omega_r^2 + \Omega_r'^2}} = \frac{2\pi}{\sqrt{\Omega_r^2 + \Omega_r'^2}}$$

για $\Omega_r = \Omega_r' \Rightarrow d_2 = \frac{1}{2}$

$$T_2 = \frac{2\pi}{\sqrt{2}\Omega_r} = \frac{1}{\sqrt{2}} \left(\frac{2\pi}{\Omega_r} \right)$$

↑
περίοδος άπλοποίησης
σταθμισμού

$$|C_2(t)|^2 = \frac{1}{2} \left(\frac{1}{2} - \frac{\cos(\sqrt{2}\Omega_r t)}{2} \right)$$

$$2\Omega = \sqrt{\Omega_r^2 + \Omega_r'^2} = \sqrt{2}\Omega_r$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\underbrace{\sqrt{2}\Omega_r t}_{2\omega_1 = \omega_2})$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\omega_2 t)$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2 \cdot 4\Lambda^2} e^{+i\Lambda t} + \frac{\Omega_R'^2}{4\Lambda^2} + \frac{\Omega_R^2}{2 \cdot 4\Lambda^2} e^{-i\Lambda t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{4\Lambda^2} \cos(\Lambda t) + \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + 2 \frac{\Omega_R^2}{4\Lambda^2} \cdot \cos(\Lambda t) \cdot \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos(\Lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos(2\Lambda t) + 1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \quad T_B = \frac{2\pi}{\Lambda} \quad \frac{T_B}{T_A} = 2 \Rightarrow \text{περιοδική κίνηση}$$

με περίοδο $T_1 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \cdot 2 = 2T_2$

$$\begin{aligned} \left| C_1\left(\frac{2\pi}{\Lambda}\right) \right|^2 &= \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos\left(2\Lambda \frac{2\pi}{\Lambda}\right) + 1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos\left(\Lambda \frac{2\pi}{\Lambda}\right) = \\ &= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_R'^2)^2}{16\Lambda^4} = 1 \end{aligned}$$

$$\begin{aligned} \left| C_1\left(\frac{2\pi}{2\Lambda}\right) \right|^2 &= \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos\left(2\Lambda \frac{2\pi}{2\Lambda}\right) + 1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos\left(\Lambda \frac{2\pi}{2\Lambda}\right) \\ &= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2} \end{aligned}$$

ή τιμή της $|C_1(t)|^2$ στο ήμισιο της περιόδου

$$\frac{d}{dt} |C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot (2 \cos(\Lambda t) \cdot (-\Lambda) \sin(\Lambda t)) + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-\Lambda) \sin(\Lambda t)$$

$$\frac{d}{dt} |C_1(t)|^2 = \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \sin(\Lambda t) \cdot [\Omega_R^2 \cos(\Lambda t) + \Omega_R'^2]$$

$$\begin{aligned} \frac{d^2}{dt^2} |C_1(t)|^2 &= \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \Lambda \cdot \cos(\Lambda t) \cdot [\Omega_R^2 \cos(\Lambda t) + \Omega_R'^2] + \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \sin(\Lambda t) \cdot \Omega_R^2 (-\Lambda) \cdot \sin(\Lambda t) \\ &= \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \Lambda \cdot \left\{ \Omega_R^2 \cos^2(\Lambda t) + \Omega_R'^2 \cos(\Lambda t) - \Omega_R^2 \sin^2(\Lambda t) \right\} \end{aligned}$$

$$\frac{d|G_1(t)|^2}{dt} = \frac{(-2\Lambda) \cdot \Omega_R^2}{16\Lambda^4} \cdot \sin(\Lambda t) \cdot [\Omega_R^2 \cdot \cos(\Lambda t) + \Omega_R'^2]$$

$$\frac{d^2|G_1(t)|^2}{dt^2} = \frac{(-2\Lambda^2 \cdot \Omega_R^2)}{16\Lambda^4} [\Omega_R^2 \sin^2(\Lambda t) - \Omega_R^2 \cos^2(\Lambda t) - \Omega_R'^2 \cos(\Lambda t)]$$

$$\frac{d|G_1(t)|^2}{dt} = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$$

\Downarrow
 $\Lambda t = n\pi, n \in \mathbb{Z}$

Π1

προβόλη, πρέπει

 $\Omega_R'^2 \leq \Omega_R^2$

Π2

Π2

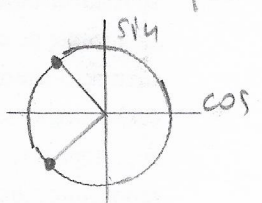
$$\frac{d^2}{dt^2} |G_1(t)|^2 = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} \left[\Omega_R^2 \cdot \sin^2(\Lambda t) - \Omega_R^2 \frac{\Omega_R'^4}{\Omega_R^4} + \Omega_R'^2 \frac{\Omega_R'^2}{\Omega_R^2} \right]$$

$$= \frac{2\Lambda^2}{16\Lambda^4} \left[\Omega_R^4 \cdot \sin^2(\Lambda t) - \Omega_R'^4 + \Omega_R'^4 \right] = \frac{2\Lambda^2}{16\Lambda^4} \cdot \Omega_R^4 \cdot \sin^2(\Lambda t) > 0$$

Θνη. στην Π2 έχουμε έλαχιστα, με τιμή μηδενική διότι τότε:

$$|G_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\Omega_R'^4}{\Omega_R^4} + \frac{\Omega_R^4}{16\Lambda^4} + \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-1) \frac{\Omega_R'^2}{\Omega_R^2} = \frac{2\Omega_R^4}{16\Lambda^4} - \frac{2\Omega_R'^4}{16\Lambda^4} = 0$$

Μάλιστα σε μία περίοδο $T_1 = \frac{2\pi}{\Lambda}$, η δυνάμει είναι και η περίοδος του όρου $\cos(\Lambda t)$, \exists 2 φορές όπου έχουμε $\cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$ δηλαδή θα έχουμε 2 μηδενισμούς σε μία περίοδο T_1 .



Όποτε τότε, $G_1 = 1$ μέγιστο ποσοστό μεταβιβάσεως maximum transfer percentage

Π1

$$\frac{d^2}{dt^2} |G_1(t)|^2 = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} \left[-\Omega_R^2 \cdot \cos^2(\Lambda t) - \Omega_R'^2 \cos(\Lambda t) \right]$$

$\sin(\Lambda t) = 0 \Rightarrow \cos(\Lambda t) = \pm 1$

Π1α

$\Lambda t = 0, 2\pi, 4\pi, \dots \cos(\Lambda t) = 1$

Π1β

$\Lambda t = \pi, 3\pi, 5\pi, \dots \cos(\Lambda t) = -1$

$\blacktriangleright t = 0, T_1, 2T_1, \dots$
 $\blacktriangleright t = \frac{T_1}{2}, \frac{3T_1}{2}, \frac{5T_1}{2}, \dots$

Π1α $\frac{d^2 |C_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} (-\Omega_R^2 - \Omega_R'^2) < 0 \Rightarrow$ τοπικό μέγιστο

13''

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot 1 + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_R'^2)^2}{16\Lambda^4} = 1$$

δηλ. είναι δίκως μέγιστο

Π1β $\frac{d^2 |C_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} (-\Omega_R^2 + \Omega_R'^2) > 0 \Rightarrow$ τοπικό ελάχιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

ή δοσκα είναι η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$d_1 = 1 - \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2} = \frac{4 \cdot \Omega_R^2 \cdot \Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

για $\Omega_R = \Omega_R'$

$$T_1 = \frac{2\pi}{\sqrt{2} |\Omega_R|} \cdot 2 = \sqrt{2} \left(\frac{2\pi}{\Omega_R} \right)$$

↓
περίοδος αντίστοιχου διασπασμού

Π2 $\Rightarrow d_1 = 1$ $\cos(\Lambda t) = -1 \Rightarrow \Lambda t = +\pi, 3\pi, \dots$ $t = \frac{\pi}{\Lambda} = \frac{2\pi}{2\Lambda} =$ το ήμισυ περιόδου
 Π1 $\Rightarrow d_1 = 1$ στο ήμισυ της περιόδου 1 φορά μηδενισμός...

δηλ. για $\Omega_R = \Omega_R'$ οι Π1, Π2 ταυτίζονται

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2} \Omega_R$$

$$4\Lambda^2 = 2\Omega_R^2$$

$$16\Lambda^4 = 4\Omega_R^4$$

π.χ. όπως στο GG, GGG

$$T_{GG} \approx 20.6783 \text{ fs}$$

$$T_{GGG} = 29.2436 \text{ fs} = \sqrt{2} T_{GG}$$

$$T_{31} = 14.6218 \text{ fs} = \frac{T_{GG}}{\sqrt{2}}$$

$$T_{32} = 29.2436 \text{ fs} = T_{GGG} = \sqrt{2} T_{GG}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4} \cdot \frac{\cos(\sqrt{2}\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} + \frac{2\Omega_R^4}{4\Omega_R^4} \cdot \cos\left(\frac{\sqrt{2}}{2}\Omega_R t\right)$$

$$|C_1(t)|^2 = \frac{1}{4} \left(\frac{1}{2} \cdot \cos(\sqrt{2} \omega_1 t) + \frac{1}{2} \right) + \frac{1}{4} + \frac{1}{2} \cos\left(\frac{\sqrt{2}}{2} \omega_1 t\right)$$

$\underbrace{\qquad\qquad\qquad}_{2\omega_1 = \omega_2}$
 $\underbrace{\qquad\qquad\qquad}_{\omega_1}$

13''

$$\frac{1}{8} \cos(2\omega_1 t) + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} \cos(\omega_1 t) =$$

$$\frac{1}{8} \left(\underbrace{\cos(2\omega_1 t) + 1 + 2 + 4 \cos(\omega_1 t)}_{2 \cos^2(\omega_1 t)} \right) = \frac{1}{8} \left(2 \cos^2(\omega_1 t) + 2 + 4 \cos(\omega_1 t) \right)$$

$$= \frac{1}{4} \left(\cos^2(\omega_1 t) + 1 + 2 \cos(\omega_1 t) \right) = \frac{1}{4} \left(\underbrace{\cos(\omega_1 t) + 1}_{2 \cos^2\left(\frac{\omega_1 t}{2}\right)} \right)^2 = \cos^4\left(\frac{\omega_1 t}{2}\right)$$

$$C_3(t) e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R}{4\Lambda} \cdot \frac{\Omega_R'}{2\Lambda} e^{+i\Lambda t} + \frac{\Omega_R}{2\Lambda} \frac{\Omega_R'}{\Omega_R} (-1) \frac{\Omega_R}{2\Lambda} + \frac{\Omega_R}{4\Lambda} \cdot \frac{\Omega_R'}{2\Lambda} e^{-i\Lambda t} \quad (14)$$

$$C_3(t) e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R \Omega_R'}{4\Lambda^2} \cos(\Lambda t) - \frac{\Omega_R \Omega_R'}{4\Lambda^2}$$

$$|C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - 2 \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$|C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \frac{\cos(2\Lambda t) + 1}{2} + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \quad T_B = \frac{2\pi}{\Lambda} \quad \frac{T_B}{T_A} = 2 \Rightarrow \text{η κίνηση είναι περιόδου } T$$

με περίοδο

$$T_3 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \cdot 2 = 2T_2 = T_1$$

$$\frac{d}{dt} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2 \cos(\Lambda t) (-\Lambda) \sin(\Lambda t) - 2 \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-\Lambda) \sin(\Lambda t)$$

$$\frac{d}{dt} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2(-\Lambda) [\cos(\Lambda t) \cdot \sin(\Lambda t) - \sin(\Lambda t)]$$

$$\frac{d}{dt} |C_3(t)|^2 = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = 1$$

$$\Rightarrow t = T_3, 2T_3, \dots$$

$$(P1) \quad \Lambda t = 0, 2\pi, 4\pi, \dots \Rightarrow \sin(\Lambda t) = 0 \text{ και } \cos(\Lambda t) = 1$$

$$(P2) \quad \Lambda t = \pi, 3\pi, 5\pi, \dots \Rightarrow \sin(\Lambda t) = 0 \text{ και } \cos(\Lambda t) = -1$$

$$\Rightarrow t = \frac{T_3}{2}, \frac{3T_3}{2}, \dots$$

$$(P1) \quad |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot 1 + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (\pm 1) = \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \mp \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4}$$

$$(P1) \Rightarrow |C_3(t)|^2 = 0 \quad \text{ελάχιστο}$$

$$(P2) \Rightarrow |C_3(t)|^2 = \frac{4\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \quad \text{μέγιστο δίνω...}$$

$$\frac{d^2}{dt^2} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2(-\Lambda) \left[\frac{1}{2} 2\Lambda \cos(2\Lambda t) - \Lambda \cos(\Lambda t) \right] = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2\Lambda^2 (\cos(\Lambda t) - \cos(2\Lambda t))$$

$$(P1) \Rightarrow (\dots) = \cos(2\pi) - \cos(4\pi) = 0$$

$$(P2) \Rightarrow (\dots) = \cos(\pi) - \cos(2\pi) = -1 - 1 = -2 \Rightarrow \frac{d^2}{dt^2} |C_3(t)|^2 < 0 \quad \text{μέγιστο}$$

Αν τα παράγωγα είναι 0, τότε

(14)

$$\frac{d^3 |C_3(t)|^2}{dt^3} = \vartheta \cdot (\Lambda(-1) \cdot \sin(\Lambda t) + 2\Lambda \sin(2\Lambda t))$$

$$= \vartheta \Lambda (2 \sin(2\Lambda t) - \sin(\Lambda t))$$

και για $\Lambda t = 2\pi \cdot \frac{\pi}{4} = \vartheta \Lambda (2 \sin(4\pi) - \sin(2\pi)) = 0$

$$\frac{d^4 |C_3(t)|^2}{dt^4} = \vartheta \Lambda (2 \cdot 2\Lambda \cos(2\Lambda t) - \Lambda \cdot \cos(\Lambda t))$$

$$= \vartheta \Lambda^2 (4 \cos(2\Lambda t) - \cos(\Lambda t))$$

και για $\Lambda t = 2\pi \cdot \frac{\pi}{4} = \vartheta \Lambda^2 (4 \cdot \underbrace{\cos(4\pi)}_1 - \underbrace{\cos(2\pi)}_1) > 0$

ΞΑΝΑ $\frac{d^2 |C_3(t)|^2}{dt^2} = \vartheta (\underbrace{\cos(\Lambda t)}_1 - \underbrace{\cos(2\Lambda t)}_1)$

για $\Lambda t = 2\pi \cdot \frac{\pi}{4} = 0$

$$\frac{d^3 |C_3(t)|^2}{dt^3} = \vartheta (-\Lambda \sin(\Lambda t) + 2\Lambda \sin(2\Lambda t))$$

για $\Lambda t = 2\pi \cdot \frac{\pi}{4} = 0$

$$\frac{d^4 |C_3(t)|^2}{dt^4} = \vartheta (-\Lambda^2 \cos(\Lambda t) + 2\Lambda \cdot 2\Lambda \cos(2\Lambda t)) = \vartheta \Lambda^2 (4 \cos(2\Lambda t) - \cos(\Lambda t))$$

για $\Lambda t = 2\pi \cdot \frac{\pi}{4} = \vartheta \Lambda^2 \cdot 3 > 0$

δηλαδή στα ελάχιστα μηδενίζονται η $|C_3(t)|^2$, καθώς και η 1η, 2η και 3η παράγωγός της!

ένω, η 4η παράγωγός της είναι θετική...

μοιάζει κάπως με ζώνη ερεψών (flat function)

[όπου μηδενίζονται σε κάποιες σημεία αλλά οι παράγωγοι]

21 Δρα

$$A_3 = \frac{4\Omega_R^2 \Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

μέγιστο ποσοστό μεταβίβασης
maximum transfer percentage

14"

για $\Omega_R' = \Omega_R$

$$T_3 = \frac{2\pi}{\sqrt{2}\Omega_R} \cdot 2 = \sqrt{2} \left(\frac{2\pi}{\Omega_R} \right)$$

$$2\omega = \sqrt{2}\Omega_R^2 = \sqrt{2}\Omega_R$$

↓ περίοδος αντίστοιχου σταθμικού

$$|G_3(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4} \frac{\cos(\sqrt{2}\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} - \frac{2\Omega_R^4}{4\Omega_R^4} \cdot \cos\left(\frac{\sqrt{2}\Omega_R t}{2}\right)$$

$$|G_3(t)|^2 = \frac{1}{4} \left(\frac{1}{2} \cos(\sqrt{2}\Omega_R t) + \frac{1}{2} + \frac{1}{4} - \frac{1}{2} \cos\left(\frac{\sqrt{2}\Omega_R t}{2}\right) \right)$$

$\underbrace{\hspace{10em}}_{2\omega_3 = 2\omega_1 = \omega_2}$
 $\underbrace{\hspace{10em}}_{\omega_1 = \omega_3}$

$$= \frac{1}{8} \cos(2\omega_3 t) + \frac{1}{8} + \frac{1}{4} - \frac{1}{2} \cos(\omega_3 t)$$

$$= \frac{1}{8} \left(\cos(2\omega_3 t) + 1 + 2 - 4 \cos(\omega_3 t) \right)$$

$$= \frac{1}{8} \left(2 \cos^2(\omega_3 t) + 2 - 4 \cos(\omega_3 t) \right)$$

$$= \frac{1}{4} \left(\cos^2(\omega_3 t) + 1 - 2 \cos(\omega_3 t) \right)$$

$$= \frac{1}{4} \left(1 - \cos(\omega_3 t) \right)^2$$

$$= \frac{1}{4} \left(2 \sin^2\left(\frac{\omega_3 t}{2}\right) \right)^2 = \sin^4\left(\frac{\omega_3 t}{2}\right)$$

A_{pe} $A_3 = \frac{4 \Omega_R^2 \Omega_{R'}^2}{(\Omega_R^2 + \Omega_{R'}^2)^2}$

μέγιστος ρυθμός μεταβιβάσεων
maximum transfer rate $1 \rightarrow 3$

$\frac{A_3}{T_3} = \frac{A_3}{T_3} = \frac{4 \Omega_R^2 \Omega_{R'}^2}{(\Omega_R^2 + \Omega_{R'}^2)^2} \frac{\sqrt{\Omega_R^2 + \Omega_{R'}^2}}{2\pi \cdot 2}$

Μέσες πιθανότητες παρουσίατος \hat{z} εκκροτικού σε κάθε στάθμη

$\langle |C_3(t)|^2 \rangle = \frac{\Omega_R^2 \Omega_{R'}^2}{16 \Lambda^4} \frac{1}{2} + \frac{\Omega_R^2 \Omega_{R'}^2}{16 \Lambda^4} = \frac{3 \Omega_R^2 \Omega_{R'}^2}{2 \cdot 16 \Lambda^4}$

$\langle |C_1(t)|^2 \rangle = \frac{\Omega_R^4}{16 \Lambda^4} \frac{1}{2} + \frac{\Omega_{R'}^4}{16 \Lambda^4} = \frac{\Omega_R^4 + 2 \Omega_{R'}^4}{2 \cdot 16 \Lambda^4}$

$\langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_{R'}^2} \frac{1}{2} = \frac{\Omega_R^2}{4 \Lambda^2} \frac{1}{2} = \frac{4 \Omega_R^2 \Lambda^2}{2 \cdot 16 \Lambda^4}$

$\langle |C_1(t)|^2 \rangle + \langle |C_2(t)|^2 \rangle + \langle |C_3(t)|^2 \rangle = \frac{3 \Omega_R^2 \Omega_{R'}^2 + \Omega_R^4 + 2 \Omega_{R'}^4 + 4 \Omega_R^2 \Lambda^2}{2 \cdot 16 \Lambda^4}$

$\frac{3 \Omega_R^2 \Omega_{R'}^2 + \Omega_R^4 + 2 \Omega_{R'}^4 + \Omega_R^2 (\Omega_R^2 + \Omega_{R'}^2)}{2 \cdot 16 \Lambda^4} = \frac{4 \Omega_R^2 \Omega_{R'}^2 + 2 \Omega_R^4 + 2 \Omega_{R'}^4}{2 \cdot 16 \Lambda^4}$

$\frac{\Omega_R^4 + 2 \Omega_R^2 \Omega_{R'}^2 + \Omega_{R'}^4}{(\Omega_R^2 + \Omega_{R'}^2)^2} = 1$

όριση t_{3mean}

$\frac{3 \Omega_R^2 \Omega_{R'}^2}{2 \cdot 16 \Lambda^4} = \frac{\Omega_R^2 \Omega_{R'}^2}{16 \Lambda^4} \cos^2(\Lambda t_{3mean}) + \frac{\Omega_R^2 \Omega_{R'}^2}{16 \Lambda^4} - \frac{2 \Omega_R^2 \Omega_{R'}^2}{16 \Lambda^4} \cos(\Lambda t_{3mean})$

ο απαιτούμενος χρόνος για να ηδύσει τη μέση τιμή

$\frac{3}{2} = \cos^2(\Lambda t_{3mean}) + 1 - 2 \cos(\Lambda t_{3mean})$

$\frac{1}{2} = \cos^2(\Lambda t_{3mean}) - 2 \cos(\Lambda t_{3mean})$

$1 = 2 \cos^2 x - 4 \cos x$

$$2\psi^2 - 4\psi - 1 = 0$$

$$\psi = \cos x$$

$$\Delta = 16 + 4 \cdot 2 = 24$$

$$\frac{4 \pm \sqrt{24}}{2 \cdot 2} = 1 \pm \frac{\sqrt{24}}{4} = 1 \pm \sqrt{\frac{24}{16}} = 1 \pm \sqrt{\frac{3 \cdot 8}{2 \cdot 8}}$$

$$= 1 \pm \frac{\sqrt{3}}{\sqrt{2}}$$

$$1 - \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2}}$$

$$1 + \frac{\sqrt{3}}{\sqrt{2}} > 1$$

ἀνοπιμέταλ

$$\cos(\Lambda t_{3mean}) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos\left(\frac{\sqrt{\Omega_R^2 + \Omega_I^2}}{2} t_{3mean}\right) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{\sqrt{\Omega_R^2 + \Omega_I^2}}{2} t_{3mean} = 1.797477 \dots$$

$$t_{3mean} = \frac{2 \cdot 1.797477}{\sqrt{\Omega_R^2 + \Omega_I^2}}$$

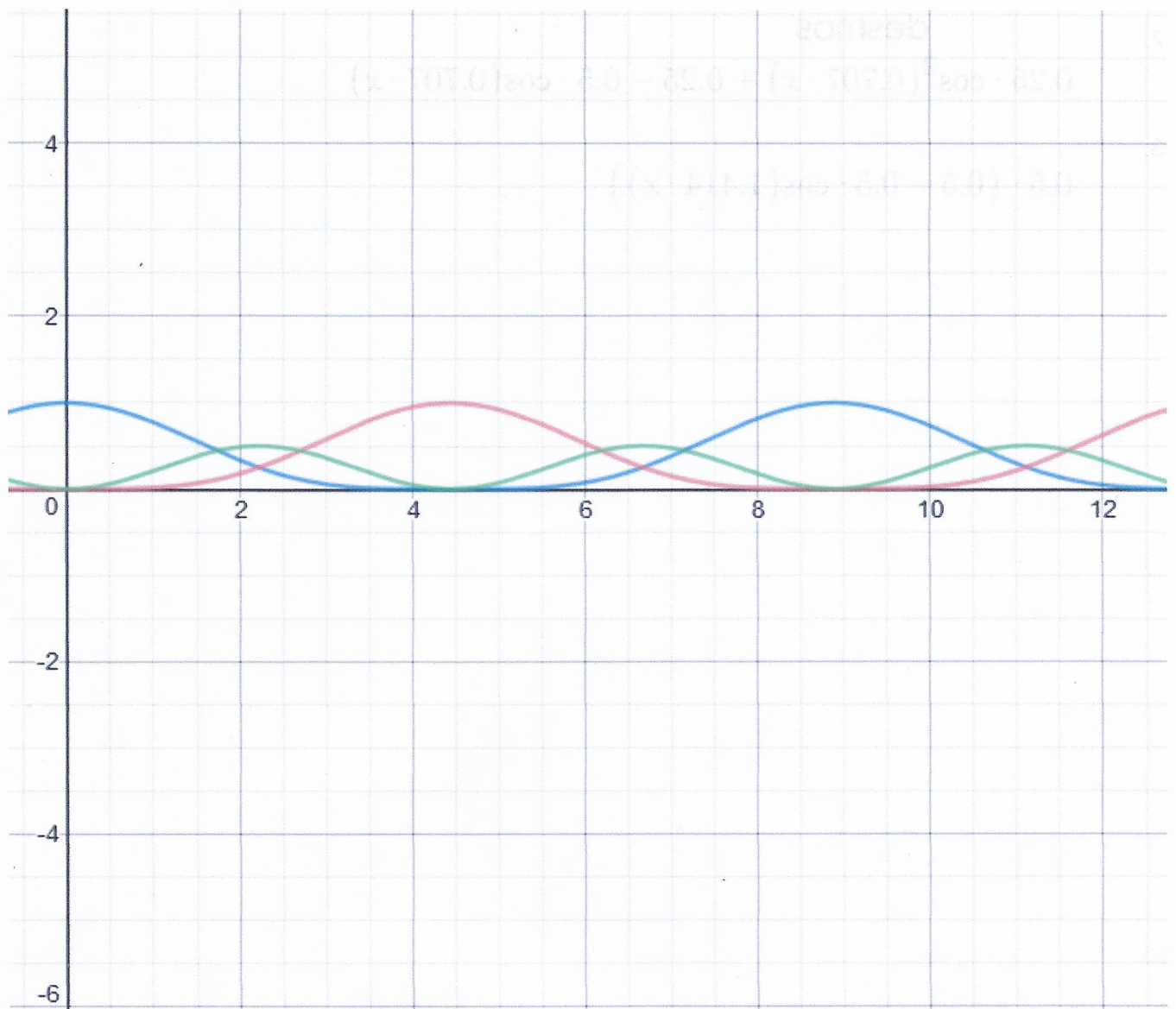
mean transfer rate
μέγος ποσότης μεταφοράς

$$k := \frac{\langle |z_3(t)|^2 \rangle}{t_{3mean}} = \frac{3 \Omega_R^2 \Omega_I^2}{2 \cdot 1.614} \frac{\sqrt{\Omega_R^2 + \Omega_I^2}}{2 \cdot 1.797477}$$

$$\frac{k}{\frac{d_3}{13}} = \frac{\cancel{3} \Omega_R^2 \Omega_I^2}{\cancel{2} (\Omega_R^2 + \Omega_I^2)^2} \frac{\sqrt{\Omega_R^2 + \Omega_I^2}}{\cancel{2} \cdot 1.797477} \frac{(\Omega_R^2 + \Omega_I^2)^2}{\cancel{4} \Omega_R^2 \Omega_I^2 \sqrt{\Omega_R^2 + \Omega_I^2}} \quad \textcircled{2} \textcircled{2}$$

$$= \frac{3 \cdot \pi}{4 \cdot 1.797477} \approx 1.21083 \dots$$

(12)



— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 + 0.5 \cdot \cos(0.707 \cdot x)$

— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 - 0.5 \cdot \cos(0.707 \cdot x)$

— $0.5 \cdot (0.5 - 0.5 \cdot \cos(1.414 \cdot x))$
 ηρέστω

για

$$\Omega_R = \Omega'_R = 1$$

$$2\Lambda = \sqrt{1^2 + 1^2} = \sqrt{2} \Rightarrow \Lambda = \frac{\sqrt{2}}{2}$$

ΑΣΚΗΣΗ (matlab)

Χρησιμοποιώντας το πρόγραμμα Oscillator.m να γίνει η γραφική παράσταση των μεταβολών $\Delta \Sigma$

με $|\Delta| = \sqrt{3} \Omega_e \Rightarrow \alpha = \frac{1}{4}, T = \frac{1}{2} \frac{2\pi}{\Omega_e}$

ΑΣΚΗΣΗ (matlab)

Να φτιαχθεί αντίστοιχο πρόγραμμα για Τρισταθμικό Σύστημα

και να γίνει η γραφική παράσταση για $\Omega_R = \Omega_I = 1 \quad \Delta = \omega$

$\Delta = 0 \quad \Delta = \Omega_R$