$$
(A)\begin{cases}
\dot{C}_1(t) = \dfrac{i\wp\mathcal{E}_0}{2\hbar} e^{i\Delta t} C_2(t) \\
\dot{C}_2(t) = \dfrac{i\wp\mathcal{E}_0}{2\hbar} e^{-i\Delta t} C_1(t)
\end{cases}
$$

$\boxed{\Delta\Sigma \text{ μετά την RWA}}$

$\Omega_R = \dfrac{\wp\mathcal{E}_0}{\hbar}$

$\Delta : \omega - \Omega$

1

Εναλλακτική λύση των διαφορικών εξισ. με Χ.Ε.

$$
(A')\begin{cases}
\dot{C}_1(t) = \dfrac{i\Omega_R}{2} e^{i\Delta t} C_2(t) \\
\dot{C}_2(t) = \dfrac{i\Omega_R}{2} e^{-i\Delta t} C_1(t)
\end{cases}
$$

$$
\ddot{C}_1(t) = \frac{i\Omega_R}{2} i\Delta e^{i\Delta t} C_2(t) + \frac{i\Omega_R}{2} e^{i\Delta t}\dot{C}_2(t)
$$

$$
= \frac{i\Omega_R}{2} i\Delta e^{i\Delta t} \frac{\dot{C}_1(t)}{\frac{i\Omega_R}{2} e^{i\Delta t}} + \frac{i\Omega_R}{2} \cancel{e^{i\Delta t}} \frac{i\Omega_R}{2} \cancel{e^{-i\Delta t}} C_1(t) \Rightarrow
$$

$$
\ddot{C}_1(t) = i\Delta \dot{C}_1(t) - \frac{\Omega_R^2}{4} C_1(t) \Rightarrow \boxed{\ddot{C}_1(t) - i\Delta \dot{C}_1(t) + \frac{\Omega_R^2}{4} C_1(t) = 0} \quad (1)
$$

$$
\ddot{C}_2(t) = \frac{i\Omega_R}{2}(-i\Delta) e^{-i\Delta t} C_1(t) + \frac{i\Omega_R}{2} e^{-i\Delta t} \dot{C}_1(t)
$$

$$
= \frac{i\Omega_R}{2}(-i\Delta) e^{-i\Delta t} \frac{\dot{C}_2(t)}{\frac{i\Omega_R}{2} e^{-i\Delta t}} + \frac{i\Omega_R}{2} \cancel{e^{-i\Delta t}} \frac{i\Omega_R}{2} \cancel{e^{i\Delta t}} C_2(t) \Rightarrow
$$

$$
\ddot{C}_2(t) = (-i\Delta)\dot{C}_2(t) - \frac{\Omega_R^2}{4} C_2(t) \Rightarrow \boxed{\ddot{C}_2(t) + i\Delta \dot{C}_2(t) + \frac{\Omega_R^2}{4} C_2(t) = 0} \quad (2)
$$

ΔΛΜ $\quad C_k(t) = U_k e^{i\mu_k t}$ $\qquad$ ΔΛΜ = Δοκιμάζω Λύσεις της Μορφής

$$
\dot{C}_k(t) = U_k i\mu_k e^{i\mu_k t}
$$

$$
\ddot{C}_k(t) = U_k (i\mu_k)^2 e^{i\mu_k t}
$$

$$
(1) \Rightarrow \cancel{U_1} (i\mu_1)^2 \cancel{e^{i\mu_1 t}} - i\Delta \cancel{U_1} i\mu_1 \cancel{e^{i\mu_1 t}} + \frac{\Omega_R^2}{4} \cancel{U_1} \cancel{e^{i\mu_1 t}} = 0 \Rightarrow
$$

$$
\boxed{-\mu_1^2 + \Delta\mu_1 + \frac{\Omega_R^2}{4} = 0} \quad \text{ΧΕ1}
$$

$\nexists$ συνθήκη για το $U_1$

$$
(2) \Rightarrow \cancel{U_2} (i\mu_2)^2 \cancel{e^{i\mu_2 t}} + i\Delta \cancel{U_2} i\mu_2 \cancel{e^{i\mu_2 t}} + \frac{\Omega_R^2}{4} \cancel{U_2} \cancel{e^{i\mu_2 t}} = 0 \Rightarrow
$$

$$
\boxed{-\mu_2^2 - \Delta\mu_2 + \frac{\Omega_R^2}{4} = 0} \quad \text{ΧΕ2}
$$

$\nexists$ συνθήκη για το $U_2$

$$\Delta_1 = \Delta^2 - 4(-1)\frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2 \qquad \lambda_{2,1} := \pm\frac{\sqrt{\Delta^2+\Omega_R^2}}{2} = \pm\lambda$$

$$\mu_1 = \frac{-\Delta \pm \sqrt{\Delta^2+\Omega_R^2}}{-2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2+\Omega_R^2}}{2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \lambda$$

$$\Delta_2 = (-\Delta)^2 - 4(-1)\frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2$$

$$\mu_2 = \frac{+\Delta \pm \sqrt{\Delta^2+\Omega_R^2}}{-2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2+\Omega_R^2}}{2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \lambda$$

Άρα, οι λύσεις θα είναι

$$\begin{cases} C_1(t) = \alpha e^{i\frac{\Delta}{2}t} e^{i\lambda t} + \beta e^{i\frac{\Delta}{2}t} e^{-i\lambda t} \\ C_2(t) = \gamma e^{-i\frac{\Delta}{2}t} e^{i\lambda t} + \delta e^{-i\frac{\Delta}{2}t} e^{i\lambda t} \end{cases}$$

$$\begin{cases} C_1(t) = e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \\ C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] \end{cases} \xRightarrow{\Delta=0} \begin{cases} C_1(t) = \alpha e^{i\lambda t} + \beta e^{-i\lambda t} \\ C_2(t) = \gamma e^{i\lambda t} + \delta e^{-i\lambda t} \end{cases}$$

Τις δοκιμάζουμε τώρα στις (A') για $\Delta = 0$, όπου $\lambda = \frac{\Omega_R}{2}$

$$\alpha i\lambda e^{i\lambda t} + \beta(-i\lambda)e^{-i\lambda t} = \frac{i\Omega_R}{2}\left(\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right) \Rightarrow \gamma = \alpha \quad \beta = -\delta$$

$$\gamma i\lambda e^{i\lambda t} + \delta(-i\lambda)e^{-i\lambda t} = i\frac{\Omega_R}{2}\left(\alpha e^{i\lambda t} + \delta e^{-i\lambda t}\right) \Rightarrow \gamma = \alpha \quad \beta = -\delta$$

$$\begin{cases} C_1(t) = \alpha e^{i\frac{\Omega_R}{2}t} + \beta e^{-i\frac{\Omega_R}{2}t} \\ C_2(t) = \alpha e^{i\frac{\Omega_R}{2}t} - \beta e^{-i\frac{\Omega_R}{2}t} \end{cases}$$

Έστωσαν οι αρχικές συνθήκες (Α.Σ.) $C_1(0) = 1$, $C_2(0) = 0$

$$\left.\begin{aligned} 1 &= \alpha + \beta \\ 0 &= \alpha - \beta \Rightarrow \beta = \alpha \end{aligned}\right\} \quad 2\alpha = 1 \Rightarrow \alpha = \frac{1}{2}$$

$$C_1(t) = \frac{1}{2}e^{i\frac{\Omega_R}{2}t} + \frac{1}{2}e^{-i\frac{\Omega_R}{2}t} \Rightarrow \boxed{C_1(t) = \cos\left(\frac{\Omega_R}{2}t\right)}$$

$$C_2(t) = \frac{1}{2}e^{i\frac{\Omega_R}{2}t} - \frac{1}{2}e^{-i\frac{\Omega_R}{2}t} \Rightarrow \boxed{C_2(t) = i\sin\left(\frac{\Omega_R}{2}t\right)}$$

Τις δοκιμάζουμε τώρα στις (Α') για $\Delta \neq 0$ $\quad \lambda = \frac{\sqrt{\Delta^2+\Omega_R^2}}{2} \Rightarrow 4\lambda^2 = \Delta^2 + \Omega_R^2$ (3)

$$i\frac{\Delta}{2} e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] + e^{i\frac{\Delta}{2}t}\left[\alpha i\lambda e^{i\lambda t} + \beta(-i\lambda) e^{-i\lambda t}\right] =$$

$$i\frac{\Omega_R}{2} e^{i\Delta t} e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] \Rightarrow$$

$$\frac{\Delta}{2}\alpha + \alpha\lambda = \frac{\Omega_R}{2}\gamma \quad \text{και} \quad \frac{\Delta}{2}\beta - \beta\lambda = \frac{\Omega_R}{2}\delta$$

$$\frac{\Delta\alpha + 2\alpha\lambda}{\cancel{2}} = \frac{\Omega_R}{\cancel{2}}\gamma \quad \text{και} \quad \frac{\Delta\beta - 2\beta\lambda}{2} = \frac{\Omega_R}{2}\delta$$

$$\boxed{\gamma = \frac{\Delta + 2\lambda}{\Omega_R}\alpha} \quad \text{και} \quad \boxed{\delta = \frac{\Delta - 2\lambda}{\Omega_R}\beta}$$

$$-i\frac{\Delta}{2} e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] + e^{-i\frac{\Delta}{2}t}\left[\gamma(i\lambda) e^{i\lambda t} + \delta(-i\lambda) e^{-i\lambda t}\right] =$$

$$i\frac{\Omega_R}{2} e^{-i\Delta t} e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \Rightarrow$$

$$-\frac{\Delta}{2}\gamma + \gamma\lambda = \frac{\Omega_R}{2}\alpha \quad \text{και} \quad -\frac{\Delta}{2}\delta - \delta\lambda = \frac{\Omega_R}{2}\beta$$

$$\frac{-\Delta\gamma + 2\gamma\lambda}{\cancel{2}} = \frac{\Omega_R}{\cancel{2}}\alpha \quad \text{και} \quad \frac{-\Delta\delta - 2\delta\lambda}{\cancel{2}} = \frac{\Omega_R}{\cancel{2}}\beta$$

$$\boxed{\alpha = \frac{-\Delta + 2\lambda}{\Omega_R}\gamma} \quad \text{και} \quad \boxed{\beta = \frac{-\Delta - 2\lambda}{\Omega_R}\delta}$$

$$\gamma = \frac{(2\lambda + \Delta)}{\Omega_R} \cdot \frac{(2\lambda - \Delta)}{\Omega_R}\gamma \Rightarrow \frac{4\lambda^2 - \Delta^2}{\Omega_R^2} = 1 \quad \text{τό οποίον ισχύει}$$

$$\delta = \frac{(\Delta - 2\lambda)}{\Omega_R} \cdot \frac{-(\Delta + 2\lambda)}{\Omega_R}\delta \Rightarrow -\frac{\Delta^2 - 4\lambda^2}{\Omega_R^2} = 1 \quad \text{τό οποίον ισχύει}$$

$$\begin{cases} C_1(t) = e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \\ C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\alpha\frac{\Delta + 2\lambda}{\Omega_R} e^{i\lambda t} + \frac{\Delta - 2\lambda}{\Omega_R}\beta e^{-i\lambda t}\right] \end{cases}$$

Έστωσαν οι αρχικές συνθήκες (Α.Σ) $C_1(0)=1$ , $C_2(0)=0$ 

$$\left.\begin{array}{l} 1=\alpha+\beta \\ 0=\alpha\dfrac{\Delta+2\lambda}{\Omega_R}+\beta\dfrac{\Delta-2\lambda}{\Omega_R} \Rightarrow \alpha(\Delta+2\lambda)=\beta(2\lambda-\Delta) \Rightarrow \\ \qquad \beta=\dfrac{2\lambda+\Delta}{2\lambda-\Delta}\alpha \end{array}\right\} \Rightarrow$$

$$1=\alpha+\frac{2\lambda+\Delta}{2\lambda-\Delta}\alpha=\alpha\frac{2\lambda-\Delta+2\lambda+\Delta}{2\lambda-\Delta}\Rightarrow(2\lambda-\Delta)=\alpha 4\lambda$$

$$\Rightarrow\boxed{\alpha=\frac{2\lambda-\Delta}{4\lambda}} \qquad \beta=\frac{2\lambda+\Delta}{2\lambda-\Delta}\,\frac{2\lambda-\Delta}{4\lambda}\Rightarrow\boxed{\beta=\frac{2\lambda+\Delta}{4\lambda}}$$

$$C_1(t)=e^{i\frac{\Delta}{2}t}\left[\frac{2\lambda-\Delta}{4\lambda}e^{i\lambda t}+\frac{2\lambda+\Delta}{4\lambda}e^{-i\lambda t}\right]$$

$$C_2(t)=e^{-i\frac{\Delta}{2}t}\left[\frac{\Delta+2\lambda}{\Omega_R}\,\frac{2\lambda-\Delta}{4\lambda}e^{i\lambda t}+\frac{\Delta-2\lambda}{\Omega_R}\,\frac{2\lambda+\Delta}{4\lambda}e^{-i\lambda t}\right]$$

$$\frac{4\lambda^2-\Delta^2}{4\Omega_R\lambda}=\frac{\Delta^2+\Omega_R^2-\Delta^2}{4\Omega_R\lambda}=\frac{\Omega_R}{4\lambda}$$

$$\frac{\Delta^2-4\lambda^2}{4\Omega_R\lambda}=\frac{\Delta^2-\Delta^2-\Omega_R^2}{4\Omega_R\lambda}=\frac{-\Omega_R}{4\lambda}$$

cos isin
−cos +isin

$$C_2(t)=e^{-i\frac{\Delta}{2}t}\left[\frac{\Omega_R}{4\lambda}e^{i\lambda t}-\frac{\Omega_R}{4\lambda}e^{-i\lambda t}\right]\Rightarrow C_2(t)=e^{-i\frac{\Delta}{2}t}\frac{\Omega_R}{4\lambda}2i\sin(\lambda t)$$

Άρα $P_2(t)=|C_2(t)|^2=\dfrac{\Omega_R^2}{\Delta^2+\Omega_R^2}\sin^2(\lambda t)$

$$P_1(t)=|C_1(t)|^2=1-\frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\sin^2(\lambda t)$$