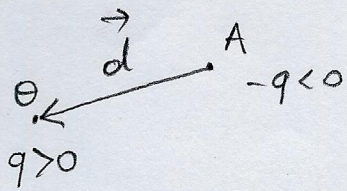
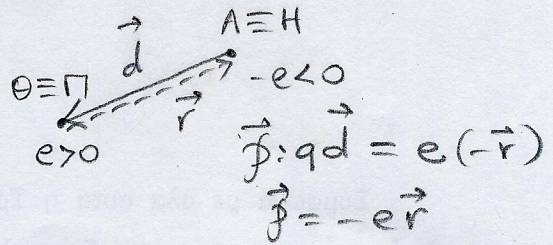


**(ΗΛΕΚΤΡΙΚΗ) ΔΙΠΟΛΙΚΗ ΡΟΠΗ** (electric) dipole moment



$$\vec{d} = A\vec{\theta}$$

$$\vec{p} = q \cdot \vec{d}$$



$$\vec{p} = q\vec{d} = e(-\vec{r})$$

$$\vec{p} = -e\vec{r}$$

$$[U] = N \cdot m = J$$

$$U = -\vec{p} \cdot \vec{E}$$

δυναμική ενέργεια (potential energy)

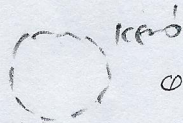
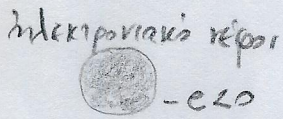
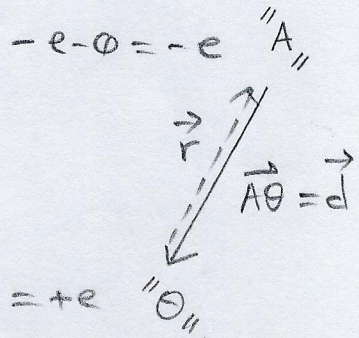
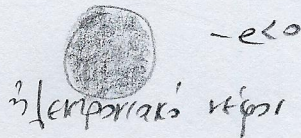
$$[\vec{\tau}] = N \cdot m$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

(μηχανική) ροπή (torque)

**ΗΛΕΚΤΡΙΚΗ ΔΙΠΟΛΙΚΗ ΡΟΠΗ ΜΕΤΑΒΑΣΕΩΣ**

transition (electric) dipole moment



$$0 - (-e) = +e \text{ "}\theta\text{"}$$

Αρχικώς

Τελικώς

Τελικώς - Αρχικώς

$$\vec{p} = e\vec{d} = -e\vec{r}$$

τελεστής (βλεκτριμής) διπολικής ροής μεταβάσεως

$$\hat{d} = \hat{p} := \sum_{i=1}^N \sum_{j=1}^N \vec{d}_{ij} |\Phi_i\rangle \langle \Phi_j|$$

$$\vec{d}_{ij} = \vec{p}_{ij} = -e \langle \Phi_i | \hat{r} | \Phi_j \rangle = \dots = -e \int d^3r \Phi_i^*(\vec{r}) \vec{r} \Phi_j(\vec{r})$$

$$\hat{r} |r\rangle = r |r\rangle$$

$$\langle \Phi_i | \hat{r} | \Phi_j \rangle = \sum_{|r'\rangle} \sum_{|r''\rangle} \langle \Phi_i | r'\rangle \langle r' | \hat{r} | r''\rangle \langle r'' | \Phi_j \rangle$$

$$= \sum_{|r'\rangle} \sum_{|r''\rangle} \Phi_i^*(r') \underbrace{r''}_{\delta_{r'r''}} \Phi_j(r'')$$

$$= \sum_{|r'\rangle} \Phi_i^*(r') r' \Phi_j(r') = \sum_{|r\rangle} \Phi_i^*(r) r \Phi_j(r)$$

$$= \int d^3r \Phi_i^*(r) r \Phi_j(r)$$

$$\underline{\underline{\Delta \Sigma}} \quad \hat{p} = \vec{d}_{11} |\Phi_1\rangle \langle \Phi_1| + \vec{d}_{12} |\Phi_1\rangle \langle \Phi_2| + \vec{d}_{21} |\Phi_2\rangle \langle \Phi_1| + \vec{d}_{22} |\Phi_2\rangle \langle \Phi_2|$$

$$\vec{d}_{11} = -e \int d^3r \Phi_1^*(r) r \Phi_1(r) = 0$$

$$\vec{d}_{12} = -e \int d^3r \Phi_1^*(r) r \Phi_2(r) \neq 0$$

$$\vec{d}_{21} = -e \int d^3r \Phi_2^*(r) r \Phi_1(r) \neq 0$$

$$\vec{d}_{22} = -e \int d^3r \Phi_2^*(r) r \Phi_2(r) = 0$$

$\vec{d}_{12} = \vec{d}_{21}$  σε  $\{\Phi_i(r)\}$  πραγματικές

$$\hat{p} = \vec{d}_{12} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} + \vec{d}_{21} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \Rightarrow \hat{p} = \vec{d}_{12} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

δυναμική ενέργεια

$$U_{\varepsilon} = - \vec{p} \cdot \vec{\varepsilon}$$

δυναμική ενέργεια με τρόπο

$$U_{\varepsilon}^m = - \vec{p} \cdot \vec{\varepsilon}^m$$

(Γ)

τελεστής δυναμικής ενέργειας με τρόπο

$$\hat{U}_{\varepsilon}^m = - \hat{p} \cdot \hat{\varepsilon}^m$$

$$\hat{U}_{\varepsilon}^m = - \sum_{i=1}^2 \sum_{j=1}^2 \vec{d}_{ij} |\Phi_i\rangle \langle \Phi_j| \cdot \hat{E}_x^m(z,t) \hat{x}$$

$$\Delta \Sigma \quad \hat{U}_{\varepsilon}^m = - \vec{d}_{12} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \hat{E}_x^m(z,t) \hat{x} = - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{E}_x^m(z,t) \vec{d}_{12} \cdot \hat{x}$$

$$\vec{d}_{12} \cdot \hat{x} = -e \int d^3r \Phi_1^*(\vec{r}) \vec{r} \Phi_2(\vec{r}) \cdot \hat{x} =$$

$$= -e \int d^3r \Phi_1^*(\vec{r}) \times \Phi_2(\vec{r}) = -e x_{12} = \mathcal{J}_{x12} := \mathcal{J}$$

"Απο  $\hat{U}_{\varepsilon}^m = e x_{12} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{E}_x^m(z,t)$

$$\hat{E}_x^m(z,t) = \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) (\hat{a}_m^{\dagger} + \hat{a}_m)$$

$$\hat{B}_y^m(z,t) = \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \frac{1}{c} \cos\left(\frac{m\pi z}{L}\right) i (\hat{a}_m^{\dagger} - \hat{a}_m)$$

$$\hat{S}_+ + \hat{S}_- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{U}_{\varepsilon}^m = e x_{12} (\hat{S}_+ + \hat{S}_-) \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) (\hat{a}_m^{\dagger} + \hat{a}_m)$$

$$\hat{U}_{\varepsilon}^m = e x_{12} \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^{\dagger} + \hat{a}_m)$$

$$\hbar g^m$$

$$\hbar g^m = e x_{12} \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) \quad (\mathcal{J} \hbar \omega_m)$$

$$\hat{U}_\varepsilon^m = \hbar g^m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m)$$

χαμηλότερη αλληλεπίδραση  
 $\Delta \Sigma$  - η τρέπου του ΗΜ πεδίου

(στην άριστη φυσική λέγεται  
 συχνά  $\hat{H}_{AF}$

AF = atom-field)

$$g^m \Rightarrow \hbar |g^m| = |\beta| \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \left| \sin\left(\frac{m\pi z}{L}\right) \right|$$

$$\Omega_R^m := 2\sqrt{\hbar} g^m$$



είναι καλύτερα να δρίσουμε  
 τη συχνότητα Rabi άκριβώς έτσι  
 ώστε να υπάρχει πλήρης αναλογία  
 με την ήμικλασική περίπτωση

$$\frac{\hbar \Omega_R}{2\sqrt{\hbar}} = |\beta| \left( \frac{\hbar \omega_m}{\varepsilon_0 V} \right)^{1/2} \left| \sin\left(\frac{m\pi z}{L}\right) \right|$$

ΤΟ ΠΑΤΙ ΘΑ ΦΑΝΕΙ ΠΑΡΑΚΑΤΩ

$$\Omega_R = \frac{|\beta|}{\hbar} \left( \frac{4\hbar \omega_m \hbar}{\varepsilon_0 V} \right)^{1/2} \left| \sin\left(\frac{m\pi z}{L}\right) \right| := \frac{|\beta| E_{0m}}{\hbar}$$

"πλάτος" ηλεκτρικού πεδίου

$$E_{0m} = \left( \frac{4\hbar \omega_m \hbar}{\varepsilon_0 V} \right)^{1/2} \left| \sin\left(\frac{m\pi z}{L}\right) \right|$$

χωρικά διαμορφωμένο

$$[E_{0m}] = \left( \frac{J}{\frac{F}{m} \cdot m^3} \right)^{1/2} = \left( \frac{C \cdot V}{\frac{C}{V} \cdot m^2} \right)^{1/2} = \frac{V}{m}$$

μονάδα ηλεκτρικού πεδίου

ΣΥΝΟΨΗ ΧΑΜΙΛΤΟΝΙΑΝΩΝ

$$\hat{H}_{HM,m} = \hbar \omega_m \left( \hat{a}_m^\dagger \hat{a}_m + \frac{1}{2} \right)$$

m τρόπος ΗΜ πεδίου  
 $\omega_m = \frac{m\pi c}{L}, m \in \mathbb{N}^*$

$$\hbar \omega_m \left( \hat{N}_m + \frac{1}{2} \right)$$

κι άγρώνεται τον  $\hbar \omega_m / 2$

$$\hat{H}_{HM,m} = \hbar \omega_m \hat{a}_m^\dagger \hat{a}_m$$

$$\hbar \omega_m \hat{N}_m$$

$$\hat{H}_{\Delta\Sigma} = E_2 \hat{S}_+ \hat{S}_- + E_1 \hat{S}_- \hat{S}_+$$

$$E_2 - E_1 = \hbar \Omega$$

δισταθμικά εθωσμεν

δέτονται  $E_1 = 0$

$$\hat{H}_{\Delta\Sigma} = \hbar \Omega \hat{S}_+ \hat{S}_-$$

χωρικά διαμορφωμένο "πλάτος"

$$E_{0m} = \left| \left( \frac{4\hbar \omega_m^2 \epsilon_m}{\epsilon V} \right)^{1/2} \sin\left(\frac{m\pi z}{L}\right) \right|$$

$$\hat{U}_\epsilon^m = \hbar g^m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m) = \hat{H}_{HM-\Delta\Sigma}$$

άλλη επιδραση m τρόπου ΗΜ πεδίου - ΔΣ

$$\Omega_R^m := 2\sqrt{n_m} g_m$$

$$\Omega_R^m := \frac{|E_{0m}|}{\hbar}$$

συχρότητα Rabi

"Αρα η δλική Χαμιλτονιανή του m τρόπου γράφεται

$$\hat{H}_{Rm} = \hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_- + \hbar g^m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m)$$

ονομάζεται συχνά Χαμιλτονιανή Rabi

ΠΡΟΣΟΧΗ

$$|\uparrow, n_m\rangle \text{ κ } |\downarrow, n_m\rangle$$

ιδιοκαταστάσεις τῆς  $(\hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_-)$

και ζῆ τῆς  $\hat{H}_{HM,m} + \hat{H}_{\Delta\Sigma} + \hat{H}_{\Delta\Sigma-HM,m}$

$$\hat{H}_{HM,m} + \hat{H}_{\Delta\Sigma}$$

As souye poretiktikotera tu Xayltoriari Adhiki spistaw  $\Delta E$  - HM nediaw  
 $\Delta E$  - Eros tponou u toS HM nediaw

$$\hat{U}_\varepsilon^m = \hbar g^m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m) =$$

$$= \hbar g^m \left( \hat{S}_+ \hat{a}_m^\dagger + \hat{S}_+ \hat{a}_m + \hat{S}_- \hat{a}_m^\dagger + \hat{S}_- \hat{a}_m \right)$$

1or  $\hat{S}_+ \hat{a}_m^\dagger$

$\implies$   
 $f_i$   
 $\omega_i$

$$\begin{pmatrix} 0 \\ \bullet \end{pmatrix}$$

npiw

$$\begin{pmatrix} \bullet \\ 0 \end{pmatrix}$$

ypetw

$\implies$   
 $f_f < f_i$   
 $\omega_f < \omega_i$

$\Delta E > 0$

av exw pollous tponous den einai napetogoi yuxanismois

2or  $\hat{S}_+ \hat{a}_m$

$$\begin{pmatrix} 0 \\ \bullet \end{pmatrix}$$

npiw

$$\begin{pmatrix} \bullet \\ 0 \end{pmatrix}$$

ypetw

"ous diatpiti tuv erepote

3or  $\hat{S}_- \hat{a}_m^\dagger$

$$\begin{pmatrix} \bullet \\ 0 \end{pmatrix}$$

npiw

$$\begin{pmatrix} 0 \\ \bullet \end{pmatrix}$$

ypetw

"ous diatpiti tuv erepote

4or  $\hat{S}_- \hat{a}_m$

$$\begin{pmatrix} \bullet \\ 0 \end{pmatrix}$$

npiw

$\implies$   
 $f_i$   
 $\omega_i$

$$\begin{pmatrix} 0 \\ \bullet \end{pmatrix}$$

ypetw

$\implies$   
 $f_f > f_i$   
 $\omega_f > \omega_i$

$\Delta E < 0$

av exw pollous tponous den einai napetogoi yuxanismois

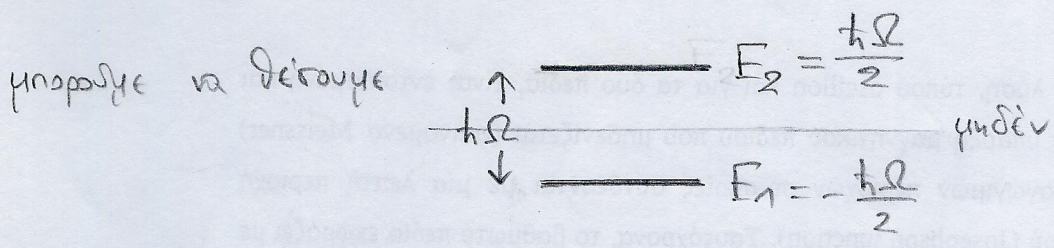
An apothisoupe ton 1o kai ton 4o opo pou o katevas yponou den diatpiti tuv erepote

$\implies \hat{U}_\varepsilon^m = \hbar g^m (\hat{S}_+ \hat{a}_m + \hat{S}_- \hat{a}_m^\dagger)$

$$\hat{H}_{JCM} = \hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_- + \hbar g_m (\hat{S}_+ \hat{a}_m + \hat{S}_- \hat{a}_m^\dagger)$$

Jaynes - Cummings  
 Xayltoriari

$$\hat{H}_{\Delta\Sigma} = E_2 \hat{S}_+ \hat{S}_- + E_1 \hat{S}_- \hat{S}_+ \quad E_2 - E_1 = \hbar\Omega$$



$$\hat{H}_{\Delta\Sigma} = \frac{\hbar\Omega}{2} \hat{S}_+ \hat{S}_- - \frac{\hbar\Omega}{2} \hat{S}_- \hat{S}_+$$

$$\left. \begin{aligned} \hat{S}_+ \hat{S}_- &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \hat{S}_- \hat{S}_+ &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned} \right\} \begin{aligned} \hat{S}_+ \hat{S}_- + \hat{S}_- \hat{S}_+ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \hat{S}_+ \hat{S}_- - \hat{S}_- \hat{S}_+ &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \hat{\sigma}_z \end{aligned}$$

"Αρα  $\hat{H}_{\Delta\Sigma} = \frac{\hbar\Omega}{2} \hat{\sigma}_z$  ή μπορούμε να  $\hat{H}_{\Delta\Sigma}$  στο  $\hat{\sigma}_z$  στο  $\hat{H}_{\Delta\Sigma}$  στο  $\hat{\sigma}_z$  Jaynes-Cummings!

ΑΣΚΗΣΗ

$\hat{a}^+ |n\rangle = \sqrt{n+1} |n+1\rangle$   $\hat{a} |n\rangle = \sqrt{n} |n-1\rangle$  (H)

Να αποδείξτε τις σχέσεις

(α)  $[\hat{a}, \hat{a}] = 0$  (β)  $[\hat{a}^+, \hat{a}^+] = 0$  (γ)  $[\hat{a}, \hat{a}^+] = 1$  (δ)  $\hat{N} |n\rangle = n |n\rangle$

(ε)  $[\hat{N}, \hat{a}] = -\hat{a}$  (ς)  $[\hat{N}, \hat{a}^+] = \hat{a}^+$  (ζ)  $\hat{N} (\hat{a} |n\rangle) = (n-1) (\hat{a} |n\rangle)$

(η)  $\hat{N} (\hat{a}^+ |n\rangle) = (n+1) (\hat{a}^+ |n\rangle)$

(α)  $[\hat{a}, \hat{a}] = \hat{a}\hat{a} - \hat{a}\hat{a} = 0$  (β)  $[\hat{a}^+, \hat{a}^+] = \hat{a}^+\hat{a}^+ - \hat{a}^+\hat{a}^+ = 0$

(γ)  $[\hat{a}, \hat{a}^+] |n\rangle = \hat{a}\hat{a}^+ |n\rangle - \hat{a}^+\hat{a} |n\rangle = \hat{a}\sqrt{n+1} |n+1\rangle - \hat{a}^+\sqrt{n} |n-1\rangle =$   
 $= \sqrt{n+1}\sqrt{n+1} |n\rangle - \sqrt{n}\sqrt{n} |n\rangle = (n+1)|n\rangle - n|n\rangle = 1 \cdot |n\rangle$

$\Rightarrow [\hat{a}, \hat{a}^+] = 1$

(ε)  $[\hat{N}, \hat{a}] = [\hat{a}^+\hat{a}, \hat{a}] = \hat{a}^+[\hat{a}, \hat{a}] + [\hat{a}^+, \hat{a}]\hat{a} = -\hat{a}$

(ς)  $[\hat{N}, \hat{a}^+] = [\hat{a}^+\hat{a}, \hat{a}^+] = \hat{a}^+[\hat{a}, \hat{a}^+] + [\hat{a}^+, \hat{a}^+]\hat{a} = \hat{a}^+$

(δ)  $\hat{N} |n\rangle = \hat{a}^+\hat{a} |n\rangle = \hat{a}^+\sqrt{n} |n-1\rangle = \sqrt{n}\sqrt{n} |n\rangle = n |n\rangle \Rightarrow \hat{N} |n\rangle = n |n\rangle$

(ζ)  $\hat{N} (\hat{a} |n\rangle) = \hat{N} \sqrt{n} |n-1\rangle = \sqrt{n} (n-1) |n-1\rangle = (n-1) \sqrt{n} |n-1\rangle = (n-1) (\hat{a} |n\rangle)$

(η)  $\hat{N} (\hat{a}^+ |n\rangle) = \hat{N} \sqrt{n+1} |n+1\rangle = \sqrt{n+1} \hat{N} |n+1\rangle = \sqrt{n+1} (n+1) |n+1\rangle =$   
 $(n+1) \sqrt{n+1} |n+1\rangle = (n+1) (\hat{a}^+ |n\rangle)$