(α')

Είχαμε δείξει πώς για την κατάσταση

$$|\psi_A(t)\rangle = c_1(t)|\downarrow n\rangle + c_2(t)|\uparrow n-1\rangle = |A\rangle$$

$$\left.\begin{aligned} \langle \hat{a}^{+}\hat{a}\rangle &= n - |c_2(t)|^2 \\ \langle \hat{a}\hat{a}^{+}\rangle &= n + |c_1(t)|^2 \end{aligned}\right\} \Rightarrow \langle \hat{a}\hat{a}^{+}\rangle - \langle \hat{a}^{+}\hat{a}\rangle = 1$$

$$\left.\begin{aligned} \langle \hat{S}_+\hat{S}_-\rangle &= |c_2(t)|^2 \\ \langle \hat{S}_-\hat{S}_+\rangle &= |c_1(t)|^2 \end{aligned}\right\} \Rightarrow \langle \hat{S}_+\hat{S}_-\rangle + \langle \hat{S}_-\hat{S}_+\rangle = 1$$

$$\langle \hat{a}^{+}\hat{a}\rangle + \langle \hat{S}_+\hat{S}_-\rangle = n$$

$$\langle \hat{S}_+\hat{a}\rangle = c_2^*(t)\, c_1(t)\sqrt{n} \qquad \langle \hat{S}_+\hat{a}^{+}\rangle = 0$$

$$\langle \hat{S}_-\hat{a}^{+}\rangle = c_1^*(t)\, c_2(t)\sqrt{n} \qquad \langle \hat{S}_-\hat{a}\rangle = 0$$

Μεταφράζοντας δηλαδή την $|\psi_A(t)\rangle = c_1(t)|\downarrow n\rangle + c_2(t)|\uparrow n-1\rangle = |A\rangle$

- $\hat{a}^{+}\hat{a}$ μετρά τον αριθμό των φωτονίων, άρα

$$\langle \hat{a}^{+}\hat{a}\rangle_{|A\rangle} = |c_1(t)|^2 n + |c_2(t)|^2 (n-1) = n\left(|c_1(t)|^2 + |c_2(t)|^2\right) - |c_2(t)|^2$$
$$= n - |c_2(t)|^2$$

- $\hat{a}\hat{a}^{+}$ μετρά τον αριθμό των φωτονίων και προσθέτει 1, άρα

$$\langle \hat{a}\hat{a}^{+}\rangle_{|A\rangle} = |c_1(t)|^2 (n+1) + |c_2(t)|^2 n = n\left(|c_1(t)|^2 + |c_2(t)|^2\right) + |c_1(t)|^2$$
$$= n + |c_1(t)|^2$$

- $\hat{S}_+\hat{S}_-$ μετρά τον αριθμό των ηλεκτρονίων στην άνω στάθμη, άρα

$$\langle \hat{S}_+\hat{S}_-\rangle_{|A\rangle} = |c_2(t)|^2$$

- $\hat{S}_-\hat{S}_+$ μετρά τον αριθμό των ηλεκτρονίων στην κάτω στάθμη, άρα

$$\langle \hat{S}_-\hat{S}_+\rangle_{|A\rangle} = |c_1(t)|^2$$

$$|\psi_A(t)\rangle = c_1(t)\,|\downarrow n\rangle + c_2(t)\,|\uparrow n-1\rangle$$

$$i\hbar\frac{\partial}{\partial t}|\psi_A(t)\rangle = \hat H\,|\psi_A(t)\rangle$$

$$\hat H = \hat H_{JCm} = \hbar\omega_m \hat a_m^{+}\hat a_m + \hbar\Omega\hat S_+\hat S_- + \hbar g_m\left(\hat S_+\hat a_m + \hat S_-\hat a_m^{+}\right)$$

Α.Σ. $c_1(0)=1, \quad c_2(0)=0$

$$A' = i\hbar\frac{\partial}{\partial t}|\psi_A(t)\rangle = \left(\dot c_1(t)\,|\downarrow n\rangle + \dot c_2(t)\,|\uparrow n-1\rangle\right)i\hbar$$

$$\Delta' = \left(\hbar\omega_m \hat a_m^{+}\hat a_m + \hbar\Omega\hat S_+\hat S_- + \hbar g_m\hat S_+\hat a + \hbar g_m\hat S_-\hat a^{+}\right)\left(c_1(t)\,|\downarrow n\rangle + c_2(t)\,|\uparrow n-1\rangle\right) =$$

$$= \hbar\omega_m c_1(t)\,n\,|\downarrow n\rangle + \hbar\omega_m c_2(t)\,(n-1)\,|\uparrow n-1\rangle +$$

$$\hbar\Omega\, c_1(t)\,|0\, n\rangle + \hbar\Omega\, c_2(t)\,|\uparrow n-1\rangle +$$

$$\hbar g_m c_1(t)\sqrt{n}\,|\uparrow n-1\rangle + \hbar g_m c_2(t)\sqrt{n-1}\,|0\, n-2\rangle +$$

$$\hbar g_m c_1(t)\sqrt{n+1}\,|0\, n+1\rangle + \hbar g_m c_2(t)\sqrt{n}\,|\downarrow n\rangle =$$

Επί $\langle\downarrow n|$

$$\left.\begin{aligned} A' &= i\hbar\,\dot c_1(t) \\ \Delta' &= \hbar\omega_m c_1(t)\,n + \hbar g_m c_2(t)\sqrt{n} \end{aligned}\right\} \quad i\cancel{\hbar}\,\dot c_1(t) = \cancel{\hbar}\omega n\, c_1(t) + \cancel{\hbar} g_m\sqrt{n}\, c_2(t)$$

Επί $\langle\uparrow n-1|$

$$\left.\begin{aligned} A' &= i\hbar\,\dot c_2(t) \\ \Delta' &= \hbar\omega_m c_2(t)(n-1) + \hbar\Omega c_2(t) + \hbar g_m\sqrt{n}\, c_1(t) \end{aligned}\right\} \Rightarrow$$

$$i\cancel{\hbar}\,\dot c_2(t) = \cancel{\hbar} g_m\sqrt{n}\, c_1(t) + \left[\cancel{\hbar}\omega_m(n-1) + \cancel{\hbar}\Omega\right]c_2(t)$$

$$i\begin{bmatrix}\dot c_1(t)\\ \dot c_2(t)\end{bmatrix} = \begin{bmatrix} n_m\omega_m & \sqrt{n_m}\,g_m \\ \sqrt{n_m}\,g_m & \Omega+(n_m-1)\omega_m\end{bmatrix}\begin{bmatrix} c_1(t)\\ c_2(t)\end{bmatrix}$$

υπενθυμίζεται πως για απλότητα "ξεχνάμε" το δείκτη m

$\omega = \omega_m$

$g = g_m$

$n = n_m$

Ορίζουμε $\Omega_n := \left[\left(\frac{\omega-\Omega}{2}\right)^2 + g^2 n\right]^{1/2} = \sqrt{\left(\frac{\Delta}{2}\right)^2 + g^2 n}$

και παραλείποντας τις πράξεις προκύπτει

$$c_1(t) = \exp\left[-i\left(n\omega + \frac{\Omega-\omega}{2}\right)t\right]\left\{\cos(\Omega_n t) + i\,\frac{\Omega-\omega}{2\Omega_n}\cdot\sin(\Omega_n t)\right\}$$

$$c_2(t) = \exp\left[-i\left(n\omega + \frac{\Omega-\omega}{2}\right)t\right]\left\{-i\,\frac{g\sqrt{n}}{\Omega_n}\cdot\sin(\Omega_n t)\right\}$$

(γ')

Άρα $\boxed{|c_2(t)|^2 = \frac{ng^2}{\Omega_n^2}\sin^2(\Omega_n t)}$ $\quad |c_1(t)|^2 = 1 - |c_2(t)|^2 = \dots$

$$\boxed{\langle \hat{a}^+\hat{a}\rangle_{Ⓐ} = n - \frac{ng^2}{\Omega_n^2}\sin^2(\Omega_n t)} \qquad \boxed{\langle \hat{S}_+\hat{S}_-\rangle_{Ⓐ} = \frac{ng^2}{\Omega_n^2}\cdot\sin^2(\Omega_n t)}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\langle \hat{a}^+\hat{a}\rangle_{Ⓐ} = n - \frac{ng^2}{2\Omega_n^2} + \frac{ng^2}{2\Omega_n^2}\cos(2\Omega_n t)$$

$$\langle \hat{S}_+\hat{S}_-\rangle_{Ⓐ} = \frac{ng^2}{2\Omega_n^2} - \frac{ng^2}{2\Omega_n^2}\cos(2\Omega_n t)$$

maximum transfer percentage
μέγιστο ποσοστό μεταβιβάσεως

$$\mathcal{A}_R = \frac{ng^2}{\Omega_n^2} = \frac{ng^2}{\frac{\Delta^2}{4} + ng^2} = \frac{4ng^2}{4ng^2 + \Delta^2}$$

περίοδος ταλαντώσεων

$$T_R = \frac{2\pi}{2\Omega_n} = \frac{\pi}{\Omega_n} = \frac{2\pi}{2\sqrt{\frac{\Delta^2}{4} + g^2 n}} = \frac{2\pi}{\sqrt{\Delta^2 + 4ng^2}}$$

$$\Omega_n = \sqrt{\frac{\Delta^2}{4} + g^2 n}$$

$$2\Omega_n = \sqrt{\Delta^2 + 4ng^2}$$

$$2\Omega_n = \sqrt{\Delta^2 + \Omega_R^2}$$

$\Omega_R := 2\sqrt{n}\, g$ (κυκλική) συχνότητα Rabi

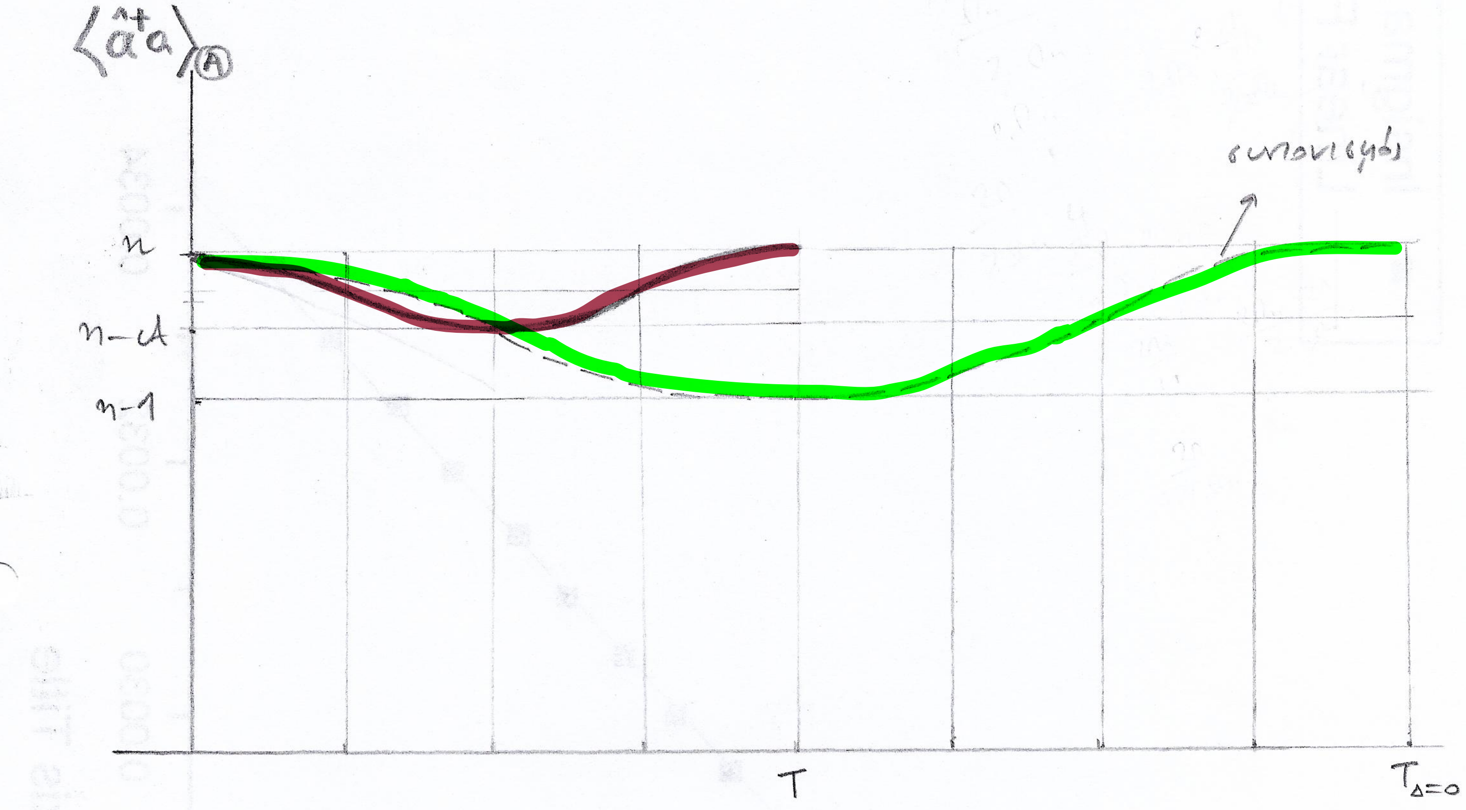
$$\mathcal{A}_R = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}$$

$$T_R = \frac{2\pi}{\sqrt{\Delta^2 + \Omega_R^2}} = \frac{1}{f_R}$$

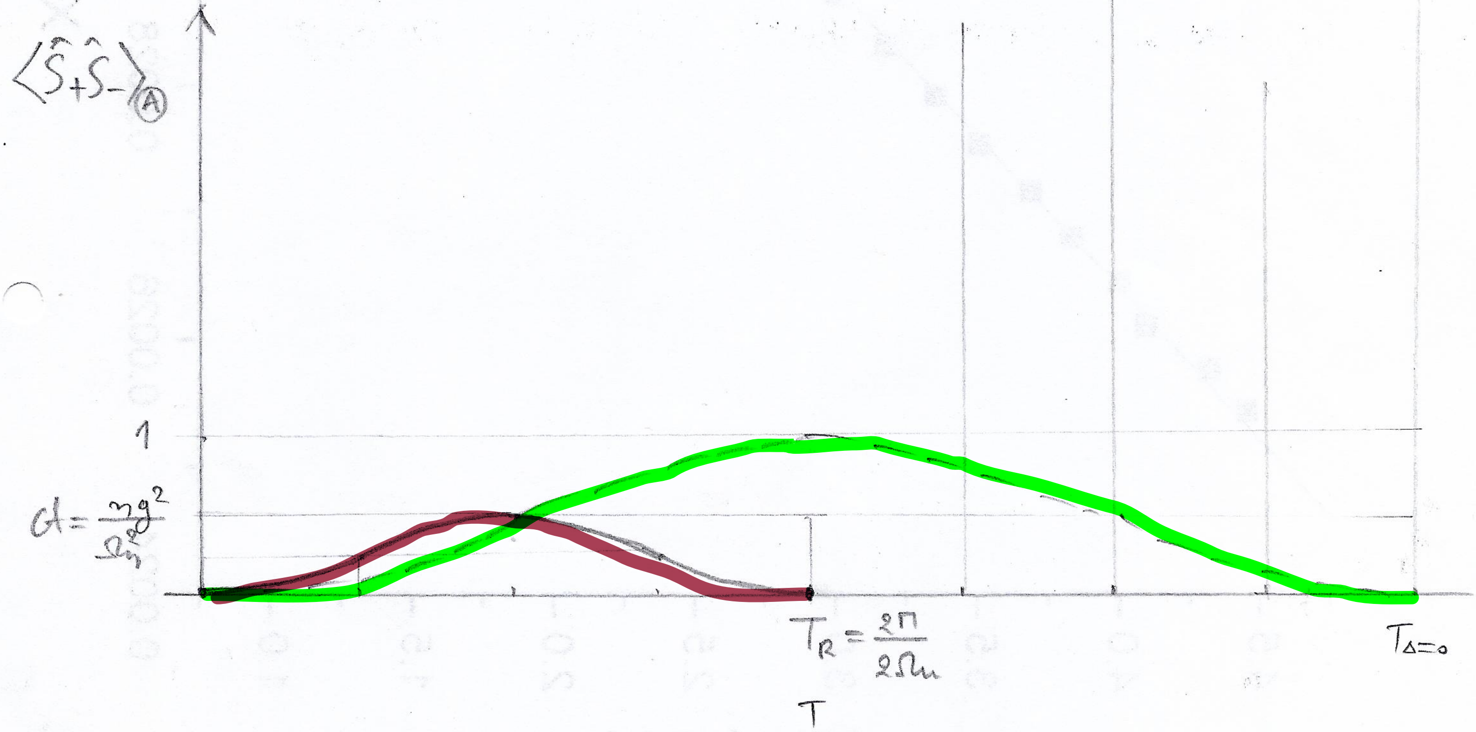
Δηλαδή η (κυκλική) συχνότητα Rabi Ω_R
και ο αποσυντονισμός Δ καθορίζουν
την περίοδο και το μέγιστο ποσοστό μεταβιβάσεως

Αν $\Delta = 0 \Rightarrow \mathcal{A}_R = 1$

$$T_R = \frac{2\pi}{\Omega_R}$$



⟨â⁺a⟩ (A)
n
n−α
n−1
συντονισμός
T
T_{Δ=0}



⟨Ŝ₊Ŝ₋⟩ (A)
1
α = ng²/Ω²
T_R = 2π/2Ω_n
T_{Δ=0}
T

$$i\begin{pmatrix}\dot c_1\\ \dot c_2\end{pmatrix}=\underbrace{\begin{pmatrix} n\omega & g\sqrt{n}\\ g\sqrt{n} & \Omega+(n-1)\omega\end{pmatrix}}_{A}\begin{pmatrix}c_1\\ c_2\end{pmatrix} \quad \vec{x}(t)=\begin{pmatrix}c_1\\ c_2\end{pmatrix} \quad \dot{\vec{x}}(t)=\begin{pmatrix}\dot c_1\\ \dot c_2\end{pmatrix}$$

(ε)

$$i\dot{\vec{x}}(t)=A\vec{x}(t)\Rightarrow \boxed{\dot{\vec{x}}(t)=-iA\vec{x}(t)}$$

$$\Delta\Lambda M \quad \vec{x}(t)=\vec{u}e^{-i\lambda t}\Rightarrow \dot{\vec{x}}(t)=\vec{u}(-i\lambda)e^{-i\lambda t} \Rightarrow \vec{u}(-i\lambda)\cancel{e^{-i\lambda t}}=-iA\vec{u}\cancel{e^{-i\lambda t}}$$

$$\Rightarrow \boxed{A\vec{u}=\lambda\vec{u}} \; (*) \Leftrightarrow A\vec{u}=\lambda I\vec{u}\Leftrightarrow (A-\lambda I)\vec{u}=0$$

πρόβλημα ιδιοτιμών

πρέπει $\det(A-\lambda I)=0$

$$\begin{vmatrix} n\omega-\lambda & g\sqrt{n}\\ g\sqrt{n} & \Omega+(n-1)\omega-\lambda\end{vmatrix}=0\Rightarrow (n\omega-\lambda)\left[\Omega+(n-1)\omega-\lambda\right]-ng^2=0$$

$$\lambda^2-\left[\Omega+(n-1)\omega+n\omega\right]\lambda+n\omega\left[\Omega+(n-1)\omega\right]-ng^2=0$$

Έστω $n=1$

ένα φωτόνιο στην κοιλότητα

$$A=\begin{pmatrix}\omega & g\\ g & \Omega\end{pmatrix} \quad \text{και} \quad \det(A-\lambda I)=0\Rightarrow$$

$$\begin{vmatrix}\omega-\lambda & g\\ g & \Omega-\lambda\end{vmatrix}=0\Rightarrow$$

$$(\omega-\lambda)(\Omega-\lambda)-g^2=0$$

$$\lambda^2-(\omega+\Omega)\lambda+\omega\Omega-g^2=0$$

$$\Delta'=(\omega+\Omega)^2-4(\omega\Omega-g^2)=$$

$$=\omega^2+\Omega^2+2\omega\Omega-4\omega\Omega+4g^2$$

$$=(\omega-\Omega)^2+4g^2\Rightarrow$$

$$\Delta'=\Delta^2+4g^2$$

$$(*)\begin{bmatrix} n\omega & g\sqrt{n}\\ g\sqrt{n} & \Omega+(n-1)\omega\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\lambda\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix} n\omega-\lambda & g\sqrt{n}\\ g\sqrt{n} & \Omega+(n-1)\omega-\lambda\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\lambda_{2,1}=\frac{(\omega+\Omega)\pm\sqrt{\Delta^2+4g^2}}{2}$$

$$\lambda_{2,1}=\frac{\omega+\Omega}{2}\pm\sqrt{\left(\frac{\Delta}{2}\right)^2+g^2}$$

$$\boxed{\lambda_{2,1}=H_1\pm\Omega_1} \qquad H_1=\frac{\omega+\Omega}{2} \qquad \Omega_1=\sqrt{\left(\frac{\Delta}{2}\right)^2+g^2}$$

$\lambda_1 = H_1 - \Omega_1$ $\quad A\vec{U}_1 = \lambda_1 \vec{U}_1 \Rightarrow \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix}\begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix} = (H_1 - \Omega_1)\begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix}$

$$\Rightarrow \begin{array}{l} \omega U_{11} + g U_{21} = (H_1 - \Omega_1) U_{11} \Rightarrow g U_{21} = (H_1 - \Omega_1 - \omega) U_{11} \\ g U_{11} + \Omega U_{21} = (H_1 - \Omega_1) U_{21} \Rightarrow g U_{11} = (H_1 - \Omega_1 - \Omega) U_{21} \end{array}$$

$$g U_{21} = \frac{(H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) U_{21}}{g}$$

$$g^2 U_{21} = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) U_{21}$$

Άρα $U_{21} = 0 \Rightarrow U_{11} = 0$

ή $g^2 = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) \Rightarrow$

$$g^2 = \left(\frac{\Omega - \omega}{2} - \Omega_1\right)\left(\frac{\omega - \Omega}{2} - \Omega_1\right) \Rightarrow$$

$$g^2 = -\left(\frac{\Delta}{2} + \Omega_1\right)\left(\frac{\Delta}{2} - \Omega_1\right) \Rightarrow g^2 = -\left[\frac{\Delta^2}{4} - \Omega_1^2\right] \Rightarrow$$

$g^2 + \frac{\Delta^2}{4} = \Omega_1^2$ που ισχύει εξ ορισμού της Ω_1

$\boxed{\text{δηλαδή το } U_{21} \text{ μπορεί να είναι οτιδήποτε π.χ. } U_{21} = 1} \Rightarrow$

$$g U_{11} = (H_1 - \Omega_1 - \Omega) \cdot 1 = \frac{\Delta}{2} - \Omega_1 \Rightarrow U_{11} = \frac{\frac{\Delta}{2} - \Omega_1}{g} \Rightarrow$$

$$\boxed{U_{11} = \frac{\Delta - 2\Omega_1}{2g}}$$

$$\vec{U}_1 = \begin{bmatrix} \dfrac{\Delta - 2\Omega_1}{2g} \\ 1 \end{bmatrix}$$

$\lambda_2 = H_1 + \Omega_1$ $\Rightarrow$ πράξεις $\Rightarrow$ $\vec{U}_2 = \begin{bmatrix} \dfrac{\Delta + 2\Omega_1}{2g} \\ 1 \end{bmatrix}$

$\lambda_2 = H_1 + \Omega_1$ (8')

$$A\vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} w & g \\ g & \Omega \end{pmatrix}\begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_1+\Omega_1)\begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix}$$

$$\Rightarrow wU_{12} + gU_{22} = (H_1+\Omega_1)U_{12} \Rightarrow gU_{22} = (H_1+\Omega_1-w)U_{12}$$

$$gU_{12} + \Omega U_{22} = (H_1+\Omega_1)U_{22} \Rightarrow gU_{12} = (H_1+\Omega_1-\Omega)U_{22}$$

$$gU_{22} = \frac{(H_1+\Omega_1-w)\,(H_1+\Omega_1-\Omega)}{g}U_{22}$$

Άρα $U_{22}=0$ $\quad (\Rightarrow U_{12}=0)$

ή

$$g^2 = (H_1+\Omega_1-w)(H_1+\Omega_1-\Omega)$$

$$g^2 = \left(\frac{w+\Omega-2w}{2}+\Omega_1\right)\left(\frac{w+\Omega-2\Omega}{2}+\Omega_1\right)$$

$$g^2 = \left(\frac{\Omega-w}{2}+\Omega_1\right)\left(\frac{w-\Omega}{2}+\Omega_1\right)$$

$$g^2 = \left(\Omega_1 - \frac{w-\Omega}{2}\right)\left(\Omega_1+\frac{w-\Omega}{2}\right) = \left(\Omega_1-\frac{\Delta}{2}\right)\left(\Omega_1+\frac{\Delta}{2}\right)$$

$$g^2 = \Omega_1^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow \Omega_1^2 = \left(\frac{\Delta}{2}\right)^2 + g^2$$

το οποίον ισχύει εξ ορισμού του Ω_1

δηλαδή το U_{22} μπορεί να είναι οτιδήποτε π.χ. $U_{22}=1$

$$gU_{12} = (H_1+\Omega_1-\Omega)\cdot 1 = \frac{w+\Omega-2\Omega}{2}+\Omega_1 = \frac{w-\Omega}{2}+\Omega_1$$

$$\boxed{U_{12} = \frac{\Delta+2\Omega_1}{2g}}$$

$$\vec{U}_2 = \begin{bmatrix} \dfrac{\Delta+2\Omega_1}{2g} \\ 1 \end{bmatrix}$$

Η γενική λύση είναι $\vec{x}(t) = \sigma_1 \vec{u}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{u}_2 e^{-i\lambda_2 t}$

$$\vec{x}(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} \sigma_1 \frac{\Delta - 2\Omega_1}{2g} e^{-i(H_1-\Omega_1)t} + \sigma_2 \cdot \frac{\Delta + 2\Omega_1}{2g} e^{-i(H_1+\Omega_1)t} \\ \sigma_1 \cdot 1 \cdot e^{-i(H_1-\Omega_1)t} + \sigma_2 \cdot 1 \cdot e^{-i(H_1+\Omega_1)t} \end{bmatrix}$$

ΑΡΧΙΚΕΣ ΣΥΝΘΗΚΕΣ $c_1(0) = 1 \quad c_2(0) = 0 \Rightarrow$

$$\left.\begin{aligned} 1 &= \sigma_1 \frac{\Delta - 2\Omega_1}{2g} + \sigma_2 \frac{\Delta + 2\Omega_1}{2g} \\ 0 &= \sigma_1 + \sigma_2 \Rightarrow \sigma_2 = -\sigma_1 \end{aligned}\right\} \Rightarrow 2g = \sigma_1(\Delta - 2\Omega_1) - \sigma_1(\Delta + 2\Omega_1)$$

$$2g = -2\Omega_1\sigma_1 - 2\Omega_1\sigma_1$$

$$g = -2\Omega_1\sigma_1 \Rightarrow \sigma_1 = -\frac{g}{2\Omega_1} = -\sigma_2$$

Άρα $c_2(t) = -\frac{g}{2\Omega_1} e^{-i(H_1-\Omega_1)t} + \frac{g}{2\Omega_1} e^{-i(H_1+\Omega_1)t} \Rightarrow$

$$c_2(t) = -\frac{g}{2\Omega_1} e^{-iH_1 t} e^{i\Omega_1 t} + \frac{g}{2\Omega_1} e^{-iH_1 t} e^{-i\Omega_1 t}$$

$$c_2(t) = \frac{g}{2\Omega_1} e^{-iH_1 t} \left\{ -\cancel{\cos(\Omega_1 t)} - i\sin(\Omega_1 t) + \cancel{\cos(\Omega_1 t)} - i\sin(\Omega_1 t) \right\}$$

$$c_2(t) = \frac{g}{2\Omega_1} e^{-iH_1 t} (-2i)\sin(\Omega_1 t) = e^{-i\frac{\omega+\Omega}{2}t} \left\{ -i\frac{g}{\Omega_1}\sin(\Omega_1 t) \right\}$$

$$\boxed{|c_2(t)|^2 = \frac{g^2}{\Omega_1^2}\sin^2(\Omega_1 t)}$$

$$|c_1(t)|^2 = 1 - |c_2(t)|^2 = 1 - \frac{g^2}{\Omega_1^2}(1 - \cos^2(\Omega_1 t))$$

$$= \left(1 - \frac{g^2}{\Omega_1^2}\right) + \frac{g^2}{\Omega_1^2}\cos^2(\Omega_1 t)$$

$$\Omega_1^2 = \frac{\Delta^2}{4} + g^2 \quad \Omega_1^2 - g^2 = \frac{\Delta^2}{4}$$

$$\boxed{|c_1(t)|^2 = \frac{\left(\frac{\Delta^2}{4}\right)}{\Omega_1^2} + \frac{g^2}{\Omega_1^2}\cos^2(\Omega_1 t)}$$

Γ' Έστω n φωτόνια στην κοιλότητα

$$A = \begin{bmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{bmatrix} \quad \text{και} \quad \det(A - \lambda I) = 0 \Rightarrow$$

$$\begin{vmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (n\omega - \lambda)\left[\Omega + (n-1)\omega - \lambda\right] - ng^2 = 0$$

$$\lambda^2 - \lambda\left[\Omega + (n-1)\omega + n\omega\right] + n\omega\left[\Omega + (n-1)\omega\right] - ng^2 = 0$$

$$\Delta' = \left[\Omega + (n-1)\omega + n\omega\right]^2 - 4\left(n\omega\left[\Omega + (n-1)\omega\right] - ng^2\right)$$

$$\Delta' = \left[\Omega + (n-1)\omega + n\omega\right]^2 - 4n\omega\left[\Omega + (n-1)\omega\right] + 4ng^2$$

$$\Delta' = \left[\Omega + (n-1)\omega - n\omega\right]^2 + 4ng^2 > 0$$

$$\lambda_{2,1} = \frac{\left[\Omega + (n-1)\omega + n\omega\right] \pm \sqrt{\left[\Omega + (n-1)\omega - n\omega\right]^2 + 4ng^2}}{2}$$

$$\lambda_{2,1} = \frac{\Omega + (n-1)\omega + n\omega}{2} \pm \sqrt{\left(\frac{\Omega + (n-1)\omega - n\omega}{2}\right)^2 + ng^2}$$

$$\lambda_{2,1} = \frac{\Omega + (n-1)\omega + n\omega}{2} \pm \sqrt{\left(\frac{\omega - \Omega}{2}\right)^2 + ng^2}$$

$$\boxed{\lambda_{2,1} = H_n \pm \Omega_n} \qquad H_n = \frac{\Omega + (n-1)\omega + n\omega}{2}$$

$$\Omega_n = \sqrt{\left(\frac{\Delta}{2}\right)^2 + ng^2}$$

$$\lambda_1 = H_n - \Omega_n \qquad A\vec{u}_1 = \lambda_1 \vec{u}_1 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega \end{pmatrix}\begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = (H_n - \Omega_n)\begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix}$$

$$n\omega\, u_{11} + g\sqrt{n}\, u_{21} = (H_n - \Omega_n)\, u_{11}$$

$$g\sqrt{n}\, u_{11} + [\Omega+(n-1)\omega]\, u_{21} = (H_n - \Omega_n)\, u_{21}$$

$$g\sqrt{n}\, u_{21} = (H_n - \Omega_n - n\omega)\, u_{11}$$

$$g\sqrt{n}\, u_{11} = \left(H_n - \Omega_n - [\Omega+(n-1)\omega]\right) u_{21}$$

$$g\sqrt{n}\, u_{21} = \frac{(H_n - \Omega_n - n\omega)\left(H_n - \Omega_n - [\Omega+(n-1)\omega]\right) u_{21}}{g\sqrt{n}}$$

"Άρα $u_{21} = 0$ ή $g^2 n = (H_n - \Omega_n - n\omega)\left(H_n - \Omega_n - [\Omega+(n-1)\omega]\right)$

$\left(\Downarrow\ u_{11} = 0\right)$

$$H_n - \Omega_n - n\omega = \frac{\Omega+(n-1)\omega+n\omega-2n\omega}{2} - \Omega_n = \frac{\Omega-\omega}{2} - \Omega_n = -\frac{\Delta}{2} - \Omega_n$$

$$H_n - \Omega_n - [\Omega+(n-1)\omega] = \frac{\Omega+(n-1)\omega+n\omega-2\Omega-2(n-1)\omega}{2} - \Omega_n = \frac{\omega-\Omega}{2} - \Omega_n = \frac{\Delta}{2} - \Omega_n$$

$$\Rightarrow g^2 n = -\left(\frac{\Delta}{2}+\Omega_n\right)\cdot\left(\frac{\Delta}{2}-\Omega_n\right) = -\left(\frac{\Delta}{2}\right)^2 + \Omega_n^2$$

$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + n g^2}$ το οποίον ισχύει εξ ορισμού του Ω_n

$\boxed{\text{δηλαδή το } u_{21} \text{ μπορεί να είναι οτιδήποτε π.χ. } u_{21} = 1}$

$$g\sqrt{n}\, u_{11} = H_n - \Omega_n - [\Omega+(n-1)\omega] = \frac{\Delta}{2} - \Omega_n$$

$$\boxed{u_{11} = \frac{\Delta - 2\Omega_n}{2g\sqrt{n}}}$$

$$\vec{u}_1 = \begin{bmatrix} \dfrac{\Delta - 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix}$$

$\boxed{\lambda_2 = H_n + \Omega_n}$ $\quad A\vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega+(n-1)\omega \end{pmatrix} \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_n+\Omega_n)\begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix}$

$$n\omega U_{12} + g\sqrt{n}\, U_{22} = (H_n+\Omega_n) U_{12}$$

$$g\sqrt{n}\, U_{12} + [\Omega+(n-1)\omega] U_{22} = (H_n+\Omega_n) U_{22}$$

$$g\sqrt{n}\, U_{22} = (H_n+\Omega_n - n\omega) U_{12}$$

$$g\sqrt{n}\, U_{12} = \Big(H_n+\Omega_n - [\Omega+(n-1)\omega]\Big) U_{22}$$

$$g\sqrt{n}\, U_{22} = \frac{(H_n+\Omega_n-n\omega)\Big(H_n+\Omega_n-[\Omega+(n-1)\omega]\Big)\, U_{22}}{g\sqrt{n}}$$

Άρα $U_{22}=0$ $\left(\Downarrow\; U_{12}=0\right)$ ή $g^2 n = (H_n+\Omega_n-n\omega)\Big(H_n+\Omega_n-[\Omega+(n-1)\omega]\Big)$

$$H_n+\Omega_n-n\omega = \frac{\Omega+(n-1)\omega+n\omega-2n\omega}{2}+\Omega_n$$

$$= \frac{\Omega-\omega}{2}+\Omega_n = -\frac{\Delta}{2}+\Omega_n$$

$$H_n+\Omega_n-[\Omega+(n-1)\omega] = \frac{\Omega+(n-1)\omega+n\omega-2\Omega-2(n-1)\omega}{2}+\Omega_n$$

$$= \frac{\omega-\Omega}{2}+\Omega_n = \frac{\Delta}{2}+\Omega_n$$

$$g^2 n = \left(\Omega_n+\frac{\Delta}{2}\right)\left(\Omega_n-\frac{\Delta}{2}\right) = \Omega_n^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow$$

$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + ng^2}$ το οποίον ισχύει εξ ορισμού του Ω_n

$\boxed{\text{δηλαδή το } U_{22} \text{ μπορεί να είναι οτιδήποτε π.χ. } U_{22}=1}$

$$g\sqrt{n}\, U_{12} = H_n+\Omega_n-[\Omega+(n-1)\omega] = \frac{\Delta}{2}+\Omega_n \Rightarrow$$

$$\boxed{U_{12} = \frac{\Delta+2\Omega_n}{2g\sqrt{n}}}$$

$$\boxed{\vec{U}_2 = \begin{bmatrix} \dfrac{\Delta+2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix}}$$

Ἡ γενικὴ λύση εἶναι $\vec{x}(t) = \sigma_1 \vec{U}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{U}_2 e^{-i\lambda_2 t}$

$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sigma_1 \begin{bmatrix} \dfrac{\Delta - 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix} e^{-i(H_n - \Omega_n)t} + \sigma_2 \begin{bmatrix} \dfrac{\Delta + 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix} e^{-i(H_n + \Omega_n)t}$$

Ἀρχικὲς Συνθῆκες $C_1(0) = 1 \quad C_2(0) = 0$

$$\left.\begin{aligned} &\sigma_1 \frac{\Delta - 2\Omega_n}{2g\sqrt{n}} + \sigma_2 \frac{\Delta + 2\Omega_n}{2g\sqrt{n}} = 1 \\ &\sigma_1 + \sigma_2 = 0 \Rightarrow \sigma_2 = -\sigma_1 \end{aligned}\right\} \Rightarrow \sigma_1 \frac{\Delta - 2\Omega_n - \Delta - 2\Omega_n}{2g\sqrt{n}} = 1$$

$$\Rightarrow \sigma_1 \frac{-4\Omega_n}{2g\sqrt{n}} = 1 \Rightarrow \sigma_1 = \frac{-g\sqrt{n}}{2\Omega_n} = -\sigma_2$$

$$C_2(t) = \frac{-g\sqrt{n}}{2\Omega_n} e^{-i(H_n - \Omega_n)t} + \frac{g\sqrt{n}}{2\Omega_n} e^{-i(H_n + \Omega_n)t}$$

$$C_2(t) = \frac{g\sqrt{n}}{2\Omega_n} e^{-iH_n t}\left[e^{-i\Omega_n t} - e^{i\Omega_n t}\right] \Rightarrow C_2(t) = \frac{g\sqrt{n}}{2\Omega_n} e^{-iH_n t}(-2i)\sin(\Omega_n t)$$

$$C_2(t) = -i\,\frac{g\sqrt{n}}{\Omega_n} e^{-iH_n t} \cdot \sin(\Omega_n t)$$

$$|C_2(t)|^2 = \frac{n g^2}{\Omega_n^2} \sin^2(\Omega_n t)$$