$$\dot{c}_1(t) = c_2(t)\,\frac{i\Omega_R}{2}\left[e^{i\Delta t} + e^{-i\Sigma t}\right]$$
$$\dot{c}_2(t) = c_1(t)\,\frac{i\Omega_R}{2}\left[e^{-i\Delta t} + e^{i\Sigma t}\right]$$

(Γ) πριν από οποιαδήποτε προσέγγιση

μέγα Ω_R / 1

Α' ΤΡΟΠΟΣ

Ας υποθέσουμε πως $e^{\cdots} \sim 1$ δηλαδή $\Delta \sim 0$, $\Sigma \sim 0$

δηλαδή πως υπερισχύει συντριπτικά ο όρος $\frac{i\Omega_R}{2}$ έξω από την τετράγωνη παρένθεση.

external ext

Τότε οι εξισώσεις γίνονται:

$$\dot{c}_1(t) = c_2(t)\,\frac{i\Omega_R}{2}\,2 = i\Omega_R\, c_2(t) \quad (\Omega_R)$$
$$\dot{c}_2(t) = c_1(t)\,\frac{i\Omega_R}{2}\cdot 2 = i\Omega_R\, c_1(t)$$

$$\ddot{c}_1(t) = i\Omega_R\,\dot{c}_2(t) = (i\Omega_R)^2 c_1(t)$$
$$\ddot{c}_2(t) = i\Omega_R\,\dot{c}_1(t) = (i\Omega_R)^2 c_2(t)$$

ΔΛΜ $c_k(t) = U_k e^{i\lambda_k t} \Rightarrow \lambda_k^2 = \Omega_R^2$

$\lambda_k = \pm\Omega_R$

$$c_1(t) = A e^{i\Omega_R t} + B e^{-i\Omega_R t} \quad (\Lambda 1)$$
$$c_2(t) = \Gamma e^{i\Omega_R t} + \Delta e^{-i\Omega_R t} \quad (\Lambda 2)$$

$(\Omega_R)\,(\Lambda 1)\,(\Lambda 2) \Rightarrow$

$$A\, i\Omega_R e^{i\Omega_R t} + B(-i\Omega_R) e^{-i\Omega_R t} = i\Omega_R\left(\Gamma e^{i\Omega_R t} + \Delta e^{-i\Omega_R t}\right) \xRightarrow{t=0}$$

$$A - B = \Gamma + \Delta \quad (3)$$

$$\Gamma\, i\Omega_R e^{i\Omega_R t} + \Delta(-i\Omega_R) e^{-i\Omega_R t} = i\Omega_R\left(A e^{i\Omega_R t} + B e^{-i\Omega_R t}\right) \xRightarrow{t=0}$$

$$\Gamma - \Delta = A + B \quad (4)$$

Α.Σ. $c_1(0) = 1$, $c_2(0) = 0$

$1 = A + B \quad (1)$

$0 = \Gamma + \Delta \Rightarrow \Delta = -\Gamma \quad (2)$

(1)(2)(3)(4) $\quad \Delta = -\Gamma \quad A = B \quad 2\Gamma = 2A \quad \Gamma = A \quad 1 = A + A = 2A$

$$A = B = \Gamma = \frac{1}{2} = -\Delta \quad (5)$$

$$c_1(t) = \frac{e^{i\Omega_R t} + e^{-i\Omega_R t}}{2} \Rightarrow c_1(t) = \cos(\Omega_R t) \Rightarrow P_1(t) = \cos^2(\Omega_R t) = \frac{1}{2} + \frac{1}{2}\cos(2\Omega_R t)$$

$$c_2(t) = \frac{e^{i\Omega_R t} - e^{-i\Omega_R t}}{2} \Rightarrow c_2(t) = i\sin(\Omega_R t) \Rightarrow P_2(t) = \sin^2(\Omega_R t) = \frac{1}{2} - \frac{1}{2}\cos(2\Omega_R t)$$

$$\frac{1}{f} = T = \frac{2\pi}{2\Omega_R} = \frac{\pi}{\Omega_R} \qquad A = 1$$

Β' τρόπος

$\dot{C}_1(t) = i\Omega_R C_2(t)$

$\dot{C}_2(t) = i\Omega_R C_1(t)$

$$\underbrace{\begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \end{bmatrix}}_{\dot{\vec{x}}(t)} = \underbrace{\begin{bmatrix} 0 & i\Omega_R \\ i\Omega_R & 0 \end{bmatrix}}_{iA} \underbrace{\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix}}_{\vec{x}(t)}$$

μέγα $\frac{\Omega_R}{2}$

ΔΔΜ $\vec{x}(t) = \vec{u} e^{i\lambda t}$

$\dot{\vec{x}}(t) = \vec{u}\, i\lambda\, e^{i\lambda t}$

$\dot{\vec{x}}(t) = iA\vec{x}(t) \Rightarrow \vec{u}\, i\lambda e^{i\lambda t} = iA\vec{u} e^{i\lambda t} \Rightarrow A\vec{u} = \lambda\vec{u}$

$(A - \lambda I)\vec{u} = \vec{0}$

$\det(A - \lambda I) = \vec{0} \Rightarrow \begin{vmatrix} -\lambda & \Omega_R \\ \Omega_R & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 = \Omega_R^2 \Rightarrow \lambda = \pm\Omega_R$

(α) $\lambda_1 = -\Omega_R$

$$\begin{bmatrix} 0 & \Omega_R \\ \Omega_R & 0 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{21} \end{bmatrix} = -\Omega_R \begin{bmatrix} u_{11} \\ u_{21} \end{bmatrix} \Rightarrow \left.\begin{aligned} \Omega_R u_{21} &= -\Omega_R u_{11} \\ \Omega_R u_{11} &= -\Omega_R u_{21} \end{aligned}\right\} \Rightarrow u_{21} = -u_{11}$$

$\vec{u}_1 = \begin{bmatrix} c \\ -c \end{bmatrix} \qquad |\vec{u}_1|^2 = 1 \Rightarrow 2|c|^2 = 1 \Rightarrow |c|^2 = 1/2$

π.χ. $c = \frac{1}{\sqrt{2}}$

$\vec{u}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(β) $\lambda_2 = \Omega_R$

$$\begin{bmatrix} 0 & \Omega_R \\ \Omega_R & 0 \end{bmatrix} \begin{bmatrix} u_{12} \\ u_{22} \end{bmatrix} = \Omega_R \begin{bmatrix} u_{12} \\ u_{22} \end{bmatrix} \Rightarrow \left.\begin{aligned} \Omega_R u_{22} &= \Omega_R u_{12} \\ \Omega_R u_{12} &= \Omega_R u_{22} \end{aligned}\right\} \Rightarrow u_{12} = u_{22}$$

$\vec{u}_2 = \begin{bmatrix} c \\ c \end{bmatrix} \qquad |\vec{u}_2|^2 = 1 \Rightarrow 2|c|^2 = 1 \Rightarrow |c|^2 = 1/2 \Rightarrow$ π.χ. $c = \frac{1}{\sqrt{2}}$

$\vec{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Γενική λύση $\vec{x}(t) = \sigma_1 \vec{u}_1 e^{i\lambda_1 t} + \sigma_2 \vec{u}_2 e^{i\lambda_2 t}$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sigma_1 \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-i\Omega_R t} + \sigma_2 \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{i\Omega_R t}$$

A.Σ. $C_1(0)=1 \qquad C_2(0)=0$

$$1 = \frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}}$$

$$0 = -\frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}} \Rightarrow \sigma_1 = \sigma_2 := \sigma$$

$$1 = 2\frac{\sigma}{\sqrt{2}} \Rightarrow \sigma = \frac{\sqrt{2}}{2}$$

μέγαλε $\frac{}{3}$

$$C_1(t) = \frac{1}{2}e^{-i\Omega_R t} + \frac{1}{2}e^{i\Omega_R t} \Rightarrow C_1(t) = \frac{e^{i\Omega_R t} + e^{-i\Omega_R t}}{2} \Rightarrow$$

$$C_2(t) = -\frac{1}{2}e^{-i\Omega_R t} + \frac{1}{2}e^{i\Omega_R t} \Rightarrow C_2(t) = \frac{e^{i\Omega_R t} - e^{-i\Omega_R t}}{2} \Rightarrow$$

$$C_1(t) = \cos(\Omega_R t) \Rightarrow P_1(t) = \cos^2(\Omega_R t) = \frac{1}{2} + \frac{1}{2}\cos(2\Omega_R t)$$

$$C_2(t) = i\sin(\Omega_R t) \Rightarrow P_2(t) = \sin^2(\Omega_R t) = \frac{1}{2} - \frac{1}{2}\cos(2\Omega_R t)$$

$$\frac{1}{f} = T = \frac{2\pi}{2\Omega_R} = \frac{\pi}{\Omega_R} \qquad A = 1$$