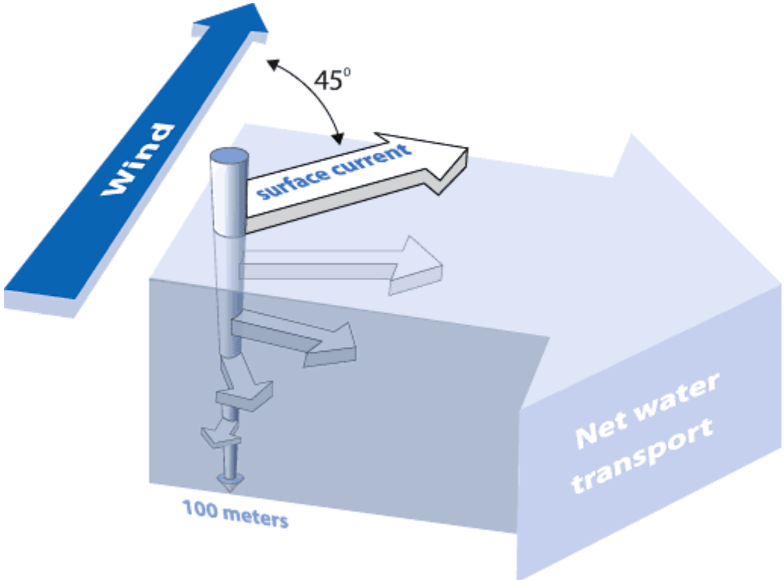
Ρεύματα παρουσία τριβής – ανεμογενής κυκλοφορία



6. Wind driven circulation

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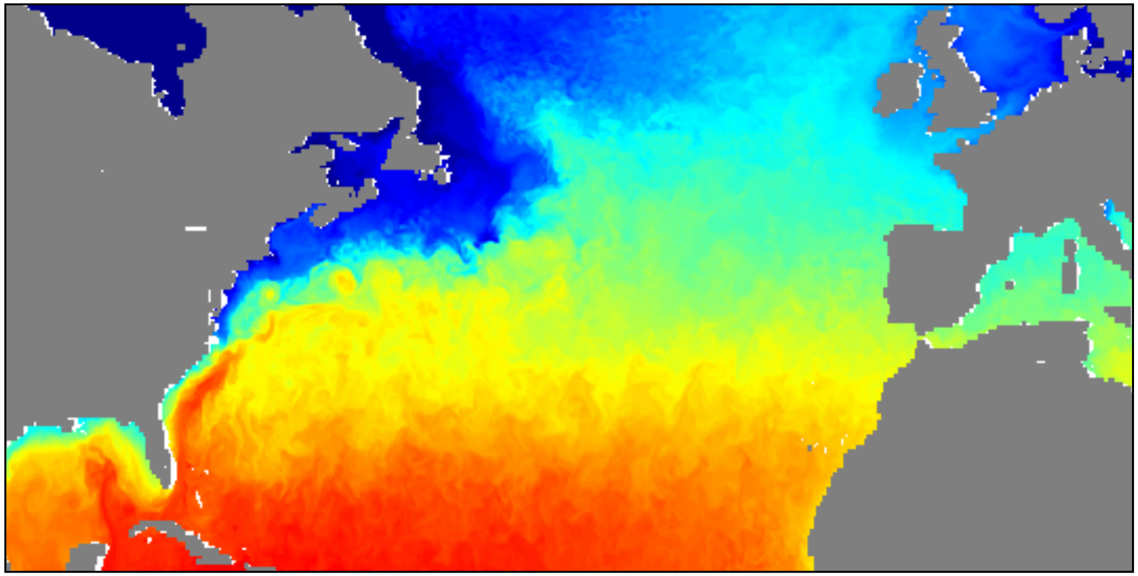
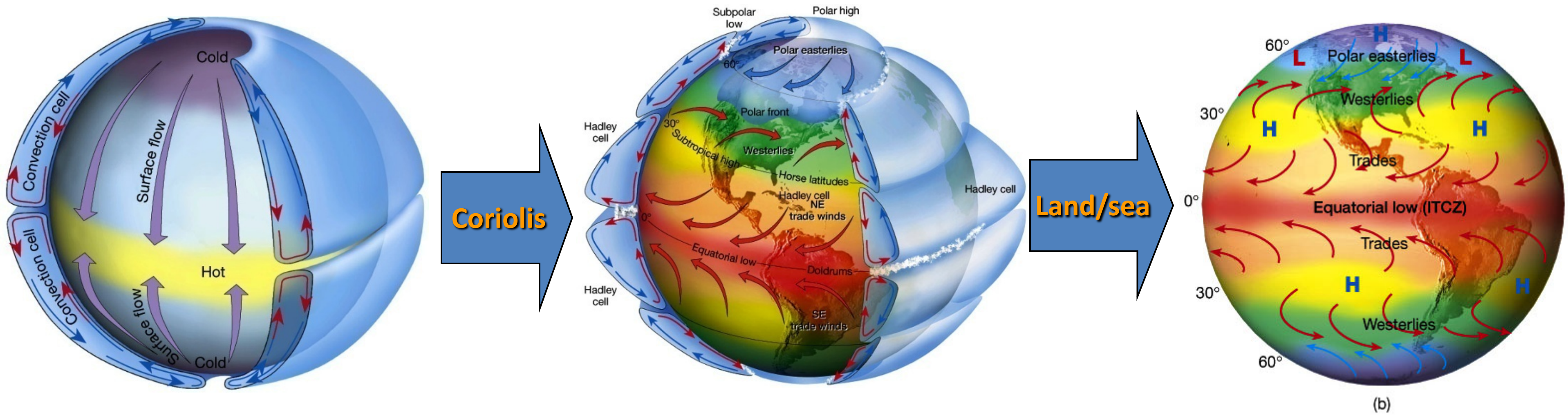
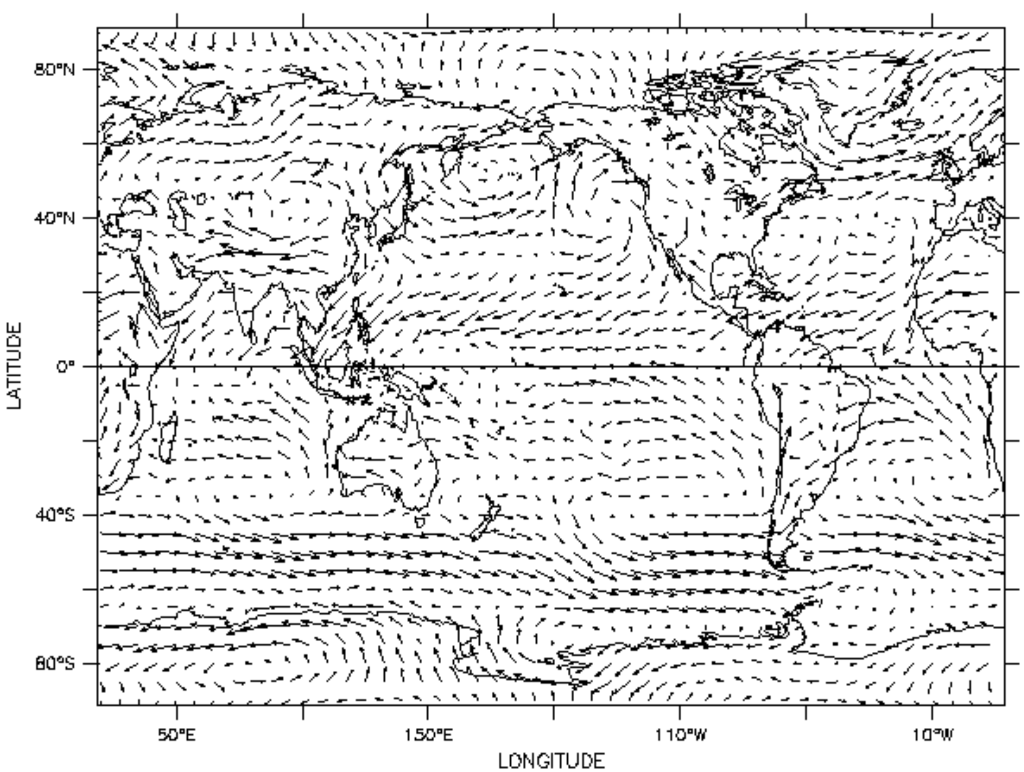
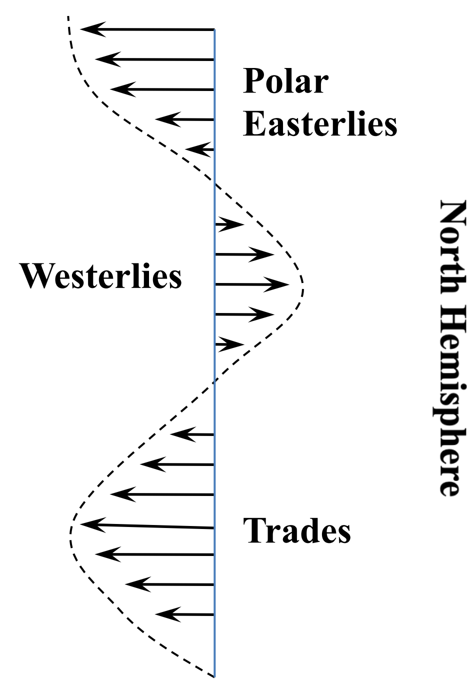


- **Ekman Theory**
- **Sverdrup theory for the wind-driven circulation**
- **Stommel and Munk theory of the western boundary intensification**

Global atmospheric circulation







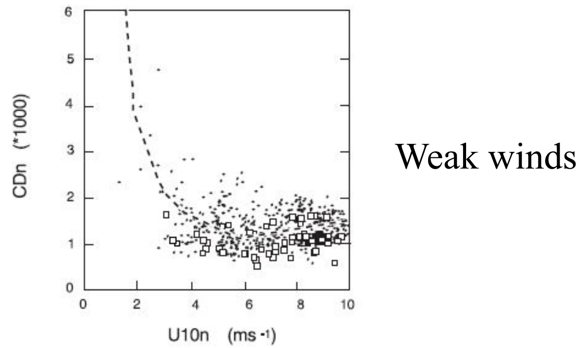
Wind stress on the sea

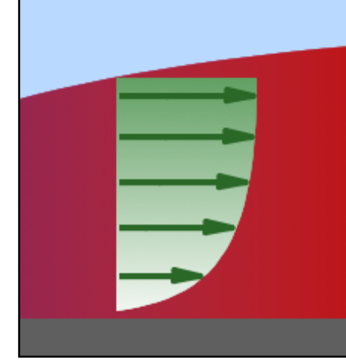
$$\vec{\tau} = c_D \rho_a \left| u_{10} \right| \vec{u}_{10}$$

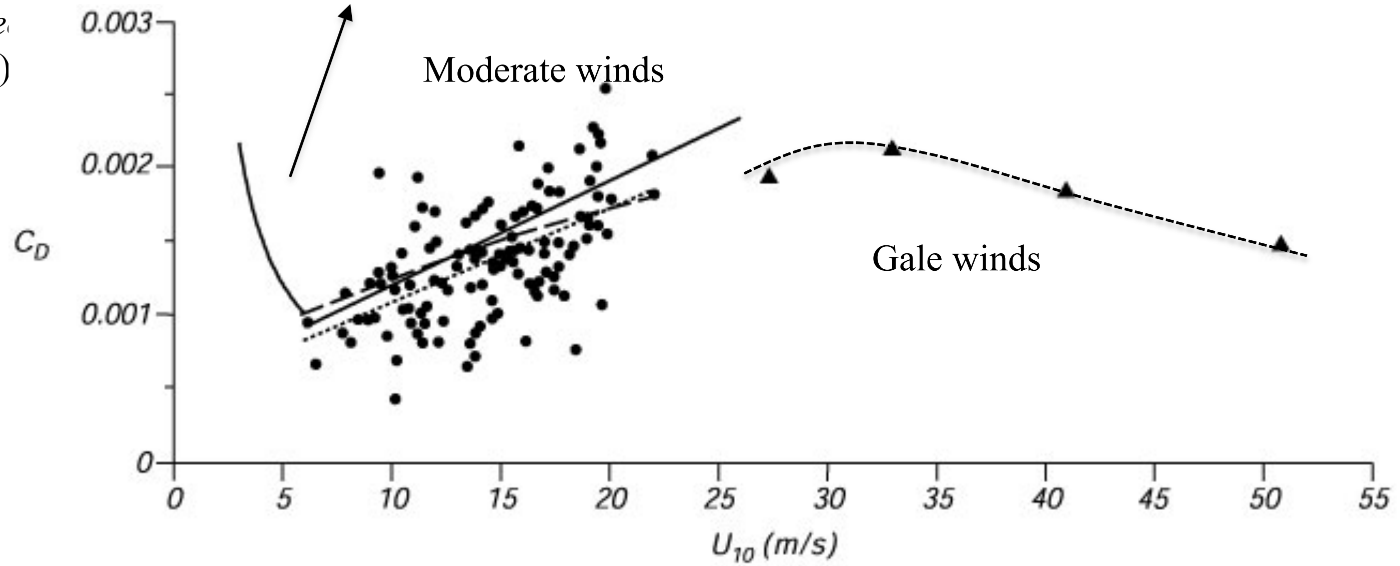
where:

ρ_α air density (1.3 kg/m^3)
u_{10} wind speed at10 m (m/se
c_D drag coefficient (unitless)

Measurements showed that c_D is related to the wind speed (not a constant). Nevertheless, we usually use a constant value for simplicity.







Examples of c_D used:

$$c_D = 1.4 \times 10^{-3}$$

$$c_D = 1.1 \times 10^{-3} \text{ ασθενείς άνεμοι}$$

$$c_D = (0.61 + 0.063u) \times 10^{-3} \text{ ισχυροί άνεμοι}$$

$$c_D = (0.29 + 3.1u_{10}^{-1} + 7.7u_{10}^{-2})10^{-3}$$

for $3m/\sec \le u_{10} \le 6m/\sec$

$$c_D = (0.60 + 0.070u_{10})10^{-3}$$

for $6m/\sec \le u_{10} \le 26m/\sec$

and many more …

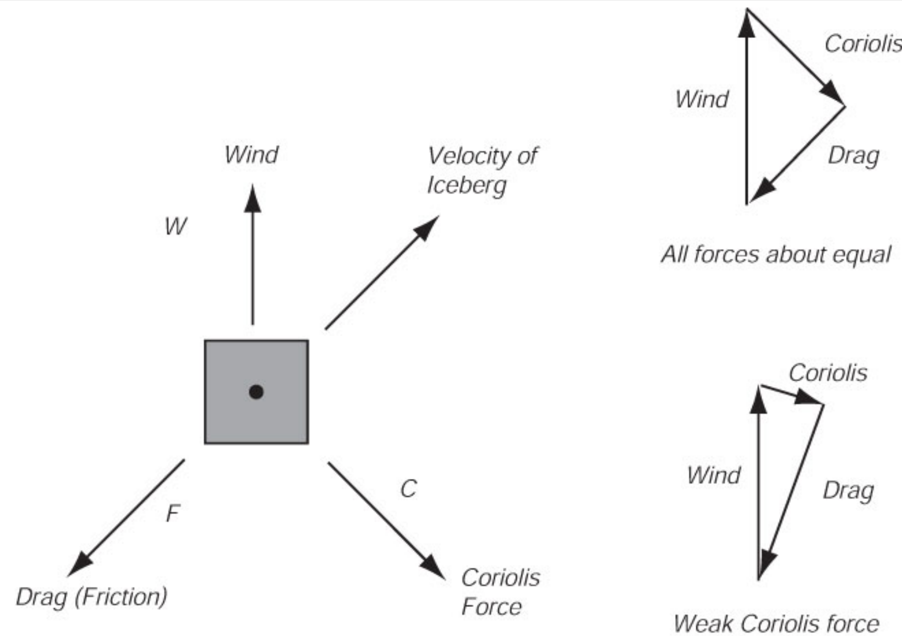
Evolution of wind-driven circulation theory:

Fridtjof Nansen	(1898)	Qualitative theory, currents transport water at an angle to the wind.
Vagn Walfrid Ekman	(1902)	Quantitative theory for wind-driven transport at the sea surface.
Harald Sverdrup	(1947)	Theory for wind-driven circulation in the eastern Pacific.
Henry Stommel	(1948)	Theory for westward intensification of wind-driven circulation (western boundary currents).
Walter Munk	(1950)	Quantitative theory for main features of the wind-driven circulation
Kirk Bryan	(1963)	Numerical models of the oceanic circulation.

The surface wind effect (Nansen theory):

Based on observations of iceberg movements

$$W + F + C = 0$$



Working Equations

x (u):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + fv + A_H\frac{\partial^2 u}{\partial x^2} + A_H\frac{\partial^2 u}{\partial y^2} + A_V\frac{\partial^2 u}{\partial z^2}$$

y (v):

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} - fu + A_H\frac{\partial^2 v}{\partial x^2} + A_H\frac{\partial^2 v}{\partial y^2} + A_V\frac{\partial^2 v}{\partial z^2}$$

z (w):

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} - g + A_H\frac{\partial^2 w}{\partial x^2} + A_H\frac{\partial^2 w}{\partial y^2} + A_V\frac{\partial^2 w}{\partial z^2}$$

Navier - Stokes equations

Continuity equation (Boussinesq approximation):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Ekman Theory

$$R_o = \frac{U}{fL} << 1$$

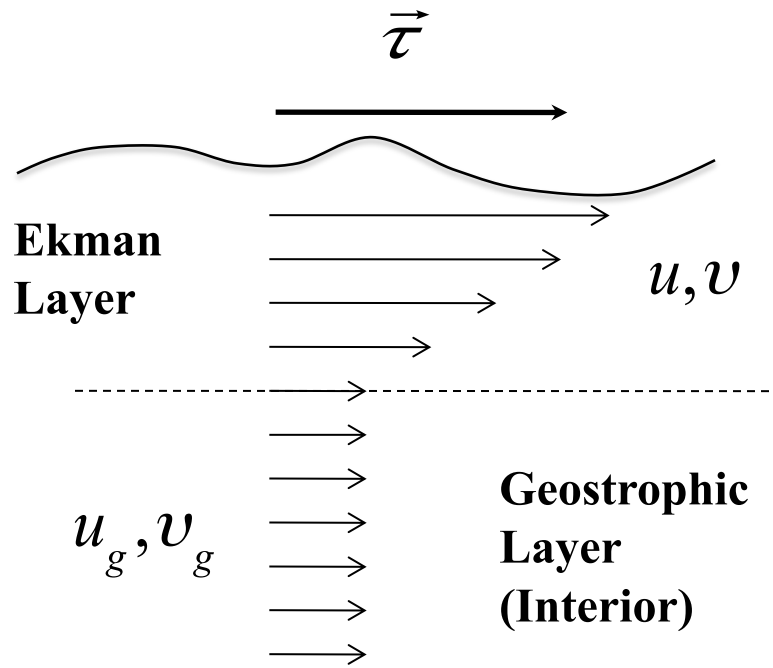
$$E_k = \frac{\text{Viscosity Terms}}{\text{Coriolis Term}} = A_H \frac{U}{L^2}\frac{1}{fU} = \frac{A_H}{fL^2} \quad \left(\frac{A_V}{fH^2}\right) \sim 1$$

Assuming that H is the thickness of the layer affected by the wind (through friction) and << of the total depth of the ocean

$$A_H \nabla_H^2 \vec{u} << A_V \frac{\partial^2 \vec{u}}{\partial z^2}$$

The Navier-Stokes equation (horizontal) become:

$$\left.\begin{aligned} -f\upsilon &= -\frac{1}{\rho}\frac{\partial P}{\partial x} + A_V \frac{\partial^2 u}{\partial z^2} \\ fu &= -\frac{1}{\rho}\frac{\partial P}{\partial y} + A_V \frac{\partial^2 \upsilon}{\partial z^2} \end{aligned}\right\} \text{(A)}$$



The geostrophic approximation still holds:

$$\left.\begin{aligned} -f\upsilon_g &= -\frac{1}{\rho}\frac{\partial P}{\partial x} \\ fu_g &= -\frac{1}{\rho}\frac{\partial P}{\partial y} \end{aligned}\right\} \text{(B)}$$

(A) - (B)

$$-f(\upsilon - \upsilon_g) = A_V \frac{\partial^2 u}{\partial z^2}$$

$$f(u - u_g) = A_V \frac{\partial^2 \upsilon}{\partial z^2}$$

Ekman Equations

$$-f\left(v-v_g\right)=A_V\frac{\partial^2 u}{\partial z^2} \quad (1)$$

$$f\left(u-u_g\right)=A_V\frac{\partial^2 v}{\partial z^2} \quad (2)$$

For simplicity, only zonal wind

Boundary conditions:

$$\begin{cases} \text{at } z=0 \mapsto \rho A_v\dfrac{\partial u}{\partial z}=\tau^x;\rho A_v\dfrac{\partial v}{\partial z}=0 \\ \text{at } z\to -\infty \mapsto u=u_g;v=v_g \end{cases}$$

Multiplying (2) by $i=\sqrt{-1}$ and adding (1):

$$\frac{d^2V}{dz^2}-\frac{if}{A_v}V=0 \quad (3) \quad \text{where } V\equiv u-u_g+i\left(v-v_g\right)$$

remember u_g,v_g not $\propto z$

The solution is: $V=Ae^{(1+i)z/\delta}+Be^{-(1+i)z/\delta}$

where $\delta=\sqrt{\dfrac{2A_v}{f}}$

B=0, since the solution can not become infinite with z becoming -∞

Using the two surface boundary conditions: $A=\dfrac{\tau^x\delta(1-i)}{2\rho A_v}$

Substituting in (3):

$$u-u_g+i\left(v-v_g\right)=\frac{\tau\delta\left(1-i\right)}{2\rho A_v}e^{\frac{(1+i)z}{\delta}}=\left(1-i\right)\frac{\tau/\rho}{\sqrt{fA_v}}\frac{\sqrt{2}}{2}e^{\frac{z}{\delta}}\left(\cos\frac{z}{\delta}+i\sin\frac{z}{\delta}\right)=$$

$$=\frac{\tau/\rho}{\sqrt{fA_v}}e^{\frac{z}{\delta}}\left(\frac{\sqrt{2}}{2}\cos\frac{z}{\delta}+\frac{\sqrt{2}}{2}\sin\frac{z}{\delta}\right)-i\frac{\tau/\rho}{\sqrt{fA_v}}e^{\frac{z}{\delta}}\left(\frac{\sqrt{2}}{2}\cos\frac{z}{\delta}-\frac{\sqrt{2}}{2}\sin\frac{z}{\delta}\right)\Rightarrow$$

$$\cos\frac{\pi}{4}\cos\frac{z}{\delta}+\sin\frac{\pi}{4}\sin\frac{z}{\delta}=\cos\left(-\frac{z}{\delta}+\frac{\pi}{4}\right) \qquad \sin\frac{\pi}{4}\cos\frac{z}{\delta}-\cos\frac{\pi}{4}\sin\frac{z}{\delta}=\cos\left(-\frac{z}{\delta}+\frac{\pi}{4}\right)$$

$$u=u_g+\frac{\tau^x/\rho}{\sqrt{fA_v}}e^{z/\delta}\cos\left(-\frac{z}{\delta}+\frac{\pi}{4}\right)$$

$$v=v_g-\frac{\tau^x/\rho}{\sqrt{fA_v}}e^{z/\delta}\sin\left(-\frac{z}{\delta}+\frac{\pi}{4}\right)$$

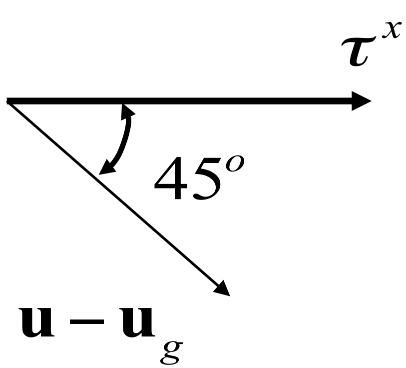
$$u = u_g + \frac{\tau^x/\rho}{\sqrt{fA_v}} e^{z/\delta} \cos\left(-\frac{z}{\delta} + \frac{\pi}{4}\right) \qquad \upsilon = \upsilon_g - \frac{\tau^x/\rho}{\sqrt{fA_v}} e^{z/\delta} \sin\left(-\frac{z}{\delta} + \frac{\pi}{4}\right)$$

II. Surface Ekman velocity:

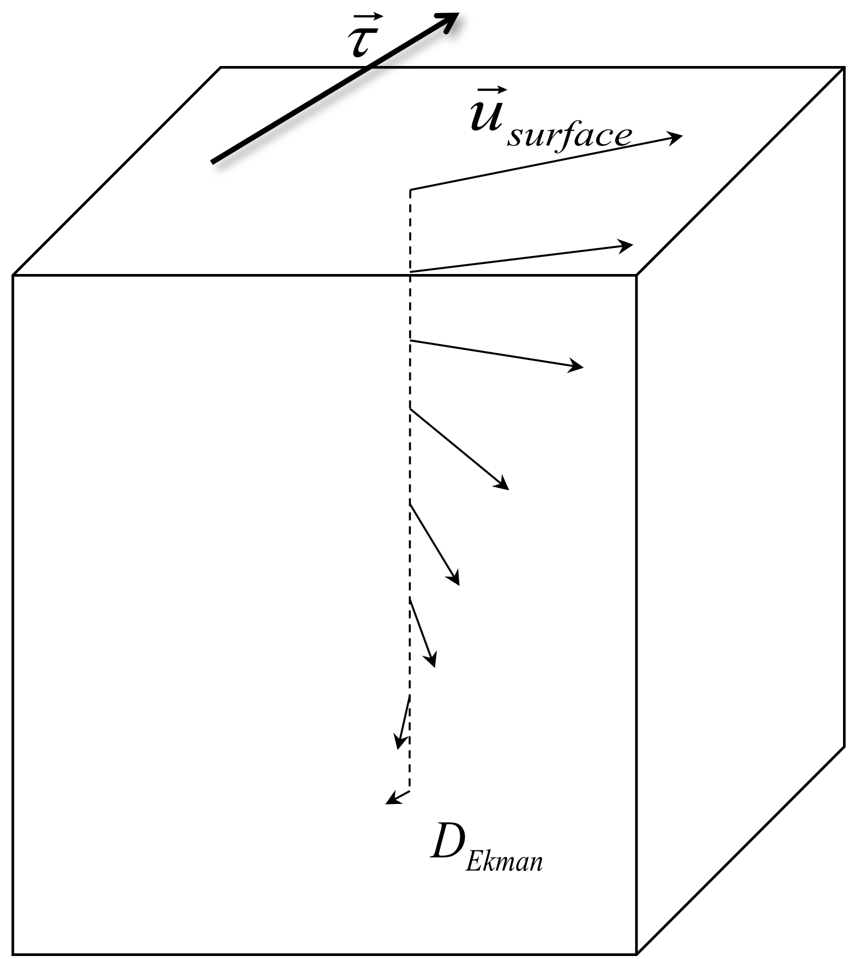
at $z = 0$

$$u_{surface} = u_g + \frac{\tau^x/\rho}{\sqrt{fA_v}} \cos\left(\frac{\pi}{4}\right)$$

$$\upsilon_{surface} = \upsilon_g - \frac{\tau^x/\rho}{\sqrt{fA_v}} \sin\left(\frac{\pi}{4}\right)$$



II. Vertical distribution of Ekman solution:



Ekman Spiral

The depth of the layer (**Ekman layer**) is defined as the depth where velocity is decreased by *e* (e-folding scale)

$$\delta = \sqrt{\frac{2A_v}{f}}$$

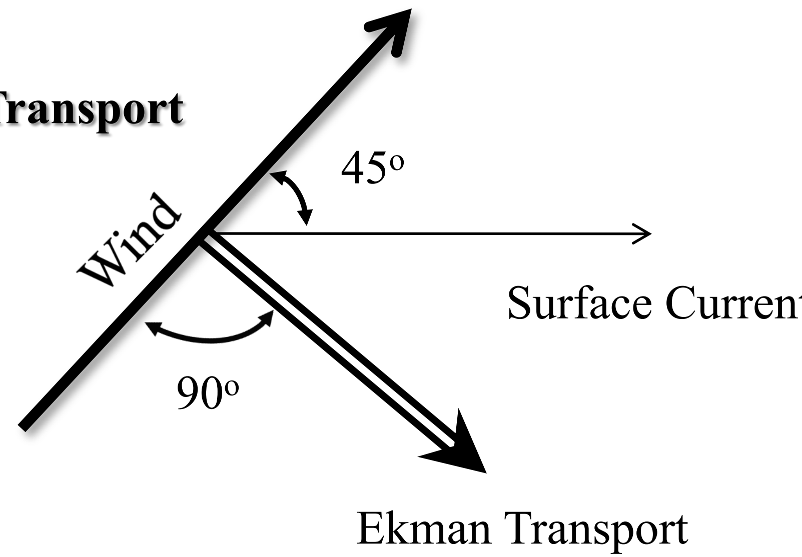
III. The vertically integrated Ekman transport:

Integrating the ocean velocity due to the direct effect of the wind $\left(\mathbf{u}-\mathbf{u}_g\right)$ from the interior to the ocean surface:

$$U = \int_{-\infty}^{0}\left(u-u_g\right)dz = \frac{1}{\rho f}\tau^y$$

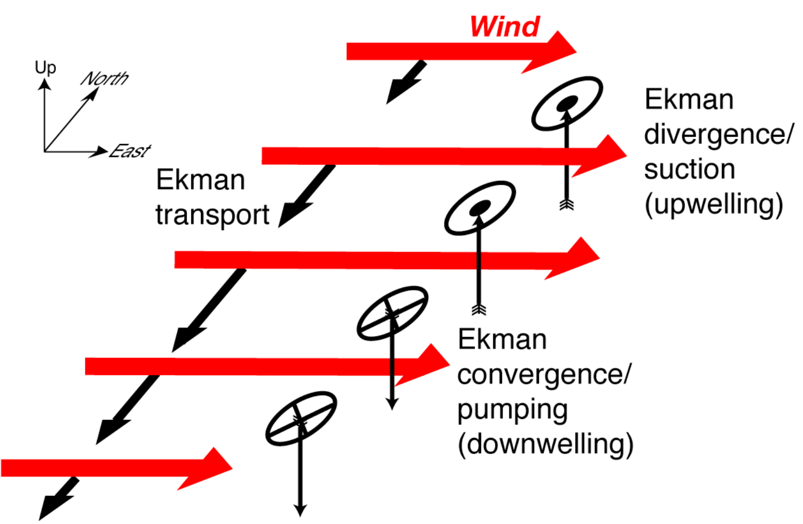
$$V = \int_{-\infty}^{0}\left(\upsilon-\upsilon_g\right)dz = -\frac{1}{\rho f}\tau^x$$

Ekman Transport

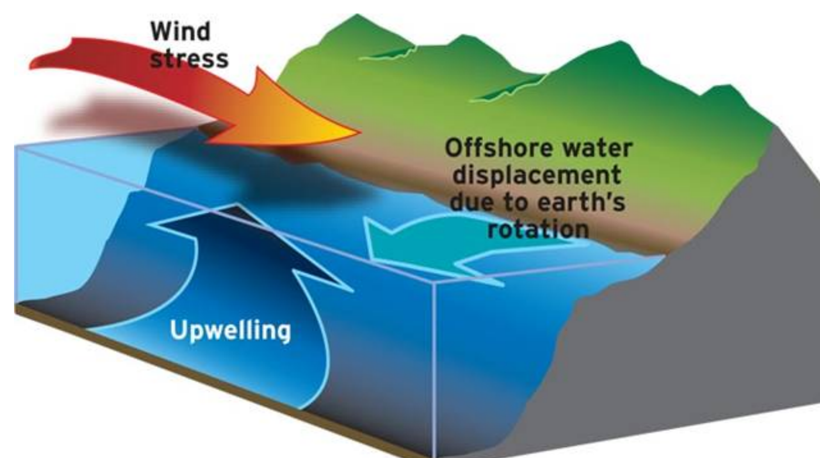


Consequences:

Open sea upwelling/downwelling

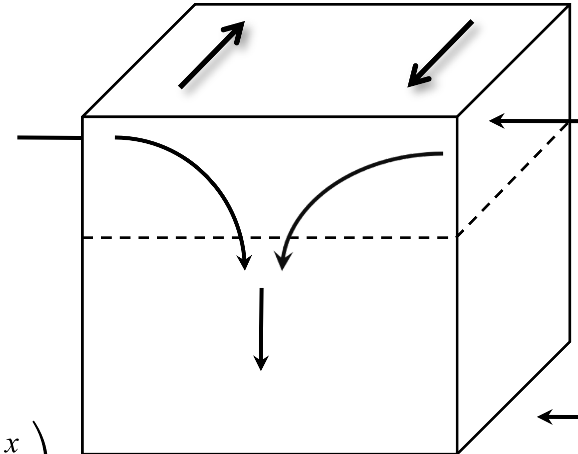


Coastal upwelling



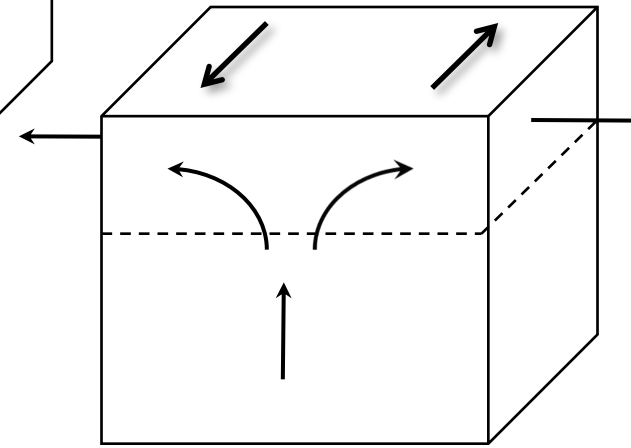
IV. Ekman pumping

From continuity: $\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} + \frac{\partial w}{\partial z} = 0$



$$-\int_{\delta}^{0} dw = \int_{\delta}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} \right) dz$$

$$\Rightarrow -\cancel{w}(o) + w(\delta) = \underbrace{\frac{1}{\rho f}\left(\frac{\partial \boldsymbol{\tau}^{y}}{\partial x} - \frac{\partial \boldsymbol{\tau}^{x}}{\partial y} \right)}_{\textbf{Wind Stress Curl}}$$



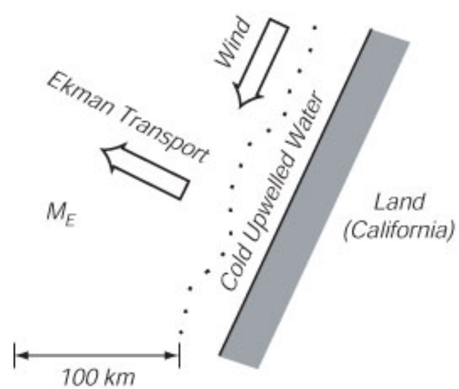
At the bottom of the Ekman layer:

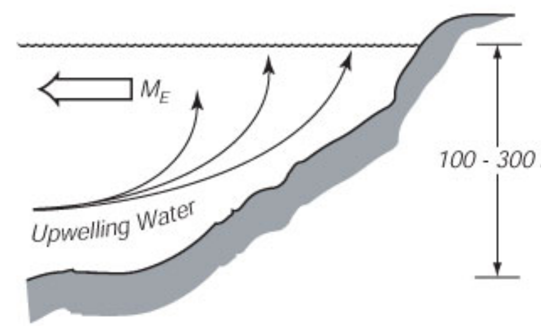
$$\boxed{w(\delta) = \frac{1}{\rho f}\left(\frac{\partial \boldsymbol{\tau}^{y}}{\partial x} - \frac{\partial \boldsymbol{\tau}^{x}}{\partial y} \right)}$$

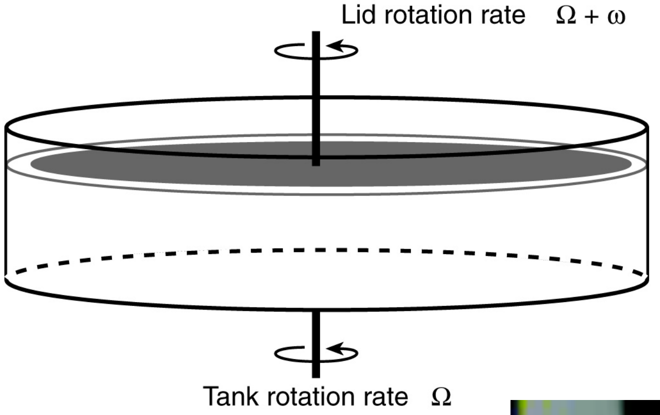
Coastal Ekman pumping

$$\boxed{w(\delta) = \frac{U}{L} = \frac{\boldsymbol{\tau}^{y}}{L\rho f}}$$

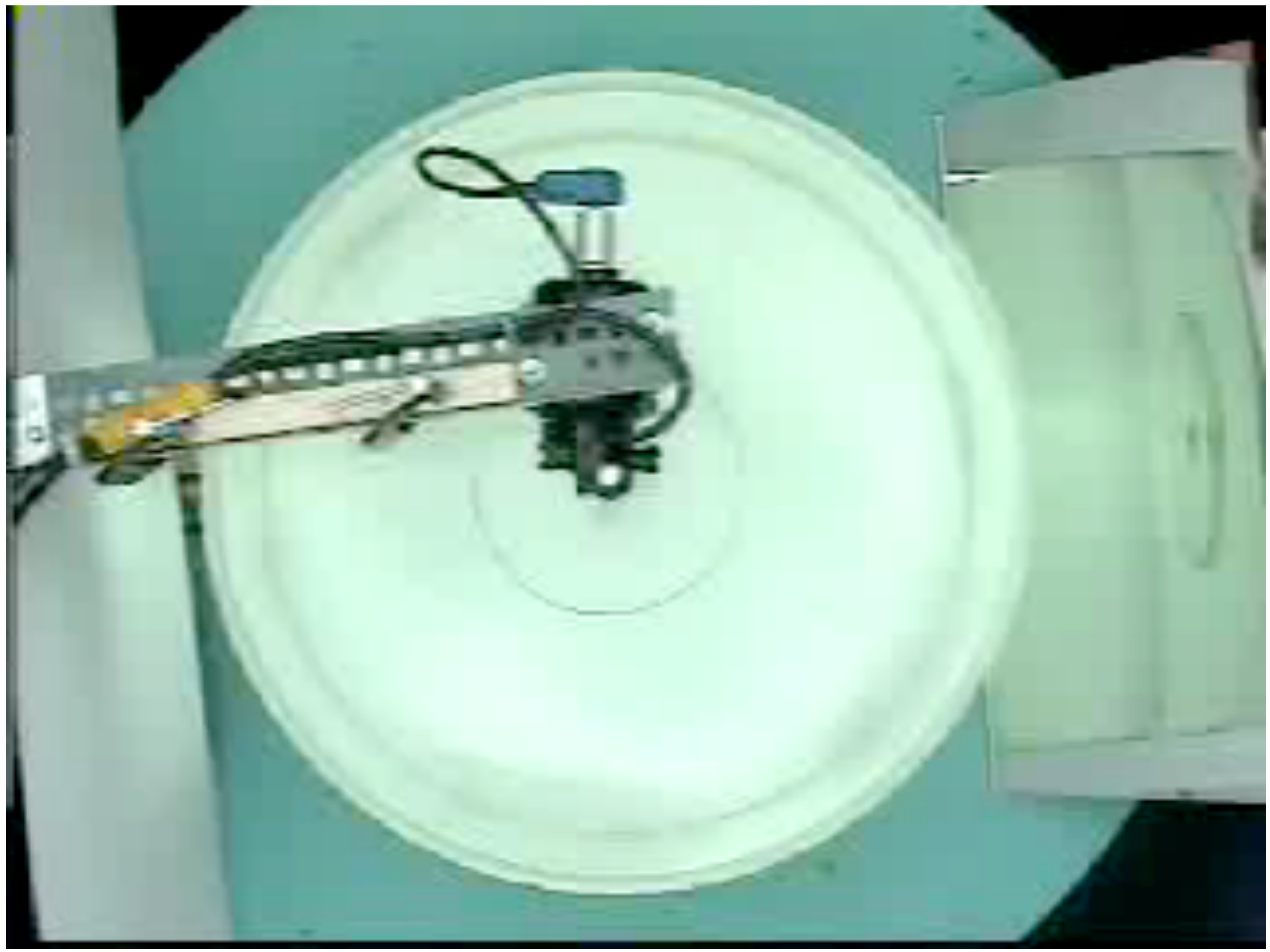
We can not define the derivative along the coastal zone







Lid rotation rate Ω + ω
Tank rotation rate Ω



Sverdrup theory for the wind-driven circulation

Starting from the same equations of motion (with friction):

$$-fv = -\frac{1}{\rho}\frac{\partial P}{\partial x} + A_V \frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$fu = -\frac{1}{\rho}\frac{\partial P}{\partial y} + A_V \frac{\partial^2 v}{\partial z^2} \qquad (2)$$

$$\frac{\partial}{\partial x}(2) - \frac{\partial}{\partial y}(1)$$

$$\cancel{u\frac{\partial f}{\partial x}} + f\frac{\partial u}{\partial x} + v\boxed{\frac{\partial f}{\partial y}} + f\frac{\partial v}{\partial y} = -\cancel{\frac{1}{\rho}\frac{\partial^2 P}{\partial x \partial y}} + \cancel{\frac{1}{\rho}\frac{\partial^2 P}{\partial x \partial y}} + \frac{1}{\rho}\frac{\partial^3 v}{\partial x \partial z^2} - \frac{1}{\rho}\frac{\partial^3 u}{\partial y \partial z^2}$$

$\frac{\partial f}{\partial y} \rightarrow \boldsymbol{\beta}$

$$f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \beta v = \frac{1}{\rho}\frac{\partial}{\partial z^2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Integrating from the bottom (-*H*) to the sea surface:

$$\rho f \int_{-H}^{0} \cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} dz + \beta \int_{-H}^{0} \rho v dz = \int_{-H}^{0} \left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right)$$

since at $z = 0 \mapsto \rho A_v \frac{\partial u}{\partial z} = \tau^x$;

$$\rho A_v \frac{\partial v}{\partial z} = \tau^y$$

Using continuity and the fact that w(0)=w(-H)=0

Meridional mass transport M_y

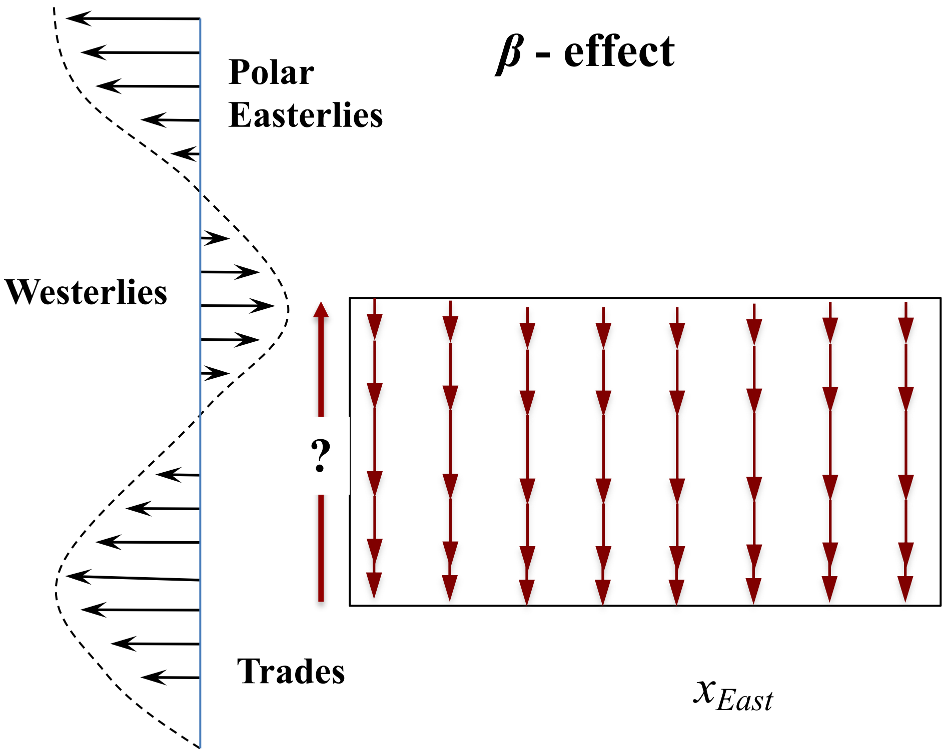
Wind stress curl

$$\beta M_y = \frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y} = \text{wind} - \text{stress curl}$$

Depth integrated Vorticity equation for the wind-driven circulation

***β* - effect**

$$\frac{\partial \tau^y}{\partial x} \to 0 \qquad (\tau^y = 0)$$



The meridional Sverdrup transport

$$M_y = -\frac{1}{\beta}\frac{\partial \tau^x}{\partial y}$$

Zonal velocity vanishes at one boundary. Selection according to vorticity conservation

$$\Delta(f+\zeta)+\zeta_\tau = 0$$

Eastern boundary:

$\Delta f \downarrow$; ζ must be +; balanced by ζ_τ –

Western boundary:

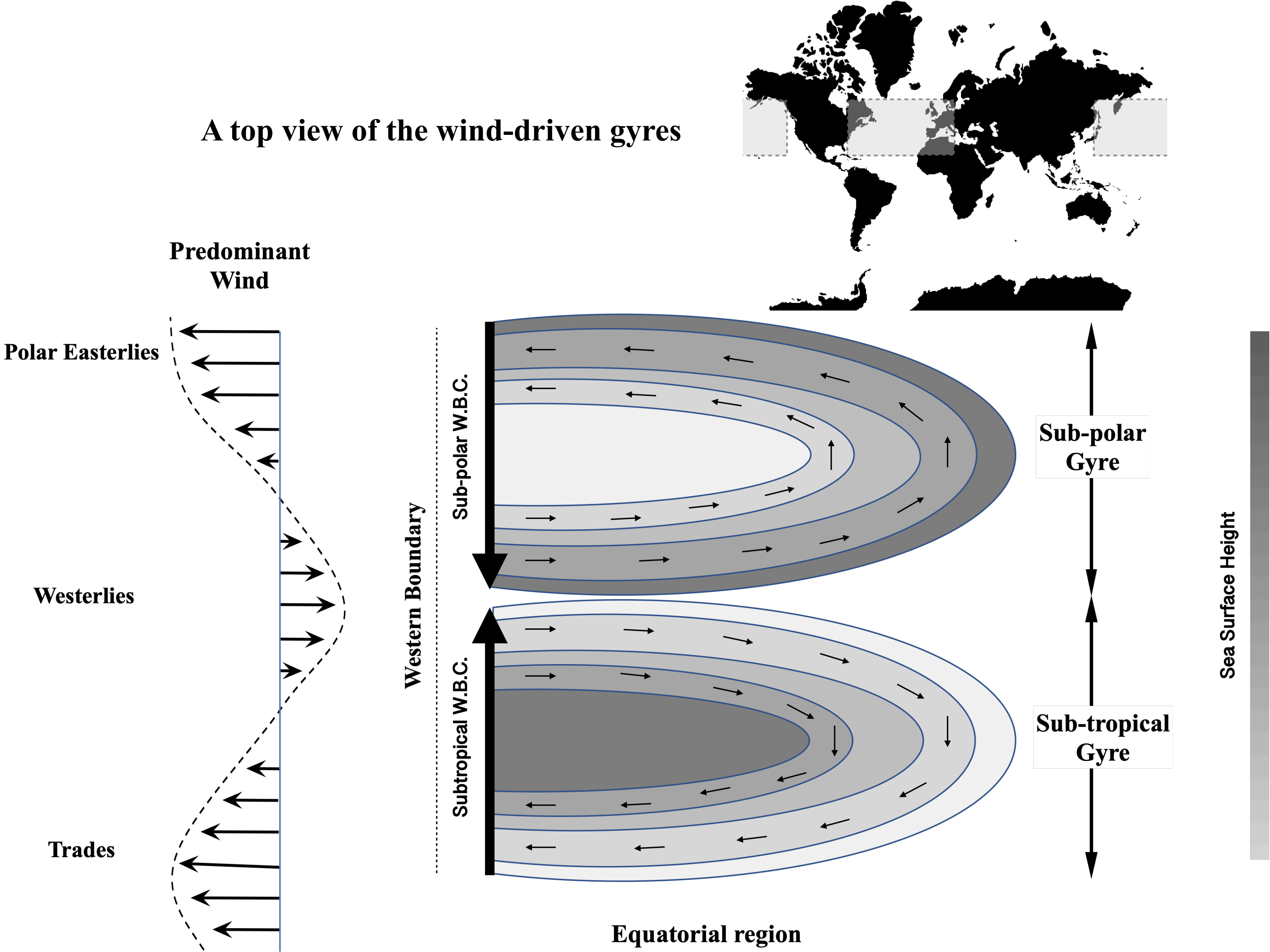
$\Delta f \uparrow$; ζ must be -; does not balanced by ζ_τ –

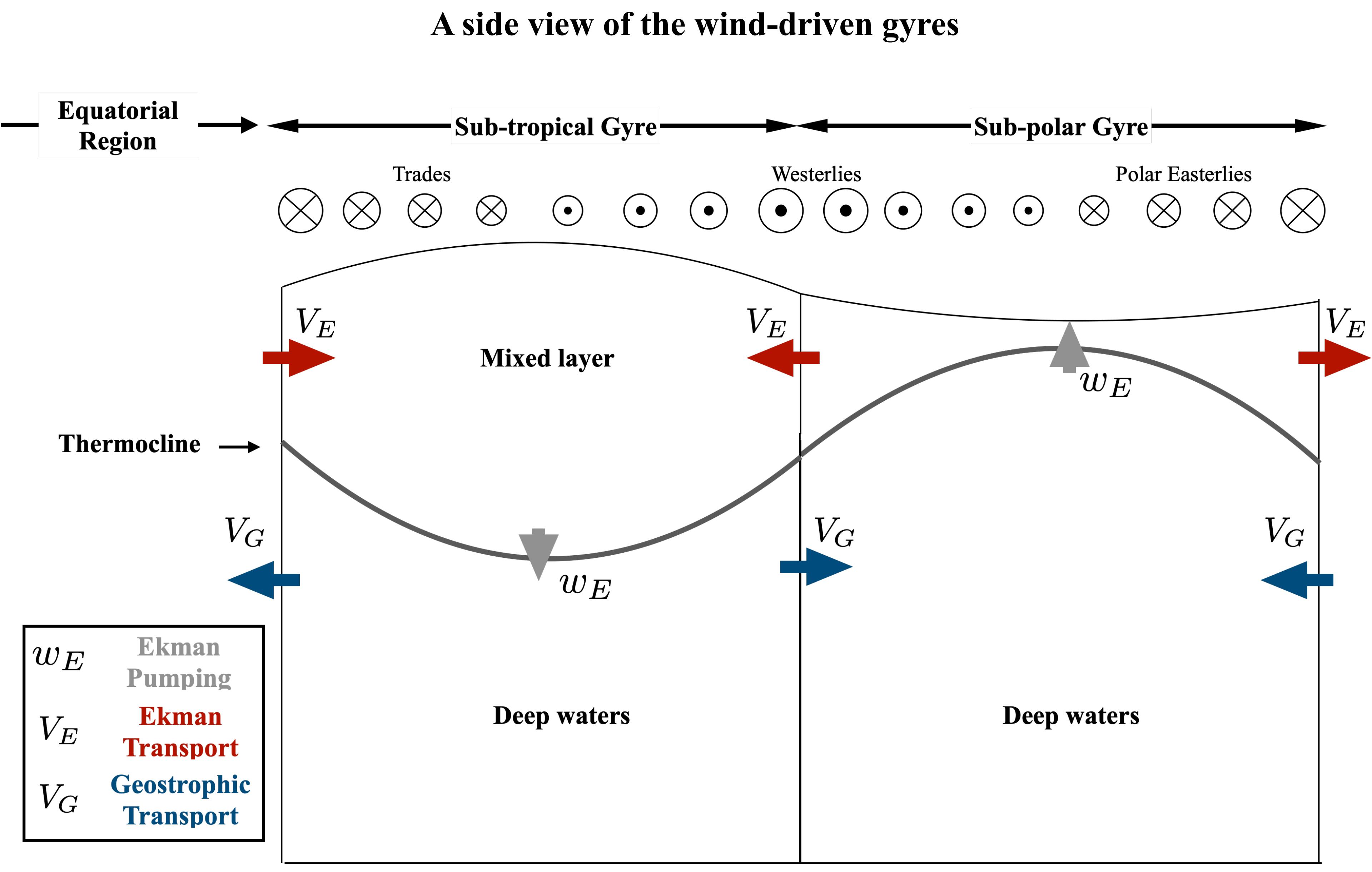
We need another vorticity term

The zonal Sverdrup transport

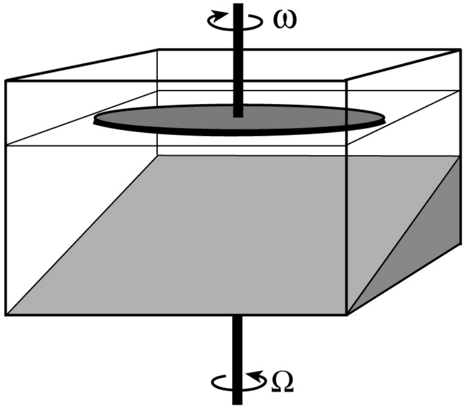
Using the depth integrated continuity equation

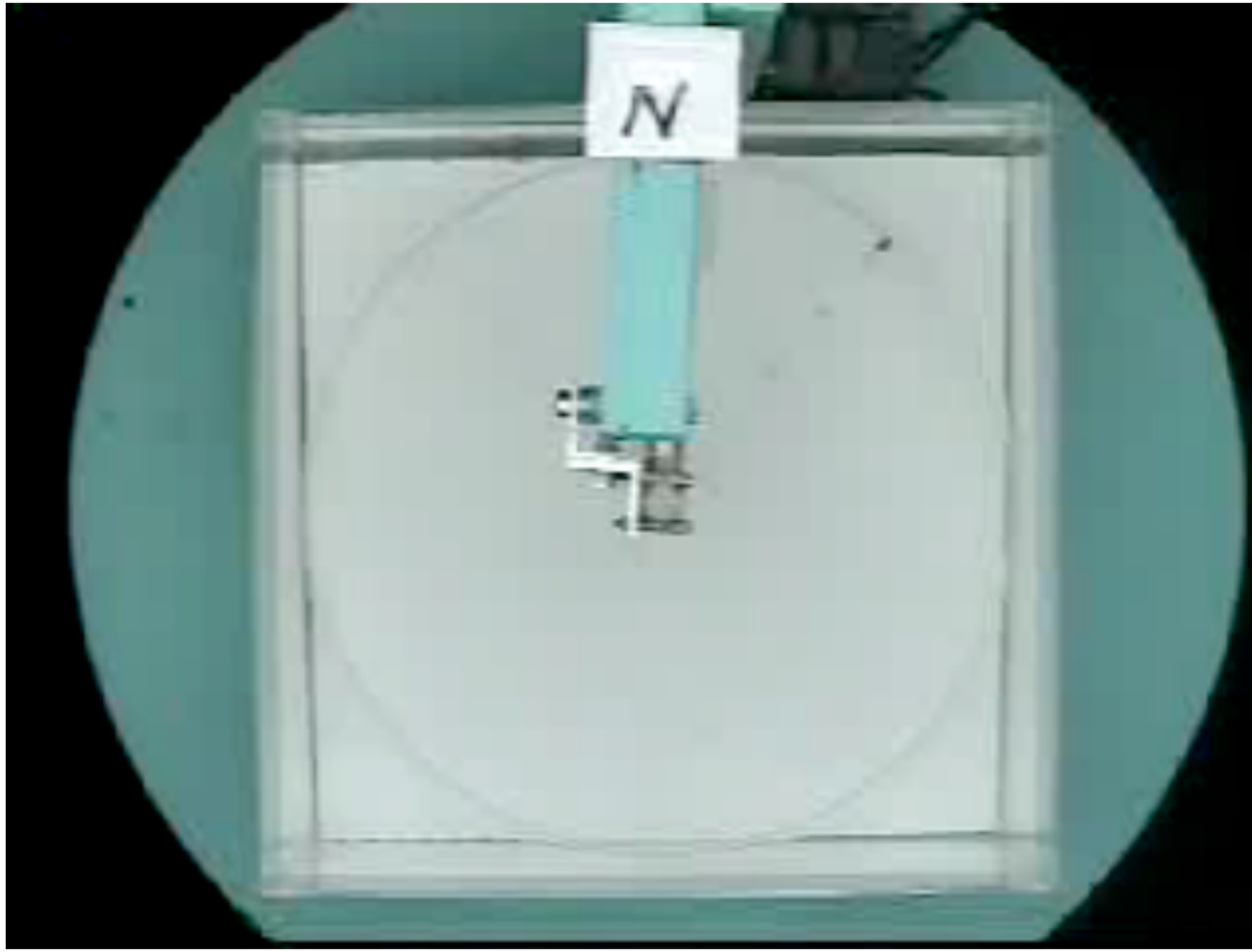
$$M_x(x) = \frac{\partial}{\partial y}\int_x^{x_{East}} M_y\,dx = -\frac{1}{\beta}\frac{\partial^2 \tau^x}{\partial y^2}\left(x_{East} - x\right)$$


A top view of the wind-driven gyres
Predominant Wind
Polar Easterlies
Westerlies
Trades
Western Boundary
Sub-polar W.B.C.
Subtropical W.B.C.
Sub-polar Gyre
Sub-tropical Gyre
Sea Surface Height
Equatorial region



A side view of the wind-driven gyres
Equatorial Region
Sub-tropical Gyre
Sub-polar Gyre
Trades
Westerlies
Polar Easterlies
V_E
V_E
V_E
Mixed layer
w_E
Thermocline
V_G
V_G
V_G
w_E
w_E Ekman Pumping
V_E Ekman Transport
V_G Geostrophic Transport
Deep waters
Deep waters





Why is there a **Western Boundary Current**?

Ekman's (and Sverdrup's) scaling is failing at the western boundary:

$\frac{\partial \tau^x}{\partial y}$ is very small at the boundaries (weak and irregular winds) ⇨ L is now much smaller than the open ocean ocean O(100 km) ⇨ $Ek_H = \frac{A_H}{fL^2} \sim 1$ horizontal friction/dissipation dominates

Stommel proposed a more general vorticity equation (including the western boundary region) that can be applied to the interior (Sverdrup) and the western limit of ocean. There is a dissipation term $-R\zeta$

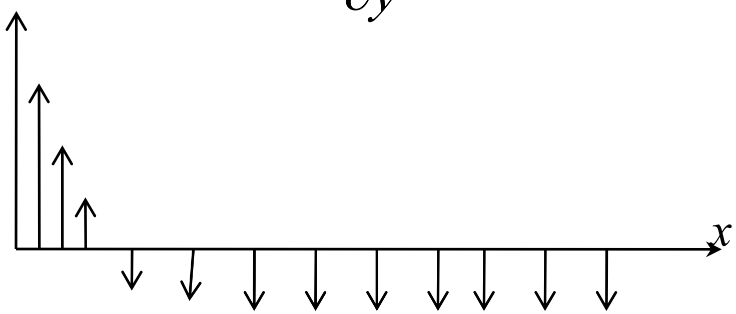
$$\beta U - R\zeta + \frac{\partial \tau^x}{\partial y} = 0$$

Interior ←

$$\beta U + \frac{\partial \tau^x}{\partial y} = 0$$

→ **WBL**

$$\beta U - R\zeta = 0$$



Munk followed Stommel's approach but the dissipation term is more complicated and appropriate for the ocean dynamics, of the form: $A_H \nabla^2 \zeta$

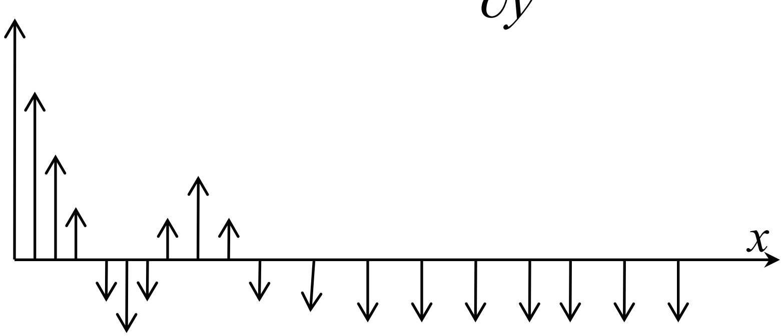
$$\beta U - A_H \nabla^2 \zeta + \frac{\partial \tau^x}{\partial y} = 0$$

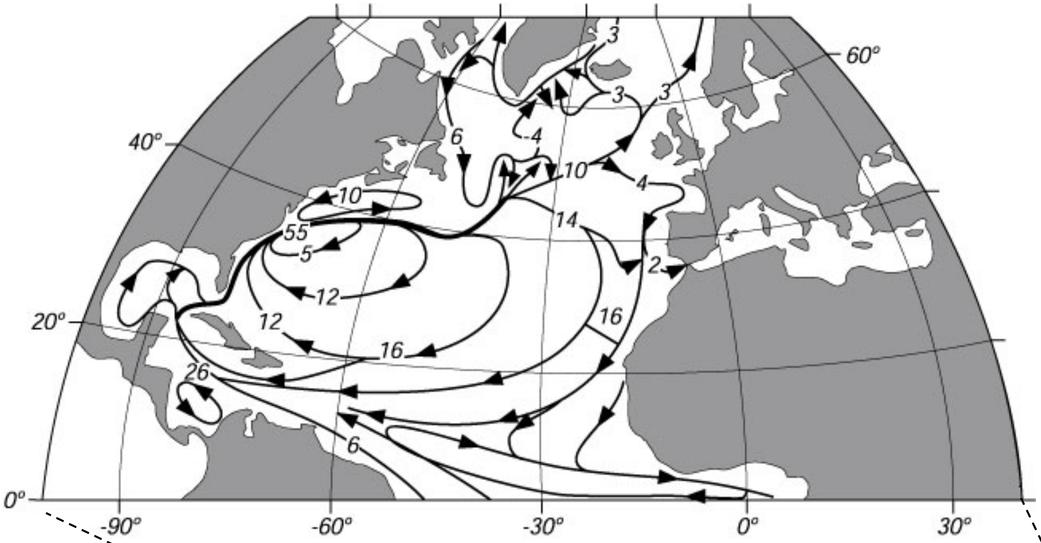
Interior ←

$$\beta U + \frac{\partial \tau^x}{\partial y} = 0$$

→ **WBL**

$$\beta U - A_H \nabla^2 \zeta = 0$$



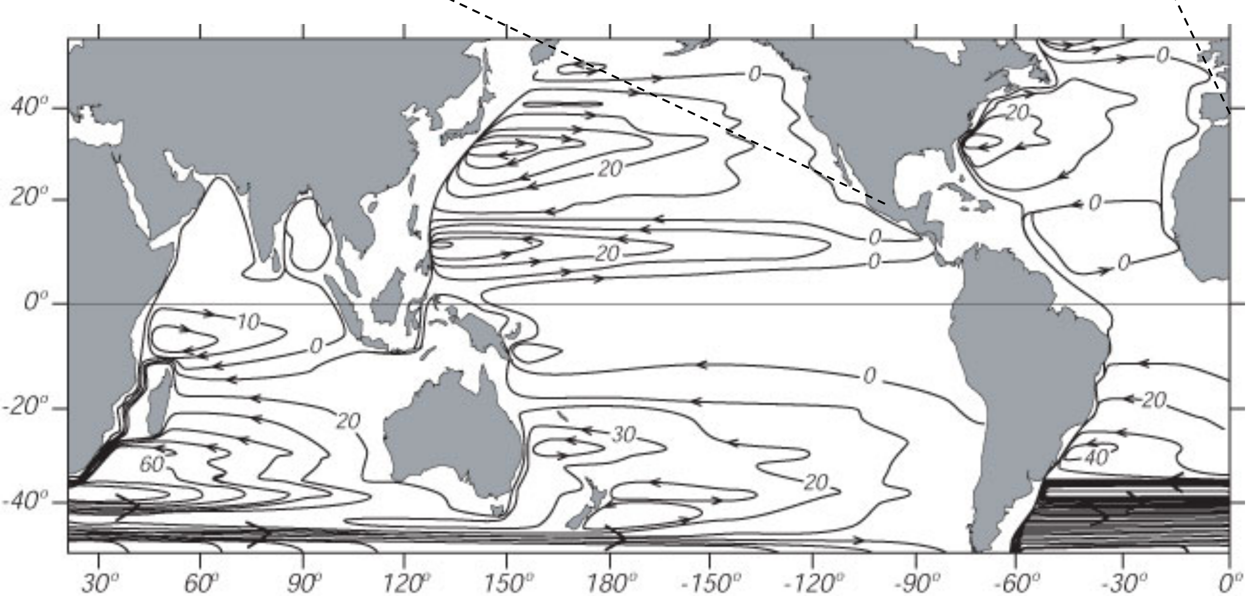


Stommel Model:

$$R\zeta \approx \beta U$$

$$\Rightarrow R\frac{U}{L} \approx \beta U$$

$$\Rightarrow L = O\left(\frac{R}{\beta}\right)$$



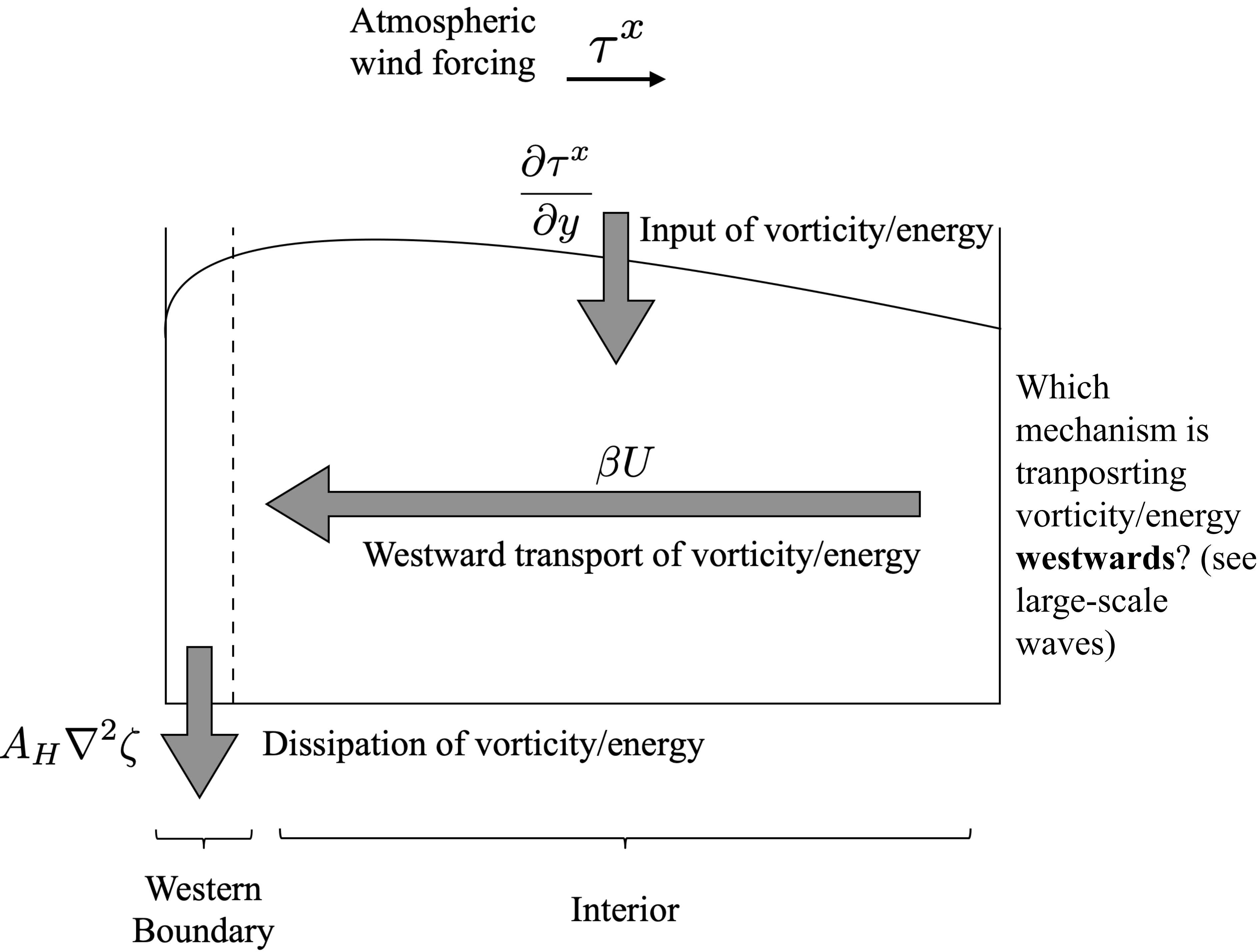
Munk Model:

$$A_H \nabla^2 \zeta \approx \beta U$$

$$\Rightarrow A_H \frac{U}{L^3} \approx \beta U$$

$$\Rightarrow L = O\left(\sqrt[3]{\frac{A_H}{\beta}}\right)$$

We can estimate L or R/A_H.


Atmospheric wind forcing
τ^x
$\frac{\partial \tau^x}{\partial y}$
Input of vorticity/energy
βU
Westward transport of vorticity/energy
Which mechanism is tranposrting vorticity/energy **westwards**? (see large-scale waves)
$A_H \nabla^2 \zeta$
Dissipation of vorticity/energy
Western Boundary
Interior