$\boxed{\text{ΚΙΝΗΜΑΤΙΚΗ}}$

K1

(1α) $\vec{V}(x,y) = 2\hat{i} + 4x\hat{j}$ $\left\{ \begin{array}{c} t=0 \\ x=0, y=0 \end{array} \right\}$

$$u_x = 2 \Rightarrow dx/dt = 2 \Rightarrow x = 2t + \cancel{c}$$

$$u_y = 4x \Rightarrow dy/dt = 4x$$

$$\frac{dy}{dt} = 4\cdot 2t = 8t \Rightarrow y = 4t^2 + \cancel{c}$$

$$\left\{ \begin{array}{l} x = 2t \\ y = 4t^2 \end{array} \right\} \Rightarrow y = (2t)^2 = x^2$$

$$\boxed{\begin{array}{l} \vec{r} = 2t\hat{i} + 4t^2\hat{j} \\ \vec{v} = 2\hat{i} + 8t\hat{j} \\ \vec{a} = 8\hat{j} \end{array}}$$

(1β) $\vec{F} = m\vec{a} = m\vec{a} = m\frac{d\vec{v}}{dt} = 8m\hat{j}$

$\vec{F} = -\nabla U \Rightarrow$ ① $F_x = -\frac{\partial U}{\partial x}$

② $F_y = -\frac{\partial U}{\partial y} \Rightarrow 8m = -\frac{\partial U}{\partial y}$

$\Rightarrow -U(x,y) = 8my + f(x)$

① $\rightsquigarrow \frac{df(x)}{dx} = 0 \Rightarrow f(x) = c$

Άρα $-U = 8my + c$

K2

(2)

$$\boxed{\begin{aligned}\vec{v} &= t\hat{i} + 2t\hat{j} \\ \vec{a} &= \hat{i} + 2\hat{j} \\ \vec{r} &= \frac{t^2}{2}\hat{i} + t^2\hat{j}\end{aligned}}$$

$$\left.\begin{aligned} x(t) &= \frac{t^2}{2} \\ y(t) &= t^2 \end{aligned}\right\} \Rightarrow \boxed{y = 2x}$$

$$\vec{v} \times \vec{a} = \vec{v} \times (\vec{a}_N + \vec{a}_T) = \vec{v} \times \vec{a}_N$$

$$\|\vec{v} \times \vec{a}\| = v \cdot a_N \Rightarrow a_N = \frac{\|\vec{v} \times \vec{a}\|}{v}$$

$$\Rightarrow \rho = \frac{v^2}{a_N} = \frac{v^3}{|\vec{v} \times \vec{a}|}$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t & 2t & 0 \\ 1 & 2 & 0 \end{vmatrix} = (2t - 2t)\hat{k} = 0\hat{k}$$

Άρα $\rho \rightarrow \infty$ (ευθεία τροχιά)

K3

(3) $a_x = -4\cos(2t)$

$a_y = -12\sin(2t)$

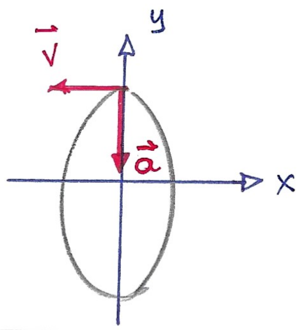
$t=0 \quad \begin{cases} v_{x_0} = 0 \\ v_{y_0} = 6 \end{cases} \quad \begin{cases} x_0 = 1 \\ y_0 = 0 \end{cases}$

(3a) $v_x = -2\sin(2t)$

$v_y = 6\cos(2t)$

$$\left.\begin{aligned} x &= \cos(2t) \\ y &= 3\sin(2t) \end{aligned}\right\} \Rightarrow \frac{x^2}{1^2} + \frac{y^2}{3^2} = 1$$

έλλειψη με $\begin{cases} \alpha = 1 \\ \beta = 3 \end{cases}$



(3β) $t = \frac{\pi}{4}$

$\vec{r} = \cos(\pi/2)\,\hat{i} + 3\sin(\pi/2)\,\hat{j} = 0\hat{i} + 3\hat{j}$

$\vec{v} = -2\sin(\pi/2)\,\hat{i} + 6\cos(\pi/2)\,\hat{j} = -2\hat{i} + 0\hat{j}$

$\vec{\alpha} = -4\cos(\pi/2)\,\hat{i} - 12\cos(\pi/2)\,\hat{j} = -12\hat{j}$

$$\rho = \frac{v^3}{|\vec{v}\times\vec{a}|} = \frac{(+2)^3}{24} = \frac{8}{24} = \frac{1}{3}$$

$$\vec{v}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 0 \\ 0 & -12 & 0 \end{vmatrix} = 24\hat{k}$$

<u>Παρατήρηση</u>

Επειδή $\vec{a} = \vec{a}_N$ $(\vec{a} \perp \vec{v})$ μπορούμε άμεσα να ισχυριστούμε:

$$\rho = \frac{v^2}{a_N} = \frac{2^2}{12} = \frac{1}{3}$$

K4

4

$$a_x = -3\sin t$$
$$a_y = -4\cos t$$

$$\begin{cases} V_x = 3\cos t + \cancel{c_1} \\ V_y = -4\sin t + \cancel{c_2} \end{cases}$$

$$\begin{cases} x = 3\sin t + c_1 \\ y = 4\cos t + c_2 \end{cases} \xRightarrow{t=0} \begin{cases} x_0 = 3\cdot 0 + c_1 = 3 \\ y_0 = 4\cdot 1 + c_2 = 8 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} c_1 = 3 \\ c_2 = 4 \end{cases}$$

Άρα $\begin{cases} x = 3\sin t + 3 \\ y = 4\cos t + 4 \end{cases} \Rightarrow \left(\frac{x-3}{3}\right)^2 + \left(\frac{y-4}{4}\right)^2 = 1$

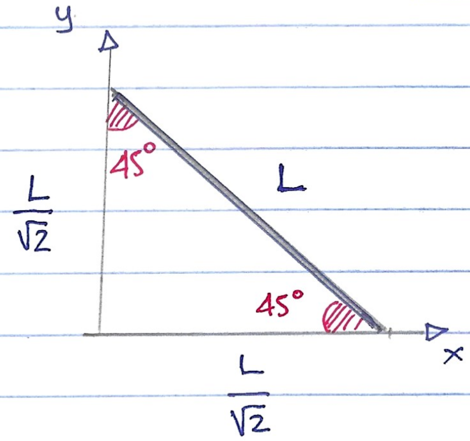
$$\Rightarrow \boxed{\frac{(x-3)^2}{3^2} + \frac{(y-4)^2}{4^2} = 1}$$

Έλλειψη με κέντρο $(x_0, y_0) = (3,4)$

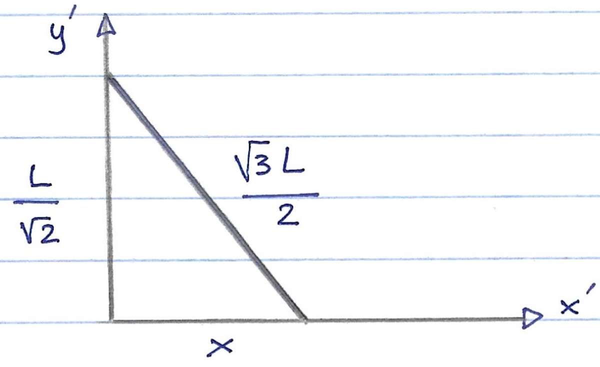
& ημιάξονες $\begin{cases} a = 3 \\ \beta = 4 \end{cases}$

ΣΧΕΤΙΚΟΤΗΤΑ

Σ1



(ακίνητος)



(κινούμενος)

Για τον ακίνητο παρατηρητή, οι δύο κάθετες πλευρές του ισοσκελούς ορθογωνίου τριγώνου είναι $\frac{L}{\sqrt{2}}$ η κάθε μία.

Κατά την κατεύθυνση y ΔΕΝ υπάρχει μεταβολή μήκους.
'Αρα:

$$x^2 + \frac{L^2}{2} = \frac{3L^2}{4} \Longrightarrow x^2 = \frac{L^2}{4} \Longrightarrow$$

$$\Longrightarrow \boxed{x = L/2}$$

Συστολή μήκους: $D = \frac{D_0}{\gamma} \Longrightarrow \frac{L}{2} = \frac{\frac{L}{\sqrt{2}}}{\gamma}$

$$\Longrightarrow \gamma = \frac{2}{\sqrt{2}} \Longrightarrow \boxed{\gamma = \sqrt{2}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Longrightarrow \gamma^2 = \frac{1}{1-\beta^2} \Longrightarrow \beta^2 = 1 - \frac{1}{\gamma^2} = \frac{\gamma^2 - 1}{\gamma^2}$$

$$\Longrightarrow \beta = \frac{\sqrt{\gamma^2-1}}{\gamma} = \frac{\sqrt{2-1}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \simeq 0.707$$

Σ2

1

$$E = 2E_0 \Downarrow \gamma_1 = 2 \Downarrow \beta_1 = \frac{\sqrt{\gamma_1^2 - 1}}{\gamma_1} = \frac{\sqrt{3}}{2}$$

2

$$\tau = 3\tau_0 \Downarrow \gamma_2 = 3 \Downarrow \beta_2 = \frac{\sqrt{\gamma_2^2 - 1}}{\gamma_2} = \frac{\sqrt{8}}{3}$$

$$V_{12} = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}\, c = \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{8}}{3}}{1 + \frac{\sqrt{24}}{6}}\, c = \frac{3\sqrt{3} + 2\sqrt{8}}{6 + \sqrt{24}}\, c$$

$$\longrightarrow \boxed{V_{12} \approx 0.996\, c}$$

Σ3 Έστω $\beta_1 = \beta_2 = x$. Τότε:

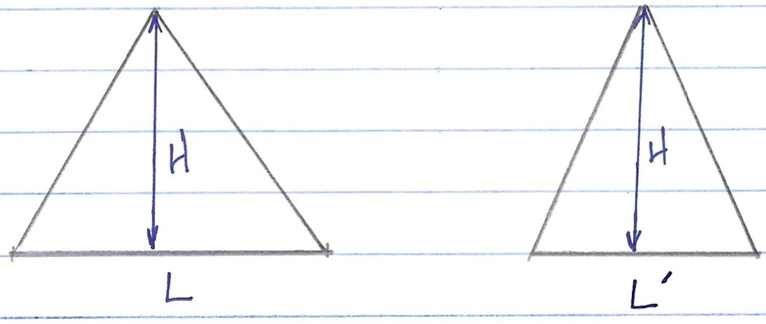
$$\frac{x + x}{1 + x^2}\, c = \frac{1}{\lambda}\, c \Longrightarrow \frac{2x}{1 + x^2} = \frac{1}{\lambda}$$

$$\Longrightarrow x^2 - 2\lambda x + 1 = 0 \Longrightarrow \boxed{x = \lambda - \sqrt{\lambda^2 - 1}}$$

(Η λύση με πρόσημο $+\sqrt{\cdots}$ δεν έχει φυσικό νόημα)

Για $\lambda = 1$ προκύπτει $\beta_1 = \beta_2 = 1$ (φωτόνια)

Σ4



Το ύψος παραμένει αμετάβλητο, ενώ για τη βάση L ισχύει:

$$L' = \frac{L}{\gamma} = \frac{L}{\frac{1}{\sqrt{1-\beta^2}}} = \sqrt{1-\beta^2}\,L = 0.8\,L$$

$$S' = \frac{1}{2}L'H = \frac{1}{2}0.8\,LH = 0.8\,S = \frac{S}{1.25}$$

ΣΤΑΤΙΚΗ - ΡΟΠΕΣ - ΚΕΝΤΡΟ ΜΑΖΑΣ

ΚΜ1 Όπως στη διαφάνεια (24) του αρχείου της επανάληψης

ΚΜ2

$m_1 : (0,0)$

$m_2 : (3,0)$

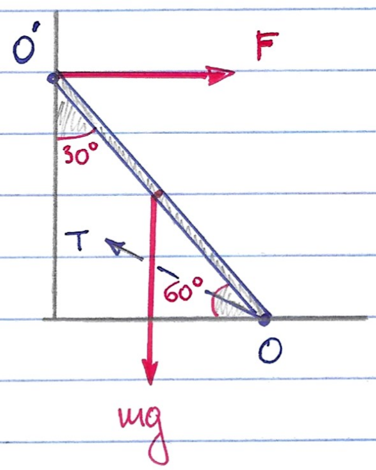
$m_3 : (3,3)$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{6m}{3m} = 2$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{3m}{3m} = 1$$

Άρα $\boxed{(x_{cm}, y_{cm}) = (2,1)}$

ΚΜ3



Επειδή η T είναι άγνωστη παίρνουμε ροπές ως προς O:

$$F \cdot (L \sin 60^\circ) = mg \left(\frac{L}{2} \cos 60^\circ \right)$$

$$\Rightarrow F \cdot \cancel{L} \cdot \frac{\sqrt{3}}{2} = mg \cdot \frac{\cancel{L}}{2} \cdot \frac{1}{2}$$

$$\Rightarrow \boxed{F = \frac{mg}{2\sqrt{3}}}$$

ΔΥΝΑΜΙΚΟ

Δ1

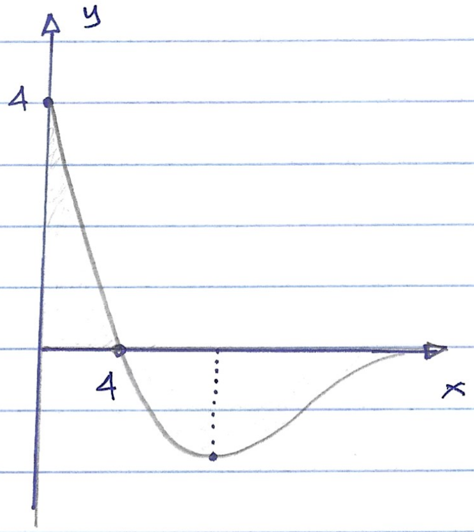
(α) $\vec{F} = -\vec{\nabla} U \Longrightarrow \vec{F} = -\frac{\partial}{\partial x}\left\{-4x\, e^{-x/4}\right\}\hat{i}$

$$\Longrightarrow F = F_x = 4\, e^{-x/4} + 4x \cdot \left(-\frac{1}{4}\right) e^{-x/4}$$

$$\Longrightarrow F = (4-x)\, e^{-x/4}$$

$$\left[\begin{array}{ll} x=0 & F=4 \\ x=4 & F=0 \\ x>4 & F<0 \\ x\to\infty & F\to 0 \end{array}\right]$$

$\boxed{a=4}$



(β) $W_{0\to a} = U(0) - U(\alpha) = -4\cdot 0 \cdot e^{0} + 4\cdot 4\cdot e^{-1} = 16/e$

$W_{a\to\infty} = U(\alpha) - U(\infty) = -\frac{16}{e} + 0 = -16/e$

άρα $\boxed{W_{0\to a} = -W_{a\to\infty}}$

(γ) Ελάχιστο της U = Μέγιστο της K

$U(x) + K(x) = E$

{ Το ελάχιστο της U είναι στο $x = \alpha = 4$ }

$K(x) = E - U(x) \Longrightarrow K(\alpha) = E - U(\alpha) = -4 + \frac{16}{e}$

$K(\alpha) \simeq 1.89$

Δ2 $\vec{F}(x,y) = a\beta xy\,\hat{i} + (ax^2 + \beta y^2)\,\hat{j}$

a $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x} \Longrightarrow a\beta x = 2ax$

$$\Longrightarrow \begin{Bmatrix} a \in R \\ \beta = 2 \end{Bmatrix}$$

β $\vec{F}(x,y) = 2axy\,\hat{i} + (ax^2 + 2y^2)\,\hat{j}$

$$\vec{F} = -\vec{\nabla} U$$

$$-\frac{\partial U}{\partial x} = F_x \Longrightarrow -\frac{\partial U}{\partial x} = 2axy \Longrightarrow$$

$$\Longrightarrow -U(x,y) = ax^2y + g(y) \quad (1)$$

$$-\frac{\partial U}{\partial y} = F_y \xrightarrow{(1)} ax^2 + g'(y) = ax^2 + 2y^2$$

$$\Longrightarrow g'(y) = 2y^2 \Longrightarrow g(y) = \frac{2}{3}y^3 + c_1$$

Άρα $\boxed{-U(x,y) = ax^2y + \frac{2}{3}y^3 + c_1}$

$$W_{A\to B} = U(A) - U(B) = a\cdot 1^2\cdot 3 + \frac{2}{3}3^3 = \underline{3a + 18}$$

A : (0,0)

B : (1,3)

Δ3 $\vec{F} = (2x+y)\hat{i} + (2y+x)\hat{j}$

$$\left.\begin{aligned} \frac{\partial F_x}{\partial y} &= 1 \\ \frac{\partial F_y}{\partial x} &= 1 \end{aligned}\right\} \Longrightarrow \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

συντηρητική

$$\vec{F} = -\vec{\nabla} U$$

$$-\frac{\partial U}{\partial x} = F_x \Longrightarrow -\frac{\partial U}{\partial x} = 2x+y$$

$$\Longrightarrow -U(x,y) = x^2 + xy + g(y) \quad (1)$$

$$-\frac{\partial U}{\partial y} = F_y \overset{(1)}{\Longrightarrow} x + g'(y) = 2y + x$$

$$\Longrightarrow g'(y) = 2y \Longrightarrow g(y) = y^2 + C_1$$

Άρα $\boxed{-U(x,y) = x^2 + xy + y^2 + C_1}$

11/1/2025