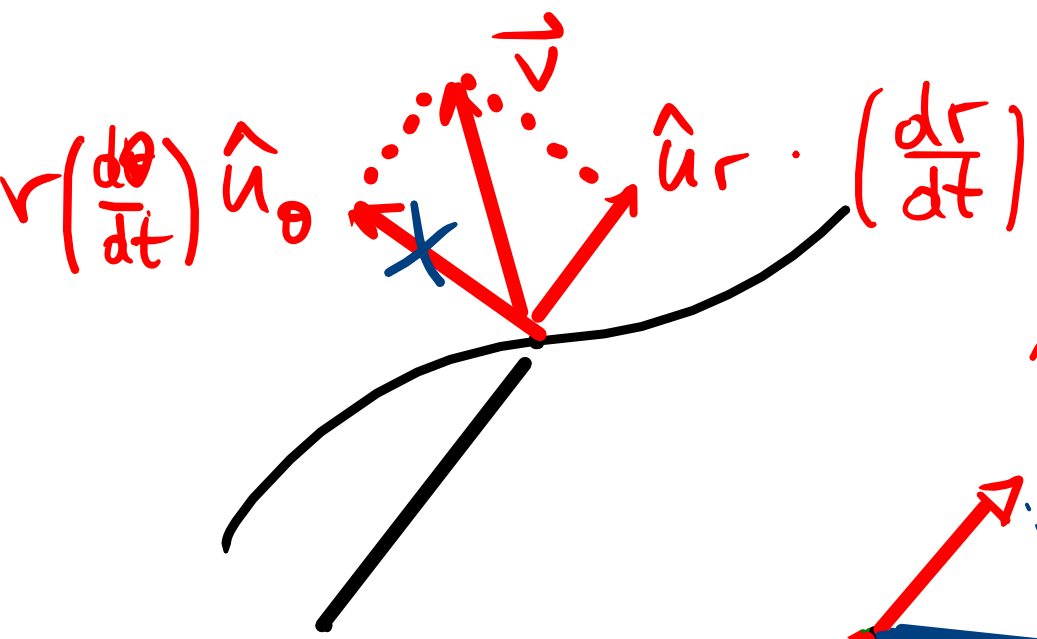
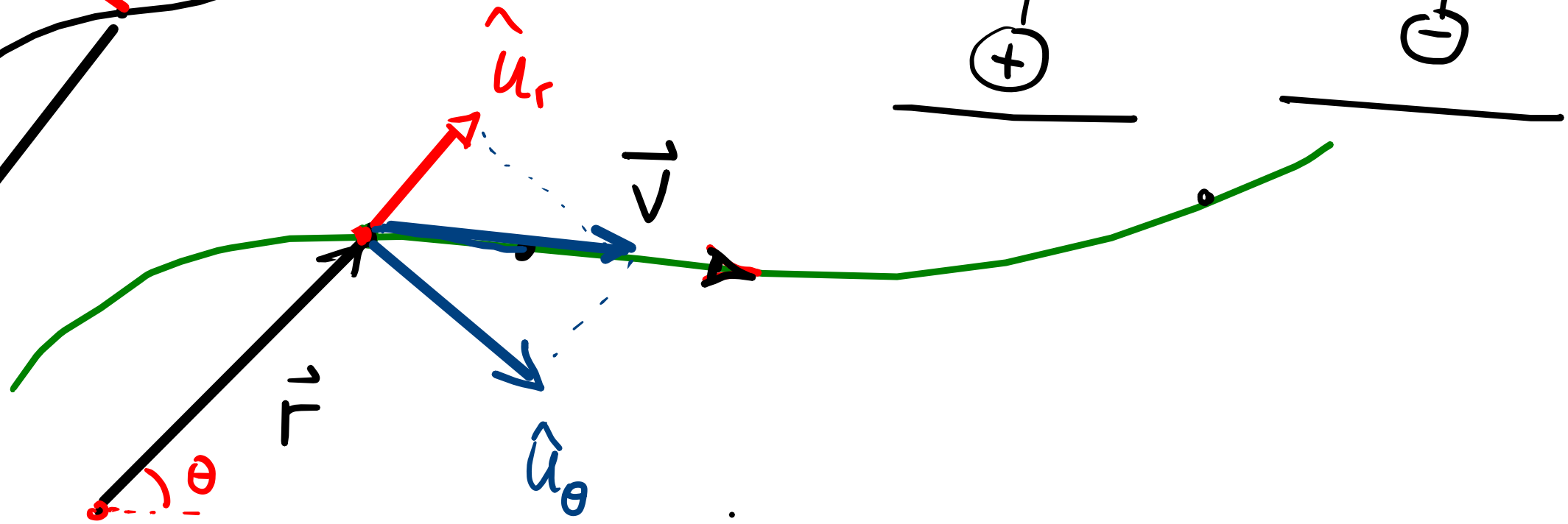
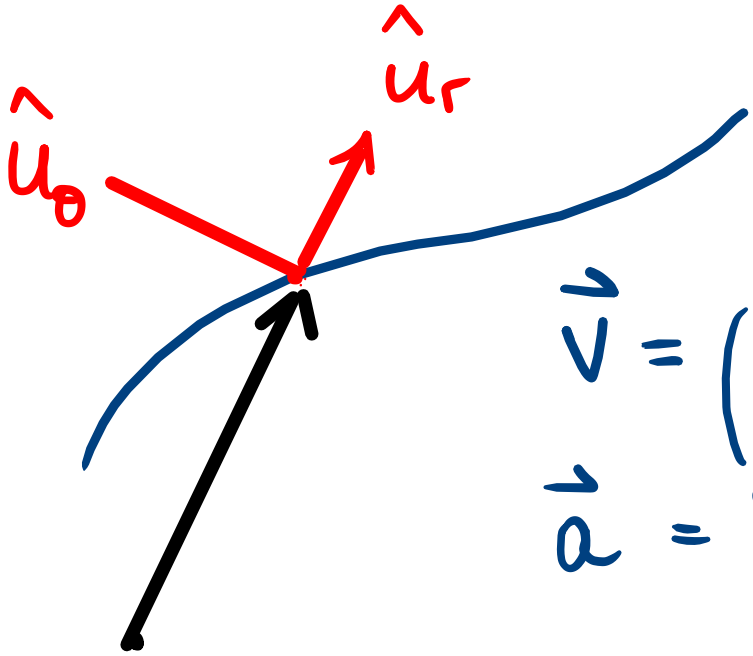
$$\vec{v} = \left(\frac{dr}{dt}\right)\hat{u}_r + r\left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$

(+) (−)







$$\vec{r} = r\,\hat{u}_r$$

$$\vec{v} = \left(\frac{dr}{dt}\right)\hat{u}_r + r\left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$

$$\vec{a} = ?$$

$$\frac{d\hat{u}_r}{dt} = \left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$

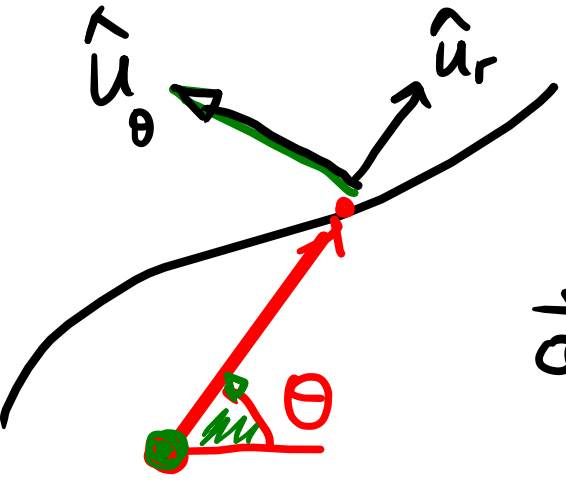
$$\frac{d\hat{u}_\theta}{dt} = -\left(\frac{d\theta}{dt}\right)\hat{u}_r$$

$$\vec{a} = \frac{d}{dt}\left\{\left(\frac{dr}{dt}\right)\hat{u}_r + r\cdot\left(\frac{d\theta}{dt}\right)\hat{u}_\theta\right\} = \left(\frac{d^2r}{dt^2}\right)\cdot\hat{u}_r + \left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$

$$+ \left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\hat{u}_\theta + r\left(\frac{d^2\theta}{dt^2}\right)\hat{u}_\theta - r\cdot\left(\frac{d\theta}{dt}\right)^2\cdot\hat{u}_r$$

$$= \left\{\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right\}\hat{u}_r + \left\{2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) + r\left(\frac{d^2\theta}{dt^2}\right)\right\}\hat{u}_\theta$$

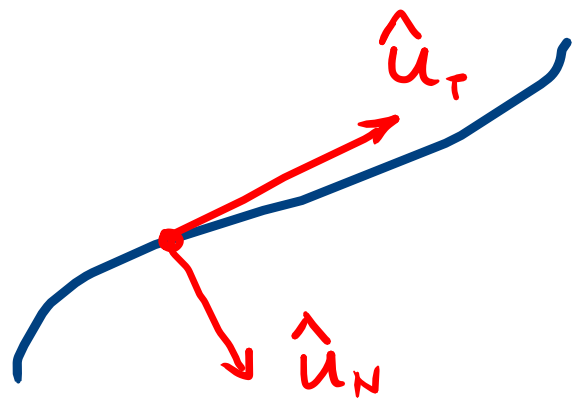
$$\vec{a} = \left(\frac{d^2r}{dt^2} - r\left[\frac{d\theta}{dt}\right]^2\right)\hat{u}_r + \left(2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) + r\frac{d^2\theta}{dt^2}\right)\hat{u}_\theta$$



$$\frac{d\theta}{dt} = \omega \qquad \text{γωνιακή ταχύτητα} = \omega(t)$$

$$\vec{a} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{u}_r + \left(2\omega\left(\frac{dr}{dt}\right) + r\frac{d\omega}{dt}\right)\hat{u}_\theta$$

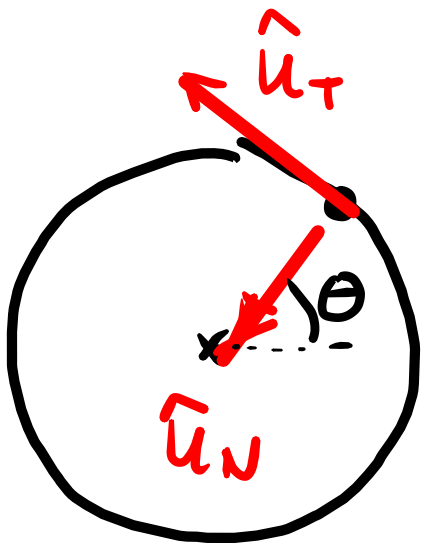
γωνιακή επιτάχυνση



$$\vec{a} = \vec{a}_T + \vec{a}_N = \left(\frac{dv}{dt}\right)\hat{u}_T + \frac{v^2}{\rho}\hat{u}_N$$

$$v = ds/dt = \frac{d^2s}{dt^2}$$

Γενικευμένες



κυκλική

κίνηση

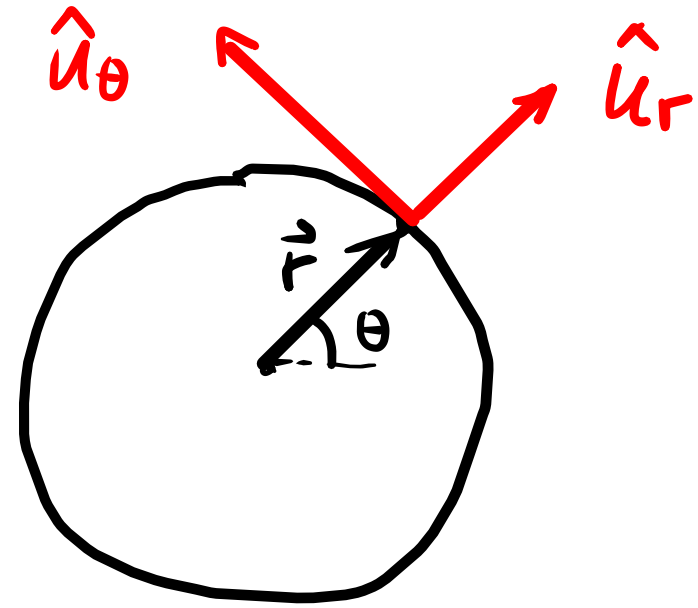
$R = r = \text{σταθερό}$

$\omega = \omega(t)$

$$\vec{v} = \frac{ds}{dt}\hat{u}_T = \frac{d(R\cdot\theta)}{dt}\hat{u}_T$$

$$= R\frac{d\theta}{dt}\hat{u}_T = R\omega\hat{u}_T$$

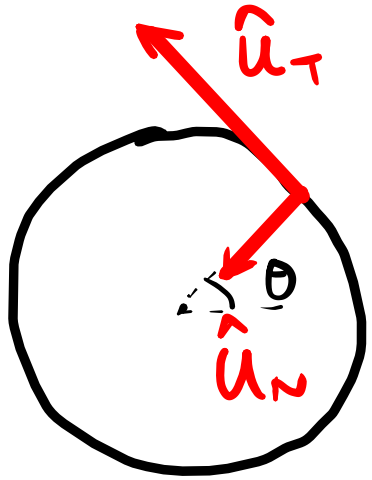
Πολικές



$$\vec{v} = \left(\frac{dr}{dt}\right)\hat{u}_r + r\left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$

$$\frac{dr}{dt} = 0$$

$$\vec{v} = r\omega\hat{u}_\theta$$



$$\vec{a} = \frac{dv}{dt}\hat{u}_T + \frac{v^2}{\rho}\hat{u}_N$$

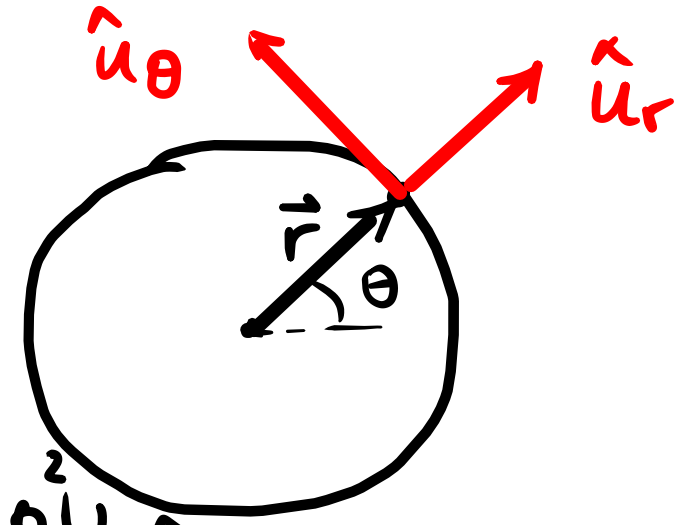
$$\frac{dv}{dt} = \frac{d}{dt}(\omega R) = R\frac{d\omega}{dt}$$

$$\vec{a} = R\frac{d\omega}{dt}\hat{u}_T + \omega^2 R\,\hat{u}_N$$

ΓΕΝΙΚΕΥΜΕΝΕΣ

$r = \text{σταθερό} = R$

$$\frac{d\theta}{dt} = \omega$$



$$\vec{a} = \left\{\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right\}\hat{u}_r + \left\{2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) + r\frac{d^2\theta}{dt^2}\right\}\hat{u}_\theta$$

$$= -r\omega^2\hat{u}_r + r\cdot\frac{d^2\theta}{dt^2}\hat{u}_\theta$$

$$= -R\omega^2\hat{u}_r + R\frac{d\omega}{dt}\hat{u}_\theta$$

ΠΟΛΙΚΕΣ

$$x = x(t)$$

$$y = y_0$$

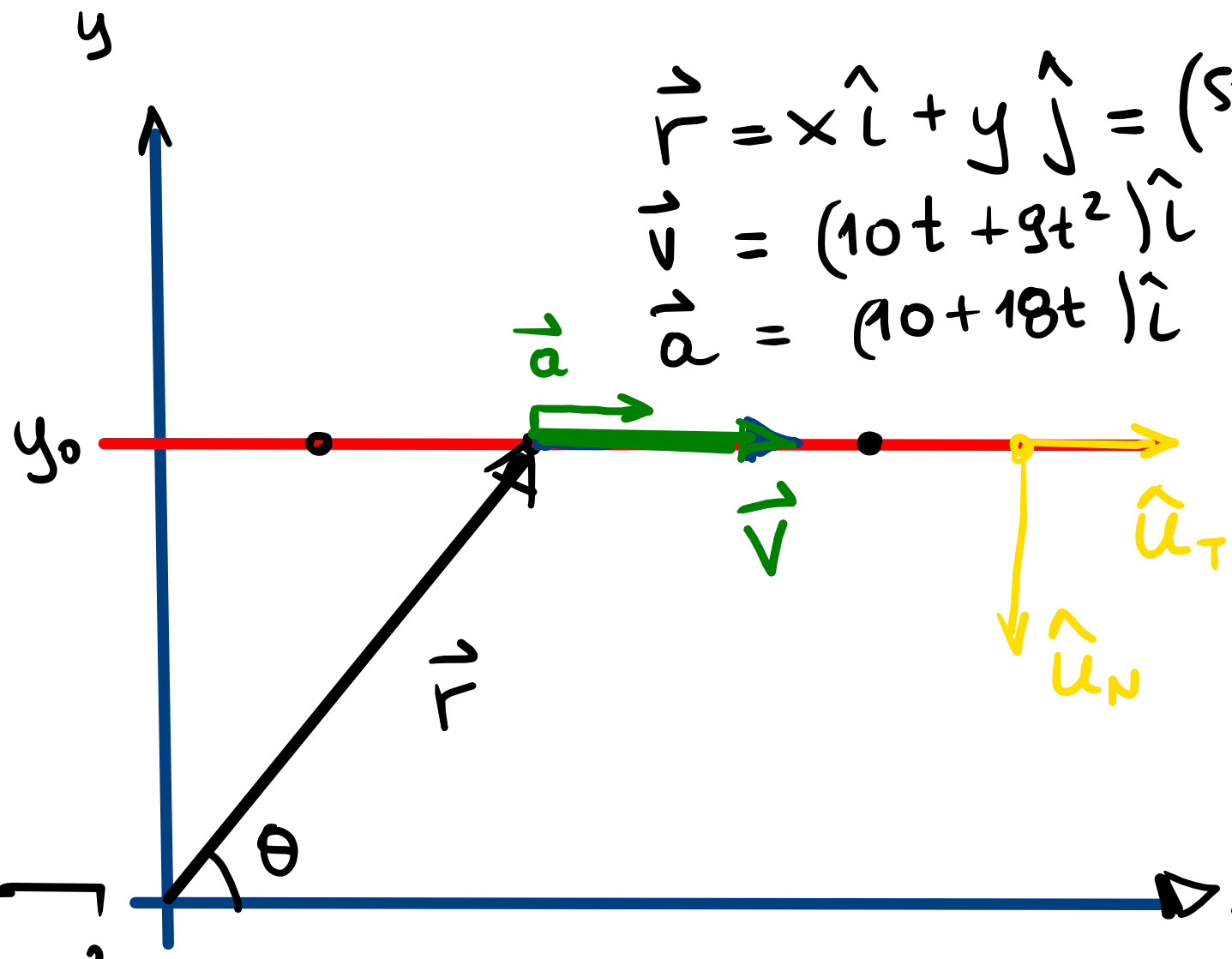
$$x = 5t^2 + 3t^3$$

$$\vec{r} = r\,\hat{u}_r$$

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(5t^2 + 3t^3)^2 + y_0^2}$$

$$\theta = \operatorname{atan} \frac{y_0}{\sqrt{x^2 + y^2}}$$



$$\vec{r} = x\hat{i} + y\hat{j} = (5t^2 + 3t^3)\hat{i} + y_0\hat{j}$$

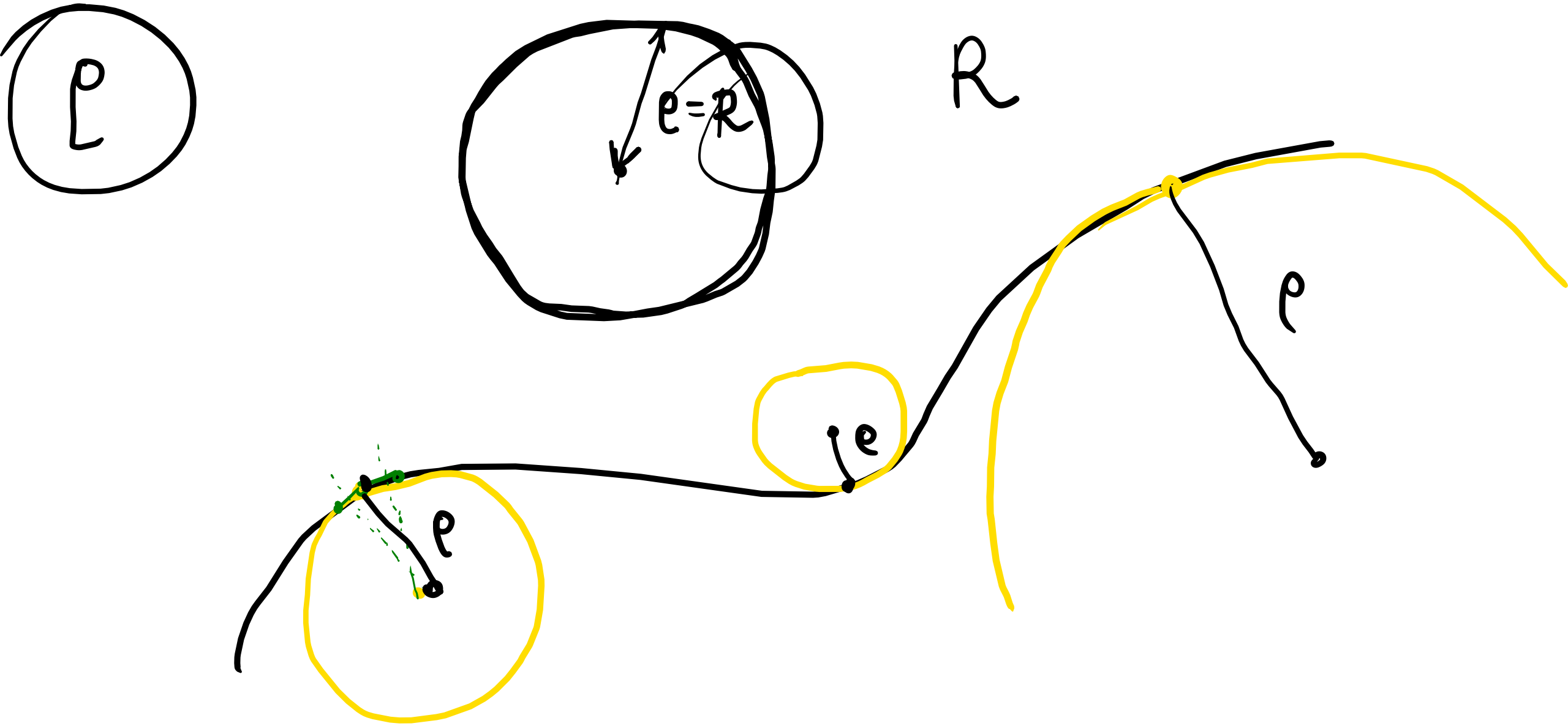
$$\vec{v} = (10t + 9t^2)\hat{i}$$

$$\vec{a} = (10 + 18t)\hat{i}$$

$$\vec{a} = \frac{dv}{dt}\hat{u}_T + \frac{v^2}{\rho}\hat{u}_N$$

$dr/dt = ?$

$d\theta/dt = ?$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