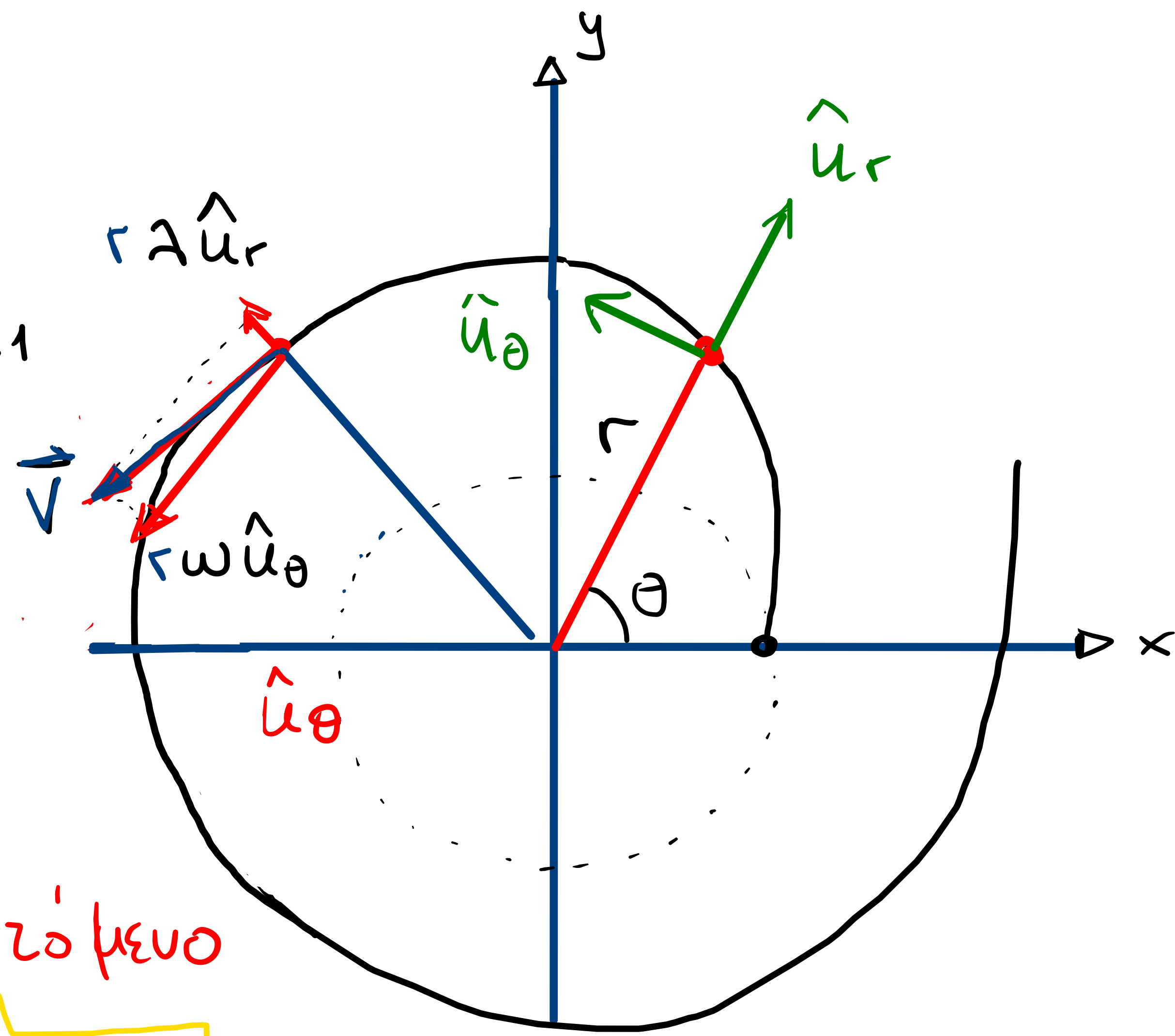
Σπειροειδής κίνηση

$$r = r_0 \cdot e^{\lambda t} \qquad [\lambda] = s^{-1}$$

$$\frac{d\theta}{dt} = \omega = \text{σταθερό} \qquad [r_0] = m$$

$$[\omega] = s^{-1}$$



$$\vec{v} = \left(\frac{dr}{dt}\right)\hat{u}_r + r \cdot \left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$

$$\frac{dr}{dt} = r_0 \cdot \lambda e^{\lambda t}$$

$\vec{v}$

εφαπτόμενο

$$\vec{v} = r_0 \cdot \lambda e^{\lambda t} + r\omega\,\hat{u}_\theta$$

$$= r\lambda\,\hat{u}_r + \underline{r}\,\omega\,\hat{u}_\theta = r\left(\lambda\,\hat{u}_r + \omega\,\hat{u}_\theta\right)$$

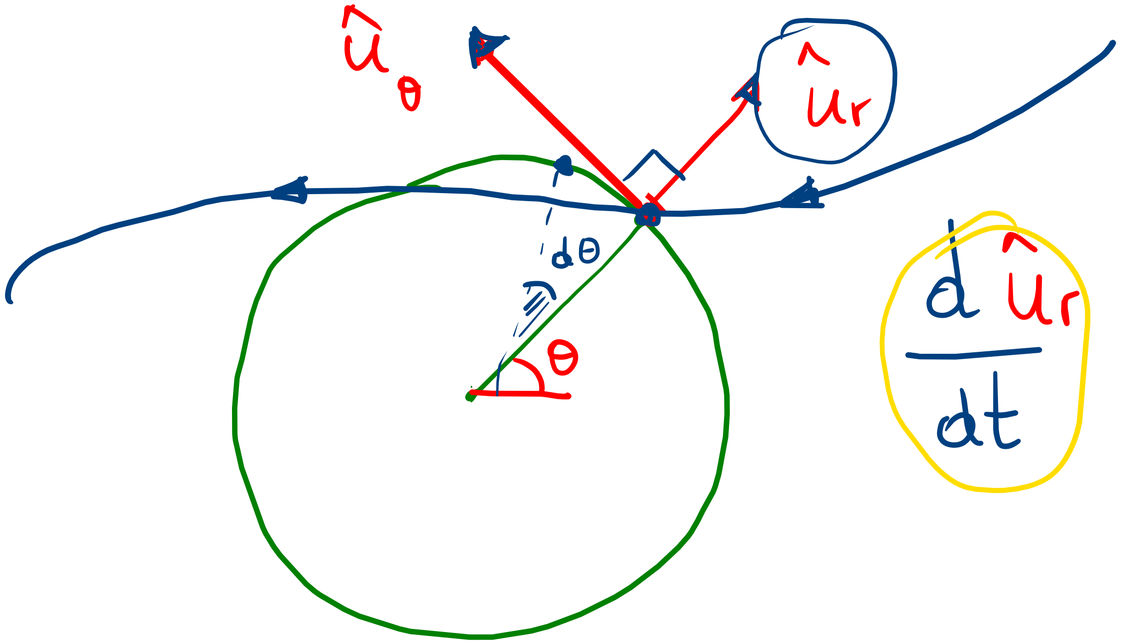
$$\vec{v} = r\cdot\lambda\,\hat{u}_r + r\cdot\omega\,\hat{u}_\theta \qquad \vec{a} = ? = \frac{d\vec{v}}{dt} \qquad r = r_0 e^{\lambda t}$$

$$\vec{a} = \left[\lambda \frac{dr}{dt}\hat{u}_r + \lambda r \frac{d\hat{u}_r}{dt}\right] + \left[\omega \frac{dr}{dt}\hat{u}_\theta + r\omega \frac{d\hat{u}_\theta}{dt}\right] \qquad \frac{dr}{dt} = r_0\cdot\lambda\cdot e^{\lambda t}$$

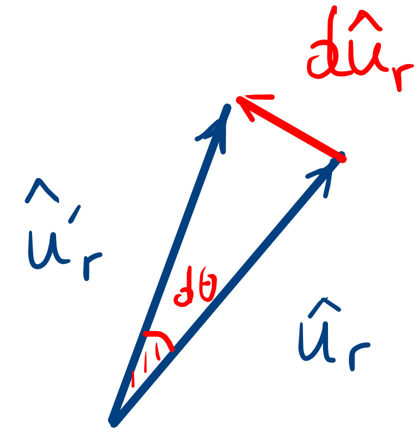
$$= \lambda r$$

$$= \left[\lambda\cdot\lambda\cdot r\,\hat{u}_r + \lambda r\omega\,\hat{u}_\theta\right] + \left[\omega\lambda r\,\hat{u}_\theta - r\omega^2\,\hat{u}_r\right]$$

$$= \underbrace{r(\lambda^2 - \omega^2)}_{m\cdot s^{-2}}\hat{u}_r + \underbrace{2\lambda\omega r}_{m\cdot s^{-2}}\hat{u}_\theta$$



$$\frac{d\hat{u}_r}{dt} = \left(\frac{d\theta}{dt}\right)\hat{u}_\theta$$



Γεωμετρική απόδειξη

$$\left.\begin{aligned} \hat{u}_r &= \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \\ \hat{u}_\theta &= -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} \end{aligned}\right\}$$

$$\frac{d\hat{u}_r}{dt} = -\sin\theta\frac{d\theta}{dt}\hat{i} + \cos\theta\frac{d\theta}{dt}\hat{j}$$

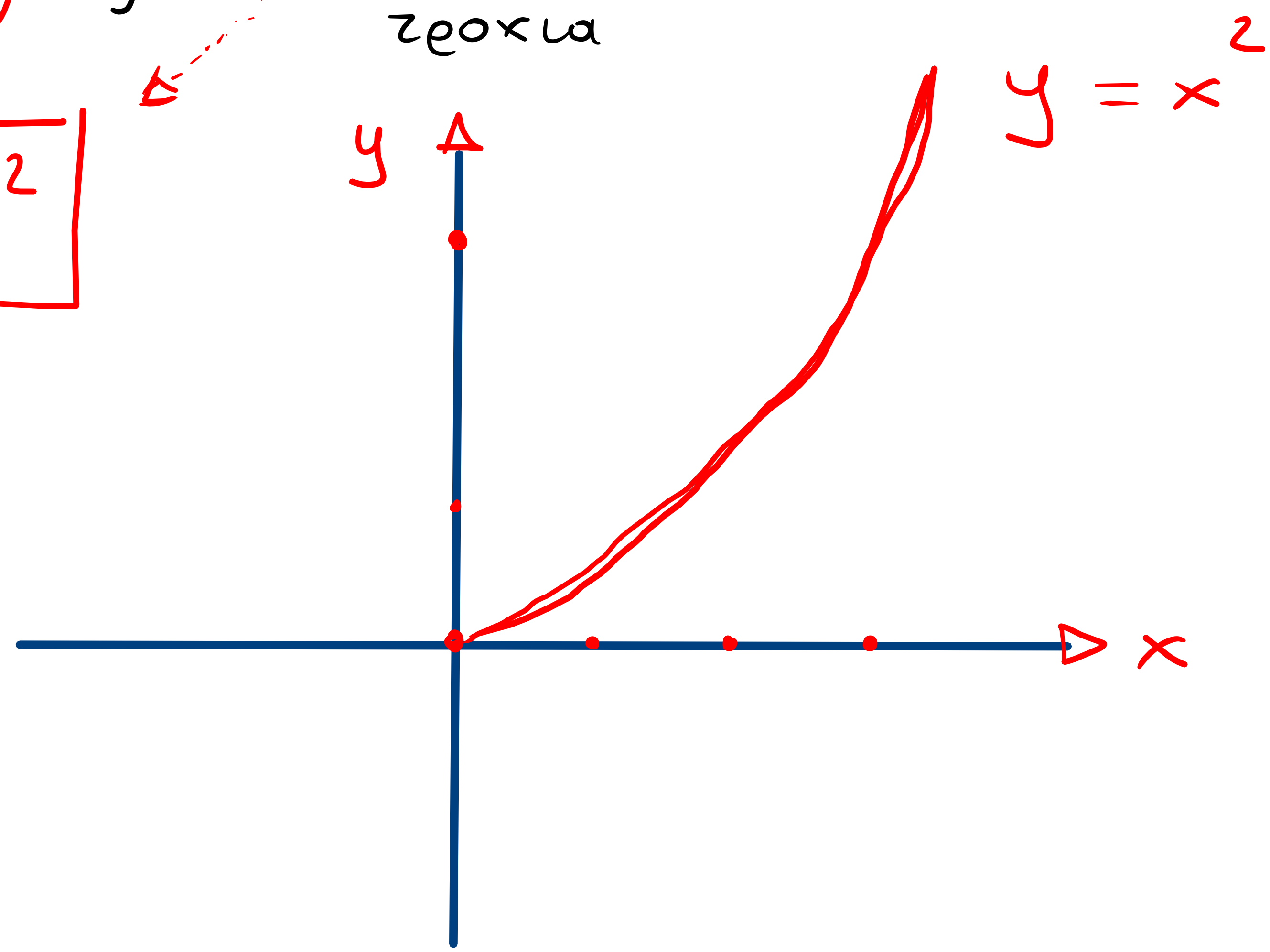
$$\frac{d\hat{u}_r}{dt} = \frac{d\theta}{dt}\cdot\hat{u}_\theta$$

– calculus –

Τροχιά

$$\left\{ \begin{array}{l} x(t) \\ y(t) \\ z(t) \end{array} \right\} \rightsquigarrow \underbrace{f(x,y,z)}_{\text{τροχιά}}$$

$$\left\{ \begin{array}{l} x = t \\ y = t^2 \end{array} \right\} \Rightarrow \boxed{y = x^2}$$



$y = f(x) \leftarrow$

$z = g(x) \leftarrow$

$$\begin{cases} x = t^2 \\ y = t^4 + 5 \end{cases} \qquad \boxed{y = x^2 + 5} \qquad y = y(x)$$

$$\begin{cases} x(t) = 2t^2 + 3 \\ y(t) = 3t^2 + 1 \end{cases} \longrightarrow \begin{aligned} 3x &= \cancel{6t^2} + 9 \\ 2y &= \cancel{6t^2} + 2 \end{aligned}$$

$$2y - 3x = -7 \Rightarrow \boxed{y = \frac{3}{2}x - \frac{7}{2}}$$

$$\begin{cases} V_x = 4t^3 + 4t \\ V_y = 4t \end{cases}$$

$$\vec{v} = (4t^3 + 4t)\hat{\imath} + 4t\,\hat{\jmath}$$

$t = 0 \longrightarrow (x_0, y_0) = (1, 2)$

$$V_x = 4t^3 + 4t \Rightarrow \int \frac{dx}{dt} = \int 4t^3 + 4t \Rightarrow x = t^4 + 2t^2 + \cancel{C_x}^{\,1}$$

$$1 = 0 + 0 + C_x$$

$$\boxed{x = t^4 + 2t^2 + 1} = (t^2+1)^2$$

$$V_y = 4t \Rightarrow \frac{dy}{dt} = 4t \Rightarrow y = 2t^2 + C_y \qquad \boxed{y = 2t^2 + 2}$$

$$2 = 0 + C_y$$

$$4x = 4t^4 + 8t^2 + 4$$

$$y^2 = 4t^4 + 8t^2 + 4$$

$$4x - y^2 = 0 \Rightarrow 4x = y^2$$

$$\Rightarrow x = \frac{y^2}{4} \Rightarrow y = \pm 2\sqrt{x}$$

ΠΑΓΙΔΑ !!!

$t_0 = 0 \longrightarrow (x_0, y_0) = (0,0)$

$\vec{v}(x,y) = 2\hat{i} + 4x\hat{j}$

$V_x = 2 \Rightarrow \frac{dx}{dt} = 2 \Rightarrow \boxed{x = 2t}$ (1)

$V_y = 4x \Rightarrow \frac{dy}{dt} = 4x \Rightarrow \cancel{y = 2x^2}$ ΛΑΘΟΣ!

(1), (2) $\rightarrow \boxed{y = x^2}$

Εναλλακτικά ?

$V_x = 2y$

$V_y = 4t$

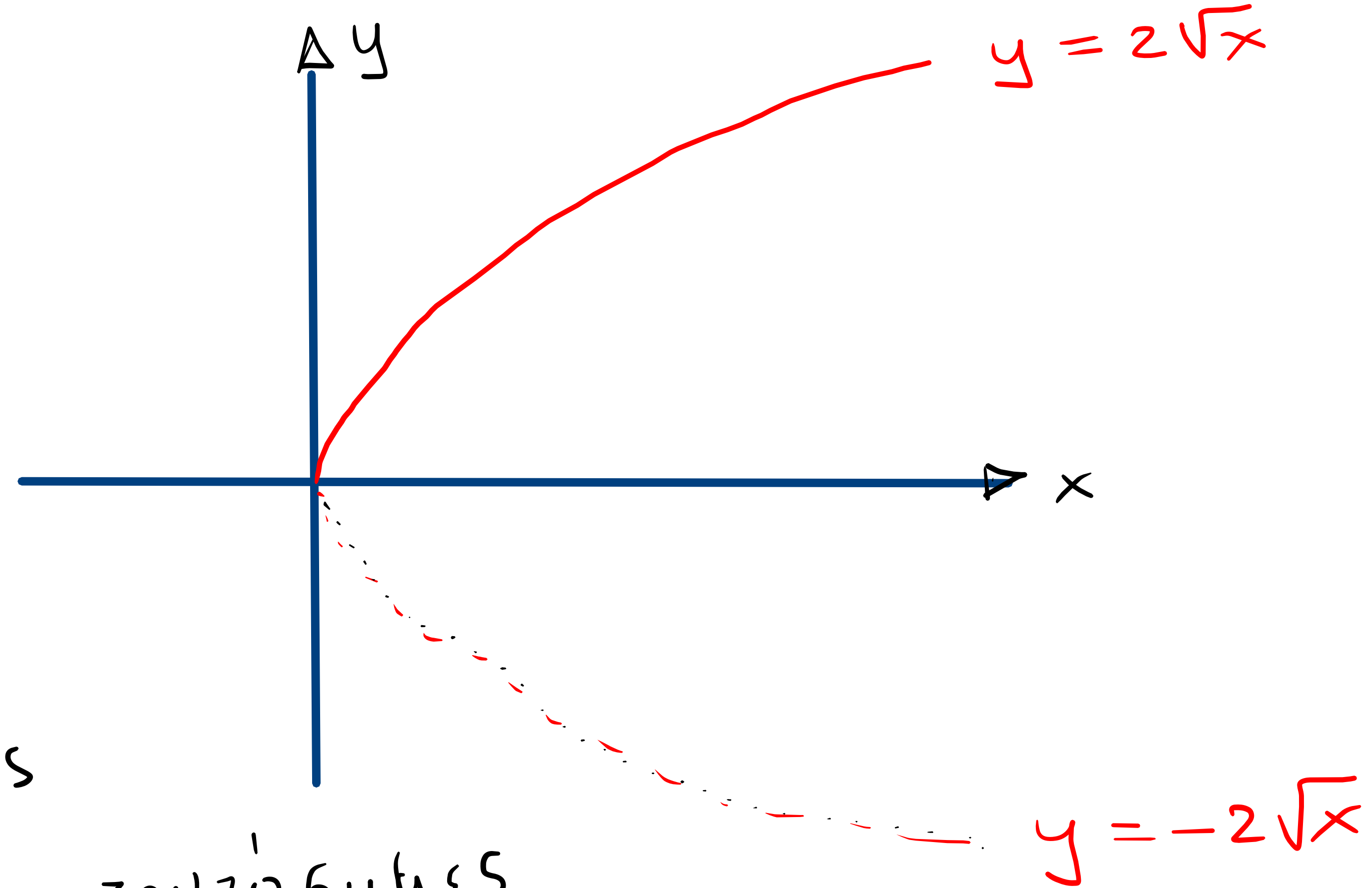
$y(t)$

$\frac{dy}{dt} = 4x \Rightarrow \frac{dy}{dt} = 4 \cdot (2t) \Rightarrow \int \frac{dy}{\cancel{dt}} = \int 8t\,dt$

$\Rightarrow \boxed{y = 4t^2}$ (2)

$$y = \pm 2\sqrt{x}$$

$$x = \frac{y^2}{4}$$



Οι δύο κινηματικές λύσεις εμπεριέχουν ταυτόσημες φυσικές έννοιες

(π.χ. για δοσμένο x η ακτίνα καμπυλότητας είναι η ίδια)

$r = r_0 e^{\lambda t}$

$\frac{d\theta}{dt} = \omega$

$x(t) = r \cdot \cos\theta = r_0 \cdot e^{\lambda t} \cdot \cos(\omega t)$

$y(t) = r\sin\theta = r_0 e^{\lambda t} \sin(\omega t)$

$$V_x = \frac{dx}{dt} = \lambda r_0 e^{\lambda t} \cos(\omega t) - r_0 e^{\lambda t} \cdot \omega \sin(\omega t)$$

$V_y = \ldots$

$$V = ? = \sqrt{V_x^2 + V_y^2} = \ldots$$