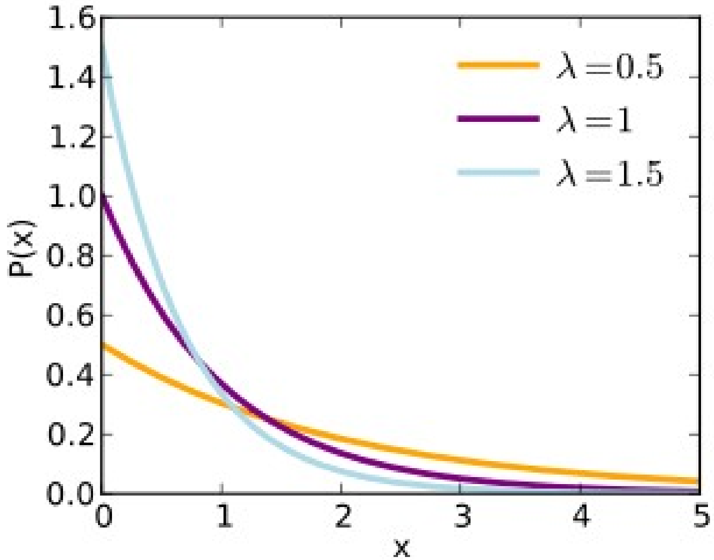
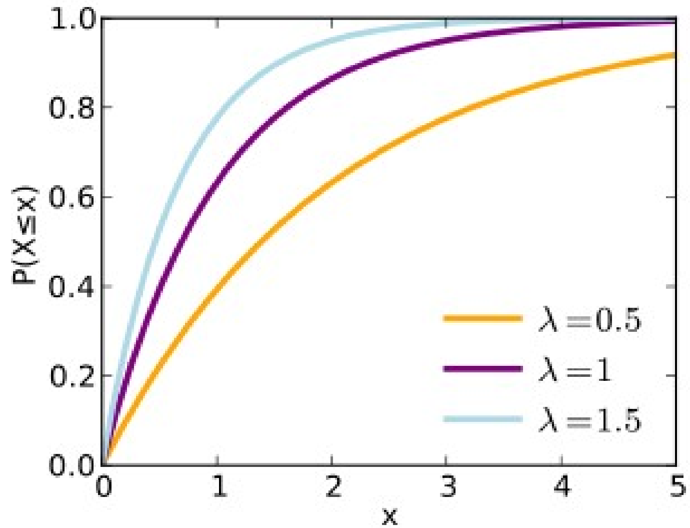
Κατανομή Gibbs

$$dN(E, E+dE) = Np(E)\,dE \qquad p(E) = \frac{\exp\left(-\frac{E}{k_B T}\right)}{k_B T}$$

$$p(x) = \lambda exp(-\lambda x), \lambda = \frac{1}{k_B T}, \; P(x \leqslant x_0) = \int_0^{x_0} p(x)\,dx = 1 - \exp(-\lambda x)$$





Κατανομή Maxwell-Boltzmann η κατανομή των ταχυτήτων των μορίων στα αέρια

$$\vec{v}=(v_x, v_y, v_z)$$

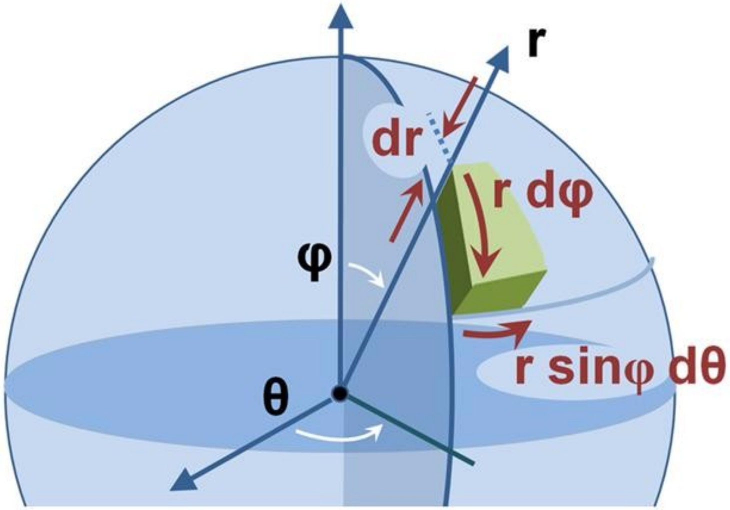
$$dN(\vec{v}, \vec{v}+d\vec{v})=Np(\vec{v})\,d\vec{v}=Np(v_x, v_y, v_z)\,dv_x\,dv_y\,dv_z$$

$$p(v_x, v_y, v_z)\propto\exp\left[-\frac{m}{2}\left(\frac{v_x^2+v_y^2+v_z^2}{k_B T}\right)\right]$$

Πρέπει να την κανονικοποιήσουμε ώστε το ολοκλήρωμά της να είναι 1.

Ολοκλήρωμα Euler-Poisson:

$$\int_{-\infty}^{\infty}\exp(-x^2)\,dx=\sqrt{\pi}$$



$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$dv_x\, dv_y\, dv_z = 4\pi\, v^2\, dv$$

$$\frac{dN(v, v + d\,v)}{N} = p_{MB}(v)\, dv = \iint_{\sigma\varphi\alpha\iota\rho\iota\kappa\acute{o}\varsigma\ \varphi\lambda o\iota\acute{o}\varsigma\ \pi\acute{\alpha}\chi o\upsilon\varsigma\ dv} p(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z$$

$$p_{MB}(v) \propto v^2 \exp\left(-\frac{m v^2}{2 k_B T}\right)$$

$$I(\lambda) = \int_{-\infty}^{\infty} \exp(-\lambda x^2)\, dx = \sqrt{\frac{\pi}{\lambda}}$$

$$-\frac{dI(\lambda)}{d\lambda} = \int_{-\infty}^{\infty} x^2 \exp(-\lambda x^2)\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}$$

$$\int_{-\infty}^{\infty} x^2 \exp(-\lambda x^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}} = 2\int_0^{\infty} x^2 \exp(-\lambda x^2)\,dx$$

$$4\sqrt{\frac{\lambda^3}{\pi}}\int_0^{\infty} x^2 \exp(-\lambda x^2)\,dx = 1$$

$$p_{MB}(v) \propto v^2 \exp\left(-\frac{m v^2}{2 k_B T}\right)$$ άρα $$\lambda = \frac{m}{2 k_B T}$$

$$p_{MB}(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{m v^2}{2 k_B T}\right)$$

Έτσι ώστε

$$\int_0^{\infty} p_{MB}(v)\,dv = 1$$

$$\int_{-\infty}^{\infty} x^2 \exp(-\lambda x^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}} = 2\int_0^{\infty} x^2 \exp(-\lambda x^2)\,dx$$

$$4\sqrt{\frac{\lambda^3}{\pi}}\int_0^{\infty} x^2 \exp(-\lambda x^2)\,dx = 1$$

$$p_{MB}(v) \propto v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) \quad \text{άρα} \quad \lambda = \frac{m}{2k_B T}$$

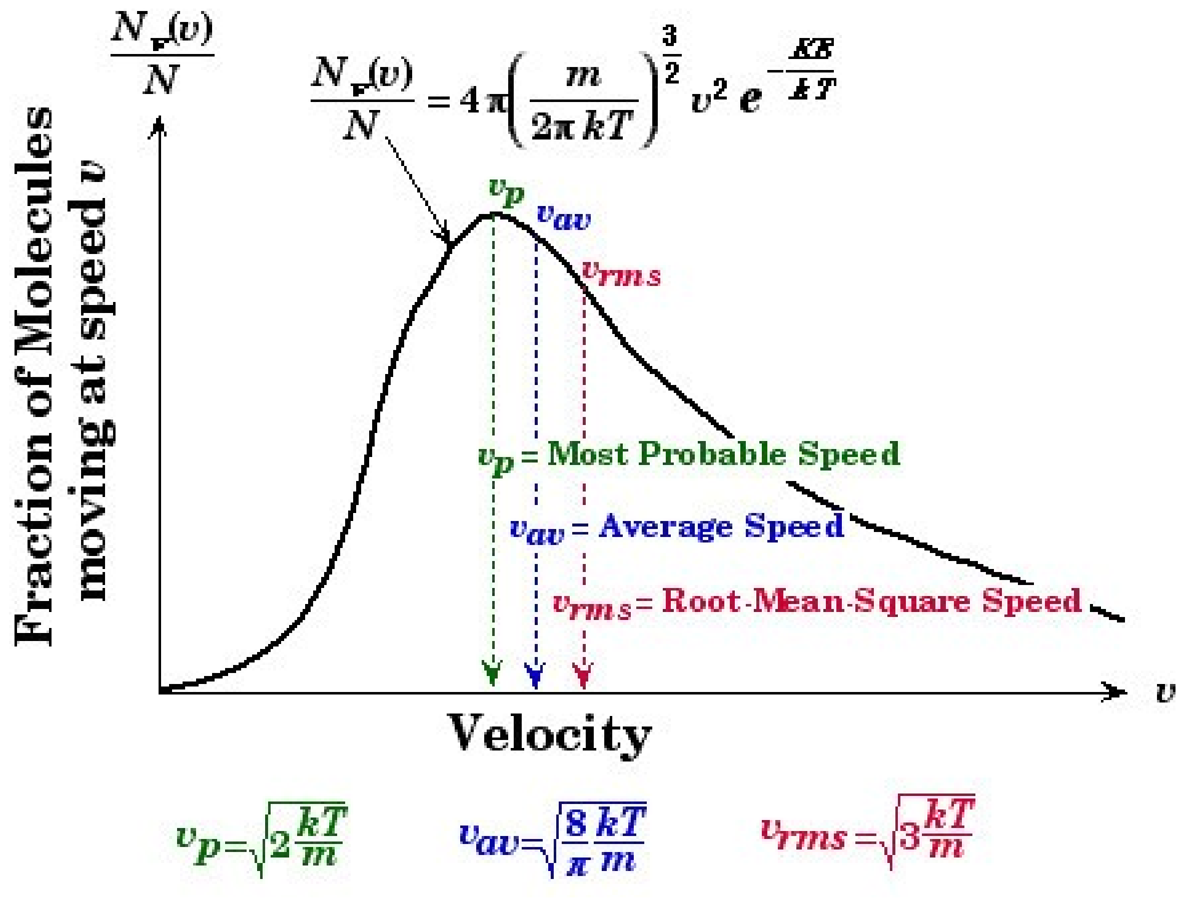
$$p_{MB}(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)$$

Έτσι ώστε

$$\int_0^{\infty} p_{MB}(v)\,dv = 1$$

Κατανομή Maxwell-Boltzmann η κατανομή των ταχυτήτων των μορίων στα αέρια

$$p_{MB}(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)$$



$$I_1(\lambda)=\int_0^\infty x^2\exp\left(-\lambda x^2\right)dx=\frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}},$$

$$-\frac{dI_1(\lambda)}{d\lambda}=\int_0^\infty x^4\exp\left(-\lambda x^2\right)dx=\frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}},$$

$$4\sqrt{\frac{\lambda^3}{\pi}}\int_0^\infty x^4\exp\left(-\lambda x^2\right)dx=\frac{3}{2\lambda},\quad \lambda=\frac{m}{2k_BT}$$

$$\langle v^2\rangle=\int_0^\infty v^2\,p_{MB}(v)\,dv=\frac{3}{2\lambda},\quad \langle v^2\rangle=\frac{3k_BT}{m}$$

$$v_{rms}=\sqrt{\langle v^2\rangle}=\sqrt{\frac{3k_BT}{m}},\quad \langle K\rangle=\left\langle\frac{mv^2}{2}\right\rangle=\frac{3}{2}k_BT$$

$$\langle v\rangle = 4\sqrt{\frac{\lambda^3}{\pi}}\int_0^\infty v^3 \exp(-\lambda v^2)\,dv = 2\sqrt{\frac{\lambda}{\pi}}\int_0^\infty v^2 \exp(-\lambda v^2)\,d(\lambda v^2)$$

$$y=\lambda v^2\,,\;\; \langle v\rangle = \frac{2}{\sqrt{\lambda\pi}}\int_0^\infty y \exp(-y)\,dy$$

$$I_2(\mu) = \int_0^\infty \exp(-\mu y)\,dy = \frac{1}{\mu}\,,\;\; -\frac{d\, I_2(\mu)}{d\mu} = \int_0^\infty y \exp(-\mu y)\,dy = \frac{1}{\mu^2}$$

$$\langle v\rangle = \frac{2}{\sqrt{\lambda\pi}} = \sqrt{\frac{8k_B T}{\pi m}}$$

$$\sigma\tau\eta\nu\;\pi\iota\acute{o}\;\pi\iota\theta\alpha\nu\acute{\eta}\;\tau\alpha\chi\acute{\upsilon}\tau\eta\tau\alpha\; v_p\,,\quad \frac{dp_{MB}(v)}{dv}\Big|_{v=v_p} = 0$$

$$p_{MB}(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) \quad \acute{\alpha}\rho\alpha$$

$$\left[2v_p \exp\left(-\frac{mv^2}{2k_B T}\right) - \frac{mv_p^3}{k_B T}\exp\left(-\frac{mv^2}{2k_B T}\right)\right] = 0,\quad v_p = \sqrt{\frac{2k_B T}{m}}$$

$$p_{MB}(v)=4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{m v^2}{2 k_B T}\right)$$

$$\langle K\rangle=\langle\frac{mv^2}{2}\rangle=\frac{3}{2}k_B T\ ,\ v_{rms}=\sqrt{\langle v^2\rangle}=\sqrt{\frac{3k_B T}{m}}\ ,\ \langle v\rangle=\sqrt{\frac{8k_B T}{\pi m}}\ ,\ v_p=\sqrt{\frac{2k_B T}{m}}$$

$$k_B=1.3806488(13)\times 10^{-23}\,J/K$$

$$N_A=6.02214129(27)\times 10^{23}/mol$$

$$R=N_A k_B=8.314\,4621(75)\,J/(K.mol)$$

$$Loschmidt\ constant\,(273.15\,K\ ,101.325\,kPa)(K.\Sigma.)$$

$$L=2.686\,7805(24)\times 10^{25}\,m^{-3}$$

$$V_{mol}\approx 22.4\times 10^{-3}\,m^3$$