# Εξισώσεις Friedmann

$$\boxed{H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2}} \rightarrow \boxed{\Omega_{tot} - 1 = \sum_i \Omega_i - 1 = \frac{k}{H^2 a^2}} \quad \left(\Omega_i = \frac{\rho_i}{\rho_c}\right)$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad (k = 0)$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)} \rightarrow \boxed{q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} = \frac{1}{2}\Omega_{tot} + \frac{3}{2}\sum_i w_i \Omega_i}$$

$$\rho \rightarrow \rho + \frac{\Lambda}{8\pi G}$$

$$p \rightarrow p - \frac{\Lambda}{8\pi G}$$

$$q = \frac{1}{2}\sum_i \Omega_i (1 + 3w_i)$$

$$q = \frac{1}{2}\Omega_M - \Omega_\Lambda + \Omega_R$$

$$\Omega_{tot} = \Omega_M + \Omega_\Lambda + \Omega_R$$

Scale factor - Redshift $\rightarrow$ $$a(z) = \frac{1}{z+1}$$

$$E(z) = \frac{H(z)}{H_0}$$

$$E^2(z) = \sum_i \Omega_i(z) - \frac{k}{H_0^2 a_0^2}(1 + z^2) = \sum_i \Omega_i(z) + \Omega_k(z)$$

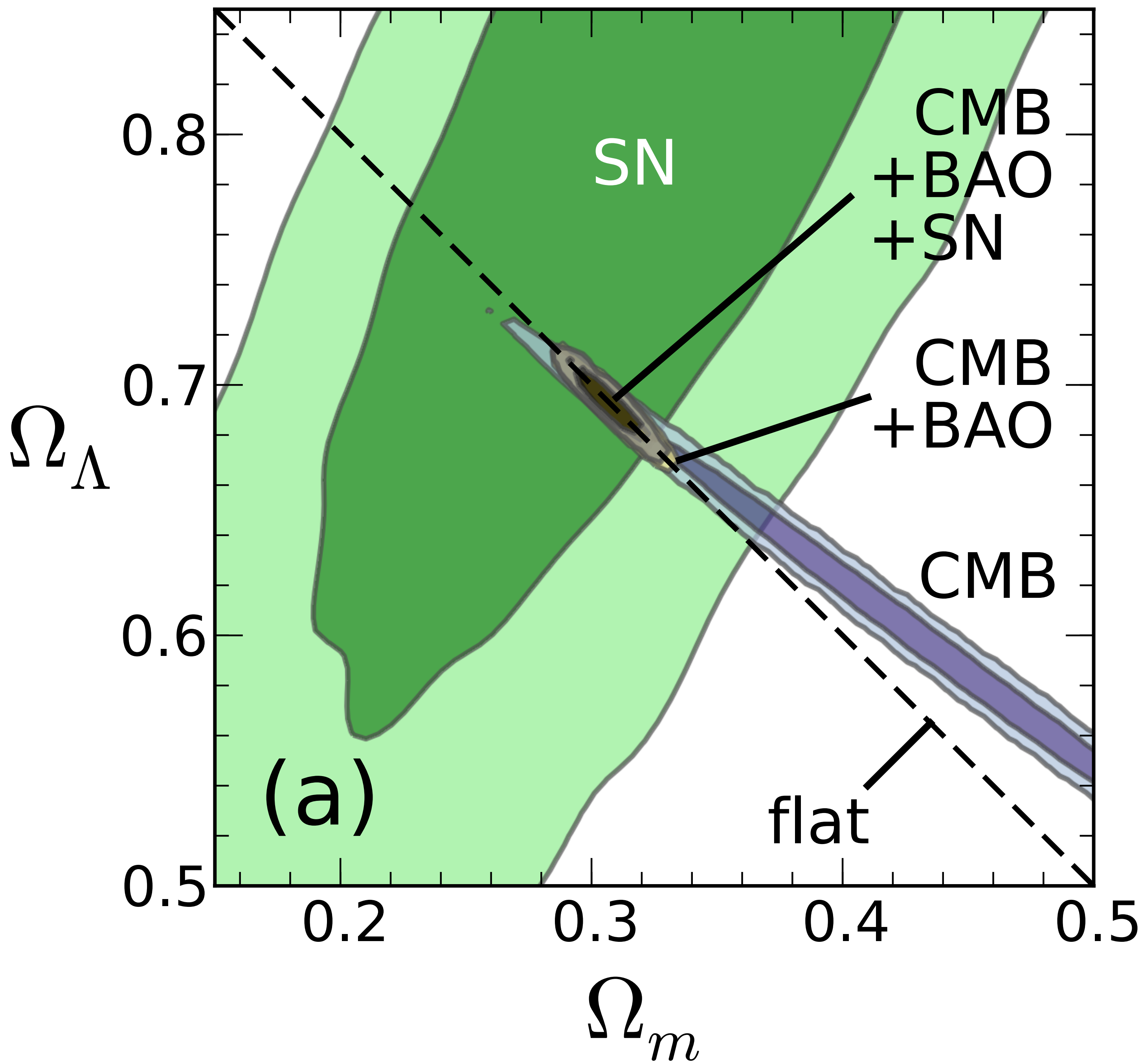
$$\boxed{E^2(z) = \sum_i \Omega_i(z) + (1 - \Omega_{tot}^0)(1 + z)^2}$$

$$p = w\rho \longrightarrow \rho \sim a^{-3(1+w)}$$

- **Matter :** $w = 0\,,\ \rho_M \sim a^{-3}$ $\quad \Omega_M(z) = \Omega_M^0(1+z)^3$
- **Radiation :** $w = \frac{1}{3}\,,\ \rho_R \sim a^{-4}$ $\quad \Omega_R(z) = \Omega_R^0(1+z)^4$
- **Dark Energy :** $w = -1\,,\ \rho_\Lambda = const.$ $\quad \Omega_\Lambda(z) = \Omega_\Lambda^0$

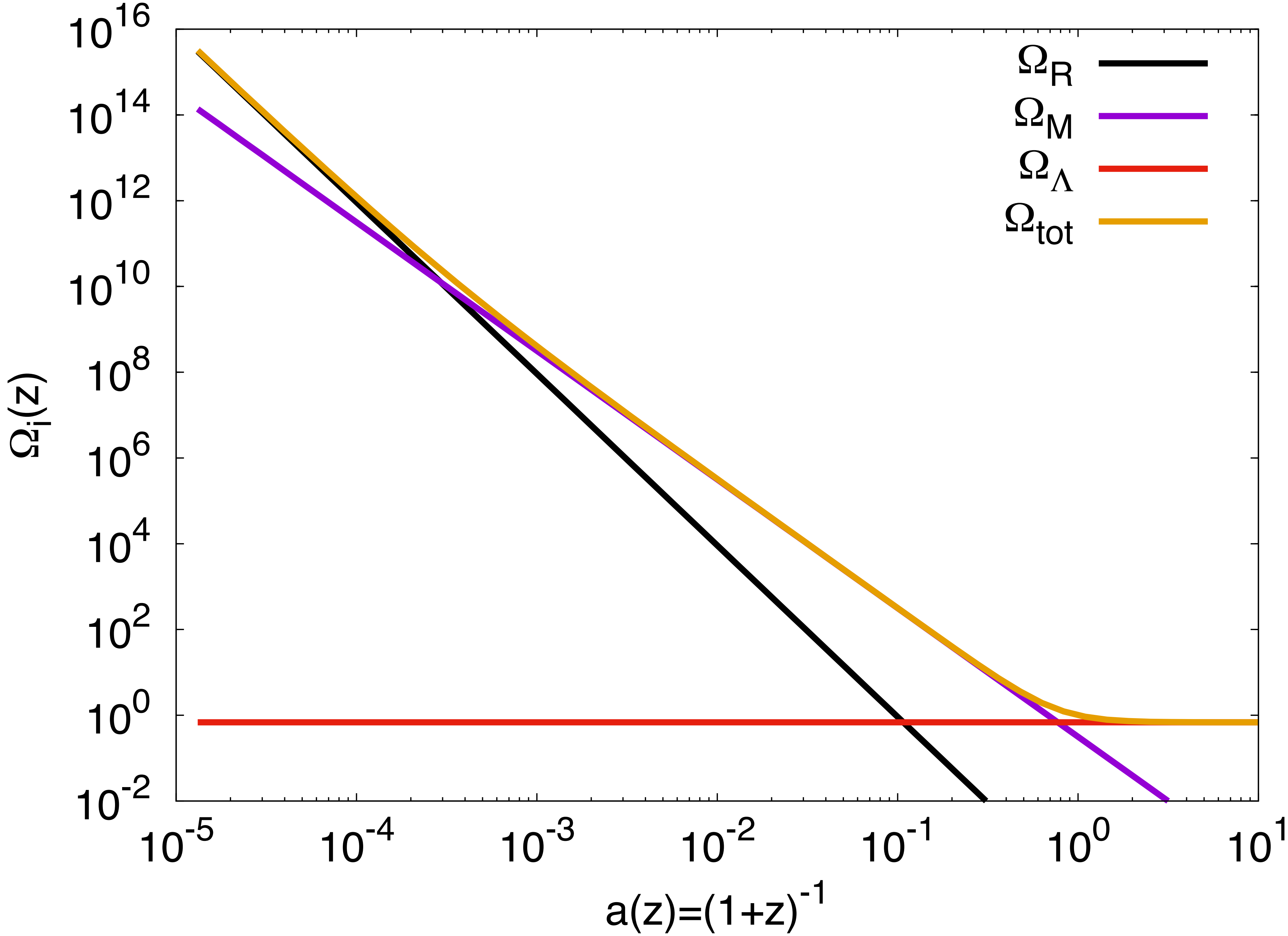
$$\boxed{w\,,\ \rho_w \sim a^{-3(1+w)} \qquad \Omega_w(z) = \Omega_w^0\,(z+1)^{3(1+w)}}$$

**ΛCDM** → $\Omega_\Lambda + \Omega_M^0 = 1 \quad (k=0)$




Πειραματικά δεδομένα → $\Omega_\Lambda = 0.685^{+0.017}_{-0.016} \qquad \Omega_M^0 = 0.315^{+0.017}_{-0.016}$

$$\Omega_\Lambda + \Omega_M^0 = 1 \quad (k=0) \quad \Omega_R^0 \simeq 9.17(19) \times 10^{-5}$$



# Ηλικία του Σύμπαντος

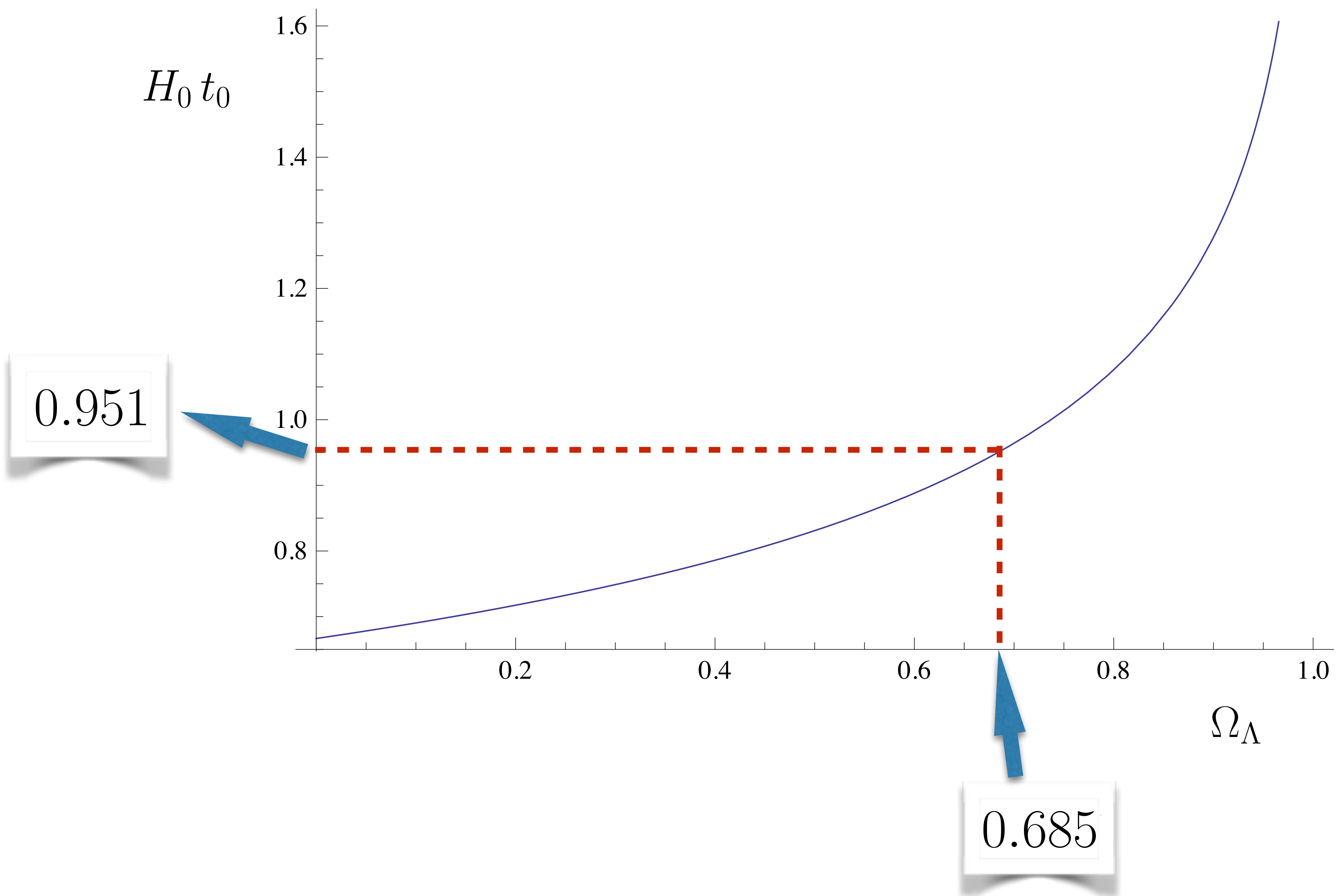
$$E^2(z) = \Omega_M^0(1+z)^3+\Omega_R^0(1+z)^4+\Omega_\Lambda+(1-\Omega_M^0-\Omega_R^0-\Omega_\Lambda)(1+z)^2$$

$$E^2(z) = (1+z^2)(1+\Omega_M\, z) - \Omega_\Lambda(z+2)z$$

$$\boxed{t_0 = \frac{1}{H_0}\int_0^\infty \frac{dz}{(1+z)E(z)}}$$

$$\Omega_\Lambda + \Omega_M^0 = 1$$

$$\boxed{H_0\, t_0 = \frac{2}{3\sqrt{\Omega_\Lambda}} \ln \frac{1+\sqrt{\Omega_\Lambda}}{\sqrt{1-\Omega_\Lambda}}}$$



$$\Omega_\Lambda = 0.685^{+0.017}_{-0.016} \qquad \Omega_M^0 = 0.315^{+0.017}_{-0.016}$$

$$\frac{1}{H_0} = \frac{1}{h_0}\, 9.78\, Gyr = 14.532\, Gyr \qquad h_0 = 0.673 \pm 0.012$$

$$\Omega_\Lambda = 0.685 \longrightarrow H_0\, t_0 = \int_0^\infty \ldots = 0.951$$

$$\boxed{t_0 = 14.532\, Gyr \times \int_0^\infty \ldots = 13.81 \pm 0.05\, Gyr}$$

# Χρονική εξάρτηση

$$H^2 = \frac{\dot{a}}{a} \sim \rho \sim a^{-3(1+w)}$$

$$a(t) \sim R(t) \sim t^{\frac{2}{3(1+w)}} \longrightarrow H \sim \frac{2}{3(1+w)t}$$

- Matter : $a(t) \sim t^{2/3}$
- Radiation : $a(t) \sim \sqrt{t}$
- Dark Energy : $a(t) \sim e^{\sqrt{\frac{\Lambda}{3}}t} \sim e^{Ht}$

# Χρονολόγιση με U - Pb

Αλυσίδα $^{238}U$

$^{238}U$ σε $^{206}Pb$ τ=4.47 Gyr

Αλυσίδα $^{235}U$

$^{235}U$ σε $^{207}Pb$ τ=704 Myr

$$\left(\frac{^{207}Pb}{^{204}Pb}\right)_P = \left(\frac{^{207}Pb}{^{204}Pb}\right)_I + \left(\frac{^{235}U}{^{204}Pb}\right)_P \left(e^{\lambda_{235} t} - 1\right)$$

$$\left(\frac{^{206}Pb}{^{204}Pb}\right)_P = \left(\frac{^{206}Pb}{^{204}Pb}\right)_I + \left(\frac{^{238}U}{^{204}Pb}\right)_P \left(e^{\lambda_{238} t} - 1\right)$$

$$\left[\frac{\left(\frac{^{207}Pb}{^{204}Pb}\right)_P - \left(\frac{^{207}Pb}{^{204}Pb}\right)_I}{\left(\frac{^{206}Pb}{^{204}Pb}\right)_P - \left(\frac{^{206}Pb}{^{204}Pb}\right)_I}\right] = \left(\frac{1}{137.88}\right)\left(\frac{e^{\lambda_{235} t} - 1}{e^{\lambda_{238} t} - 1}\right)$$

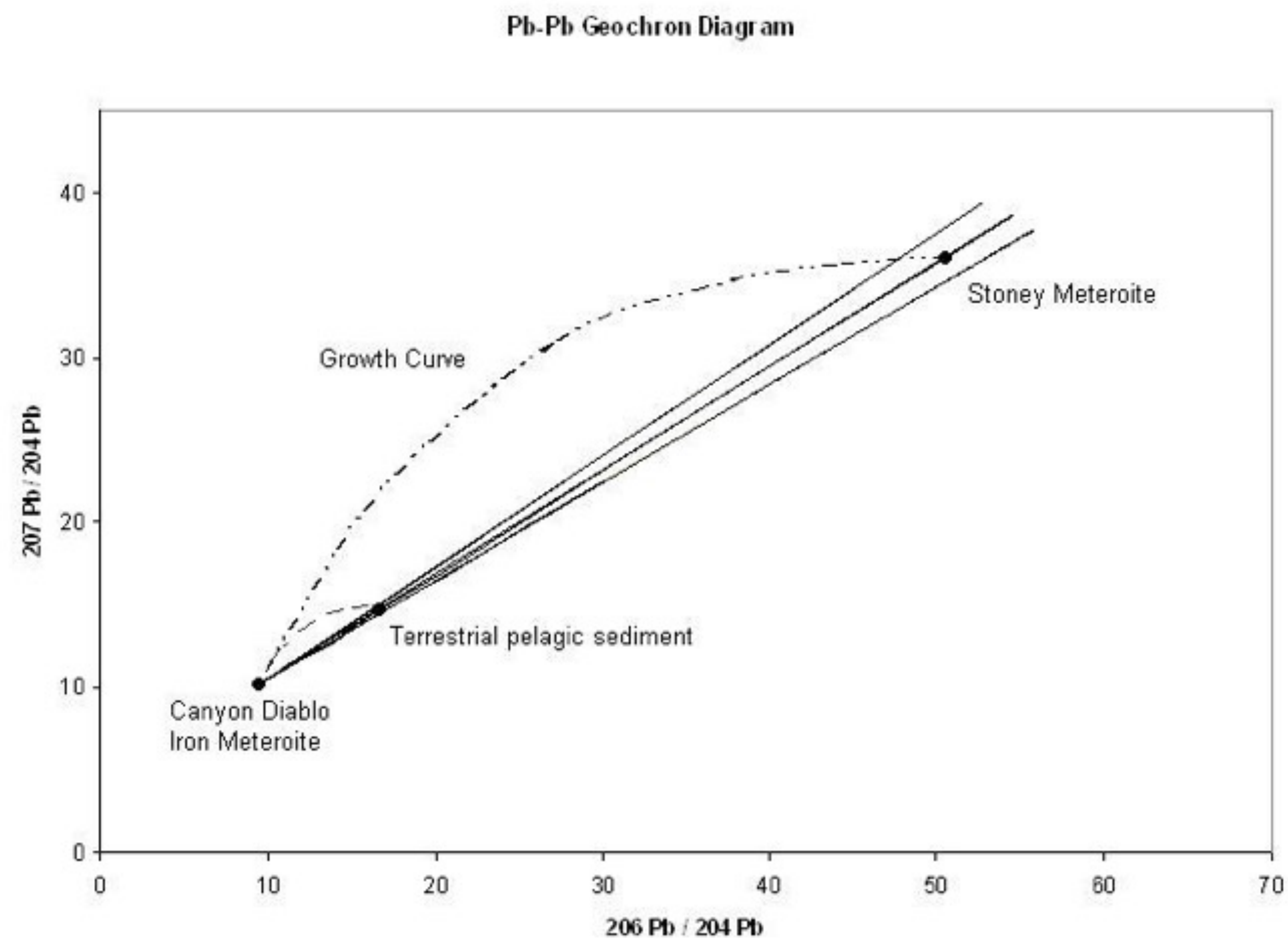
Clair C. Patterson, 1956

Ηλικία Γης = 4.55  Gyr (±70 Myr)

**Τωρινή εκτίμηση: 4.54 ± 0.05 Gyr**

Meteor Crater, Arizona

Canyon-diablo


Pb-Pb Geochron Diagram
Growth Curve
Stoney Meteroite
Terrestrial pelagic sediment
Canyon Diablo
Iron Meteroite
207 Pb / 204 Pb
206 Pb / 204 Pb
0
10
20
30
40
50
60
70