Οι εκφωνήσεις των ασκήσεων που λύσαμε/συζητήσαμε στις διαδικτυακές συναντήσεις. Παρακάτω θα βρείτε ιδιατέρως συνοπτικές απαντήσεις. Προσοχή! Οι απαντήσεις δίνονται μόνο ως επιπλέον βοήθεια, προφανώς μία πλήρη απάντηση απαιτεί τις κατάλληλες επεξηγήσεις/ πράξεις.

9.1 Evaluate the energy of the blackbody photons inside your eye. Compare this with the visible energy inside your eye while looking at a 100-W light bulb that is 1 m away. You can assume that the light bulb is 100% efficient, although in reality it converts only a few percent of its 100 watts into visible photons. Take your eye to be a hollow sphere of radius 1.5 cm at a temperature of 37°C. The area of the eye's pupil is about 0.1 cm^2. Why is it dark when you close your eyes?

9.8 In Example 9.2.3, suppose that only two measurements of the specific intensity, I_1 and I_2, are available, made at angles θ_1 and θ_2. Determine expressions for the intensity $I_{\lambda,0}$ of the light above Earth's atmosphere and for the vertical optical depth of the atmosphere, $\tau_{\lambda,0}$, in terms of these two measurements.

9.12 If the temperature of a star's atmosphere is *increasing* outward, what type of spectral lines would you expect to find in the star's spectrum at those wavelengths where the opacity is greatest?

9.19 Using the results for the plane-parallel gray atmosphere in LTE, determine the ratio of the effective temperature of a star to its temperature at the top of the atmosphere. If $T_e = 5777$ K, what is the temperature at the top of the atmosphere?

9.21 Consider a horizontal plane-parallel slab of gas of thickness L that is maintained at a constant temperature T. Assume that the gas has optical depth $\tau_{\lambda,0}$, with $\tau_\lambda = 0$ at the top surface of the slab. Assume further that no radiation enters the gas from outside. Use the general solution of the transfer equation (9.54) to show that when looking at the slab from above, you see blackbody radiation if $\tau_{\lambda,0} \gg 1$ and emission lines (where j_λ is large) if $\tau_{\lambda,0} \ll 1$. You may assume that the source function, S_λ, does not vary with position inside the gas. You may also assume thermodynamic equilibrium when $\tau_{\lambda,0} \gg 1$.

9.26 The two solar absorption lines given in Table 9.3 are produced when an electron makes an upward transition from the ground state orbital of the neutral Na I atom.

(a) Using the general curve of growth for the Sun, Fig. 9.22, repeat the procedure of Example 9.5.5 to find N_a, the number of absorbing sodium atoms per unit area of the photosphere.

(b) Combine your results with those of Example 9.5.5 to find an average value of N_a. Use this value to plot the positions of the four sodium absorption lines on Fig. 9.22, and confirm that they do all lie on the curve of growth.

10.9 Prove that the energy corresponding to the Gamow peak is given by Eq. (10.34).

10.10 Calculate the ratio of the energy generation rate for the pp chain to the energy generation rate for the CNO cycle given conditions characteristic of the center of the present-day (evolved) Sun, namely $T = 1.5696 \times 10^7$ K, $\rho = 1.527 \times 10^5 \text{ kg m}^{-3}$, $X = 0.3397$, and $X_{\text{CNO}} = 0.0141$.[31] Assume that the pp chain screening factor is unity ($f_{pp} = 1$) and that the pp chain branching factor is unity ($\psi_{pp} = 1$).

10.11 Beginning with Eq. (10.62) and writing the energy generation rate in the form

$$\epsilon(T) = \epsilon'' T_8^{\alpha},$$

show that the temperature dependence for the triple alpha process, given by Eq. (10.63), is correct. ϵ'' is a function that is independent of temperature.
Hint: First take the natural logarithm of both sides of Eq. (10.62) and then differentiate with respect to $\ln T_8$. Follow the same procedure with your power law form of the equation and compare the results. You may want to make use of the relation

$$\frac{d \ln \epsilon}{d \ln T_8} = \frac{d \ln \epsilon}{\frac{1}{T_8} d T_8} = T_8 \frac{d \ln \epsilon}{d T_8}.$$

10.12 The Q value of a reaction is the amount of energy released (or absorbed) during the reaction. Calculate the Q value for each step of the PP I reaction chain (Eqs. 10.37–10.39). Express your answers in MeV. The masses of ${}^{2}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$ are 2.0141 u and 3.0160 u, respectively.

10.13 Calculate the amount of energy released or absorbed in the following reactions (express your answers in MeV):
(a) ${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma$
(b) ${}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + 2\,{}^{4}_{2}\mathrm{He}$
(c) ${}^{19}_{9}\mathrm{F} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He}$
The mass of ${}^{12}_{6}\mathrm{C}$ is 12.0000 u, by definition, and the masses of ${}^{16}_{8}\mathrm{O}$, ${}^{19}_{9}\mathrm{F}$, and ${}^{24}_{12}\mathrm{Mg}$ are 15.99491 u, 18.99840 u, and 23.98504 u, respectively. Are these reactions exothermic or endothermic?

10.14 Complete the following reaction sequences. Be sure to include any necessary leptons.
(a) ${}^{27}_{14}\mathrm{Si} \rightarrow {}^{?}_{13}\mathrm{Al} + e^{+} + \underline{?}$
(b) ${}^{?}_{13}\mathrm{Al} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{24}_{12}\mathrm{Mg} + {}^{4}_{?}\underline{?}$
(c) ${}^{35}_{17}\mathrm{Cl} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{36}_{18}\mathrm{Ar} + \underline{?}$

10.20 **(a)** On the same graph, plot the density structure of stars of polytropic indices $n = 0$, $n = 1$, and $n = 5$. *Hint:* You will want to plot ρ_n/ρ_c vs. r/λ_n.
(b) What can you conclude about the concentration of density with radius for increasing polytropic index?
(c) From the trend that you observe for the analytic solutions to the Lane–Emden equation, what would you expect regarding the density concentration of an adiabatically convective stellar model compared to a model in radiative equilibrium?

(d) Explain your conclusion in part (c) in terms of the physical processes of convection and radiation.

10.21 Estimate the hydrogen-burning lifetimes of stars near the lower and upper ends of the main sequence. The lower end of the main sequence[32] occurs near 0.072 $M_\odot$, with $\log_{10} T_e = 3.23$ and $\log_{10}(L/L_\odot) = -4.3$. On the other hand, an 85 $M_\odot$ star[33] near the upper end of the main sequence has an effective temperature and luminosity of $\log_{10} T_e = 4.705$ and $\log_{10}(L/L_\odot) = 6.006$, respectively. Assume that the 0.072 $M_\odot$ star is entirely convective so that, through convective mixing, all of its hydrogen, rather than just the inner 10%, becomes available for burning.

10.22 Using the information given in Problem 10.21, calculate the radii of a 0.072 $M_\odot$ star and a 85 $M_\odot$ star. What is the ratio of their radii?

10.23 **(a)** Estimate the Eddington luminosity of a 0.072 $M_\odot$ star and compare your answer to the main-sequence luminosity given in Problem 10.21. Assume $\overline{\kappa} = 0.001\ \text{m}^2\ \text{kg}^{-1}$. Is radiation pressure likely to be significant in the stability of a low-mass main-sequence star?

(b) If a 120 $M_\odot$ star forms with $\log_{10} T_e = 4.727$ and $\log_{10}(L/L_\odot) = 6.252$, estimate its Eddington luminosity, assuming the opacity is due to electron scattering. Compare your answer with the actual luminosity of the star.

12.11 Show that the Jeans mass (Eq. 12.14) can also be written in the form

$$M_J = \frac{c_J v_T^4}{P_0^{1/2} G^{3/2}} \tag{12.33}$$

where the isothermal sound speed, v_T, is given by Eq. (12.18), P_0 is the pressure associated with the density ρ_0 and temperature T, and $c_J \simeq 5.46$ is a dimensionless constant.

12.12 By invoking the requirements of hydrostatic equilibrium, explain why the assumption of a constant gas pressure P_0 in Eq. (12.33) cannot be correct for a static cloud without magnetic fields. What does that imply about the assumptions of constant mass density in an isothermal molecular cloud having a constant composition throughout?