# The Implications of Experimental Violation of Bell's Inequalities for Local Realism

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Bell's Theorem highlights the incompatibility between predictions of quantum mechanics and local hidden variable theories (local realism). Through the formalism of Bell's Inequalities, which are satisfied by local realism but violated by quantum mechanics, the conflict between the two can be put to tests by experiments. The apparent violation of Bell's Inequalities in many independent experiments performed between 1970s and the present provided strong evidence in support of quantum mechanics as the description of the universe. Though there are persistent loopholes in these experiments which gave hope to supporters of local realism, the more recent experimental efforts are on their way to achieve fully loophole-free violation of Bell's Inequalities. Nevertheless, the experimental evidence to date does not preclude the possibility of a nonlocal hidden variable underlying quantum mechanical phenomena.

## I. INTRODUCTION

In 1935, Einstein, Podolsky, and Rosen[1] (hereafter referred to as EPR) put forth an argument against one of the postulates of the standard "Copenhagen interpretation" of quantum mechanics as advocated by Bohr and Heisenberg: that the wave function gives a *complete* description of reality. According to EPR's definition, a physical theory is "complete" if every element of the physical reality have a counterpart in the theory. On the other hand, EPR believed that only physical theory that are both *local* and *realistic* are worthy of consideration, where by "local" they meant there is no "spooky action at a distance", and by "realistic" they meant that measurable quantities have values independent of the act of measurement. It turned out that these two requirements, known as *local realism*, lead to an conclusion that quantum mechanics is ultimately incomplete. This ignited a debate between Einstein and Bohr for whether a local hidden variable is required to make quantum mechanics a complete theory of reality. It was not until Bell's involvement in 1964[2] that this debate became quantified in the form of Bell's Inequalities, which were then put to test by experimentalists.

In this paper, I will first present the derivation of Bell's Inequalities from local realistic assumptions, and demonstrate the their theoretical violation by two quantum mechanically entangled particles in the singlet state in Sec. II. I will then review efforts put forth since the 1970s to demonstrate experimental violation of Bell's Inequalities in Sec. III. Lastly, the precise implications for local realism due to the experimental evidence to date of violation of Bell's Inequalities is discussed in Sec. IV.

## II. LOCAL REALISM AND BELL'S THEOREM

### A. Bell's Inequalities

In 1964, J. S. Bell was able to re-formulate EPR's assumptions of a complete, local, and realistic physical theory into a class of simple inequalities known as Bell's Inequalities[2]. I will now derive these inequalities in a tradition similar to that in Ref. [3]

Consider two particles that have been prepared in a state that move in two distinguishable direction towards two different measuring devices at location 1 and 2. These devices measure some property of the particles with adjustable detector settings $\hat{a}$ and $\hat{b}$, respectively; for instance, the detector setting can be interpreted as the direction in which you measure the spin. Now suppose that there is the initial state of the two particles can be completely described in terms of a hypothetical hidden variable $\lambda$ which has a probability distribution $\rho(\lambda)$. And let $A$ and $B$ be the measurement outcome at location 1 and 2, so that $A, B \in \{\pm 1\}$. In the most general form,

$$A = A(\hat{a}, \hat{b}, \lambda),\ B = B(\hat{a}, \hat{b}, \lambda) \tag{1}$$

For the EPR notion of locality to be true, Bell postulated that the measurement outcome $A$ at location 1 should not depend on what happens at location 2, which includes the measurement setting $\hat{b}$. Similarly, measurement outcome $B$ should not depend on $\hat{a}$. So to preserve *locality*,

$$A = A(\hat{a}, \lambda),\ B = B(\hat{b}, \lambda) \tag{2}$$

More generally, the measuring devices could contain hidden variables themselves that could influence the outcomes. But this issue can be solved by averaging over these hidden variables, assuming that the two instruments are far enough that they don't interfere due to locality. The measurement outcomes will then be $\bar{A}(\hat{a}, \lambda) = \langle A(\hat{a}, \lambda)\rangle_{\text{inst}}$, $\bar{B} = \langle B(\hat{b}, \lambda)\rangle_{\text{inst}}$ averaged over the instrumental hidden variables that are independent of $\hat{b}$, $\hat{a}$ respectively. Note $\bar{A}, \bar{B} \in [-1, 1]$. We can think of $\bar{A}$ and $\bar{B}$ simply as generalized, continuous analogues of $A$ and $B$.

Now, the inter-particle correlation is measured by the expected value of the product $\bar{A}\bar{B}$, which is

$$E(\hat{a}, \hat{b}) = \langle \bar{A}(\hat{a}, \lambda)\bar{B}(\hat{b}, \lambda)\rangle = \int d\lambda \rho(\lambda)\bar{A}(\hat{a}, \lambda)\bar{B}(\hat{b}, \lambda) \tag{3}$$

Let $\hat{a}', \hat{b}'$ be alternative settings of the measurement devices. Then we have

$$\begin{aligned} &E(\hat{a},\hat{b}) - E(\hat{a},\hat{b}') \\ &= \int d\lambda \rho(\lambda)[\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b},\lambda) - \bar{A}(\hat{a},\lambda)\bar{B}(\hat{b}',\lambda)] \\ &= \int d\lambda \rho(\lambda)\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b},\lambda)[1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b}',\lambda)] \\ &- \int d\lambda \rho(\lambda)\bar{A}(\hat{a},\lambda)\bar{B}(\hat{b}',\lambda)[1 \pm \bar{A}(\hat{a}',\lambda)\bar{B}(\hat{b},\lambda)] \end{aligned} \quad (4)$$

Then using the fact that $|\bar{A}| \leq 1$ and $|\bar{B}| \leq 1$, we can simplify above into an inequality

$$|E(\hat{a},\hat{b}) - E(\hat{a},\hat{b}')| \leq 2 \pm [E(\hat{a}',\hat{b}') + E(\hat{a}',\hat{b})] \quad (5)$$

Rearranging, we have

$$S \equiv |E(\hat{a},\hat{b}) - E(\hat{a},\hat{b}')| + |E(\hat{a}',\hat{b}') + E(\hat{a}',\hat{b})| \leq 2 \quad (6)$$

where we have defined $S$ as the left-hand-side of the inequality. If we set $\hat{a}' = \hat{b}'$, and assuming $E(\hat{b}',\hat{b}') = -1$ (which is true for some two-particle states such as the singlet state, as seen by Eq. (9) in the following section), we can rewrite (6) as

$$|E(\hat{a},\hat{b}) - E(\hat{a},\hat{b}')| \leq 1 + E(\hat{b},\hat{b}') \quad (7)$$

Eq. (7) is the original *Bell's Inequality*[2]. The more general result of (6) is the *Clauser-Horne-Shimony-Holt (CHSH) Inequality*[3][4], which appeared to more suitable for certain experimental tests. There are also other versions of Bell's Inequalities, all of which can be derived from very general assumptions of local realism. In the discussion that follows, however, we'll concern ourselves mostly with CHSH Inequality, and sometimes a version without the absolute values, which is equally valid.

### B. Bell's Theorem: Quantum Mechanics is Inconsistent with Local Realism

The CHSH Inequality can be used to highlight the difference between a local realistic theory and quantum mechanics. Consider two spin-$\frac{1}{2}$ particles in the singlet state, whose wave function is given as

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (8)$$

The left part of the tensor product corresponds to particle travelling to location 1, and the right part corresponds to the particle travelling to location 2. Quantum mechanically, we can straightforwardly derive the expected correlation function $E(\hat{a},\hat{b})$ as

$$E(\hat{a},\hat{b}) = {}_{12}\langle\psi| (\vec{\sigma}\cdot\hat{a} \otimes \vec{\sigma}\cdot\hat{b}) |\psi\rangle_{12} = -\hat{a}\cdot\hat{b} \quad (9)$$

where $\vec{\sigma}\cdot\hat{k} = (\sigma_x,\sigma_y,\sigma_z)\cdot(k_x,k_y,k_z) = k_x\sigma_x + k_y\sigma_y + k_z\sigma_z$, with $|\hat{k}| = 1$.

Note that the dot product between two unit vectors is just the cosine of the angle between them: $\hat{a}\cdot\hat{b} = \cos\theta_{a,b}$. Let us denote $\theta_{a,b} \equiv \phi_1$, $\theta_{a,b'} \equiv \phi_2$, and $\theta_{a,a'} \equiv \theta$, and assume all of $\hat{a},\hat{b},\hat{a}',\hat{b}'$ lie in the same plane, then the LHS of (6) is just

$$\begin{aligned} S &= |-\cos\phi_1 + \cos\phi_2| + |-\cos(\phi_2-\theta) - \cos(\phi_1-\theta)| \\ &= |\cos\phi_2 - \cos\phi_1| + |\cos(\phi_2-\theta) + \cos(\phi_1-\theta)| \end{aligned} \quad (10)$$

Maximizing S on $\phi_1,\phi_2,\theta \in [0,2\pi)$, we can find multiple global maxima, one of which occurs at $(\phi_1,\phi_2,\theta) = (3\pi/4, \pi/4, \pi/2)$, and

$$\max_{\phi_1,\phi_2,\theta} S = \left|\cos\frac{\pi}{4} - \cos\frac{3\pi}{4}\right| + \left|\cos -\frac{\pi}{4} + \cos\frac{\pi}{4}\right| = 2\sqrt{2} > 2 \quad (11)$$

which clearly violates the CHSH inequality (6).

This violation means that the assumptions of local realism lead to a constraint violated by the predictions of quantum mechanics. It is for this reason that the singlet state is sometimes also called the Bell state. Hence, the fact that our theory of a local hidden variable cannot reproduce all the experimental predictions of quantum mechanics, is known as *Bell's Theorem.* In other words, quantum mechanics is inconsistent with the local realistic picture that EPR had in mind.

Bell's Theorem is remarkable in that it shifted the Einstein-Bohr debate about the completeness of quantum mechanics from a general, largely qualitative argument to a simple, quantitative, and empirically testable inequality. If the violation of Bell's Inequalities is verified experimentally, quantum mechanics will be favored over local realism as the description of the universe.

## III. EXPERIMENTAL TEST OF BELL'S INEQUALITIES

### A. The early experiments of Clauser *et al.*

In 1969, J. F. Clauser *et al.* first proposed testing Bell's Inequalities with the polarization correlation of a pair of photons such as those produced during positronium annihilation or J=0→J=1→J=0 cascades of atoms[4]. In the atomic cascade case, since the initial and final state have the same total angular momentum, the emitted photon pair must have zero total angular momentum, just like the singlet state. They proposed using single-channel polarizers as detectors, as seen in Fig. 1. This posed a problem, since they were only able to detect the positive +1 outcome as a count, while the −1 outcome was treated identical to non-detection event. Another problem with any experiments with photons was the low detector efficiency, which meant a large number of photons produced will go undetected. This made it difficult to calculate correlation probabilities, since the normalizing factor depends on the total number of particles produced, which cannot be found easily.

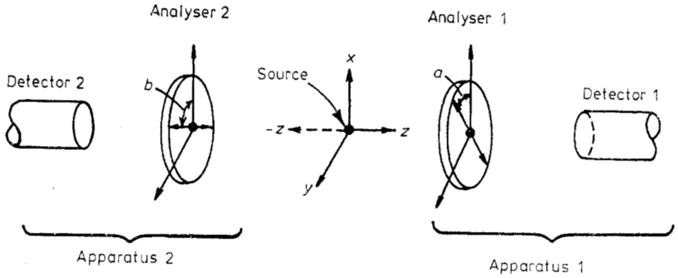


FIG. 1: Schematic diagram of the Clauser *et al.*'s experiment test of Bell's Inequalities with single-channel polarizer, adopted from Ref. [6].

Clauser *et al.* addressed both problems by first invoking two assumptions:

1. CHSH assumption[6]: the joint probability of detecting a pair of photons emerging from the polarizers is independent of the polarizers' orientations.
2. No-Enhancement or CH assumption[5][6]: the probability of a count is not increased when a polarizer is inserted between the source and the detector for any emission with hidden variable $\lambda$.

They then rewrote the CHSH Inequality as

$$\begin{aligned} -p(\infty,\infty) \leq p(\hat{a},\hat{b}) - p(\hat{a},\hat{b}') + p(\hat{a}',\hat{b}) \\ +p(\hat{a}',\hat{b}') - p(\hat{a}',\infty) - p(\infty,\hat{b}) \leq 0 \end{aligned} \quad (12)$$

where $p(\hat{a},\hat{b})$ denotes probability of detecting simultaneous counts when the orientation of polarizers are respectively $\hat{a}$ and $\hat{b}$, while a "direction" of $\infty$ simply means the polarizer is removed. Note that this inequality is homogeneous where no numbers other than 0 appears, thus the normalizing factor from the total number particles is cancelled.

In the early 1970s, Clauser *et al.* went ahead and performed a series of experiments that utilized photon pairs produced in cascades of calcium and mercury[6]. Their experimental data demonstrated excellent agreement with predictions of quantum mechanics, while confirming a violation of Eq. (12); these results supported quantum mechanics while contradicted local hidden variable theories[6].

Nevertheless, the cascade-photon experiments of Clauser *et al.* had flaws due to their reliance on the two assumptions mentioned above. While both assumptions are reasonable, they could nevertheless potentially be false. According to Clauser and Shimony, while the CHSH assumption seems testable in experiments, those tests are meaningless from certain perspectives such as the semiclassical radiation theory; the No-Enhancement or CH assumption, also seemingly reasonable, can be complicated by the fact the it has to be true for all $\lambda$. The difficulty of proving these assumption as well as the low detector efficiency of their experiment were considered

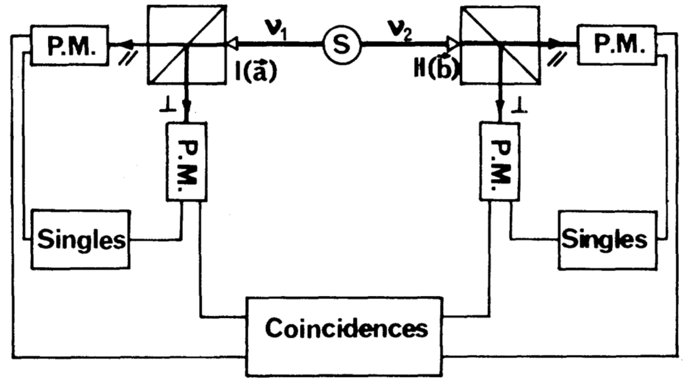


FIG. 2: Schematic diagram of am experimental test of the CHSH Inequality with double-channel polarizer, adopted from Ref. [8]. The two-channel polarizers I and II are each able to separate two orthogonal linear polarization. They are followed by photomultipliers (P.M.) and coincidence counters.

loopholes of their experimental test of Bell's Inequalities. Additionally, the fact that the polarizers were kept at fixed orientation for a non-negligible amount of time meant there could conceivably be some signal between them so that some hidden mechanism was responsible for producing the strong inter-particle correlation predicted by quantum mechanics[6].

### B. The experiments of Aspect *et al.*

In 1981–1982, A. Aspect *et al.* performed a series of experiments[7][8] that improved upon Clauser *et al.*'s results with the use of double-channel polarizers (see Fig. 2) that allowed for the detection of both +1 and −1 outcomes, so that they could test the CHSH Inequality of Eq. (6) directly. Similar to Clauser *et al.*, Aspect *et al.* utilized photon pairs produced in atomic cascades of calcium.

In these experiments, Aspect *et al.* were able to empirically measure the correlation functions in the CHSH Inequality (6) as

$$E(\hat{a},\hat{b}) = \frac{N_{++}(\hat{a},\hat{b}) + N_{--}(\hat{a},\hat{b}) - N_{+-}(\hat{a},\hat{b}) - N_{-+}(\hat{a},\hat{b})}{N_{++}(\hat{a},\hat{b}) + N_{--}(\hat{a},\hat{b}) + N_{+-}(\hat{a},\hat{b}) + N_{-+}(\hat{a},\hat{b})} \quad (13)$$

where $N_{\pm\pm}$ denotes the simultaneous count of detecting ±1 outcome at location 1, and ±1 outcome at location 2, respectively, with the specified polarizer orientations. Though also plagued by low detector efficiency, they simply ignored the no-detection outcome and normalized by the total number of photons detected. Five runs of their 1982 experiment yielded an average result of $S_{expt} = 2.697 \pm 0.015$[8], which is much greater than the upper-bound of 2 set by local realistic predictions in the CHSH Inequality of Eq. (6).

According to Aspect *et al.*, only two loopholes remained exploitable by advocates of local realistic theories. The first loophole was still the fact that detector efficiency are very low, since the sample of events detected could in principle be not random, and undetected events can theoretically tip the statistics in the opposite direction of the violation of the inequality[8]; this loophole can be easily closed as long as we assume the sampling is fair and random. The second loophole, perhaps more physically interesting, was the fact that there can conceivably be signals between the two detectors, since the detectors' spatial separation is small enough (up to 6.5 m in 1981[7]) for potential inter-detector signals at the speed to light to be responsible for the great observed inter-particle correlations. Both of these loopholes were also present in the experiments of Clauser *et al.* Nevertheless, both of these loopholes are addressed in later experiments that we will now discuss.

### C. Later experiments

G. Weihs *et al.*'s experimental test of Bell's Inequalities performed in 1998 was the first to eliminate the loophole of insufficient relativistic separation between the two distant detectors, using photon pairs generated by pumping a nonlinear BBO crystal with Ar ion laser[9]. The first improvement they made was the use of optical fibers, which allowed these tests to be performed with much greater spatial separation (400 m) of the detectors. The second improvement was the rapid randomization of the detectors' orientations, which made any potential communication between the detectors insignificant to the experimental results. This was made possible with local atomic clocks at the detectors as well as individual measurement time far less than 1.3 $\mu$s. Their typical measurement result was $S = 2.73 \pm 0.02$ for 14700 coincidence events, which corresponds to a violation of CHSH Inequality by $30\sigma$ assuming only statistical error. This is a very significant result in support of quantum mechanics over local realism, especially since every pair of detection events is relativistically separated. With a detector efficiency of merely 5%, however, Weihs *et al.*'s experiment was not entirely loophole-free, having to rely on the assumption of fair sampling.

In 2001, M. A. Rowe *et al.* were able to perform a version of the experimental test of Bell's Inequalities with efficient detection[10]. Using a complete set of measurements based on massive entangled particles of $^9Be^+$ ions as opposed to massless photons, they were able to achieve high detection efficiency, where bright ions are misidentified only 2% of the times. As a result, they did not have to invoke the assumption of fair sampling required by earlier photon-based experiments. They measured $S = 2.25 \pm 0.03$, which was still a very good violation of the CHSH Inequality. Nevertheless, Rowe *et al.*'s experiment still suffered from the loophole that their detectors were not relativistically separated, so that signaling could potentially occur between detectors and some hidden mechanism could possibly explain violation of Bell's Inequality.

Furthermore, a very recent 2012 experiment done by M. Giustina *et al.* uses photons and high-efficiency superconducting detectors while closing the loopholes of the fair-sampling assumption[11]. They tested against Eberhard's version of Bell Inequality[12], which takes into account of undetected events, which means no fair-sampling assumption is needed, and only requires a minimum efficiency of 66.7%. In addition to using photon pairs created by pumping nonlinear crystals with laser as well as optical fibers for transmission like Weihs *et al.*, Giustina *et al.* employed transition-edge sensors operating at superconducting transition which have reported efficiency of up to 98%; their measured efficiencies were $(73.77\pm0.07)\%$ in one arm and $(78.59\pm0.08)\%$ in the other, beyond the minimum requirement for Eberhard's Inequality. Their experiment show a violation of Eberhard's Inequality by $69\sigma$, an very significant result that once again supported quantum mechanics over local realistic theories. However, it isn't apparent from their report that Giustina *et al.*'s experiment provided sufficient relativistic separation between detection events. But due to the similarity of their setup to Weihs *et al.*, it is very possible that they can realize a fully loophole-free experimental tests of Bell's Inequalities in the near future.

## IV. IMPLICATIONS FOR LOCAL REALISM

Almost all the experiments performed to date, just like ones mentioned in the previous section, supported the predictions of quantum mechanics over local realism. Nevertheless, there are many loopholes in these experiments, due to either detector efficiency (No-Enhancement or fair-sampling assumption) or lack of relativistic separation of detection events. Though great efforts have been made to eliminate these loopholes, none of the experimental violation of Bell's Inequalities demonstrated so far are completely loophole-free. While some have independently and successfully eliminated loopholes due to detector efficiency or insufficient relativistic separation, no experiment have reportedly eliminated both simultaneously. This means that supporters of local realism is not without hope, though their hope is diminishing due to efforts of Giustina *et al.*[11]. If we can indeed show an experimental violation truly without loophole, we will have then demonstrated that EPR's requirement of locality and realism cannot be both true since they are inconsistent with real phenomena.

In any case, the violation of Bell's Inequalities is not a death sentence on realism per se, since there could potentially be *nonlocal* hidden variables underlying quantum mechanical phenomena. For example, the de Broglie-Bohm's Pilot Wave Theory, which postulates the physical existence of a pilot wave that guides the particle around, contains a nonlocal hidden variable in the form of the spatially-extended wave field[13][14]. Nonetheless, there are theoretical restrictions on what a valid nonlocal hidden variable theories can be. Similar to Bell's Inequality for local hidden variables, Leggett's Inequality[15] potentially ruled out a class of such hidden variable theories known as "crypto-nonlocal". There are experimental evidence of violation of Leggett's Inequality, per experiments done by Gröblacher *et al*[16].

## V. CONCLUSION

The experimental efforts of Clauser[6], Aspect[7][8], Weihs[9], Rowe[10], and Giustina[11] *et al.* have illuminated the Einstein-Bohr debate by demonstrating unequivocal violations of Bell's Inequalities. As a result, there are strong evidence in favor of quantum mechanics over any local hidden variable theory. Nevertheless, the loopholes of detector efficiency and insufficient relativistic separation were never simultaneously eliminated in the experimental Bell's test performed to date, which means that there is still a non-negligible possibility of local realism being true. In any case, recent experimental progress suggests that a truly loophole-free experimental test of Bell's Inequality is imminent. The experimental violation of Bell's Inequalities, however, does not necessarily preclude the possibility of all hidden variable theories, as there can potentially still be "nonlocal realism". Therefore, the question of whether a hidden variable underlies quantum mechanics remains open, but Bell's Theorem and its mostly robust experimental verification demonstrate that whatever this hidden variable may be, it must not be local as EPR originally thought.

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