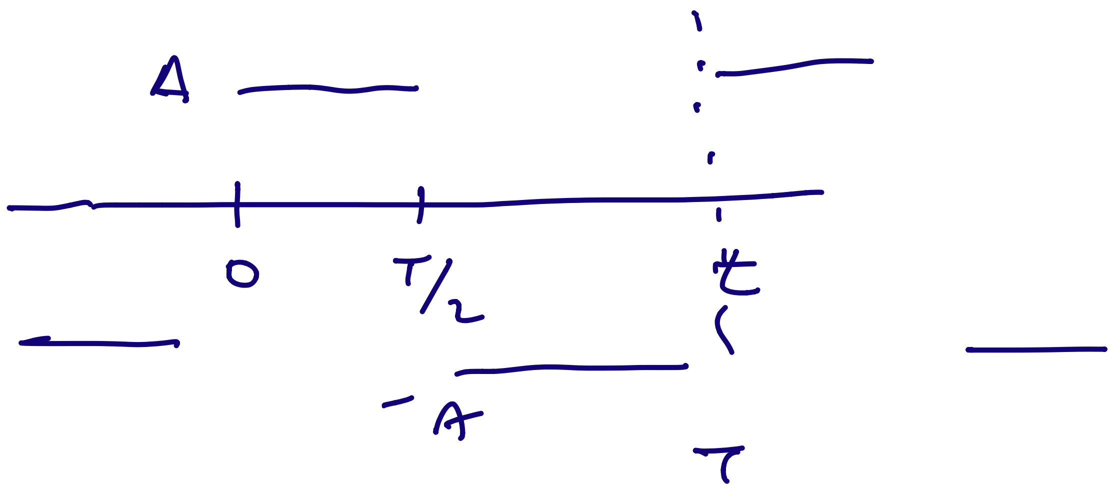
Τρίτη 13 Απριλίου

Ασκ. 13

$$\dot{x} + x = \begin{cases} A & 0<t<T/2 \\ -A & \frac{T}{2}<t<T \end{cases}$$



x_0 $\quad t=0 \quad$ t

$$x(T) = e^{-t} x_0 + \int_0^t e^{-(t-s)} F(s)\,ds$$

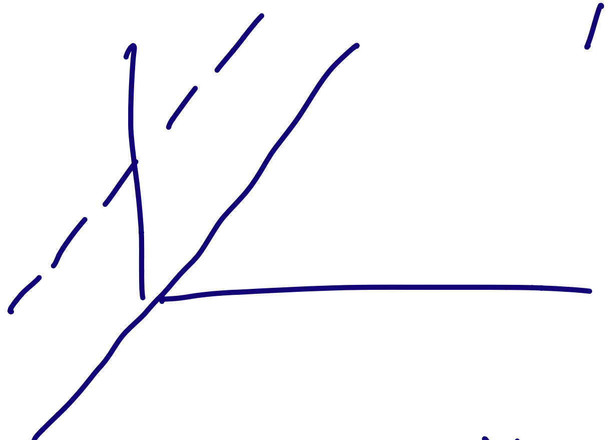
$$P(x) = e^{-T} x - A\left(1-e^{-T/2}\right)^2$$

Ευσταθία

$F(t) \qquad P(x) = \alpha x + \beta$

αυτά είναι γενικά

T=0 $\qquad P(x) \to x$



/ αφηρημένη

ή καθαρή

ο σταθερός $\qquad$ ισχύει $\alpha \qquad |\alpha| < 1$

είναι ευσταθής

ουσιαστικά $\quad \alpha = e^{-T} \leq 1$

είναι ευσταθή

$$\dot{x} - x = F(t) \qquad \alpha = e^{T}$$

ασταθές.

είναι ασταθής

$$P(x) = e^{T} x - A\left(1 - e^{T/2}\right)^2$$

$$= x$$

$$x_* = A \tanh(T/4)$$

[illegible]

$$\dot{x} = x^2 - 1 - \cos t$$

-1.37 $\quad$ 1.37

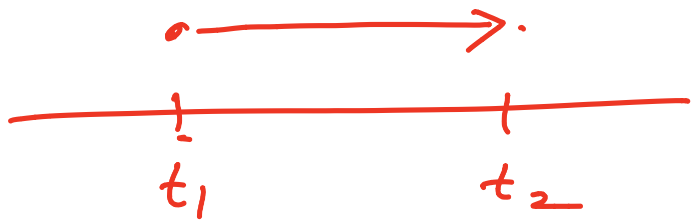
$T \to 0 \qquad x_*$

$T \to \infty \qquad x_* = -A$

Αρχ. 14/1

Διαδότης

$$\dot{x} = v(x, t)$$



$$\phi_{t_2 t_1}(x)$$

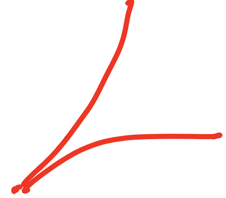
αυτόνομο σύστημα
1ης
τάξης

$$\dot{x} = \alpha x$$

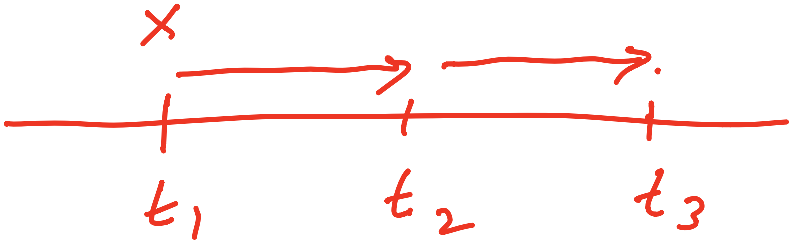
$$\phi_{t,0}(x) = e^{\alpha t} x$$

$$\phi_{t_2 t_1}(x) = e^{\alpha (t_2 - t_1)} x$$

συνάρτηση τού x

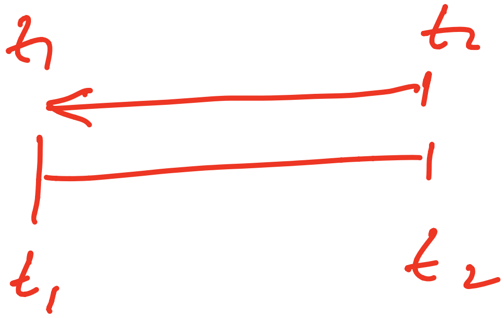


εφόσον έχω
μοναδική λύση
που επεκτείνεται
σε όλο τον χρόνο



$$\phi_{t_3 t_1}(x) = \phi_{t_3 t_2}\left(\phi_{t_2 t_1}(x)\right)$$

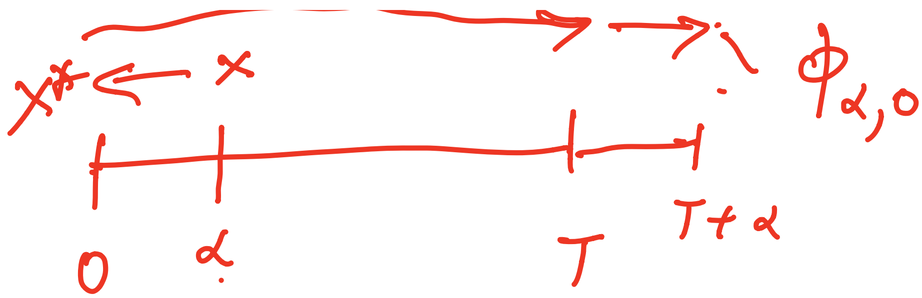
$$\phi_{t_3 t_1} = \phi_{t_3 t_2} \circ \phi_{t_2 t_1}$$



$$\phi_{t_1 t_1}(x) = x \qquad \phi_{t_1 t_1} = I$$

$$\phi_{t_1 t_2} = \phi^{-1}_{t_2 t_1}$$

$P(x^*)$



$$P(x) = \phi_{T,0}(x)$$

$$P_\alpha(x) = \phi_{T+\alpha,\alpha}(x)$$

$$P_\alpha(x) = \phi$$

$$\phi_{\alpha,0} = \phi_{T+\alpha,\alpha}$$ λόγω περιοδικότητας

$$P_\alpha(x) = \phi_{\alpha,0}\left(P\left(\phi_{0,\alpha}(x)\right)\right)$$

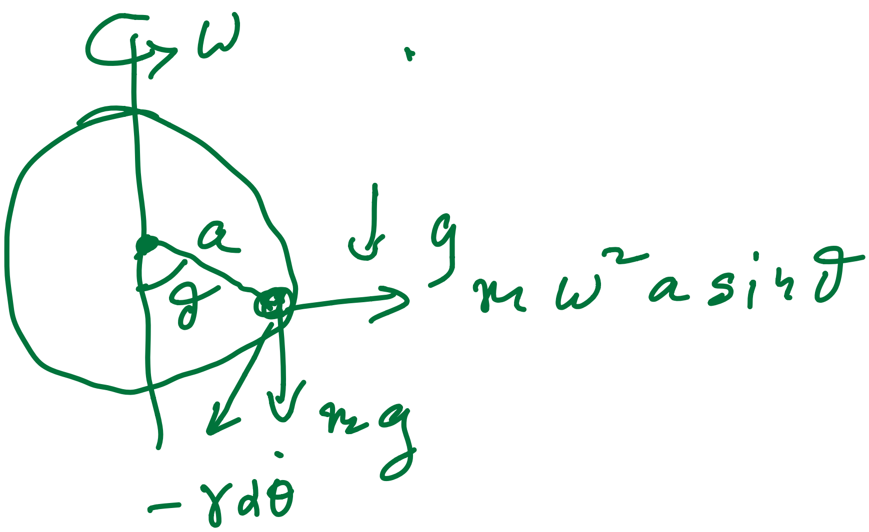
x_α πάλλει στη

$$P_0(x) = x$$ x_0 πάλλει

$X_\alpha \neq x_0$ αλλά έχω την ίδια συνάρτηση.

$$P'_\alpha(X_\alpha) = P'(X_0)$$

$$\phi_{\alpha,0} \circ \phi_{0,\alpha} = I$$



$$m\ddot\theta = -\gamma a\dot\theta - mg\sin\theta + m\omega^2 a\sin\theta\cos\theta$$

$$t = T\tau$$

$$\frac{1}{gT^2}\frac{d^2\theta}{d\tau^2} = -\frac{\gamma\alpha}{mgT}\frac{d\theta}{d\tau} - \sin\theta$$
$$+ \underbrace{\frac{\omega^2\alpha}{g}}_{=r}\sin\theta\cos\theta$$

$$T = \frac{\gamma\alpha}{mg}$$

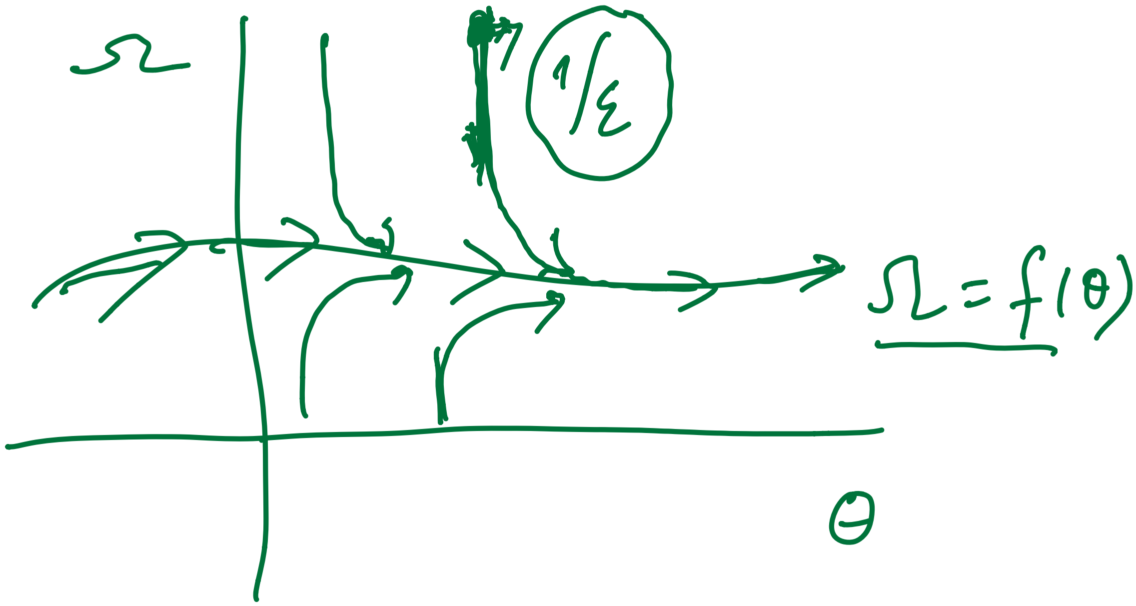
$$\underbrace{\frac{m^2 g}{\alpha^2\gamma^2}}_{\varepsilon}\frac{d^2\theta}{d\tau^2} = -\frac{d\theta}{d\tau} - \sin\theta + r\sin\theta\cos\theta$$

$$f(\theta) = -\sin\theta + r\sin\theta\cos\theta$$

$$\boxed{\varepsilon\ddot{\theta} = -\dot{\theta} + f(\theta)}$$

$\varepsilon \ll 1$

$$\dot{\theta} = \Omega$$
$$\dot{\Omega} = (f(\theta) - \Omega)/\varepsilon$$



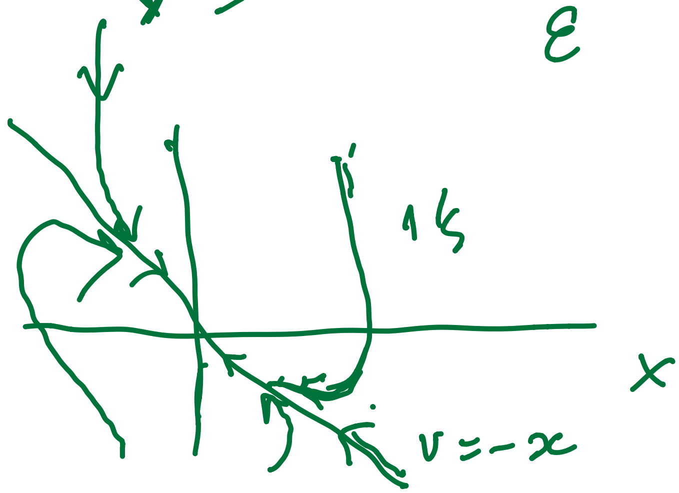
$$\varepsilon \ddot{x} + \dot{x} + x = 0$$

$$m \ddot{x} + 2\gamma \dot{x} + \kappa x = 0$$

$$x(0), \ \dot{x}(0) \neq 0$$

$$\dot{x} = y$$

$$\dot{y} = -\frac{x + y}{\varepsilon}$$



$$v = -x$$

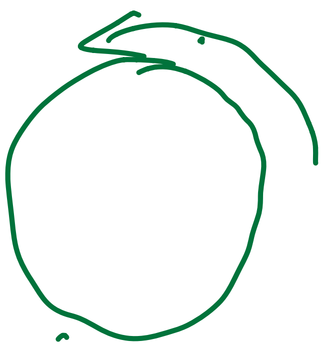
$$\dot{x} = -x$$

$$x = e^{-t} x_0$$

$t \gg 1/\varepsilon$

$$\dot{\theta} = f(\theta)$$

$$\dot{\theta} = -\sin\theta + r\sin\theta\cos\theta$$
$$= -\sin\theta(1 - r\cos\theta)$$
$$= -r\sin\theta\left(1/r - \cos\theta\right)$$



$\dot{\theta} = \omega(\theta) > 0$

μπορί να έχει περιοδική λύση

$\theta = 0, \quad \theta = \pi$

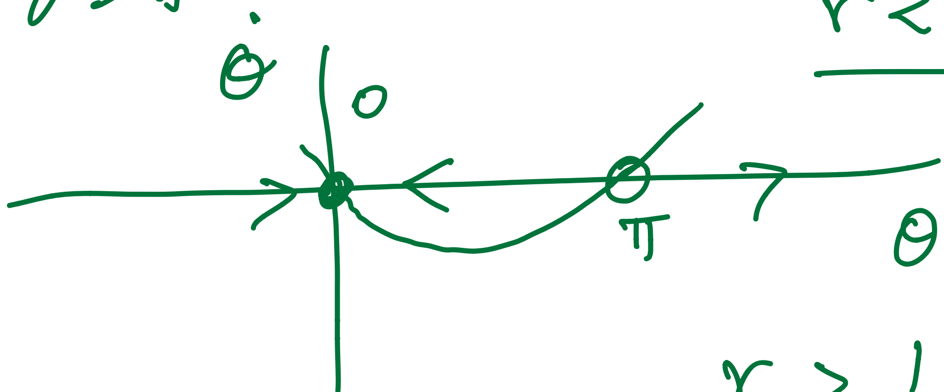
$$\dot{\theta} = -r\sin\theta\left(\frac{1}{r} - \cos\theta\right)$$

$r<1$ $\quad \frac{1}{r} - \cos\theta > 0 \quad \rightarrow f(\theta)$

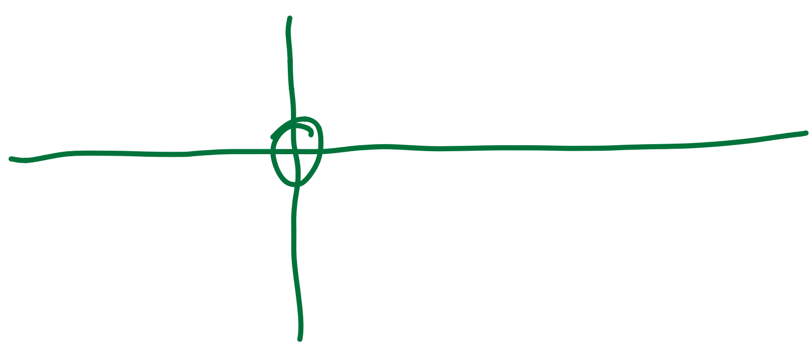
$\theta = 0$ είναι ευσταθή

$\theta = \pi$ είναι ασταθή

$r<1$

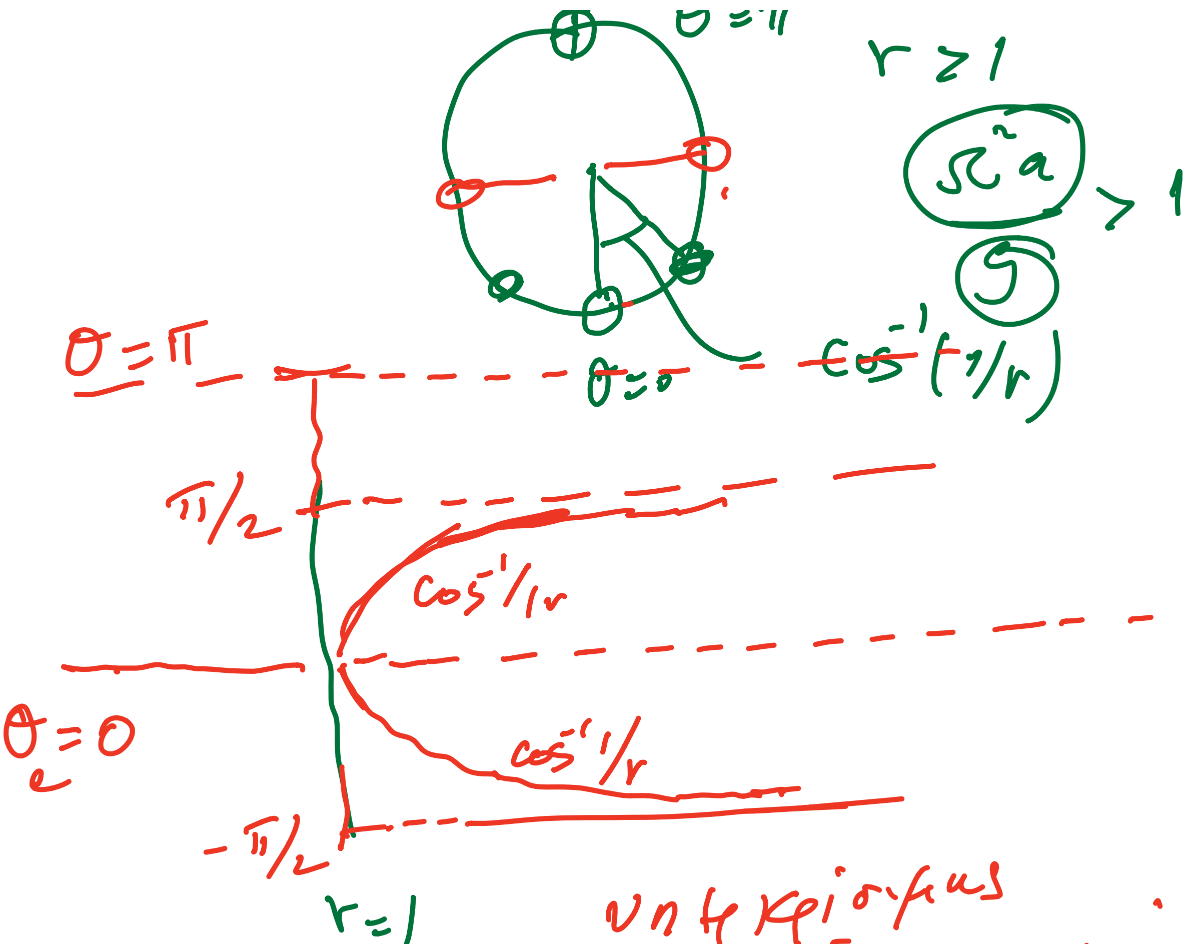


$r>1$



$\cos\theta = 1/r$

$\theta = \pm\cos^{-1}(1/r)$



υπερκρίσιμες
δικρανικές
διακλαδώσεις

Δεν έχω τριβή
και έχω ι N η μάζα

$$\varepsilon \ddot{\theta} = -\sin\theta + r\sin\theta\cos\theta$$

τί θα συμβαίνει σε Ντετερμινιστικό περιβάλλον

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - h$$

α) $h \cdot N$

$(r-h)N$

$r > h$ N είναι

β) $\dfrac{hN}{A+N}$ Holling II αναλογ. είναι επαρκή

$N \to \infty$

γ) $\dfrac{hN^2}{A^2+N^2}$ Holling III $\rightarrow$ h

$N =$ ο πάντοτε απαλθεί

$$\dot{N} = rN\left(1 - \frac{N}{K}\right) + \frac{hN^2}{A^2+N^2}$$

δε ακανίσθει
μπορεί να ό πηγαίνουσι!

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H\frac{N}{A+N}$$

$$x = \frac{N}{K} \qquad t = T\tau$$

$$\frac{K}{Tr}\frac{dx}{d\tau} = Kx(1-x) - \frac{H}{r}\frac{x}{\frac{A}{K}+x}$$

$r = 1/T$, $T = 1/r$

$$\dot{x} = x(1-x) - h\,\frac{x}{\alpha + x}\,,$$

$$h = \frac{H}{rK}\,, \quad \alpha = \frac{A}{K}$$

$x = 0$ σημείο

$x \ll 1 \qquad \dot{x} = x$

$$\frac{x}{\alpha + x} = \frac{x}{\alpha}\,\frac{1}{1 + x/\alpha} \simeq \frac{x}{\alpha}\left(1 - \frac{x}{\alpha} \ldots\right)$$

$$\dot{x} = x - \frac{h}{\alpha}\,x = \left(1 - \frac{h}{\alpha}\right)x + \ldots$$

$\frac{h}{\alpha} < 1$ τότε $x = 0$ είναι ασταθές

;

$\frac{h}{\alpha} > 1 \qquad x$ είναι ευσταθές

και μπορεί να έως αφανισμό.

$$\dot{x} = x\left(\underbrace{1 - x - \frac{h}{x+\alpha}}\right)$$

$$x + \alpha - x^2 - \alpha x - h = 0$$

$$x^2 + x(\alpha - 1) + (h - \alpha) = 0$$

$$x_{\pm} = \frac{1-\alpha}{2} \pm \sqrt{\underbrace{\frac{(1-\alpha)^2}{4} - (h-\alpha)}}$$

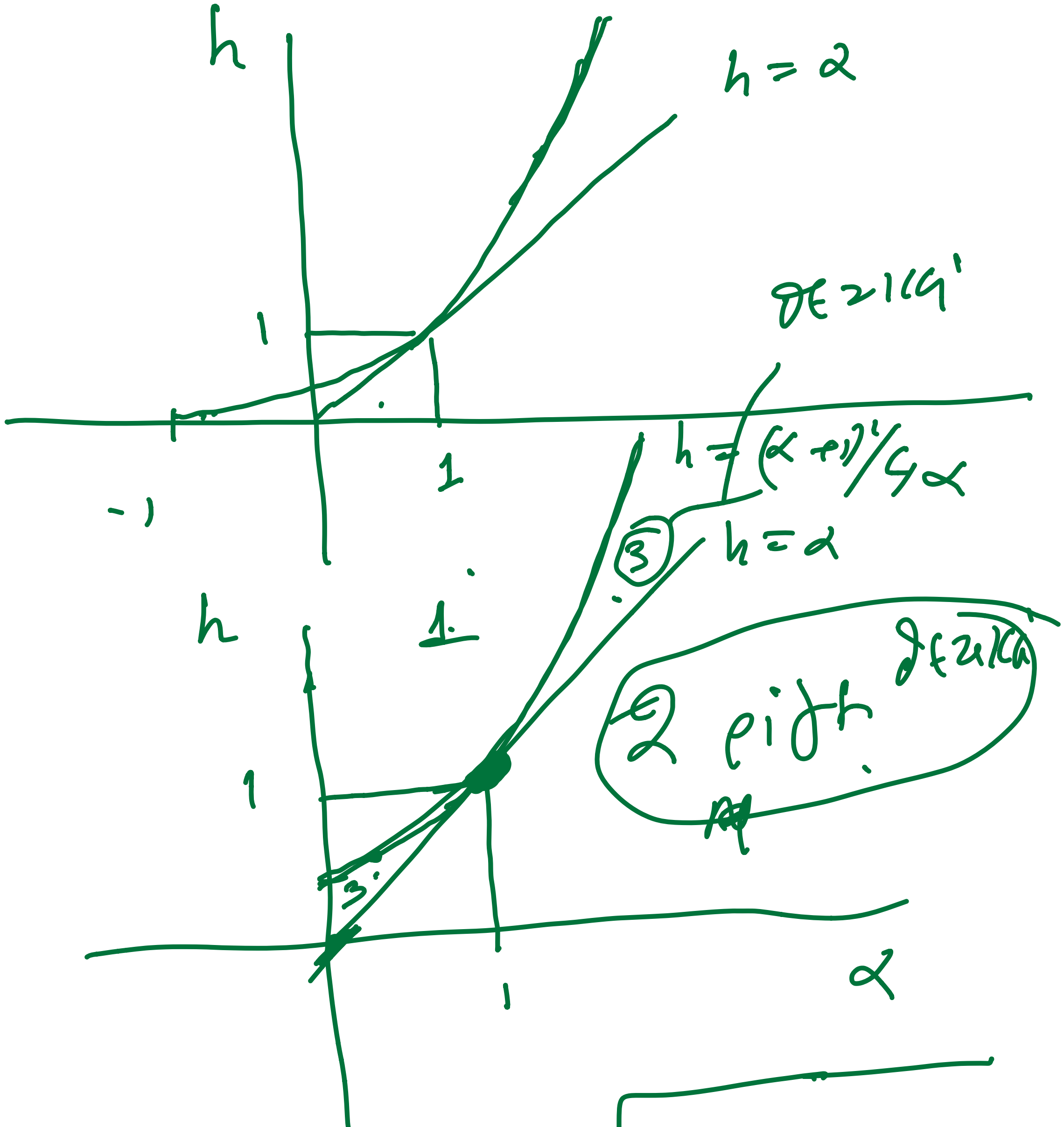
$$\frac{(1-\alpha)^2}{4} - h + \alpha = \frac{1 - 2\alpha + \alpha^2 - 4h + 4\alpha}{4}$$

$$= \frac{\cancel{\alpha^2}\,(\alpha+1)^2 - 4h}{4}$$

απαιτεί για 3 λύσεις

$$\frac{(\alpha+1)^2}{4} > h$$

$\alpha > 0$



$$x_\pm = \frac{1-\alpha}{2} \pm \frac{1}{2}\sqrt{(1-\alpha)^2 - 4(h-\alpha)}$$

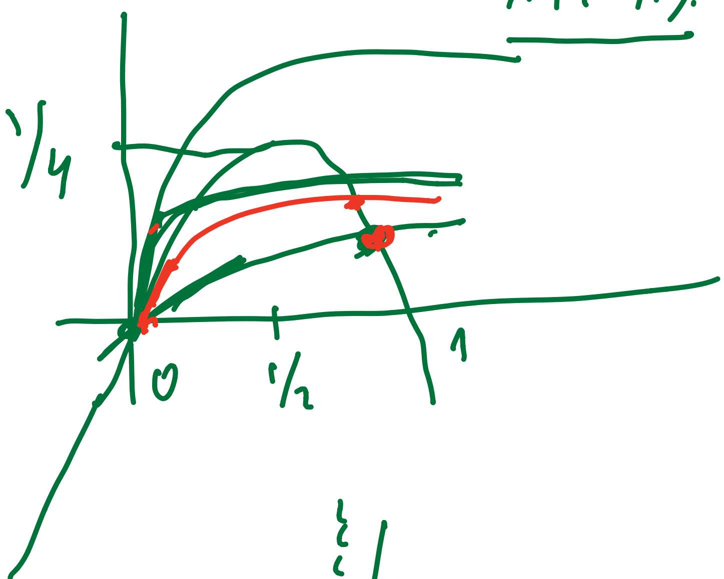
$\alpha < 1$ $\qquad \frac{1-\alpha}{2} > 0$

$h < \alpha$

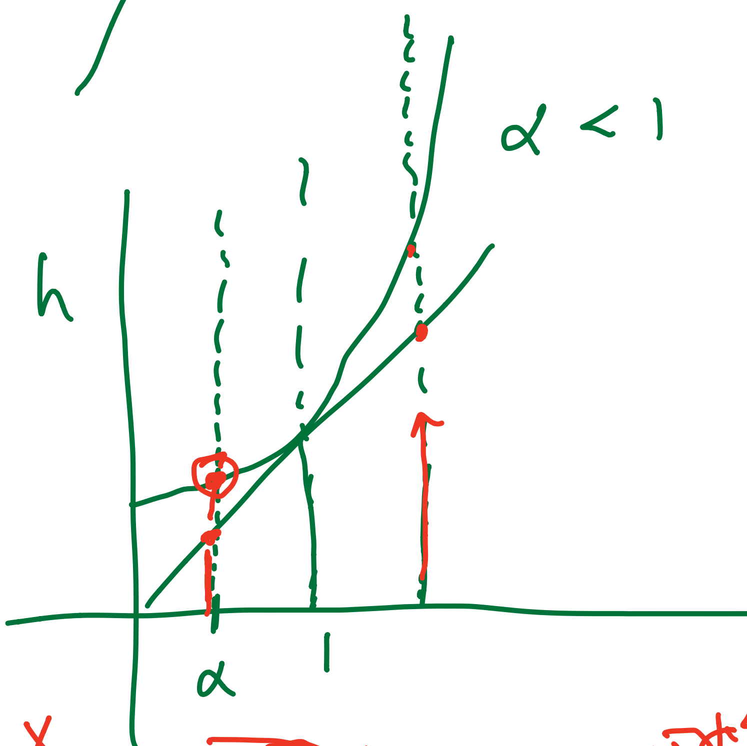
$$x_\pm = \pm\frac{1}{2}\sqrt{-4(h-\alpha)}$$

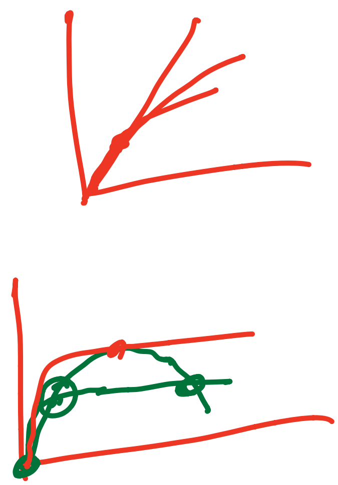
$$x(1-x) - h\frac{x}{x+\alpha}$$

$h/\alpha < 1$



$\alpha < 1$

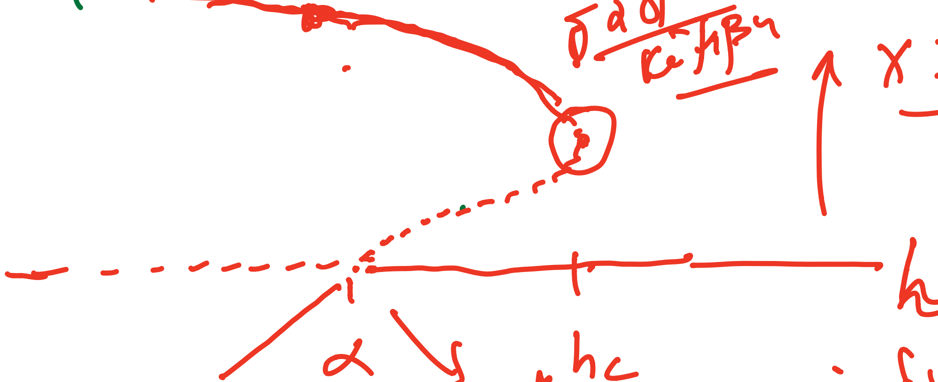




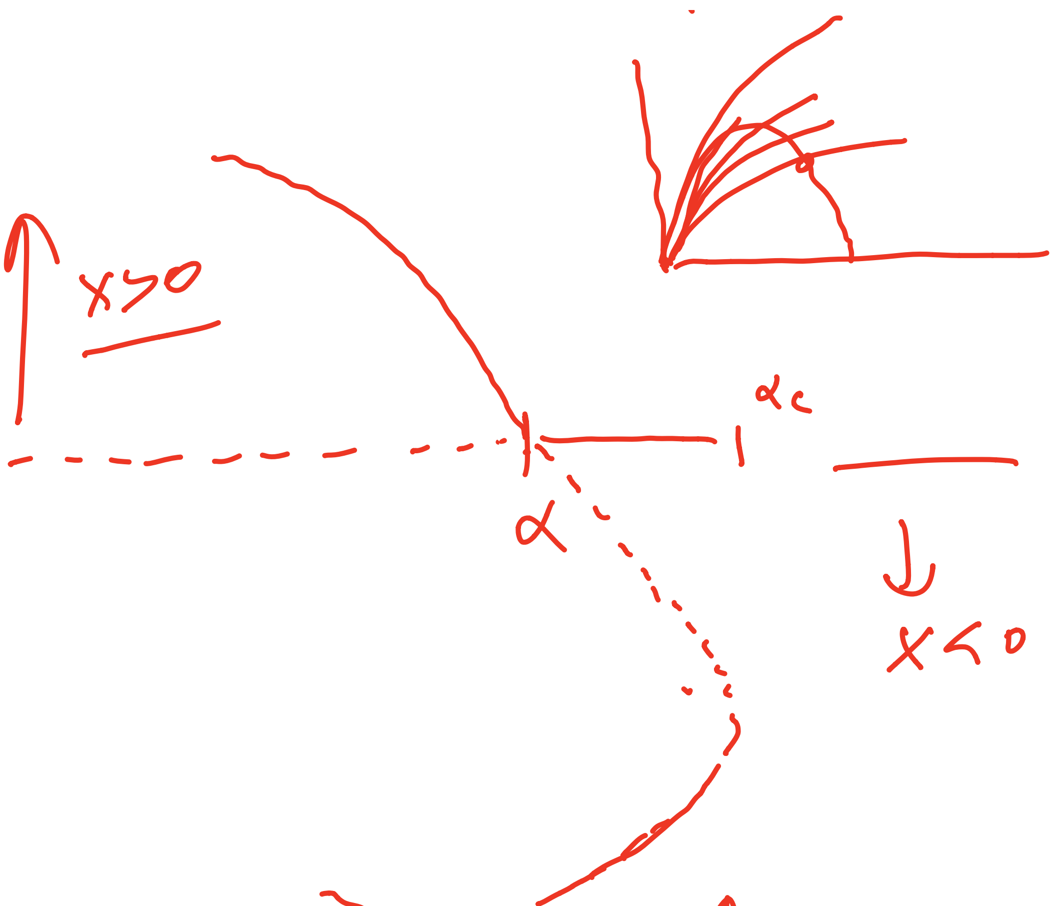
$\alpha < 1$

$$h_c = \frac{(1+\alpha)^2}{4}$$

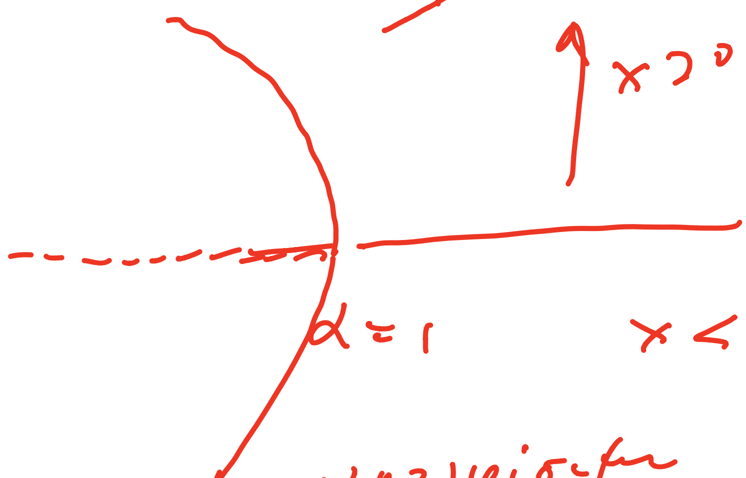
$x > 0$



$\alpha > 1$



$\alpha = 1$



υπολογίσιμη
δικρανική