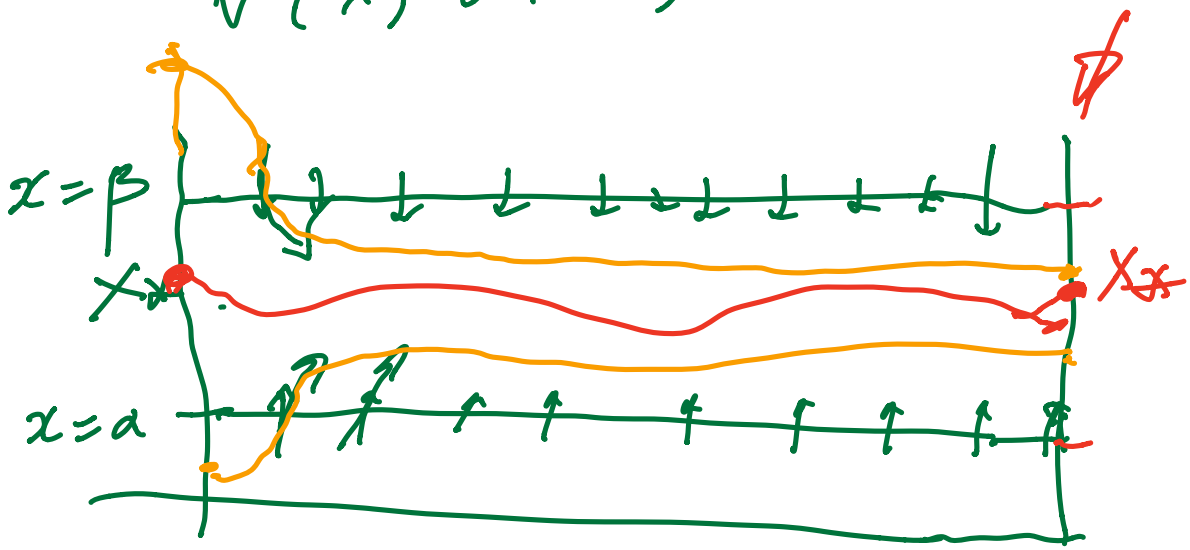
Τρίτη 6 Απριλίου

$$\dot{x} = v(x,t)$$

$$v(x, t+2\pi) = v(x,t)$$



$\beta > \alpha$

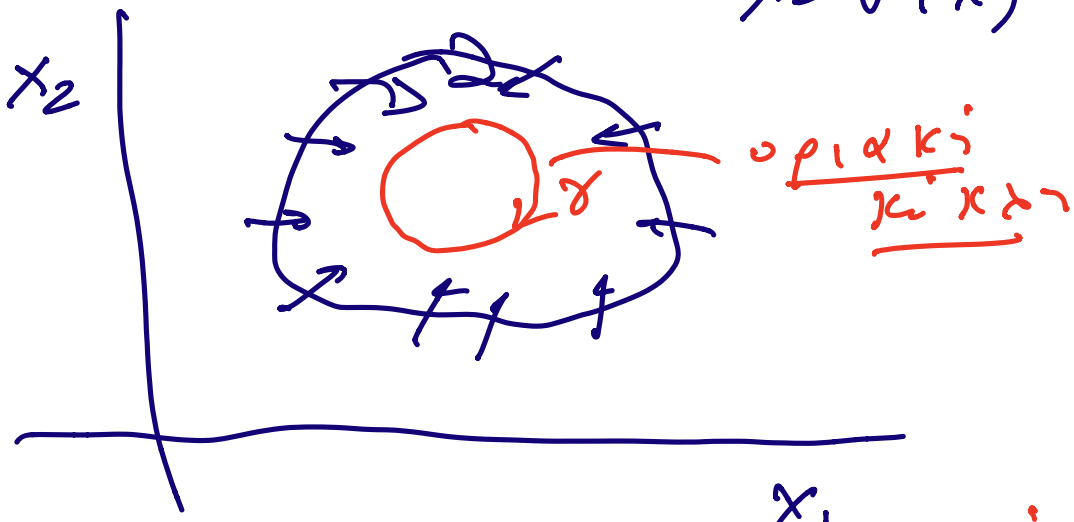
$$v(\alpha,t) > 0$$

$$v(\beta,t) < 0$$

Τότε αναγκαστικά υπάρχει περιοδική τροχιά για κάθε $\alpha < x^* < \beta$ $\quad x^* = P(x^*)$

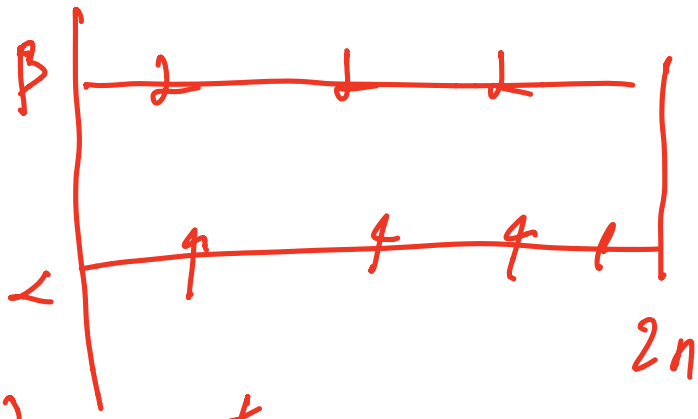
Το υλικό μας: $\vec{v} \in \mathbb{R}^2$

$\dot{\vec{x}} = \vec{V}(\vec{x})$



Δεν υπάρχει γιατί οι 2-διάστατες τροχιές

Απόδειξη



$\dot{x} = v(x,t)$

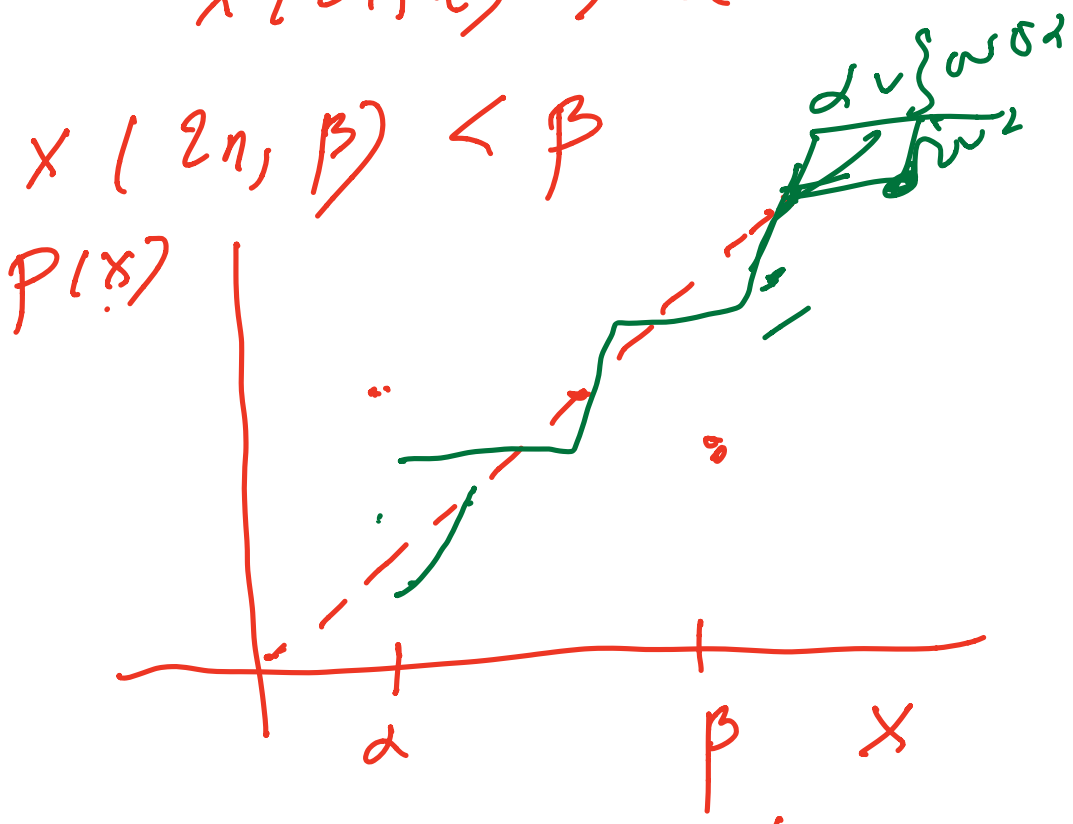
$$x = x(0) + \int_0^t v(x,s)\,ds$$

$x(0) = \alpha$ $\quad x(2\pi) = \alpha + \int_0^t v(\alpha,s)\,ds$

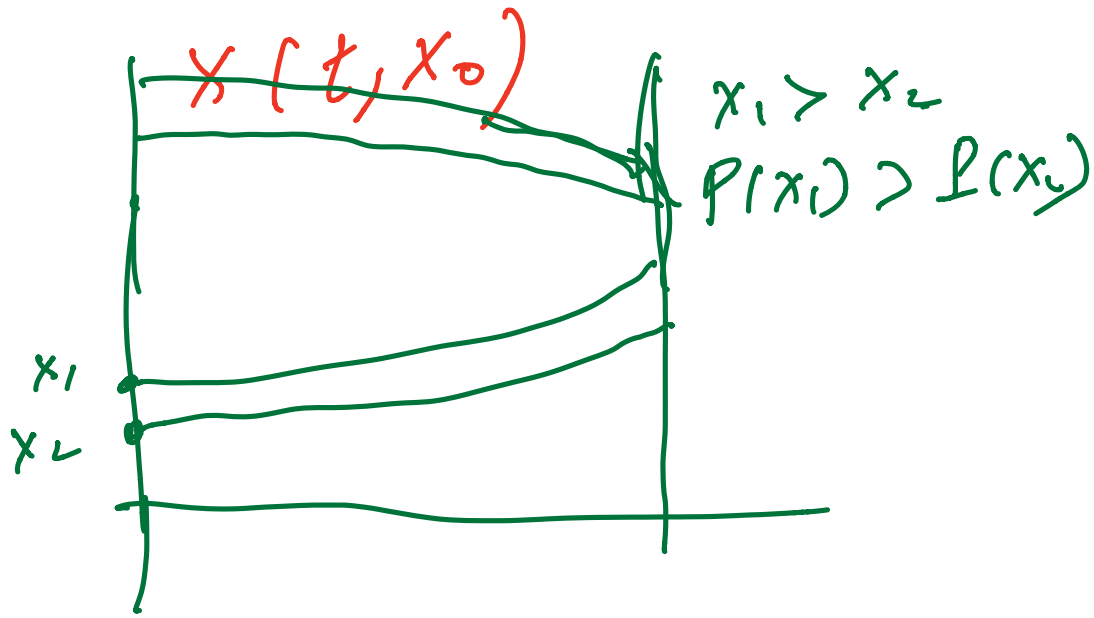
$> \alpha$

$X(2\eta, \alpha) > \alpha$

$X(2\eta, \beta) < \beta$



$V(x,t) \in C'$

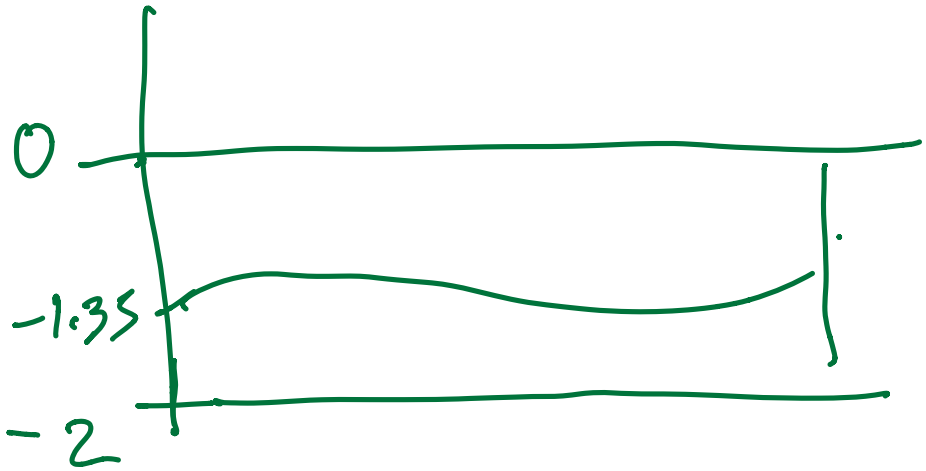


Η δυναμική κατάταξη

$$\dot{x} = x^2 - 1 - \cos t = V(x,t)$$

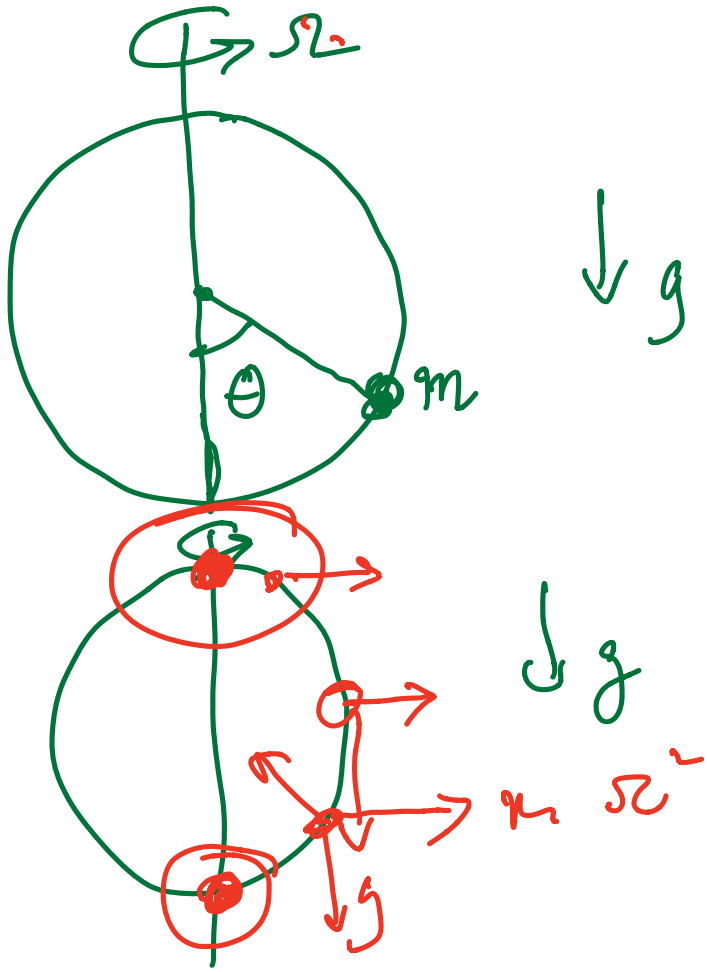
$x = 0 \quad -1 - \cos t \le 0$

$x = -2 \quad 3 - \cos t > 0$

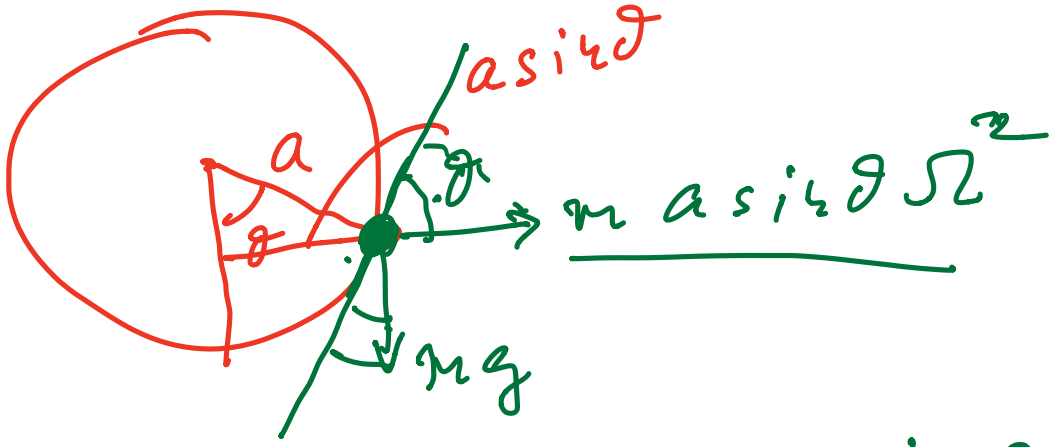


$$x^2 - 1 - \cos t = 0$$

$$x = \pm\sqrt{1 + \cos t}$$



$\Theta_E(\Omega, g, m)$



$$m a \ddot{\theta} = -\gamma a \dot{\theta} + m a \Omega^2 \sin\vartheta \cos\vartheta - m g \sin\vartheta$$

$T \qquad \tilde{t} = t/T \qquad t = \tilde{t}\,T$

$$\theta' = \frac{d\theta}{dt} = \frac{d\theta}{d\tilde{t}}\frac{d\tilde{t}}{dt} = \frac{1}{T}\frac{d\theta}{d\tilde{t}}$$

$$\frac{ma}{T^2}\theta'' = -\frac{\gamma a}{T}\theta' + ma\Omega^2 \sin\vartheta\cos\vartheta - mg\sin\vartheta$$

$$\frac{a}{gT^2}\theta'' = -\frac{\gamma a}{Tmg}\theta' + \frac{a\Omega^2}{g}\sin\vartheta\cos\vartheta - \sin\vartheta$$

$$\varepsilon = \frac{a\Omega^2}{g}$$

$$\frac{\gamma a}{Tmg} \gg \frac{a}{gT^2}$$

$$\gamma \gg \frac{m}{T}$$

$$T m g = \gamma a$$

$$T = \frac{\gamma a}{m g}$$

$$\gamma \gg \frac{m m g}{\gamma a} = \frac{m^2 g}{\gamma a}$$

$$\gamma^2 \gg \frac{m^2 g}{a}$$

$$\boxed{\gamma \gg m\sqrt{g/a}}$$

$$\frac{a}{g T^2} = \frac{a m^2 g^2}{g \gamma^2 a^2} = \frac{m^2 g}{\gamma^2 a}$$

$$\Omega^2 a/g$$

$$\boxed{\frac{m^2 g}{\gamma^2 a}\,\theta'' = -\theta + \varepsilon \sin\vartheta\cos\vartheta \uparrow - \sin\vartheta \qquad \ll 1}$$

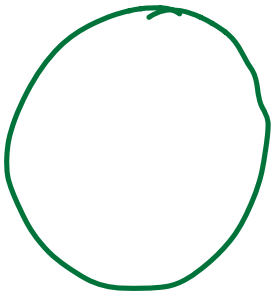
$$\left(\frac{m^2 g}{\gamma^2 \alpha}\right)\theta'' = -\theta' + \left(\frac{\Omega^2 \alpha}{g}\right)\sin\vartheta\cos\vartheta - \sin\theta$$

$$\frac{m^2 g}{\gamma^2 \alpha} \alpha < 1$$

Newtonian $\rightarrow$ Aristotelian

$$\theta' = \left(\frac{\Omega^2 \alpha}{g}\right)\sin\vartheta\cos\vartheta - \sin\theta$$

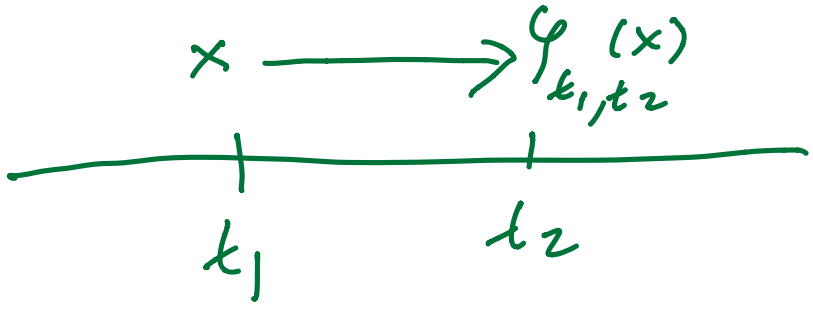
ε

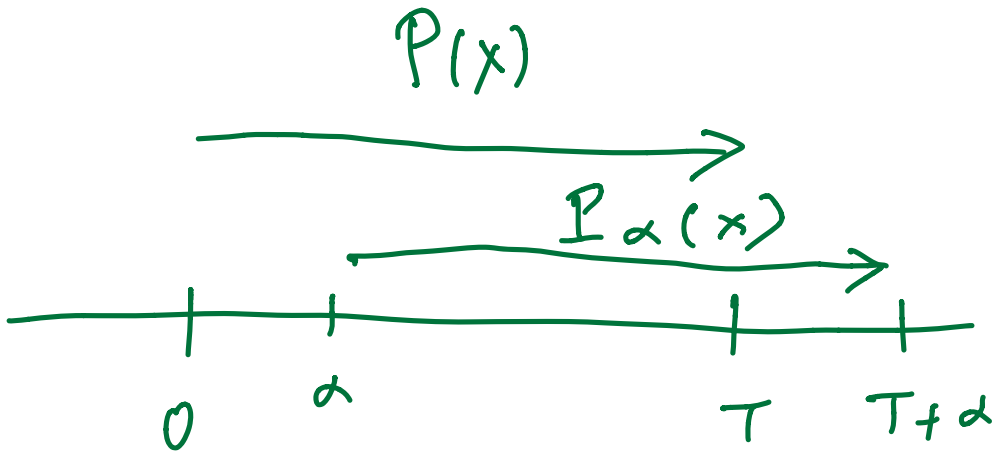


$g \to 0$

$\Omega \to \infty$

$\theta \to \pi/2$


x
$\varphi_{t_1,t_2}(x)$
t_1
t_2


P(x)
$P_\alpha(x)$
0
α
T
$T+\alpha$