Τρίτη 16 Μαρτίου

$\varepsilon \ll 1 \quad f(\varepsilon) \sim$

2-διάστατα

$$\dot{\vec{x}} = \vec{v}(\vec{x})$$

$\mathbb{R}^2$

Αυτόνομο

$$\dot{\vec{x}} = \vec{v}(\vec{x}, t)$$

μη αυτόνομο

t

$$\vec{y} = (\vec{x}, s)$$

R^4

$$\dot{\vec{x}} = \vec{v}(\vec{x}, s)$$

$$\frac{ds}{dt} = 1 \qquad \frac{d\vec{y}}{dt} = \begin{bmatrix} \vec{v}(\vec{x}, s) \\ 1 \end{bmatrix}$$

R^{n+1}

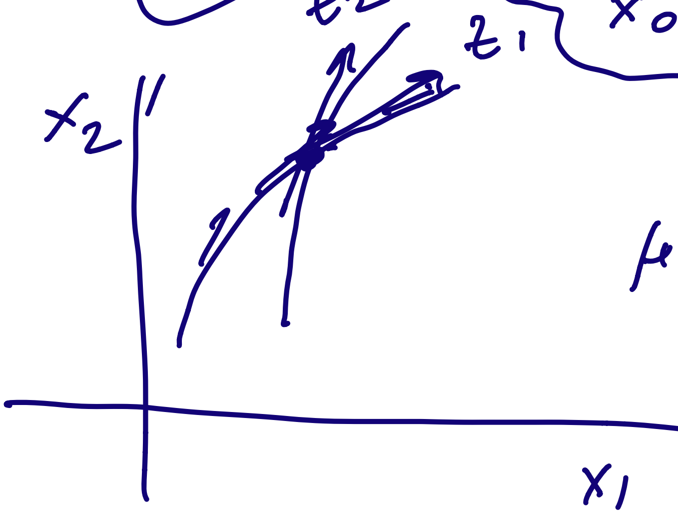
1 - διάστατα

$$\dot{x} = v(x)$$

$$x(0) = x_0$$

$$\frac{dx}{v(x)} = dt$$

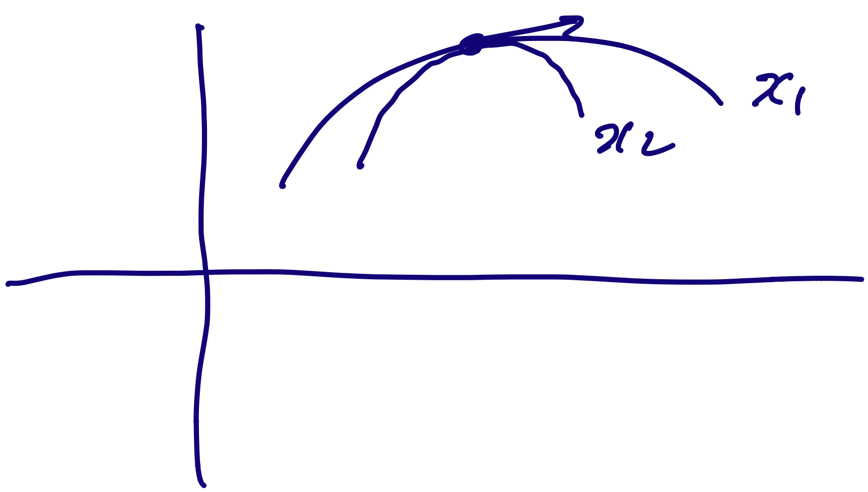
$$t - t_0 = \int_{x_0}^{x} \frac{dx}{v(x)}$$



$$\dot{\vec{X}} = \vec{v}(x)$$

μπορούν να τμηθούν

αδύνατο



μπορεί να γίνει αυτό?

$\dot{x} = v(x)$ 1-διάστατη

$v(x)$ είναι συνεχής

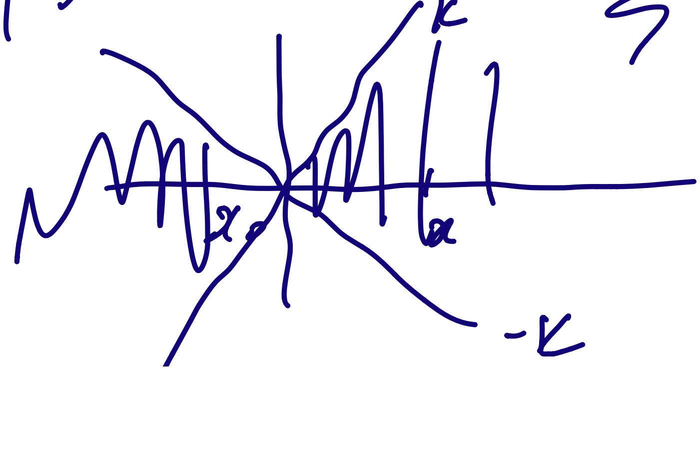
$v(x) \neq 0$ $v(x_0) \neq 0$

αποδείξτε ότι εάν $v(x)$ είναι συνεχής $v(x_0) \neq 0$ υπάρχει στη περιοχή του x_0 λύση μοναδική (Peano)

$$\boxed{\begin{aligned} \dot{x} &= v(x) \\ x &= x_0 \end{aligned}}$$

Lipschitz

$$|f(x) - f(x_0)| < K |x - x_0|$$

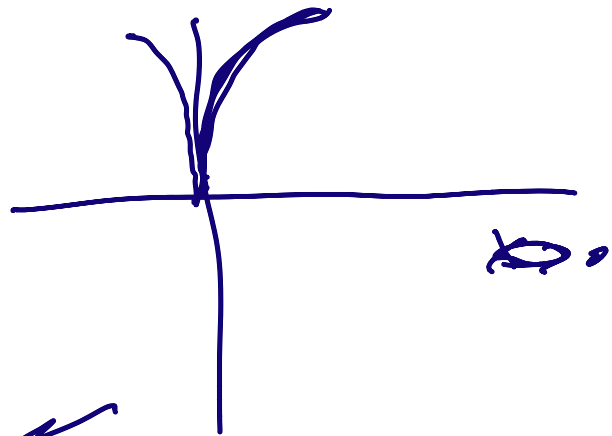


$|x|$ $\quad K=1$

$\underline{x=0}$

$\sqrt{|x|}$

$\underline{x_0=0}$



$$\boxed{\dot{x}=\sqrt{x}}\,, \qquad \underline{x>0}$$

$$x(0)=0$$

$x(t)=0 \quad \forall t, \quad \dot{x}=0,$ Ciuxl diia!

$$x=\alpha t^{\beta}$$

$$\dot{x}=\alpha\beta t^{\beta-1} \qquad \sqrt{x} \quad \sqrt{\alpha}\, t^{\beta/2}$$

$$\alpha\beta=\sqrt{\alpha}, \qquad \beta-1=\beta/2, \quad \beta/2=1$$

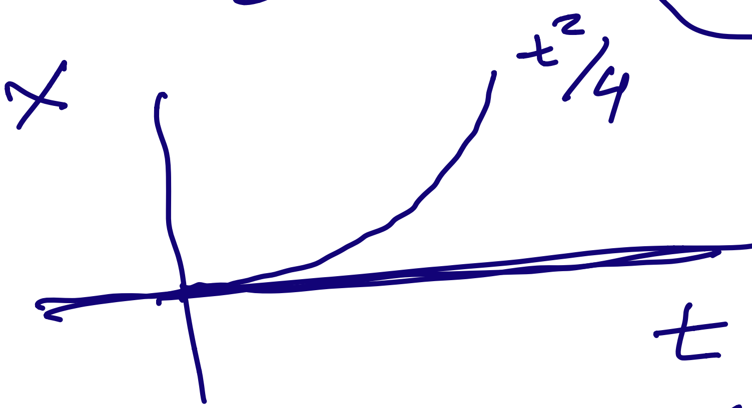
$$\beta=2$$

$$2\alpha=\sqrt{\alpha}, \quad \sqrt{\alpha}=1/2 \quad \alpha=1/4$$

$$x(t) = \frac{t^2}{4} \longrightarrow \dot{x} = \sqrt{x}, \quad x(0) = 0$$

$$\dot{x} = \frac{t}{2}$$

$$\dot{x} = \sqrt{x}$$



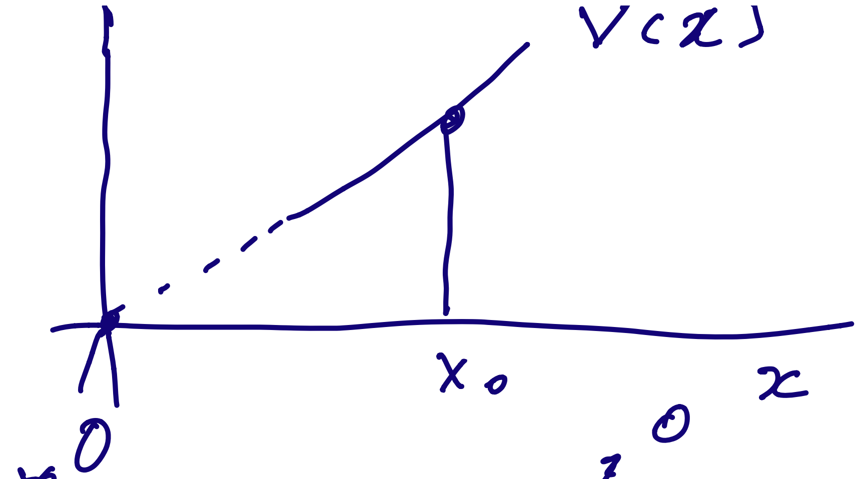
$$x(t) = 0 \qquad 0 \le t \le t_*$$

$$x(t) = \frac{1}{4}(t - t_*)^2 \qquad t > t_*$$

και αυτή είναι λύση

Γιατί η Lipschitz ?

$$t - t_0 = \int_{x_0}^{x} \frac{dx}{v(x)}$$



$$|V(x) - \cancel{V(0)}^{\,0}| < K|x - 0|$$

$$|V(x)| < K|x|$$

$$\frac{1}{|V(x)|} > \frac{1}{K\,|x|}$$

$$|t - t_0| = \left|\int_{x_0}^{x} \frac{dx}{V(x)}\right| \geq \int_{x_0}^{x} \frac{dx}{Kx}$$

$$\geq \frac{1}{K} \log\left(\frac{x}{x_0}\right)$$

Ex— like Lipscit

$x \to 0 \qquad t \to \infty$

Έστω $\quad \dot{x} = v(x)$

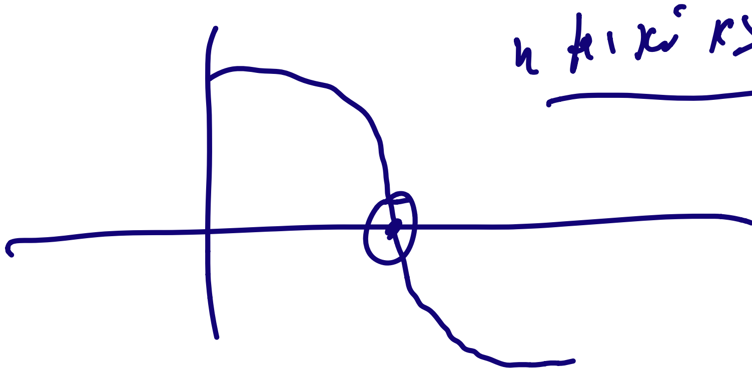
$x(0) = x_0$

υπάρχει μία και μοναδική λύση εάν η $v(x)$ είναι Lipschitz.

$$\dot{x} = x(1-x)$$

$$x(0) = 1 \Rightarrow x(t) = 1$$

η [illegible]



$$\dot{x} = x^{1+\varepsilon}, \quad \varepsilon > 0$$

η λύση εκρήγνυται

σε πεπερασμένο χρόνο

Cauchy

$$\dot{x} = v(x)$$

$x(0) = x_0$ $\quad v(x)$

αναλυτική συνάρτηση

$$\boxed{x(t) = x_0 + t\,\dot{x}(0) + \frac{t^2}{2!}\ddot{x}(0) + \dots}$$

$+ \dots$

$$\dot{x}(0) = v(x(0)) = v(x_0)$$

$$\ddot{x} = \frac{\partial v}{\partial x}\dot{x} \quad , \quad \ddot{x}(0) = \left.\frac{\partial v}{\partial x}\right|_{x_0} v(x_0)$$

$\dddot{x}(0) \dots$ $\quad x^{(n)}(0)$

$x(t)$

$$\boxed{\dot{x} = x + 2}$$

$$\left.\begin{aligned} \dot{x} &= x^2 \\ x(0) &= x_0 \end{aligned}\right\} \text{Cauchy}$$

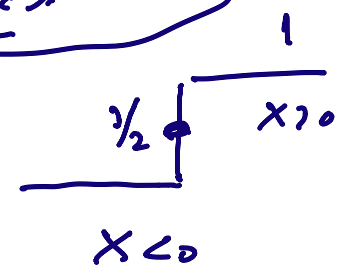
Picard-Lindelöf

weak solution

$$\dot{x} = v(x) \qquad x(0) = x_0$$

$$x(t) = x_0 + \int_0^t v(x(s))\,ds$$

$$v(x) = \theta(x)$$



$$x(t) = 0 \qquad t \le 0$$
$$x(t) = t \qquad t > 0$$

$$x_1 = x_0$$

$$x_{n+1} = x_0 + \int_0^t v(x_n(s))\,ds$$

$$\lim_{n\to\infty} x_n(t) \to x$$

Cau Lipschitz

[illegible]

[illegible]

$$\dot{x} = \alpha x \qquad x(0) = x_0$$

$$x_1 = x_0$$

$$x_{n+1} = x_0 + \int_0^t \alpha x_n \, ds$$

$$x_1 = x_0$$

$$x_2 = x_0 + \int_0^t \alpha x_0 \, ds$$

$$= x_0 + \alpha x_0 t$$

$$x_3 = x_0 + \int_0^t \alpha (x_0 + \alpha x_0 t) \, dt$$

$$= x_0 + \alpha x_0 t + \alpha^2 x_0 \frac{t^2}{2!}$$

$$x_4 = x_0 + \alpha x_0 t + \frac{\alpha^2 x_0 t^2}{2!} + \frac{\alpha^3 x_0 t^3}{3!}$$

$$x_n = \left(1 + \alpha t + \frac{\alpha^2 t^2}{2!} + \cdots + \frac{\alpha^{n-1} t^{n-1}}{(n-1)!}\right) x_0$$

$$\underline{n \to \infty} \qquad \boxed{x(t) = e^{\alpha t} x_0}$$

$$\dot{x} = \lambda x$$

$$x(t) = e^{\lambda t} x_0$$

$$e^{At} = I + At + \frac{A^2 t^2}{2!} + \dots + \frac{A^k t^k}{k!} + \dots$$

$$\boxed{\dot{x} = x(1-x)}$$

$x > 0$

επαναληπτική μέθοδο

$$x^2 + \varepsilon x - 1 = 0$$

$\varepsilon << 1$ $\quad$ $\varepsilon = 0$ $\quad$ $x = \pm 1$

$$\boxed{x = \pm\sqrt{1 - \varepsilon x}}$$

$$\boxed{x_{n+1} = \sqrt{1 - \varepsilon x_n}}$$

$x_1 = 1$

$x_2 = \sqrt{1 - \varepsilon} =$

$$x_3 = \sqrt{1 - \varepsilon \sqrt{1-\varepsilon}}$$

$$\boxed{\begin{aligned} \varepsilon x^2 + x - 1 = 0 \\ x = 1 \end{aligned}}$$

$$\dot{x} = v(x, \alpha)$$

$$x(0) = x_0$$

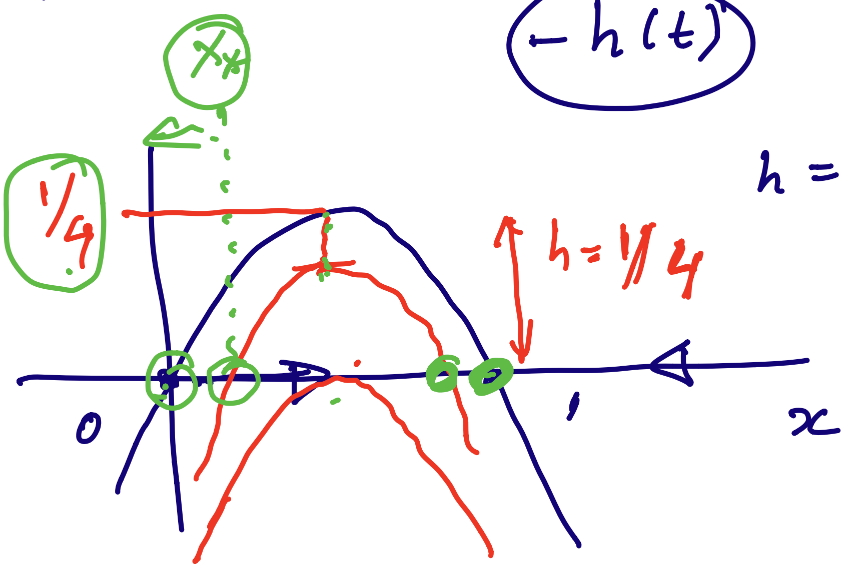
$$x(t) = x(t, \alpha, x_0)$$

εάν $v(x, \alpha)$ είναι
συνεχ. παραγωγ. x, α
και

Τότε η $x(t, \alpha, x_0)$
είναι συνεχής και
παραγωγίσιμη
συνάρτηση του
α και x_0

$$\boxed{\dot{x} = x(1-x) - h} \qquad h \geq 0$$

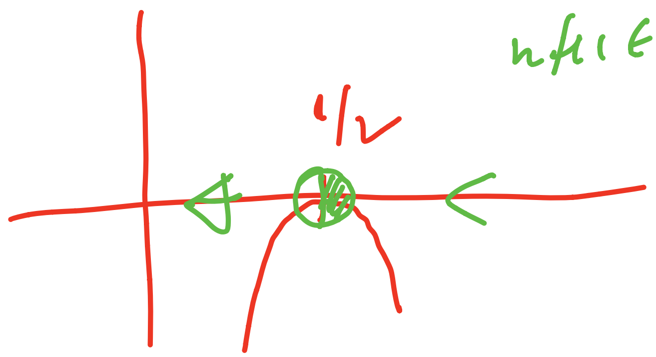
$$-h(t)$$



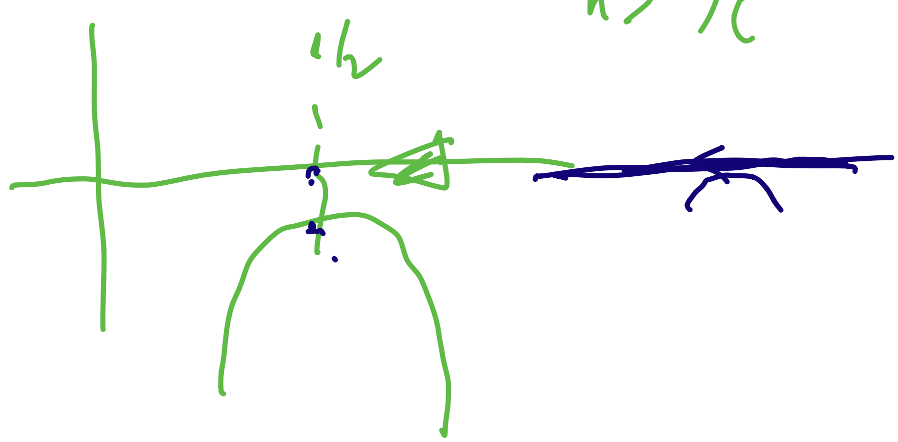
$h = 0$

α $t \to -\infty$

ω $t \to \infty$

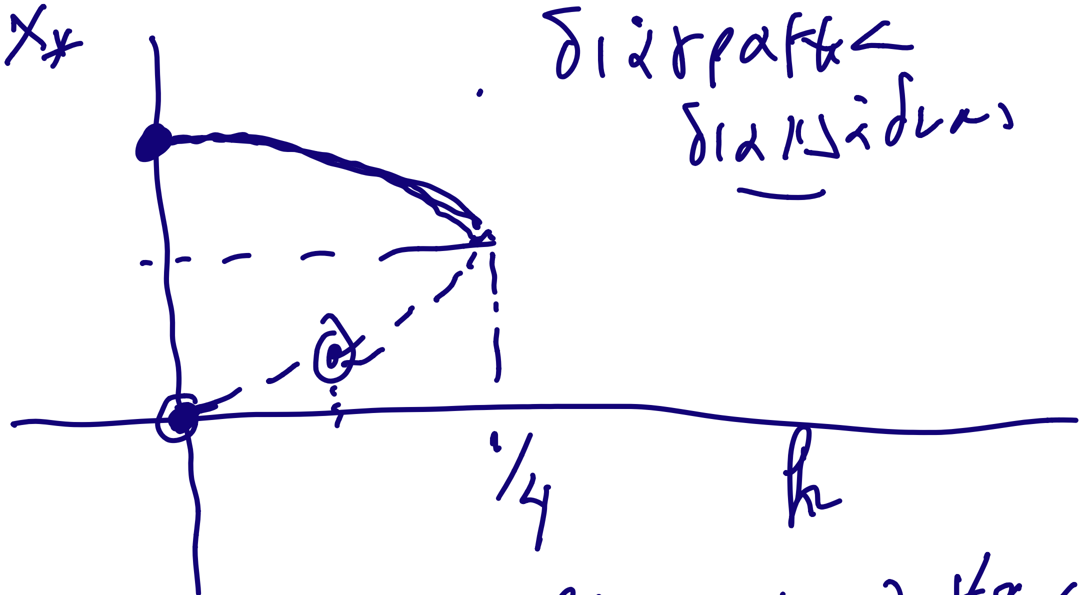


ημιευσταθή σημείο



$h > 1/4$

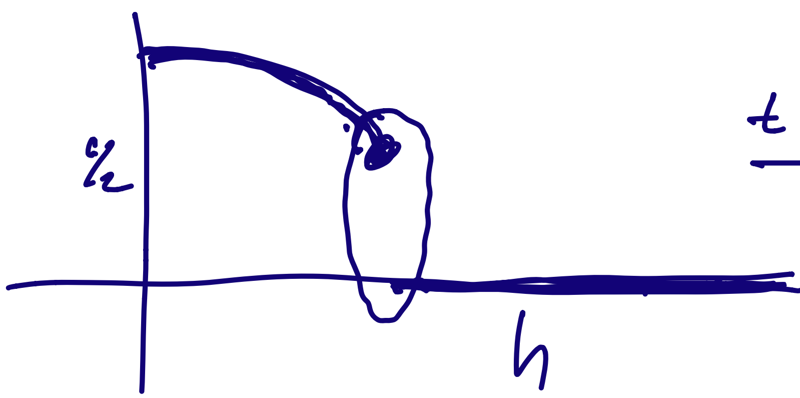
διάγραμμα διακλάδωσης



$$\omega: \{\lim_{t\to\infty} x(t,x_0),\ \forall x_0 \in \mathbb{R}^n\}$$

$$\alpha: \{\lim_{t\to-\infty} x(t,x_0),\ \forall x_0 \in \mathbb{R}^n\}$$

$x_0 > 1/2$



$t \to \infty$

Τι γίνεται στη διακλάδωση