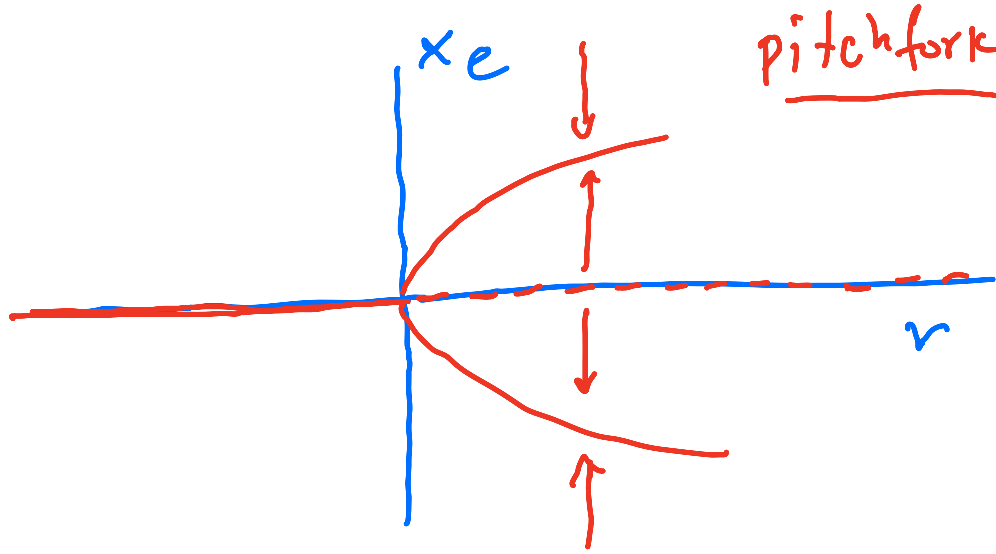
Τρίτη 10 Μαΐου

$$\dot{x} = rx - x^3 = x(r - x^2)$$

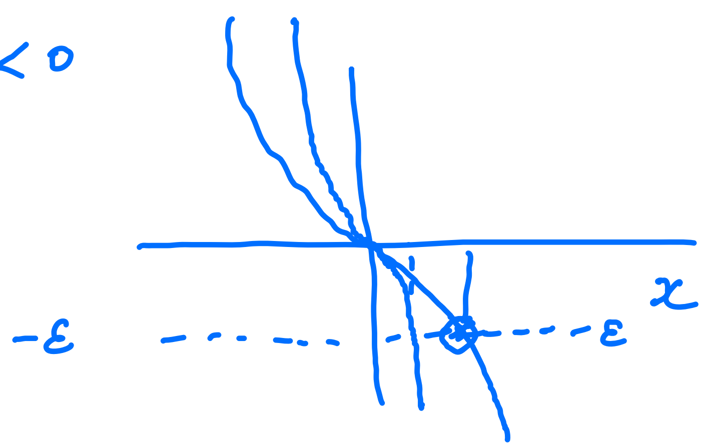
pitchfork



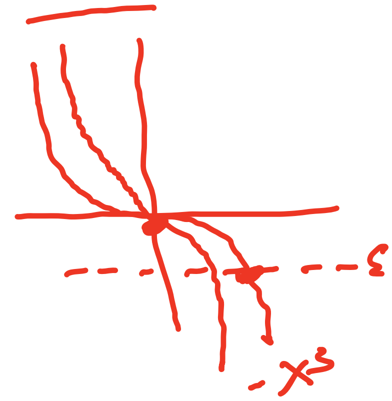
$$\dot{x} = \varepsilon + rx - x^3 \quad ,$$

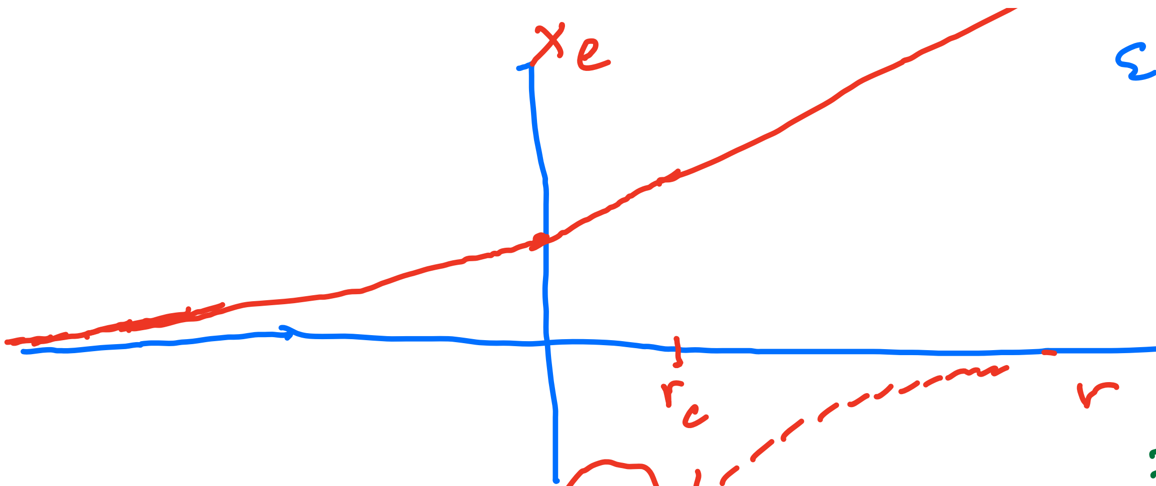
$x_{eq}(\varepsilon, r)$ $\quad$ ε $\quad$ $\varepsilon > 0$

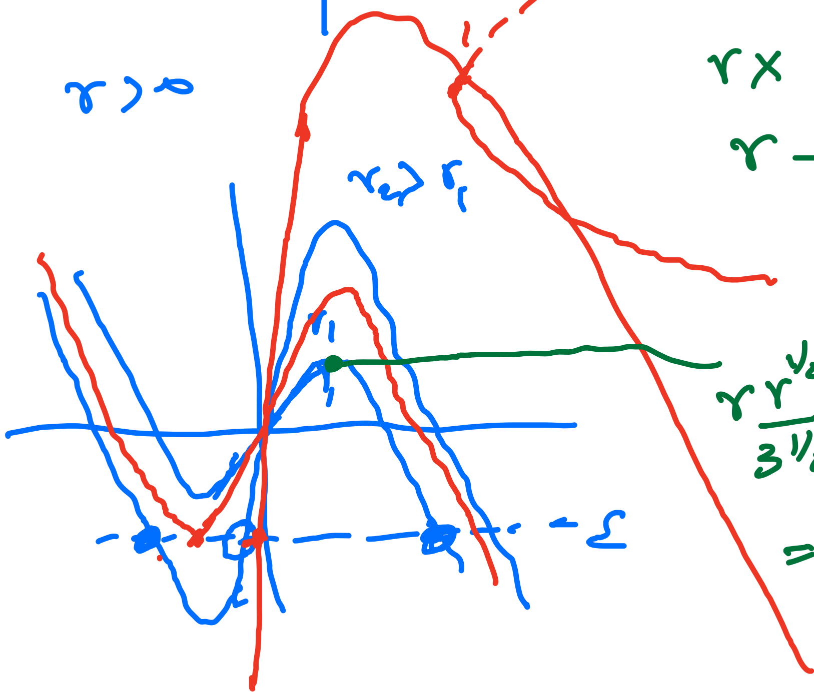
$r < 0$



$r = 0$





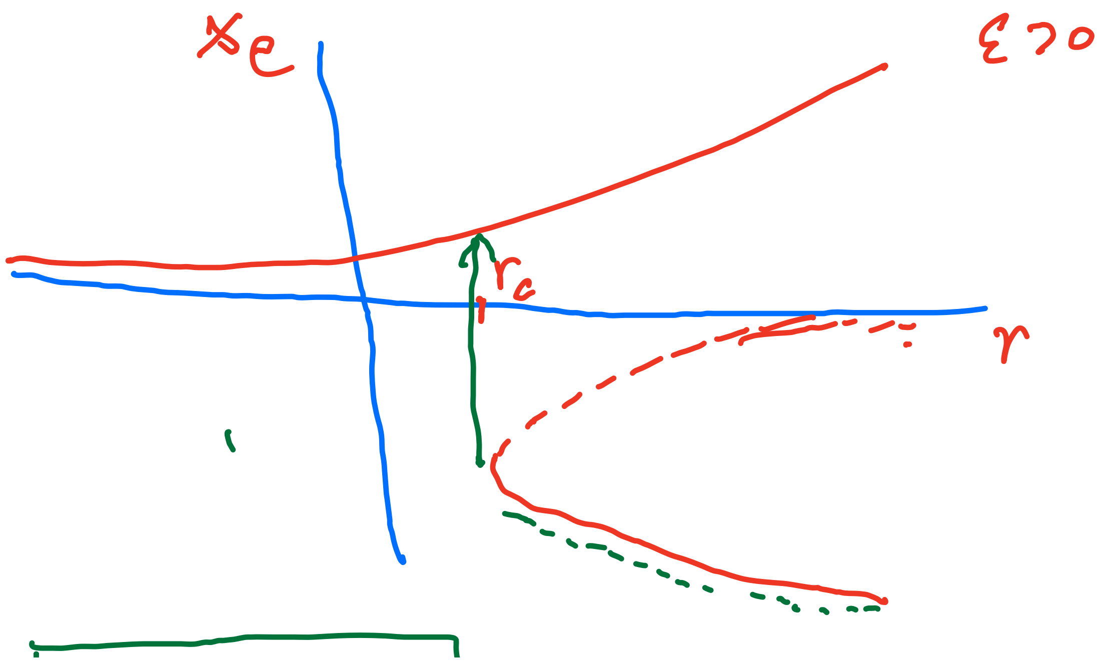


$$rx - x^3$$

$$r - 3x^2$$

$$x = \pm\sqrt{\frac{r}{3}}$$

$$\frac{r\,r^{1/2}}{3^{1/2}} - \frac{r\,r^{1/2}}{3\cdot 3^{1/2}} = \sqrt{\frac{r}{3}}\left(\frac{2r}{3}\right)$$



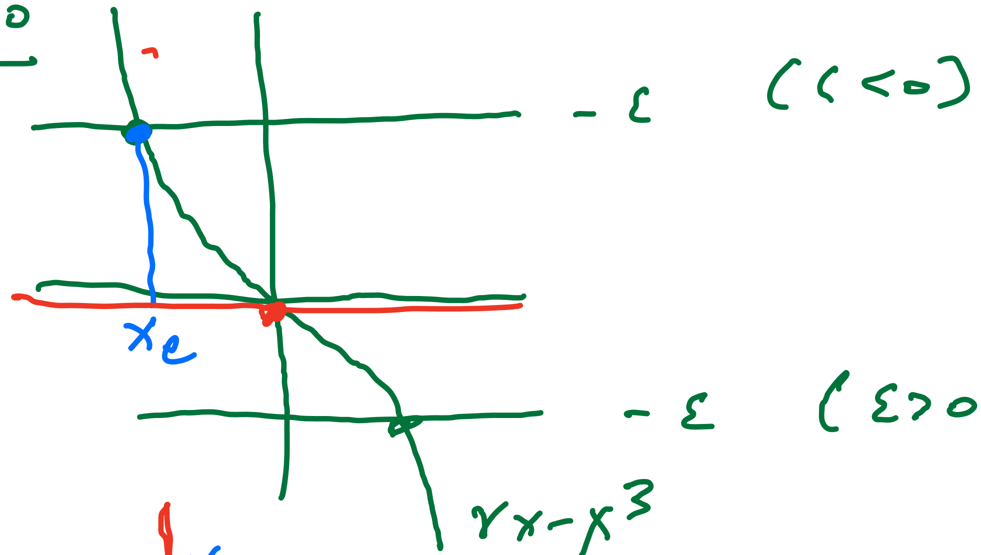
$$\boxed{\varepsilon = \frac{2 r_c^{3/2}}{3^{3/2}}}$$

r σταθερό

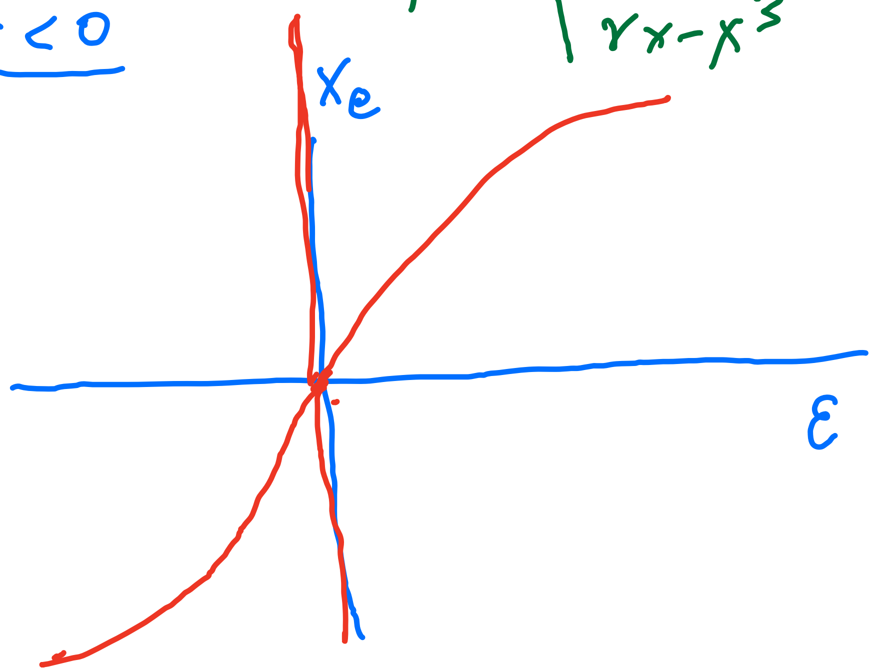
ε

$\varepsilon + r x - x^3 = 0$

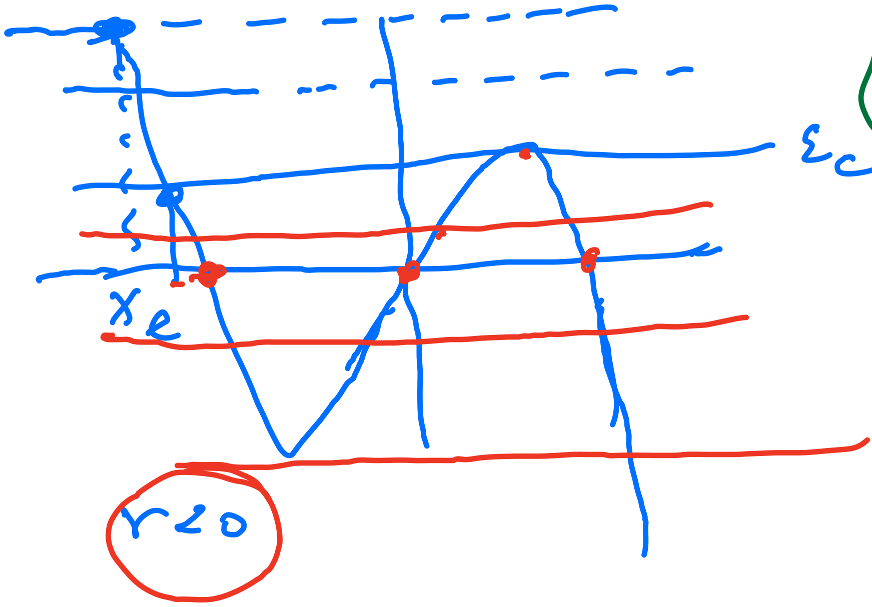
<u>$r < 0$</u>



<u>$r < 0$</u>



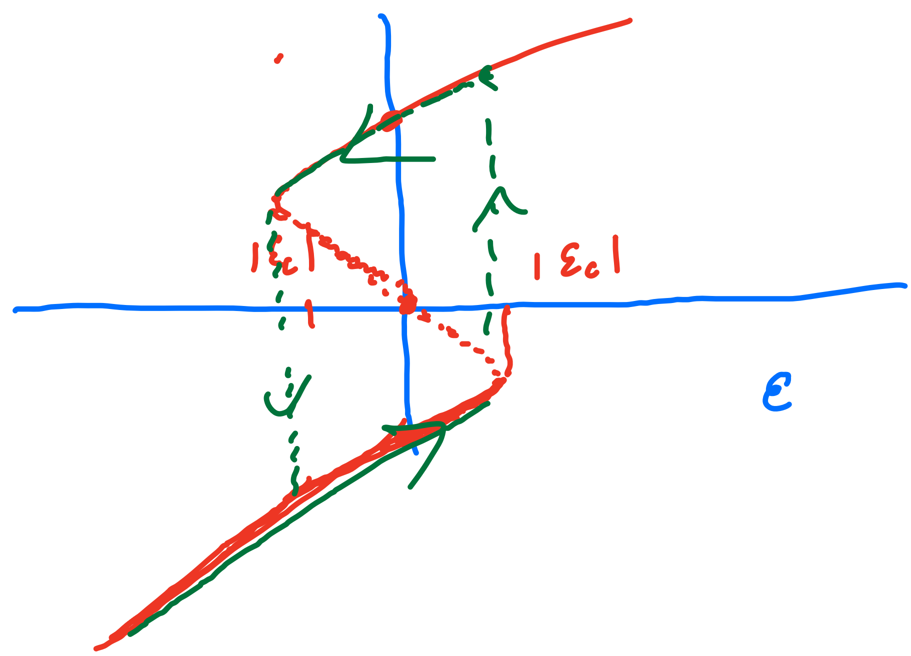
<u>$r > 0$</u>

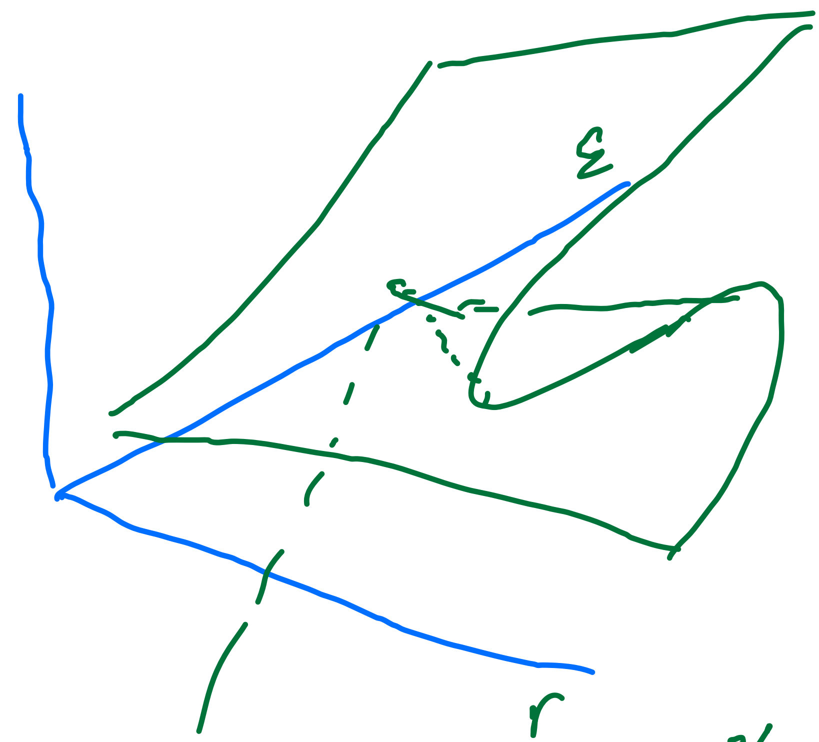


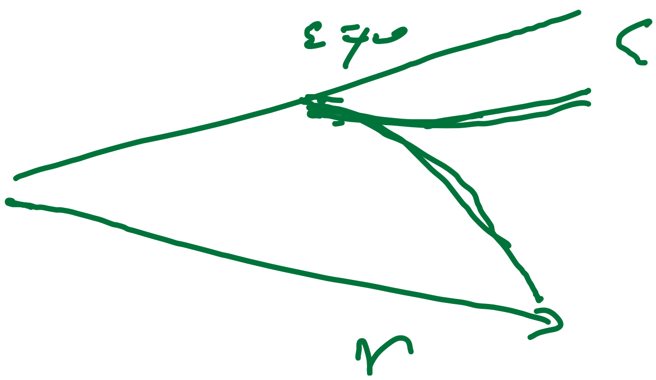
$\varepsilon < 0$

$|\varepsilon| > \frac{2r^{3/2}}{3}$

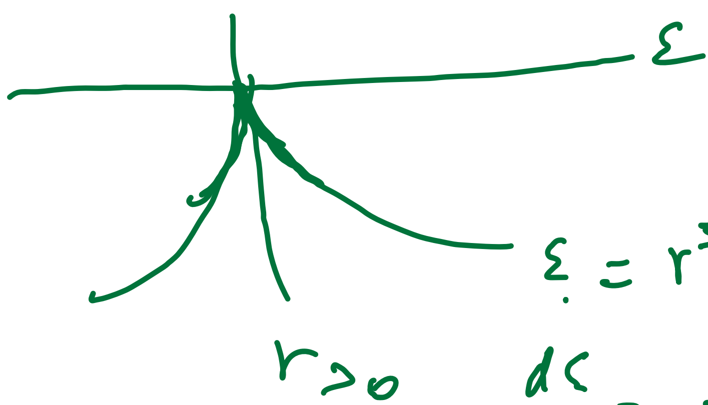
$\varepsilon_c = \frac{2r^{3/2}}{3}$







$\varepsilon = r^{3/2}$



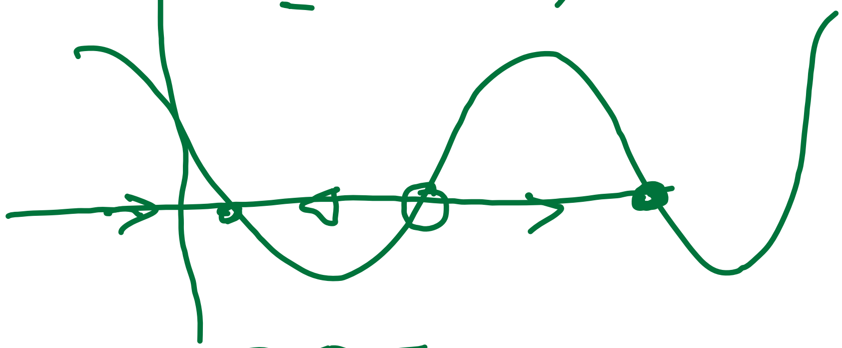
$\varepsilon = r^{3/2}$

$\frac{dr}{d\varepsilon} \to \infty$

$\varepsilon \to 0$

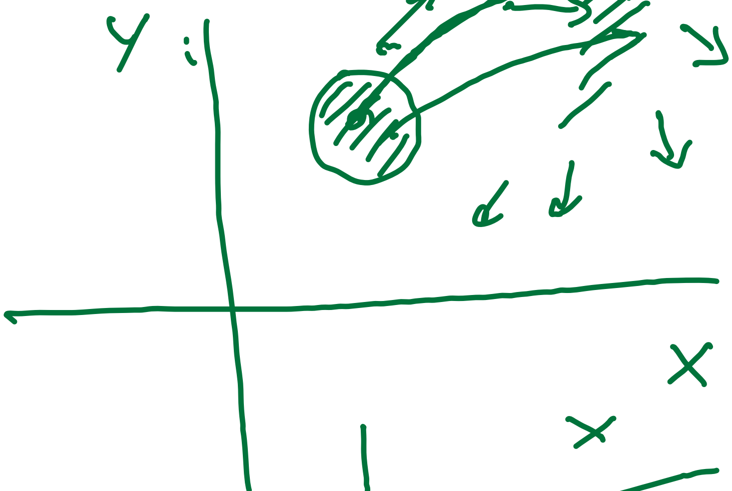
$\frac{d\varepsilon}{dr} = r^{1/2}$

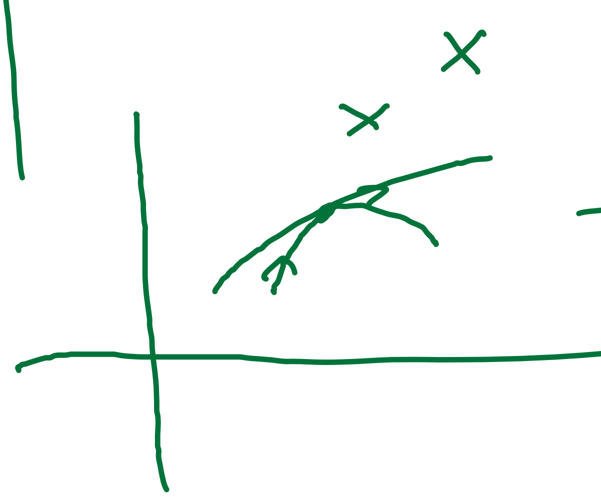
$$\dot{x} = v(x)$$



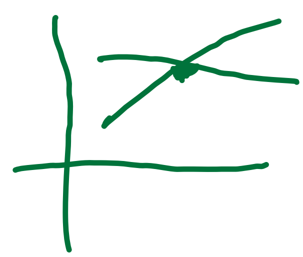
$$\boxed{\begin{aligned} \dot{x} &= v_x(x, y) \\ \dot{y} &= v_y(x, y) \end{aligned}}$$

y :





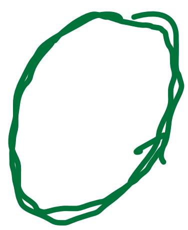
αδιλε



Poincaré – Bendixson

Ελκυστική εστία

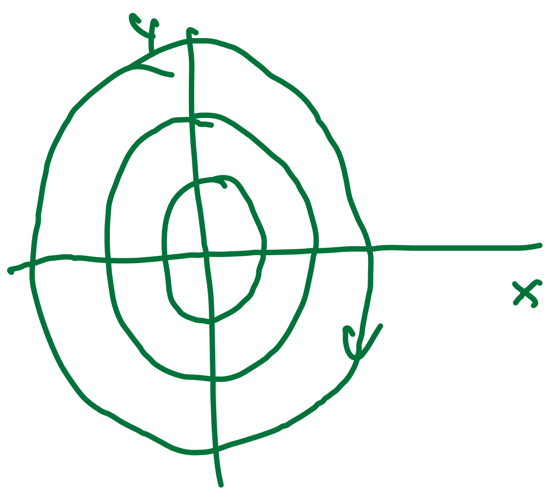
οριακός κύκλος



$$\ddot{x} = -x$$

$$\dot{x} = y$$

$$\dot{y} = -x$$



χάος απαιτεί
σε δυναμικά συστήματα 3
διαστάσεων

Ισορροπία

$$\dot{x} = V_x(x,y)$$
$$\dot{y} = V_y(x,y)$$

$$V_x(x_e, y_e) = 0$$
$$V_y(x_e, y_e) = 0$$

$$x = x_e + \xi$$
$$y = y_e + \eta$$

$$\dot{\xi} = V_x(x_e + \xi, y_e + \eta)$$
$$\dot{\eta} = V_y(x_e + \xi, y_e + \eta)$$

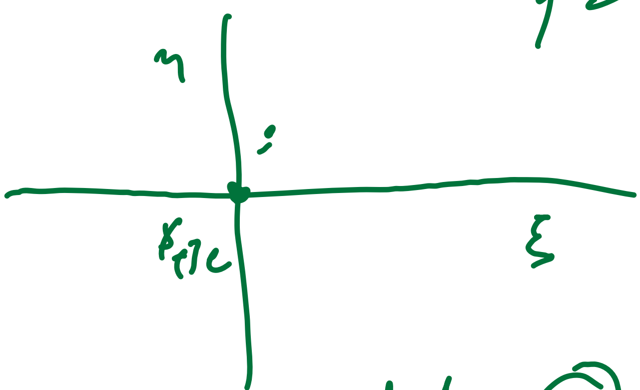
Γραμμικοποίηση γύρω από (x_e, y_e)

$$\dot{\xi} = \cancel{V_x(x_e, y_e)}^{0} + \xi \left.\frac{\partial V_x}{\partial x}\right|_{x_e, y_e} + \eta \left.\frac{\partial V_x}{\partial y}\right|_{x_e, y_e} + O(\xi, \eta)$$

$$\dot{\eta} = \cancel{V_y(x_e, y_e)}^{0} + \xi \left.\frac{\partial V_y}{\partial x}\right|_{x_e, y_e} + \eta \left.\frac{\partial V_y}{\partial y}\right|_{x_e, y_e} + O(\xi, \eta)$$

$$\boxed{\frac{d}{dt}\begin{pmatrix}\xi\\ \eta\end{pmatrix} = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}}$$

$$\psi = \begin{pmatrix}\xi\\ \eta\end{pmatrix}$$



$$\frac{d\psi}{dt} = A\psi$$

$$\psi = e^{tA}\psi(0)$$

$$e^{tA} = I + tA + \frac{t^2A^2}{2!} + \cdots \frac{t^4A^4}{4!} + \cdots$$

$$A\psi_1 = \sigma_1\psi_1 \qquad \underline{\lambda \in \mathbb{R},} \ \mathbb{C}.$$

$$A\psi_2 = \sigma_2\psi_2$$

ψ_1, ψ_2 Eigvec of Matr.

$$\psi = a\psi_1 + b\psi_2 \qquad A^2\psi_1 = \sigma_1^2\psi_1$$

$$\underline{\psi(0)=\psi_1} \qquad \psi(t) = e^{tA}\psi_1$$

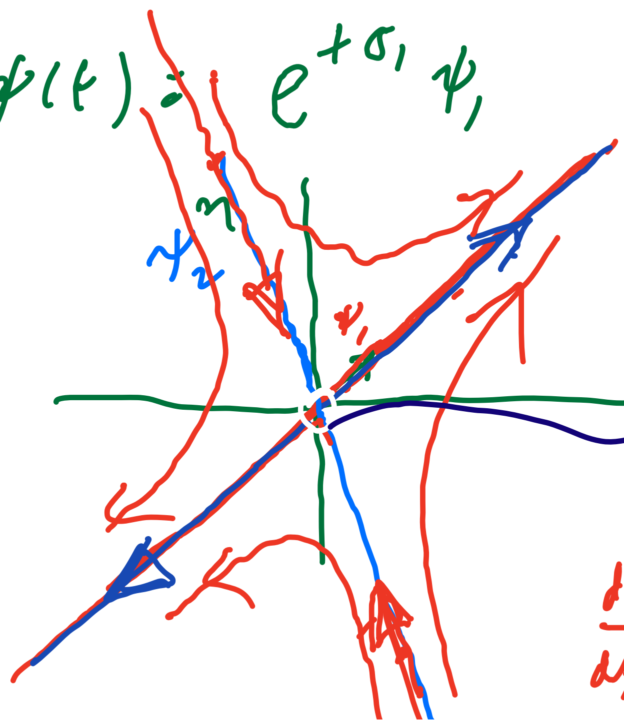
$$= \left[I + tA + t^2\frac{A^2}{2!} + \cdots\right]\psi_1$$

$$= \psi_1 + t\sigma_1\psi_1 + \frac{t^2\sigma_1^2}{2!}\psi_1 + \cdots + \frac{t^4\sigma_1^4}{4!}\psi_1$$

$$= \underbrace{\left(1 + t\sigma_1 + \frac{t^2\sigma_1^2}{2!} + \cdots + \frac{t^4\sigma_1^4}{4!} + \cdots\right)}_{e^{t\sigma_1}}\psi_1$$

$t \gg 1$ מתנהג $e^{\sigma_1 t}$

$$\psi(t) = e^{t\sigma_1}\psi_1 \qquad \psi(0) = \psi_1$$



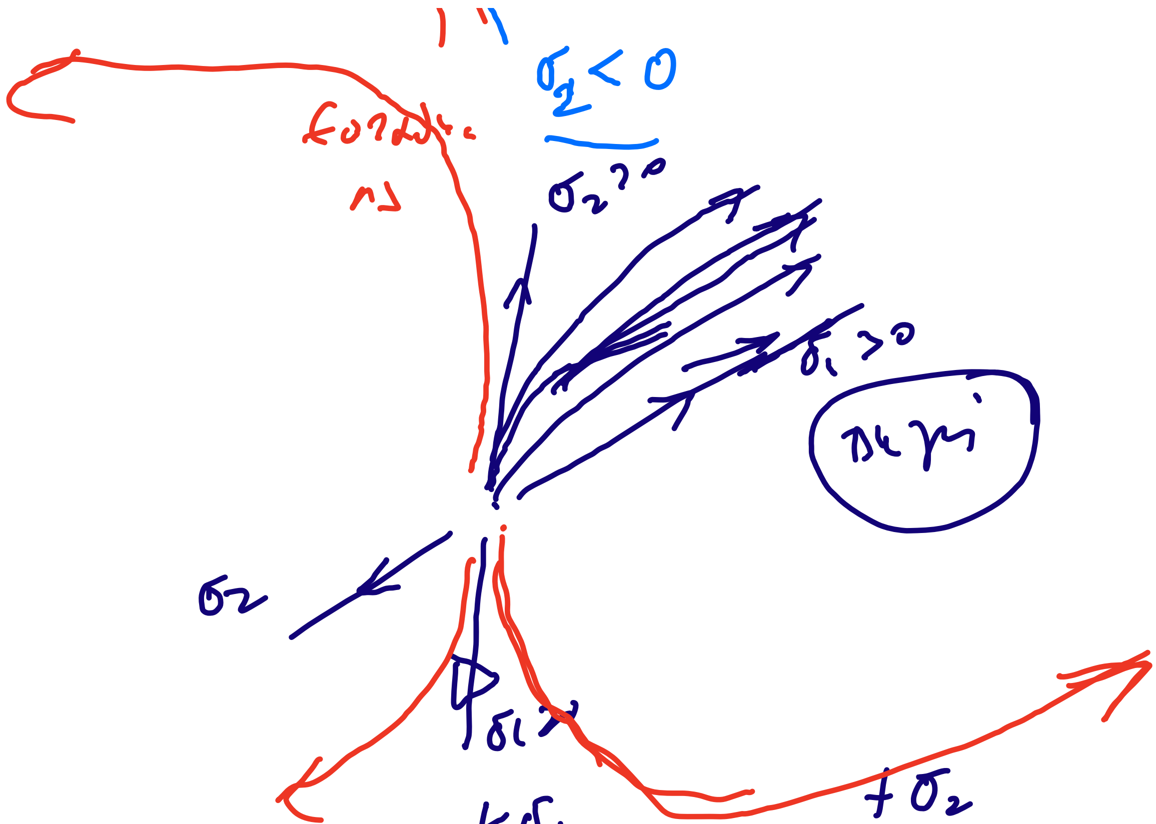
$\sigma_1 > 0$

[illegible]

[illegible]

$\psi_1 \in \mathbb{R}^2$

$$\frac{d\psi}{dt} = A\psi$$



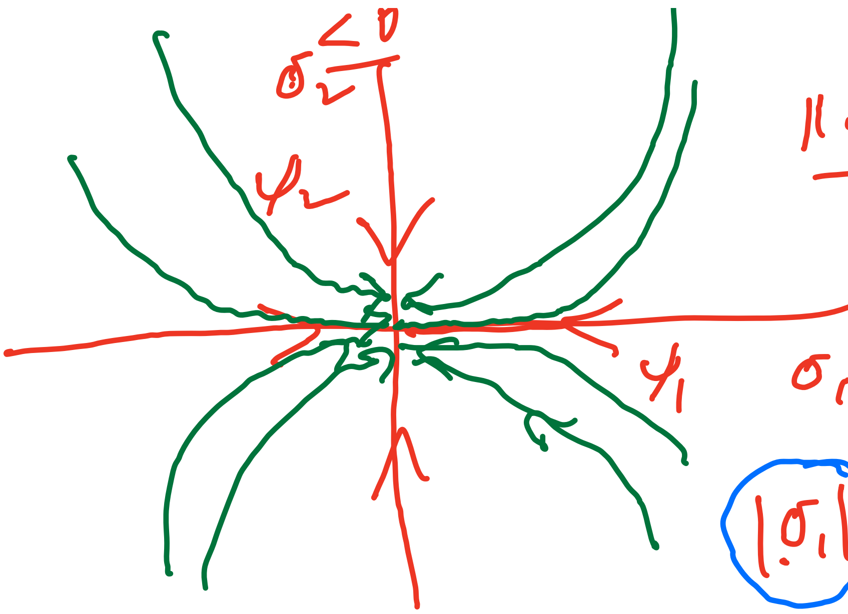
$$\psi(t) = \lambda e^{t\sigma_1} \psi_1 + \mu e^{t\sigma_2} \psi_2$$

$$\psi(0) = \lambda \psi_1 + \mu \psi_2$$

$$\sigma_1 > \sigma_2$$

$$t \gg 1 \qquad \psi(t) \sim \psi_1$$

$$\sigma_1 , \sigma_2 < 0$$



Καταβόθρα

$\sigma_1 < 0$

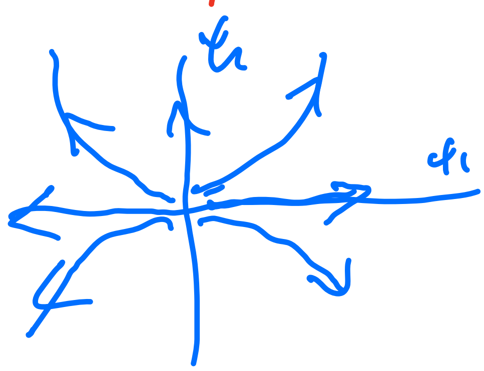
$|\sigma_1| < |\sigma_2|$

$$\psi = \lambda e^{\sigma_1 t}\psi_1 + \lambda e^{\sigma_2 t}\psi_2$$

$t \to \infty$

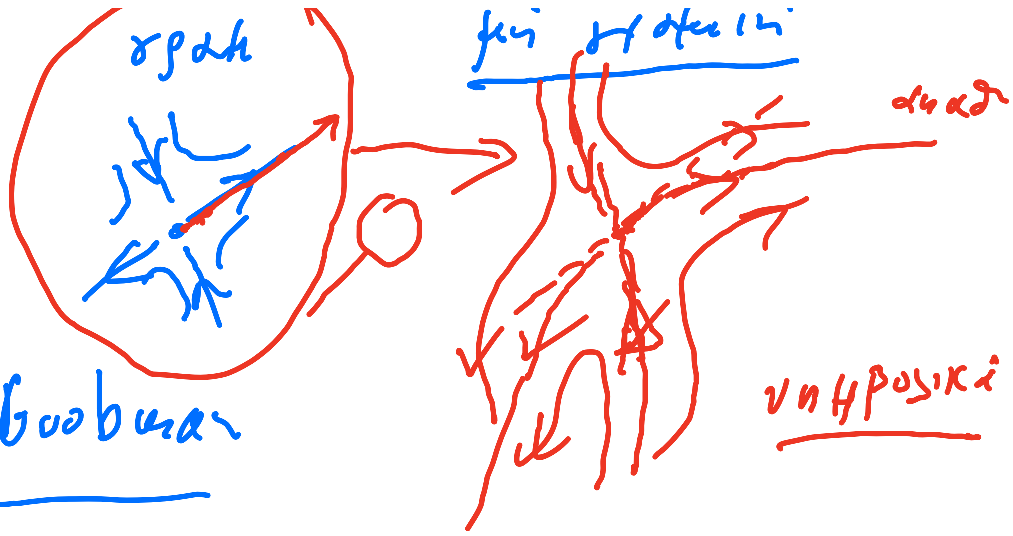
Εάν αναστρ. $\psi \sim$

$t \to -t$

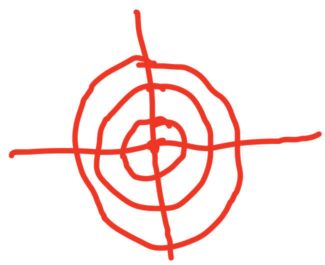


ψ_1, ψ_2 πραγματικοί διανύσμ. βάση

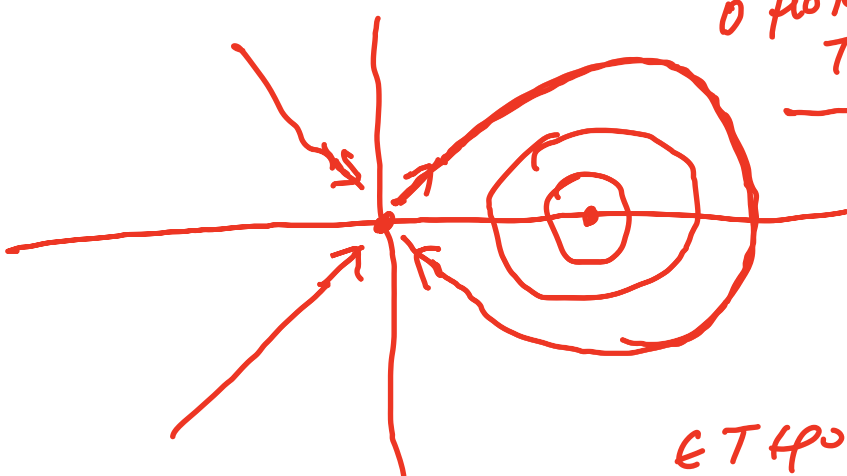
$Re(\sigma_1),(\sigma_2) \neq 0$



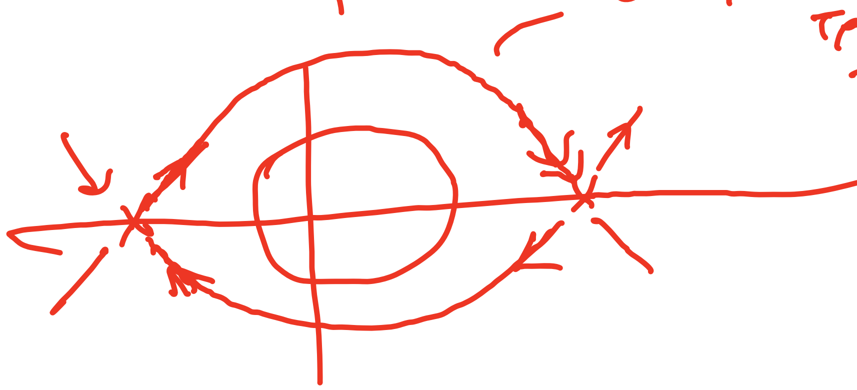
Hartman-Grobman

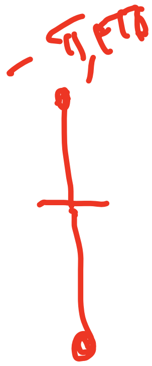


ομοκλινική τροχιά



ετεροκλινική τροχιά





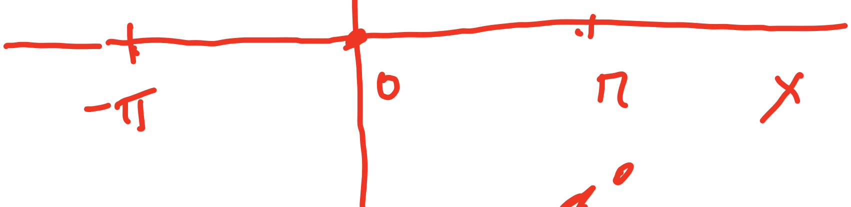
$$\frac{d^2\theta}{dt^2} = -\sin\theta$$

$x = \theta$

$y = 0$

$x = 0,\ x = \pm\pi$

$$\boxed{\begin{aligned} \frac{dx}{dt} &= y. \\ \frac{dy}{dt} &= -\sin x \end{aligned}}$$



$x = \pi$

$$\sin(\pi + \xi) = \cancel{\sin\pi}^{\,0} + \xi\cos\pi + O(\xi^2)$$

$$\left.\begin{aligned} x &= \pi + \xi \\ y &= \eta \end{aligned}\right| \qquad \left.\begin{aligned} \frac{d\xi}{dt} &= \eta \\ \frac{d\eta}{dt} &= \xi \end{aligned}\right)$$

$$\frac{d}{dt}\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{A}\begin{pmatrix} \xi \\ \eta \end{pmatrix}$$

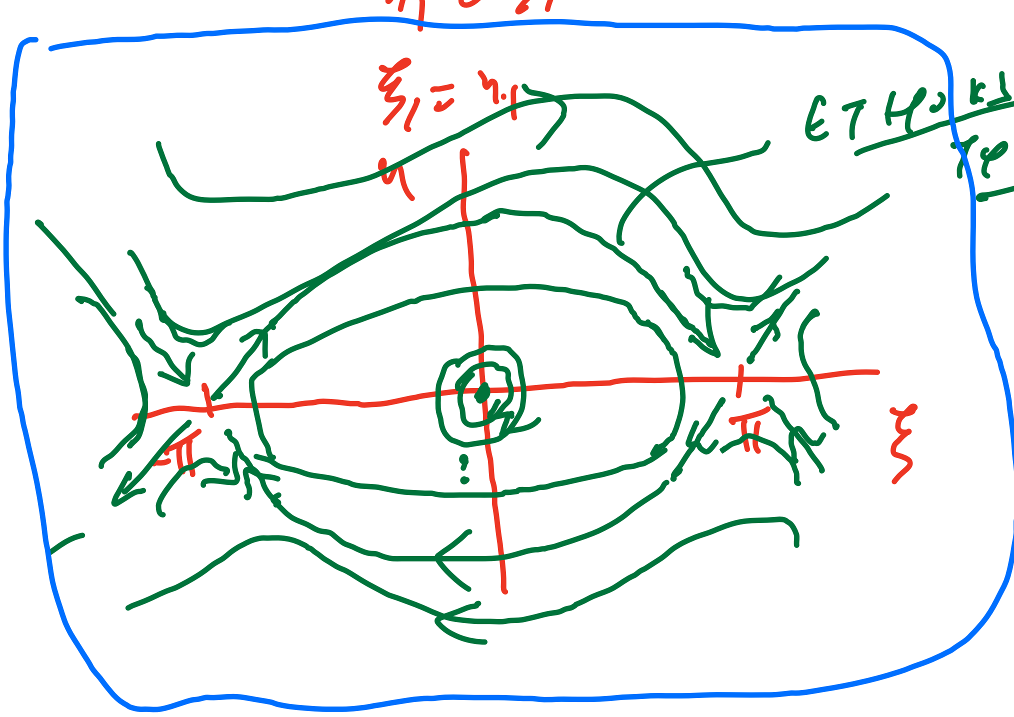
$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \qquad \lambda^2 = 1 \qquad \lambda = \pm 1$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi_1 \\ \eta_1 \end{pmatrix} = +1 \begin{pmatrix} \xi_1 \\ \eta_1 \end{pmatrix}$$

-1 $\quad \eta_1 = -\xi_1$

$\eta_1 = \xi_1$



γύρω από το (0,0)

$$\frac{d\xi}{dt} = \eta \qquad \frac{d}{dt}\begin{pmatrix}\xi\\ \eta\end{pmatrix} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}$$

$$\frac{d\eta}{dt} = -\xi$$

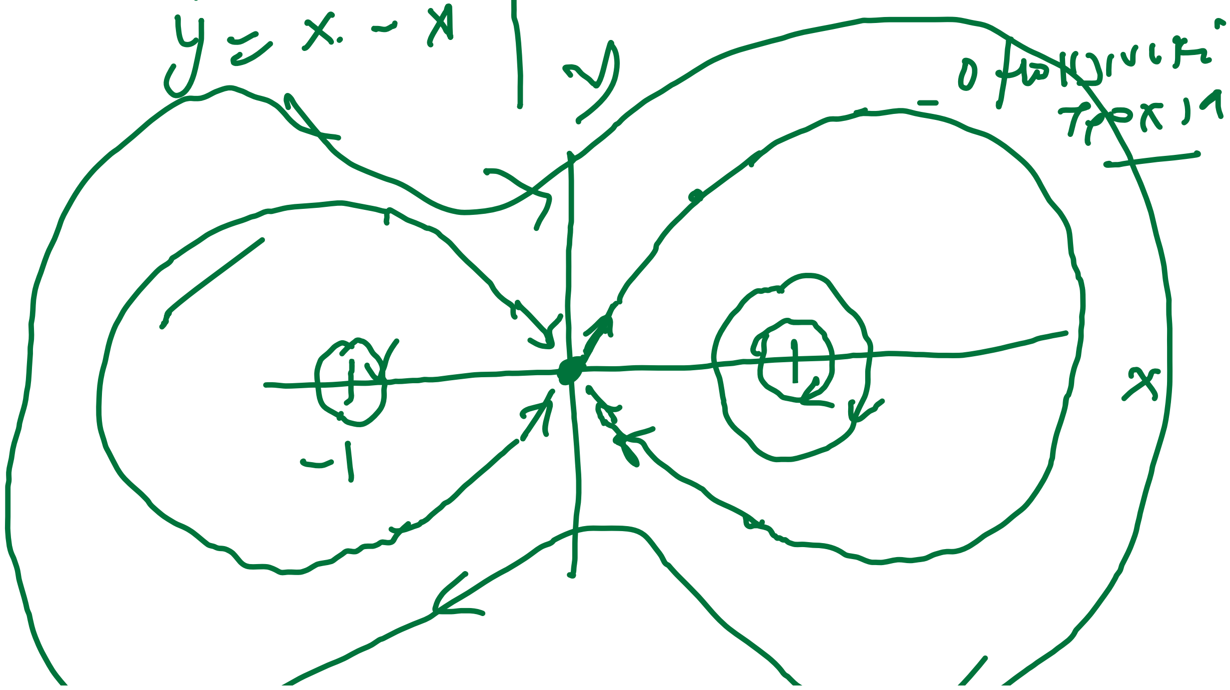
$$\begin{vmatrix}-\lambda & 1\\ 1 & -\lambda\end{vmatrix} = 0 \qquad \lambda^2+1=0$$

$$\lambda = \pm i$$

$$\ddot{x} = x - x^3$$

$$\dot{x} = y \qquad y = 0$$

$$\dot{y} = x - x^3 \qquad x = \pm 1,\ x = 0$$



$\underline{x=0}$ $\qquad$ $\dot{x} = y$

$\dot{y} = x$

$-\; x = 1+\xi$ $\qquad$ $\dot{\xi} = \eta$

$y = \eta$ $\qquad$ $\dot{\eta} = 1+\xi - (1+\xi)^3$

$= -2\xi + O(\xi^2)$

$\dot{\xi} = \eta$ $\qquad$ $A = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$

$\dot{\eta} = -2\xi$

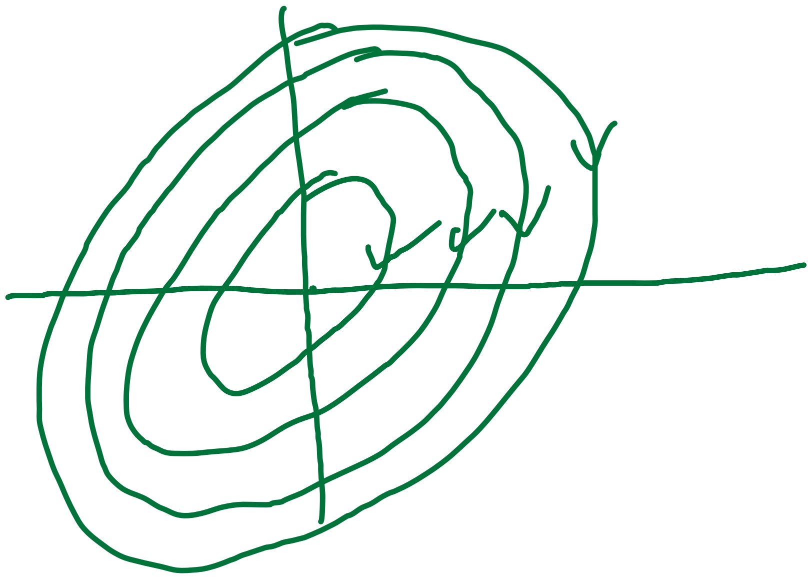
$\lambda^2 = -2$ $\qquad$ $\boxed{\lambda = \pm\sqrt{2}\, i}$

$\sigma_1 = \sigma_r + i\sigma_i$ $\qquad \psi$

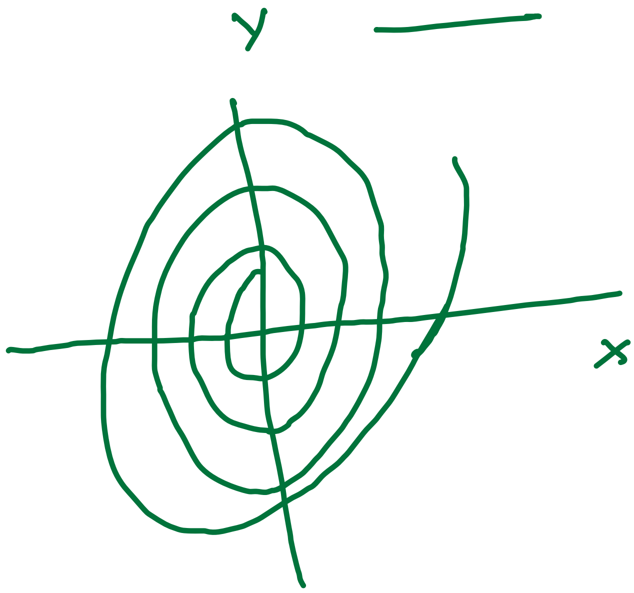
$\sigma_2 = \sigma_r - i\sigma_i$ $\qquad \psi^*$

$\psi = \lambda e^{\sigma_r t} e^{i\sigma_i t}\psi + \lambda^* e^{\sigma_r t} e^{-i\sigma_i t}\psi^*$

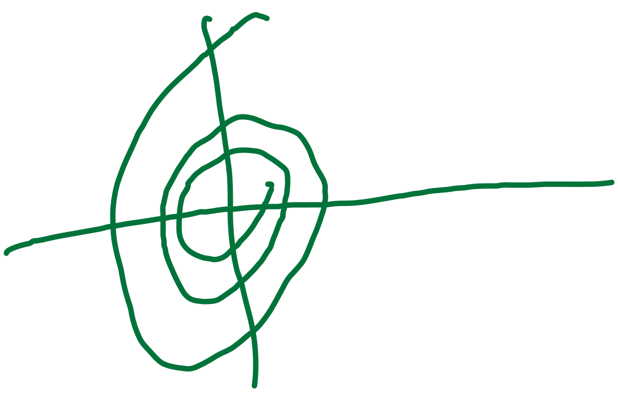
$\psi \in R$ $\qquad$ $\underline{\sigma_r = 0}$



$\underline{\sigma_{xy} > 0}$



$\sigma_{xy} < 0$

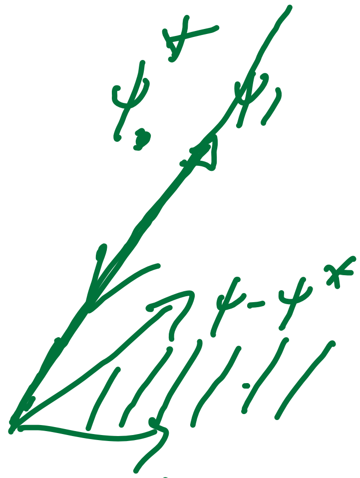


ψ_1 ψ_2 ψ_3^* ψ_1

$\psi + \psi^*$

$\psi - \psi^*$

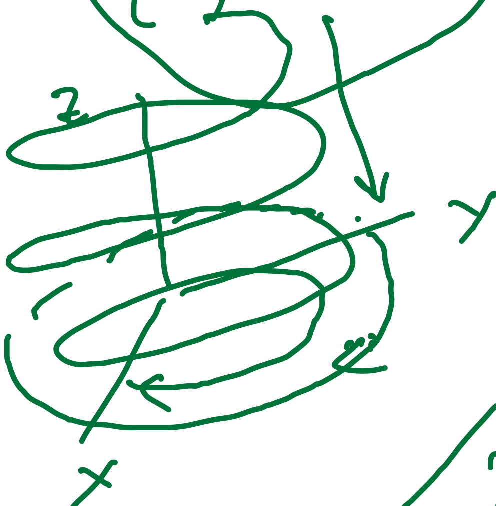
node

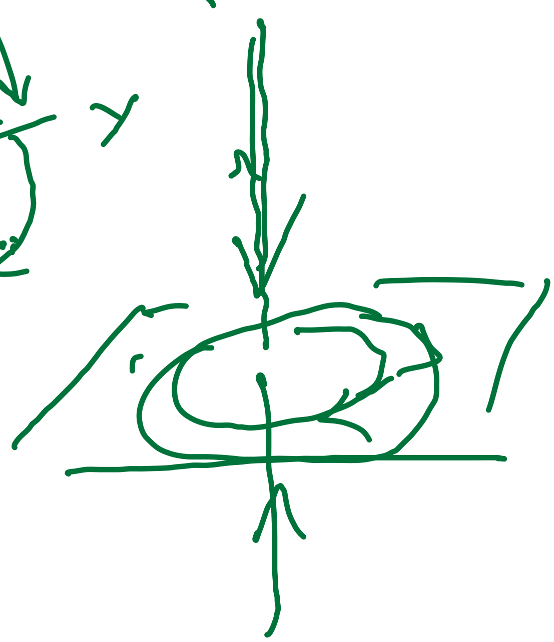


$$\begin{cases} \dot z = -z \\ \dot x = y \\ \dot y = -x \end{cases}$$

$(0, 0, z_1)$

$(x_1, y_1, 0)$





$$\dot{x} = y$$

$$\dot{y} = -x - y$$

$$\dot{z} = z$$