

Data Structures & Algorithms

Finite State Machines-FSMs

Abstract model for coin operated telephone system



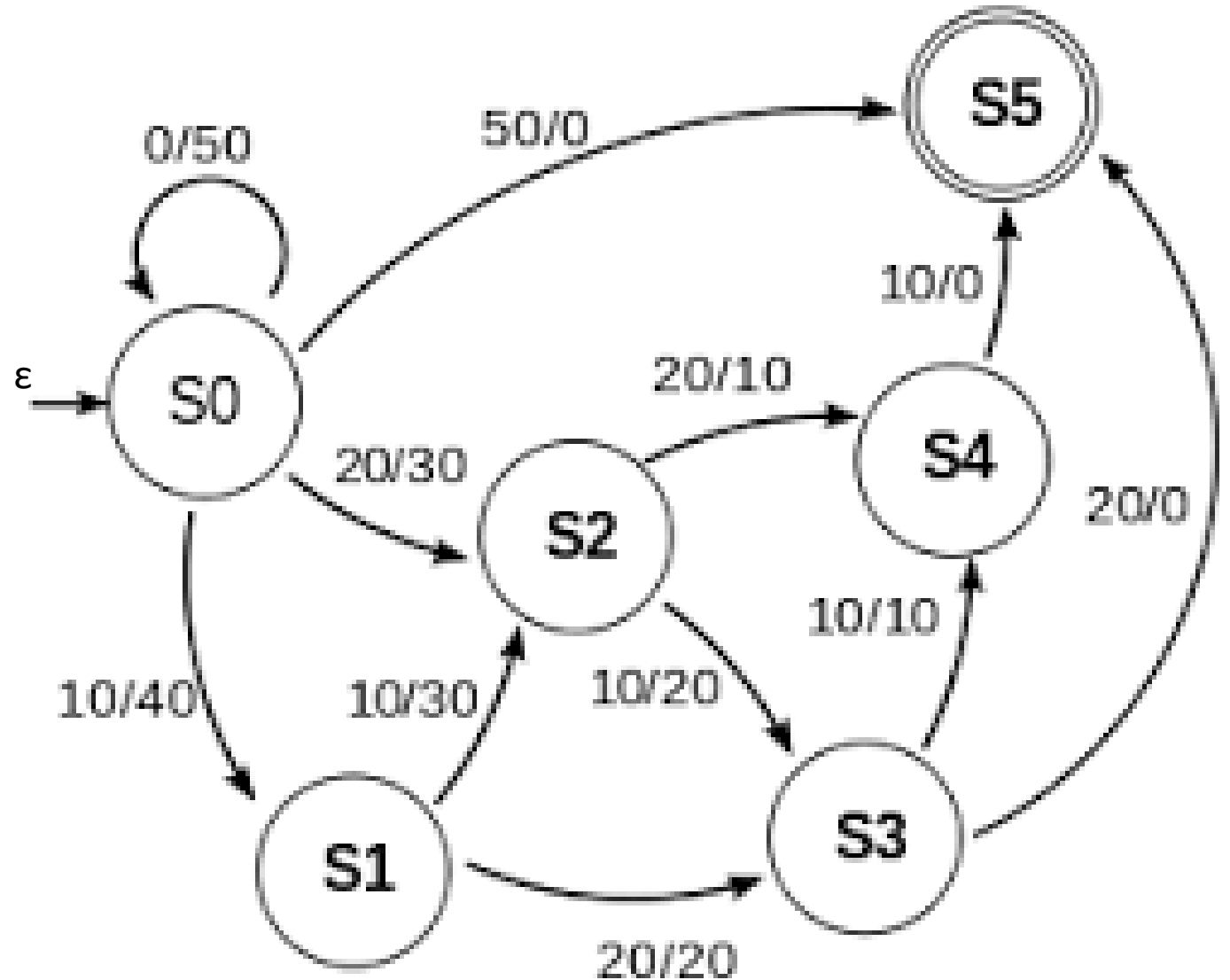
State



Transition
Input/output

ϵ void

Set of symbols
{ ϵ , 10, 20, 50}



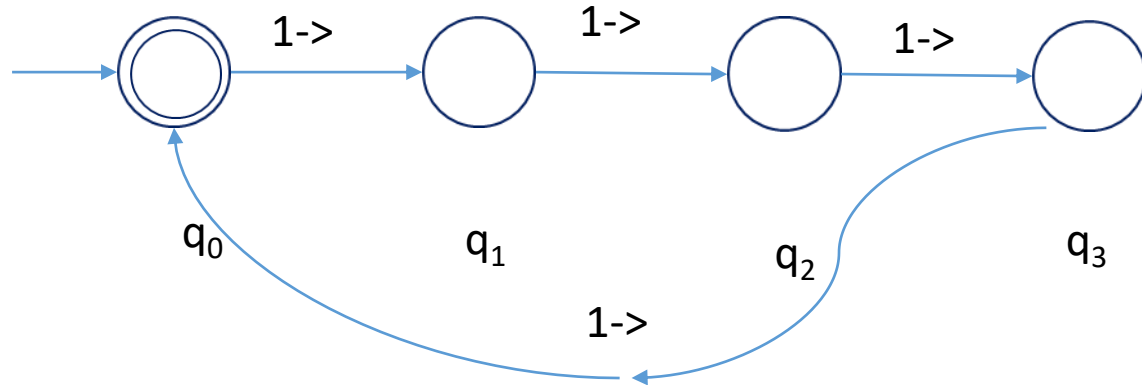
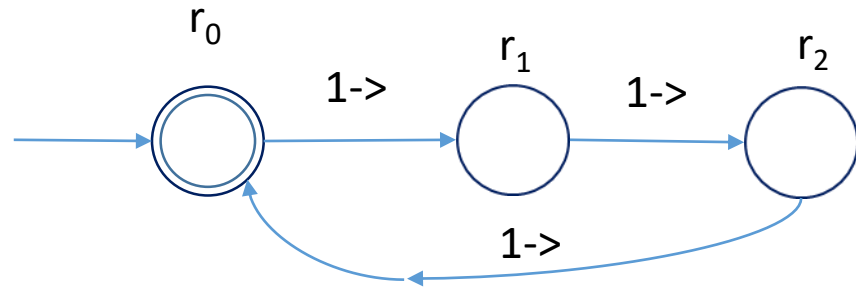
Finite State Machines-FSMs

- **FSM** is a 4-tuple $(Q, q_0, Next, Out)$
- Q is a finite set of possible states
- q_0 is the initial state
- **Next** is a function mapping the $\langle \text{state}, \text{input symbols} \rangle$ to states
- **Out** is a function mapping the $\langle \text{state}, \text{input symbols} \rangle$ to output symbols

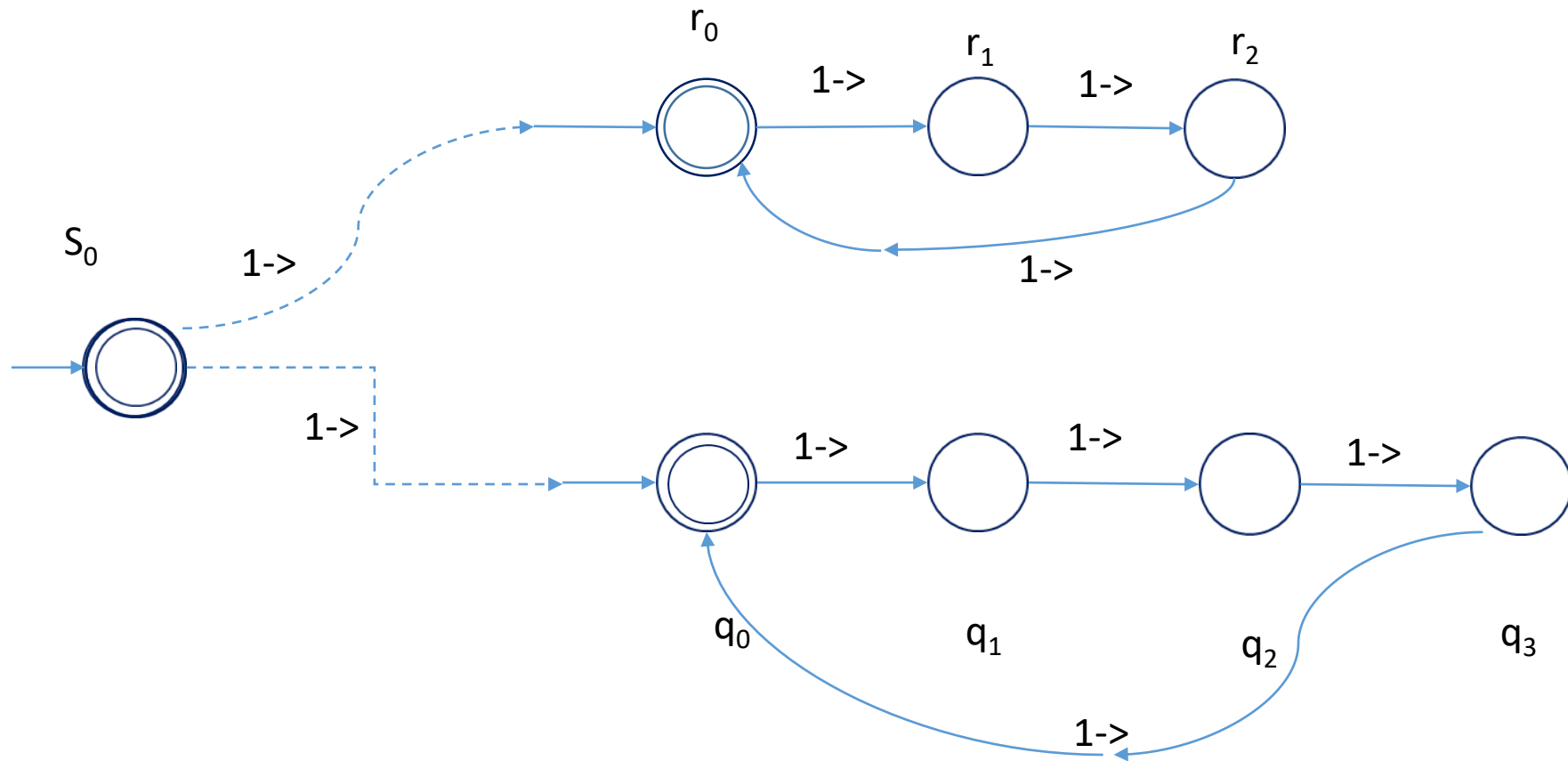
There are Finite State **Recognizers** (FSRs) that get a limited number of input sentences and we call the input set as *alphabet*

There are also the Finite State **Generators** (FSGs) that produce certain output strings (a string/sentence = a set of output symbols, produced one-symbol/stage)

Non Deterministic FSM



Non Deterministic FSM



Non Deterministic FSM

- NDFSM is a 4-tuple $(Q, q_0, Next, F)$
- Q is a finite state of states
- q_0 is the initial state
- F is a set of final states $F \subseteq Q$
- **Next** is a function defined on certain pairs (q, a) of states and input symbols (a can be ϵ) and yields sets of possible next states (subsets of Q). If Next is defined for (q, ϵ) then it is undefined for (q, b) , b another input symbol.

\forall NDFSM \exists DFSM

Proof by construction: Let NDFSM $(Q, q_0, Next, F)$. Construct $(Q', q_0', Next', F')$ so that:

- $Q' = 2^Q$
- $q_0' = \{q_0\}$
- $Next'(\{q_1, \dots, q_r\}, a) = Next(q_1, a) \cup \dots \cup Next(q_r, a)$. The $Next'(\{q_1, \dots, q_r\}, a)$ provides a transition to the state that represents the set $\{q_1, \dots, q_r\}$.
- $F' = \{q' \subseteq Q' \mid q' \cap F \neq \{\}\}$. Final state is any state that contains a final state of the original NDFSM