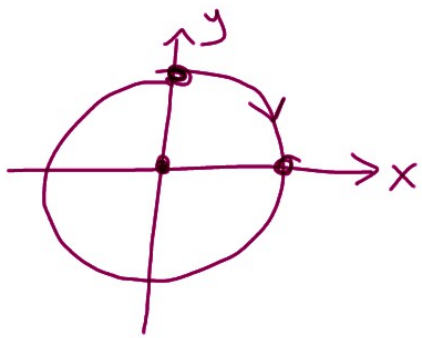
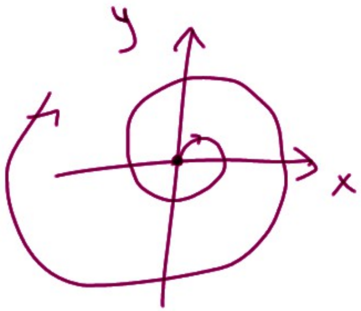
1. Αν $\bar{r} = t\sin t\,\hat{x} + t\cos t\,\hat{y} + t\,\hat{z}$

ποια η τροχιά; Ποια η L_z;

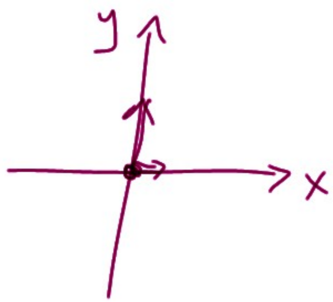
<u>Λύση</u>: $x = t\sin t$, $y = t\cos t$, $z = t$



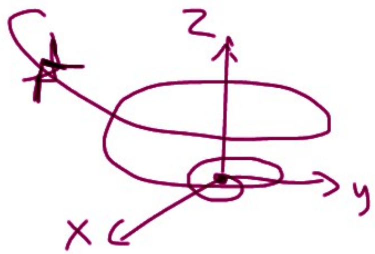


$x^2 + y^2 = t^2$

$$\bar{v} = (\sin t + t\cos t)\,\hat{x} + (\cos t - t\sin t)\,\hat{y} + \hat{z}$$

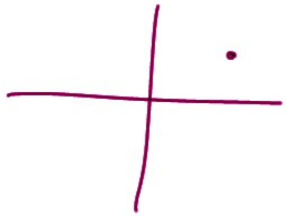


$$\bar{\alpha} = (2\cos t - t\sin t)\,\hat{x} + (-2\sin t - t\cos t)\,\hat{y}$$



$$\bar r = \underline{t\sin t}\,\hat x + t\cos t\,\hat y + t\,\hat z$$

Ποια τα $\varpi(t)$, $\varphi(t)$, $\underbrace{z(t)}_{t}$;



$$x = \varpi\cos\varphi$$

$$\varpi\cos\varphi = t\sin t$$

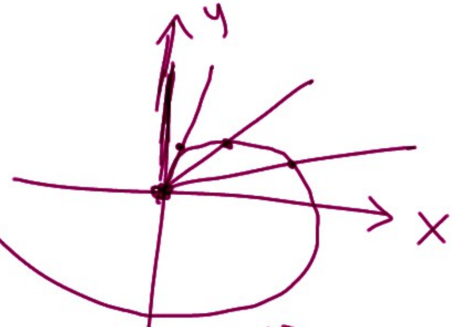
$$\varpi = \sqrt{x^2+y^2} = \sqrt{t^2} = \boxed{t = \varpi}$$

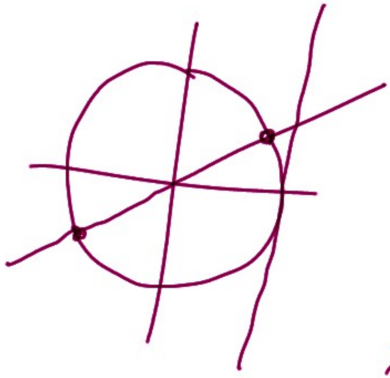
$$x = \varpi\cos\varphi \Leftrightarrow \cos\varphi = \sin t = \cos\left(\frac{\pi}{2} - t\right)$$

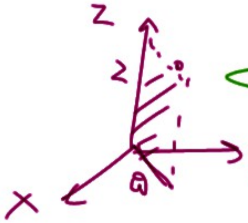
$$y = \varpi\sin\varphi \Leftrightarrow \sin\varphi = \cos t = \sin\left(\frac{\pi}{2} - t\right)$$

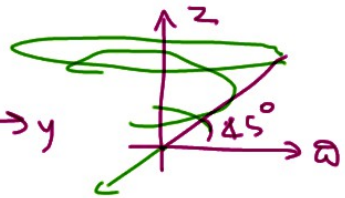
$$\boxed{\varphi = \frac{\pi}{2} - t}$$

$$\begin{cases} \cos\varphi = \dfrac{x}{\sqrt{x^2+y^2}} \\ \sin\varphi = \dfrac{y}{\sqrt{x^2+y^2}} \end{cases}$$

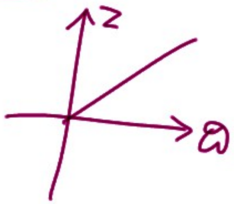








$$\underline{\varpi = z}$$



Η ταχύτητα σε κυλινδρικές

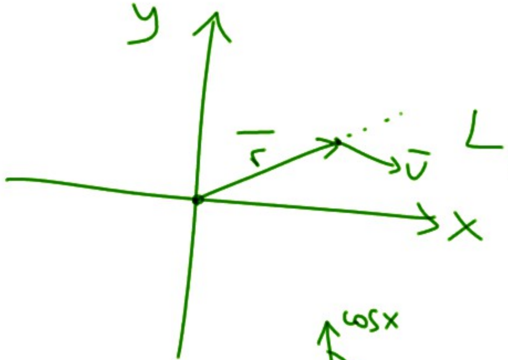
$$\bar{v} = \underbrace{\dot{\varpi}}_{1}\hat{\varpi} + \underbrace{\varpi\dot{\varphi}}_{-t}\hat{\varphi} + \underbrace{\dot{z}}_{1}\hat{z} = \hat{\varpi} - t\hat{\varphi} + \hat{z}$$

$v_\varpi = 1$

$v_\varphi = -t$

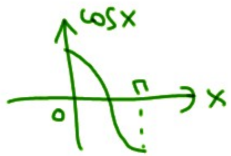
$v_z = 1$

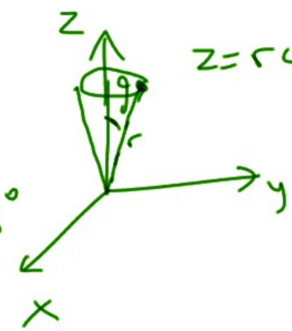
$$\bar{L}_z = \left(\bar{r} \times m\bar{v}\right)^{\hat{z}} = m\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & \\ \dot{x} & \dot{y} & \end{vmatrix} \cdot \hat{z} = m\left(x\dot{y} - y\dot{x}\right)$$



$$L_z = m\varpi^2\dot{\varphi} = -mt^2$$

Σφαιρικές συντεταγμένες;





$$r = \sqrt{x^2 + y^2 + z^2} = t\sqrt{2}, \quad \vartheta = \arccos\frac{z}{r} = \arccos\frac{1}{\sqrt{2}} = \frac{\pi}{4} = 45^\circ$$

$$\varphi = \frac{\pi}{2} - t$$

$$\bar{v} = \dot{r}\hat{r} + r\dot{\vartheta}\hat{\vartheta} + r\sin\vartheta\,\dot{\varphi}\hat{\varphi} = \sqrt{2}\,\hat{r} - t\hat{\varphi}$$

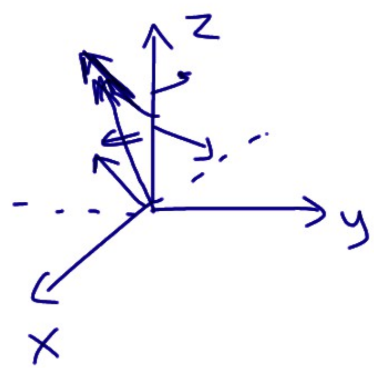
2. $\bar{r} = 3\hat{x} - 4\hat{y} + 3\hat{z}$ σε κυλινδρικές;

Ποια τα ϖ, φ, z του σώματος;

$\varpi = \sqrt{x^2+y^2} = 5$, $z = 3$, $\left.\begin{array}{l} \sin\varphi = \dfrac{-4}{5} \\ \cos\varphi = \dfrac{3}{5} \end{array}\right\}$ $\varphi \in \left(\dfrac{3\pi}{2}, 2\pi\right)$

$\varphi = 2\pi - \arcsin\dfrac{4}{5}$

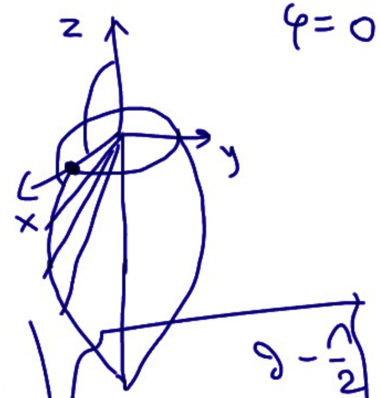
$\bar{r} = \varpi\hat{\varpi} + z\hat{z} = 5\underline{\underline{\hat{\varpi}}} + 3\hat{z}$



3. Σε σφαιρικές $\underline{v_r = v_\theta = v_\varphi}$. Αρχικά $\underbrace{\vec{r}\big|_{t=0} = r_0 \hat{x}}_{\text{δηλ. για } t=0,\ r=r_0,\ \theta=\pi/2,\ \varphi=0}$.

(α) Τροχιά;

(β) Αν $\varphi = \omega t$ κίνηση σώματος;



Λύση:

(α) $\dot{r} = r\dot{\theta} = r\sin\theta\,\dot{\varphi}$

$$\frac{dr}{\cancel{dt}} = r\frac{d\theta}{\cancel{dt}} \Leftrightarrow \int\frac{dr}{r} = \int d\theta \Leftrightarrow \ln r = \theta + C$$

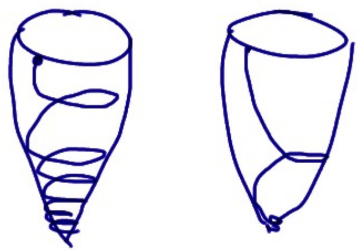
αρχικές $\ln r_0 = \frac{\pi}{2} + C$

$$\boxed{r = r_0 e^{\theta - \frac{\pi}{2}}}$$

$$\cancel{r}\frac{d\theta}{\cancel{dt}} = \cancel{r}\sin\theta\frac{d\varphi}{\cancel{dt}} \Leftrightarrow \int d\varphi = \int\frac{d\theta}{\sin\theta} \Leftrightarrow \varphi = \ln\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} + \cancel{D}$$

(β)

$$\cos\theta = -\tanh(\omega t)$$



4. $\varpi = 2R\sin\varphi$, $0 < \varphi < \pi$, $R > 0$

(α) $\hat{\varepsilon}$, $\hat{n}$, R , θεωρήστε $\dot{\varphi} > 0$.

(β) Σχήμα τροχιάς;

(γ) Για ποιο $\varphi(t)$ είναι $\bar{\alpha} \parallel \bar{r}$;

Λύση:

(α) $\bar{\upsilon} = \underset{\substack{\uparrow \\ 2R\cos\varphi\,\dot{\varphi}}}{\dot{\varpi}}\hat{\varpi} + \varpi\dot{\varphi}\hat{\varphi} = 2R\dot{\varphi}\left(\cos\varphi\,\hat{\varpi} + \sin\varphi\,\hat{\varphi}\right)$

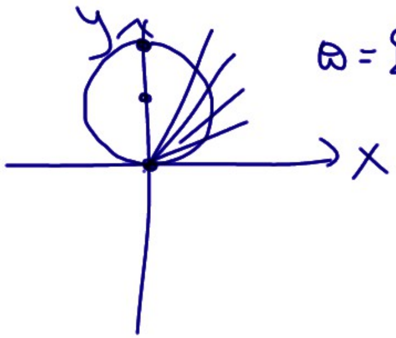
$$\hat{\varepsilon} = \frac{\bar{\upsilon}}{|\bar{\upsilon}|} = \frac{\bar{\upsilon}}{2R|\dot{\varphi}|} \xRightarrow{\dot{\varphi}>0} \cos\varphi\,\hat{\varpi} + \sin\varphi\,\hat{\varphi}$$

$$\bar{\alpha} = \Big(\underset{\substack{\downarrow \\ 2R\cos\varphi\ddot{\varphi} - 2R\sin\varphi\dot{\varphi}^2}}{\ddot{\varpi}} - \varpi\dot{\varphi}^2\Big)\hat{\varpi} + \left(2\dot{\varpi}\dot{\varphi} + \varpi\ddot{\varphi}\right)\hat{\varphi} = \left(2R\ddot{\varphi}\cos\varphi - 4R\dot{\varphi}^2\sin\varphi\right)\hat{\varpi} +$$
$$+ \left(2R\ddot{\varphi}\sin\varphi + 4R\dot{\varphi}^2\cos\varphi\right)\hat{\varphi}$$

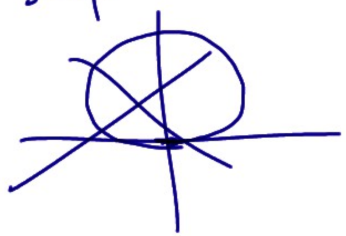
$$\bar{\alpha}_\varepsilon = \left(\bar{\alpha}\cdot\hat{\varepsilon}\right)\hat{\varepsilon} = 2R\ddot{\varphi}\,\hat{\varepsilon} \ , \quad \bar{\alpha}_k = \bar{\alpha} - \bar{\alpha}_\varepsilon = 4R\dot{\varphi}^2\left(-\sin\varphi\,\hat{\varpi} + \cos\varphi\,\hat{\varphi}\right)$$

$$\hat{n} = \frac{\bar{\alpha}_k}{|\bar{\alpha}_k|} = -\sin\varphi\,\hat{\varpi} + \cos\varphi\,\hat{\varphi} \ , \quad |\bar{\alpha}_k| = \frac{\upsilon^2}{\mathcal{R}} \Leftrightarrow \mathcal{R} = \frac{\upsilon^2}{|\bar{\alpha}_k|} = R = \text{σταθερό!}$$

$$\left.\begin{aligned} x &= \varpi\cos\varphi = 2R\sin\varphi\cos\varphi = R\sin(2\varphi) \\ y &= \varpi\sin\varphi = 2R\sin^2\varphi = R - R\cos(2\varphi) \end{aligned}\right\} \Rightarrow x^2 + (y-R)^2 = R^2$$



$\varpi = 2R\sin\varphi$



$$\alpha_\varphi = 0 \Leftrightarrow \frac{1}{\varpi}\frac{d}{dt}\left(\varpi^2\dot\varphi\right) = 0 \Leftrightarrow \varpi^2\dot\varphi = \sigma\tau\alpha\theta = \frac{L}{m}$$

$$4R^2\sin^2\varphi\,\dot\varphi = \frac{L}{m} \Leftrightarrow \int \sin^2\varphi\,d\varphi = \frac{L}{4mR^2}\int dt$$

$$\boxed{\frac{\varphi}{2} - \frac{\sin(2\varphi)}{4} = \frac{L}{4mR^2}t + D}$$