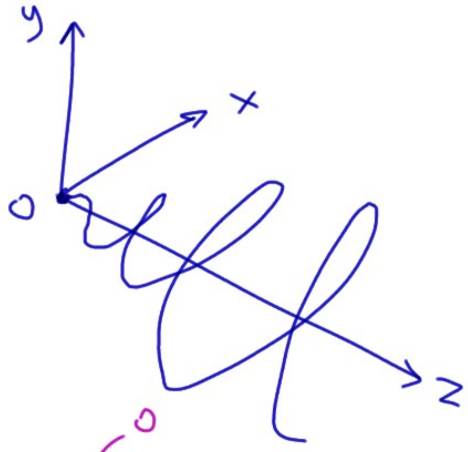
Θέμα 1 από εξέταση 13/2/2013

Κωνική έλικα $\varpi = z,\ \varphi = z,\ z \geq 0$



$m = 1$

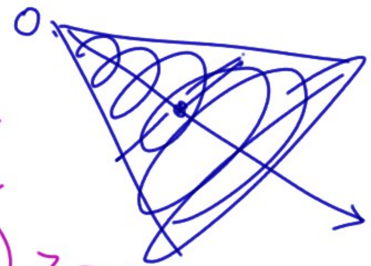
τριβές $= 0$

$\bar g = -\hat y$

Ψάχνω το $z(t)$.

$\cancel{m\bar\alpha = \underbrace{\bar N}_{\perp \bar v} + m\bar g}$

$t = \int_{0.07}^{0} \frac{dz}{\dot z} =$



$\ddot z + (\ )z = 0$

$\bar v = \dot\varpi\hat\varpi + \varpi\dot\varphi\hat\varphi + \dot z\hat z =$

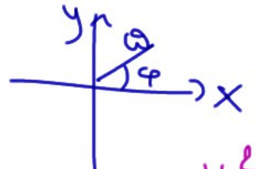
$= \dot z\hat\varpi + z\dot z\hat\varphi + \dot z\hat z$

$\frac{mv^2}{2} + V = E \Leftrightarrow$

$V = mgy \quad (-m\bar g\cdot\bar r)$

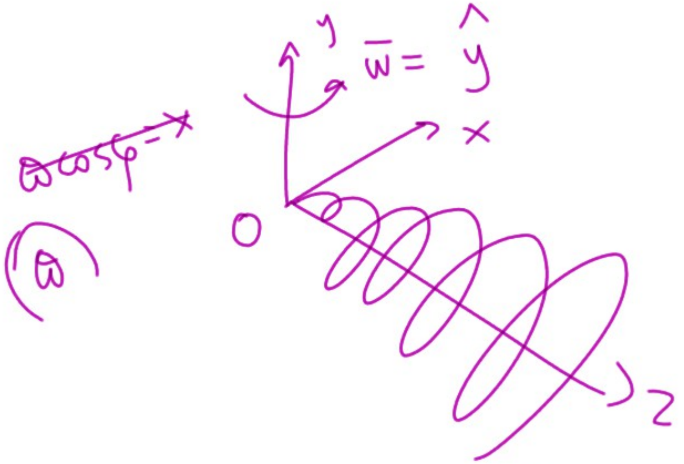
$= mg\varpi\sin\varphi =$

$= mgz\sin z$



$$\boxed{\frac{m}{2}\dot z^2(2+\cancel{z^2}) + mgz\underbrace{\sin z}_{z} = E} \quad \begin{pmatrix} m=1 \\ g=1 \end{pmatrix}$$

Αρχικά $z=0$, $\frac{mv^2}{2}=\frac{mv_0^2}{2}$ άρα $E = v_0^2/2$. Όταν σταματά $mgz_{max}\sin z_{max} = E \Leftrightarrow z_{max}\sin z_{max} = \frac{v_0^2}{2}$



Η έλικα περιστρέφεται γύρω από τον $\hat{y}$ άξονα με $\bar{w}=\hat{y}$.

Σε πόσο χρόνο γυρνά αν $z_{max}=0.07$;

(αρχικά $v=v_0$ και σταματά στο z_{max})

Λύση:

$$\bar{v}_\sigma = \dot{z}\hat{\varpi} + z\dot{z}\hat{\varphi} + \dot{z}\hat{z}$$

$$m\bar{\alpha}_\sigma = \underbrace{m\bar{g}+\bar{N}} - \cancel{m\bar{\alpha}_0} - \cancel{m\dot{\bar{\omega}}\times\bar{r}} - 2m\bar{\omega}\times\bar{v}_\sigma \underbrace{- m\bar{\omega}\times(\bar{\omega}\times\bar{r})}_{m\omega^2\vec{r}_\perp \,=\, -\nabla V_\varphi} , \quad V_\varphi = -\frac{m\omega^2 r_\perp^2}{2}$$

$$\frac{m v_\sigma^2}{2} + \underbrace{mgy - \frac{m\omega^2 r_\perp^2}{2}}_{V_{o\lambda}} = E , \quad r_\perp^2 = x^2+z^2 = (\varpi\cos\varphi)^2 + z^2 = z^2\cos^2 z + z^2$$

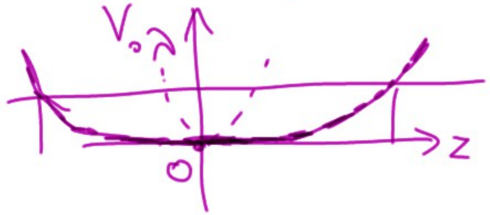
$$(\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}) \rightarrow \vec{r}_\perp$$

$$V_{o\lambda} = z\sin z - \frac{z^2(\cos^2 z+1)}{2}$$

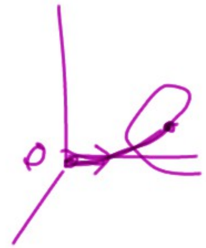
Για $|z| \ll 1$, $V_{o\lambda} = z^2 - z^2 \neq 0 \ldots$

$$V_{o\lambda}(z) \approx \underbrace{V_{o\lambda}(0)}_{0} + \underbrace{V'(0)}_{0}\, z + \frac{1}{2}\underbrace{V''(0)}_{0}\, z^2 + \frac{1}{3!}\underbrace{V'''(0)}_{0}\, z^3 + \frac{1}{4!}\underbrace{V^{(4)}(0)}_{8}\, z^4$$

Άρα $V_{o\lambda}(z) \approx \dfrac{z^4}{3}$



δηλ. $\left(\dfrac{2+z^2}{2}\right)\dot z^2 + \dfrac{z^4}{3} = E = \dfrac{v_0^2}{2} = \dfrac{z_{max}^4}{3}$



$$t = \int_{0.07}^{0} \frac{dz}{\dot z} \quad \text{με} \quad \dot z = \pm\sqrt{\frac{v_0^2 - \frac{2}{3}z^4}{2+\cancel{z^2}}}$$

$$= -\int_{0.07}^{0} \frac{dz}{\sqrt{\dfrac{v_0^2 - \frac{2}{3}z^4}{2}}}$$

$z = \left(\dfrac{3v_0^2}{2}\right)^{1/4} \xi$

$v_0 = \sqrt{\dfrac{2}{3} z_{max}^4}$

32.44

$$\int_0^1 \frac{d\xi}{\sqrt{1-\xi^4}} = 1.311$$

$\xi = \sqrt{u}$

$d\xi = \dfrac{du}{2\sqrt{u}}$

$$\int \frac{du}{2\sqrt{u}\sqrt{1-u^2}}$$