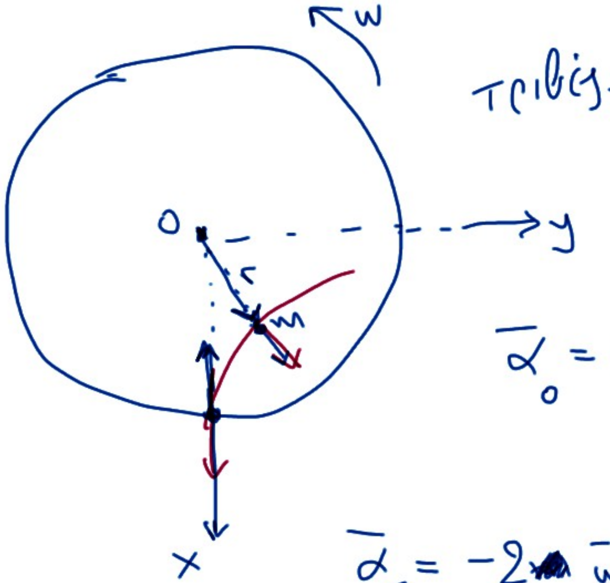
Άσκηση 2 από 4η εργασία 2010-2011



Τριβές=0

$$\bar{\alpha}_0 = 0 \qquad \bar{\alpha}_\varphi = \omega^2 \bar{r}_\perp = \begin{cases} \omega^2 (x\hat{x} + y\hat{y}) \\ \omega^2 \varpi \hat{\varpi} \end{cases}$$

$$\bar{\alpha}_c = -2\bar{\omega}\times\bar{v}_\sigma = -2\omega\hat{z}\times(\dot{x}\hat{x}+\dot{y}\hat{y})$$
$$= -2\omega\hat{z}\times(\dot{\varpi}\hat{\varpi}+\varpi\dot{\varphi}\hat{\varphi})$$

$$-m\cancel{\dot{\bar{\omega}}}\times\bar{r}$$

$$m\bar{\alpha}_\sigma = m\omega^2\bar{r}_\perp - 2m\bar{\omega}\times\bar{v}_\sigma$$

$$-\nabla V_\varphi = +m\omega^2\bar{r}_\perp \rightarrow V_\varphi = -\frac{m\omega^2 r_\perp^2}{2}$$

Αλλιώς:

$$\boxed{\frac{m v_\sigma^2}{2} + V_\varphi = E = \text{σταθ}} \Leftrightarrow \boxed{\frac{m}{2}\left(\dot{\varpi}^2 + \varpi^2\dot{\varphi}^2\right) - \frac{m\omega^2}{2}\varpi^2 = E}$$

$\dot{L}_z = 0$ αν $F_\varphi = 0$ (οπότε $\bar{T}_z = (\bar{r}\times\bar{F})\cdot\hat{z} = 0$)

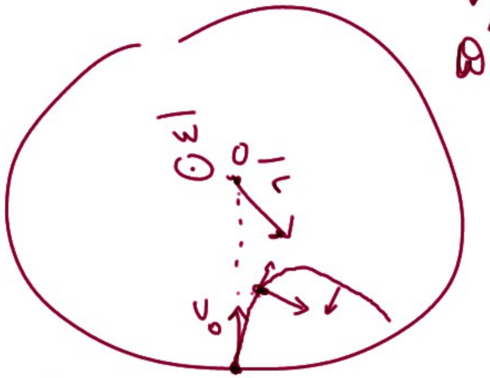
$$\dot{\bar L} = \bar r \times (\Sigma \bar F) = \underbrace{\bar r \times (m\omega^2 \bar r_\perp)}_{0} + \bar r \times (-2m\bar\omega \times \bar v_6) =$$

$$= -2m\bar\omega(\bar r \cdot \bar v_6) + 2m\bar v_6 \cancel{(\bar r \cdot \bar\omega)}^{\,0} = -2m\omega\hat z\,\varpi\dot\varpi$$

$$\bar r \to \varpi\hat\varpi \qquad \bar v_6 \to \dot\varpi\hat\varpi + \varpi\dot\varphi\hat\varphi$$

$$\bar L = \bar r \times m\bar v = \begin{vmatrix} \hat\varpi & \hat\varphi & \hat z \\ \varpi & 0 & 0 \\ m\dot\varpi & m\varpi\dot\varphi & 0 \end{vmatrix}$$

$$L_z = m\varpi^2\dot\varphi$$



$$\frac{d}{dt}(m\varpi^2\dot\varphi) = \underbrace{-2m\omega\varpi\dot\varpi}_{-\frac{d}{dt}(m\omega\varpi^2)} \Leftrightarrow m\varpi^2(\dot\varphi+\omega) = \text{σταθ}$$

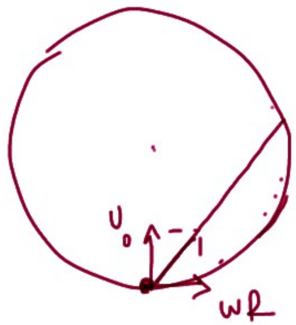
$$= mR^2(0+\omega)$$

$$\Leftrightarrow \boxed{\dot\varphi = \frac{R^2\omega}{\varpi^2} - \omega} \quad (2)$$

$$v_\varphi\Big|_{t=0} = 0 \Leftrightarrow \varpi\dot\varphi = 0$$

$$\frac{m(\dot\varpi^2 + \varpi^2\dot\varphi^2)}{2} - \frac{m\omega^2\varpi^2}{2} = E = \frac{mv_0^2}{2} - \frac{m\omega^2R^2}{2} \quad (1)$$

$$\overset{(2)}{\Longleftrightarrow} \quad \dot\varpi^2 + \frac{R^4\omega^2}{\varpi^2} = v_0^2 + R^2\omega^2$$



$$\bar v_6 = \bar v_\alpha - \bar\omega \times \bar r$$