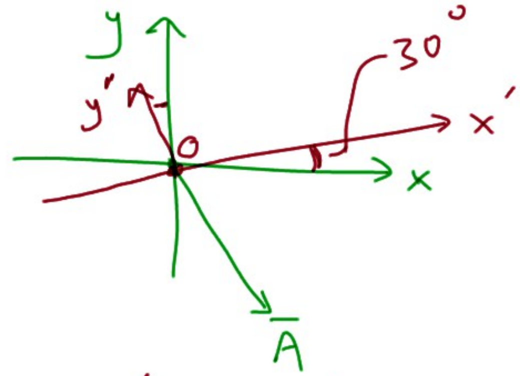
1) $\bar{A} = 3\hat{x} - 4\hat{y}$



Αν στρίψω το σύστημα κατά 30° ποιο το $\bar{A}$ στο $Ox'y'$;

$\cancel{A^{\mu'} = C^{\mu'}{}_{\nu} A^{\nu}}$

Λύση:

$$\bar{A} = (\bar{A}\cdot\hat{x}')\hat{x}' + (\bar{A}\cdot\hat{y}')\hat{y}' =$$

$$\cancel{\bar{A} = (\quad)\hat{x}' + (\quad)\hat{y}'}$$

$$\bar{A}\cdot\hat{x}' = 3\,\underbrace{\hat{x}\cdot\hat{x}'} - 4\,\underbrace{\hat{y}\cdot\hat{x}'} = 3\cdot\cos 30^\circ - 4\sin 30^\circ = \frac{3\sqrt{3}}{2} - 2$$

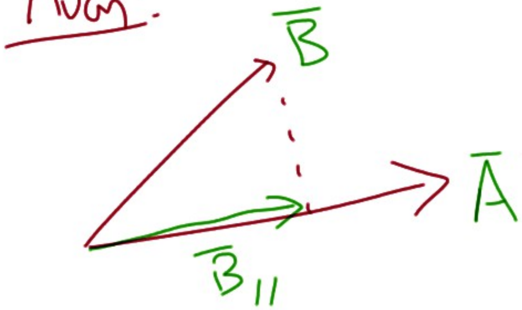
$$\bar{A}\cdot\hat{y}' = 3\,\hat{x}\cdot\hat{y}' - 4\,\hat{y}\cdot\hat{y}' = 3\cos(90^\circ + 30^\circ) - 4\cdot\cos 30^\circ =$$

$$= -3\cdot\frac{1}{2} - 2\sqrt{3}$$

$$\bar{A} = \left(\frac{3\sqrt{3}}{2} - 2\right)\hat{x}' - \left(\frac{3}{2} + 2\sqrt{3}\right)\hat{y}'$$

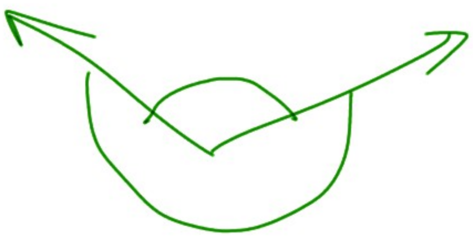
2. Αναλύστε το $\bar{B} = 2\hat{x} + 3\hat{y}$ πάνω και κάθετα στο $\bar{A} = 3\hat{x} - 4\hat{y}$

Λύση:

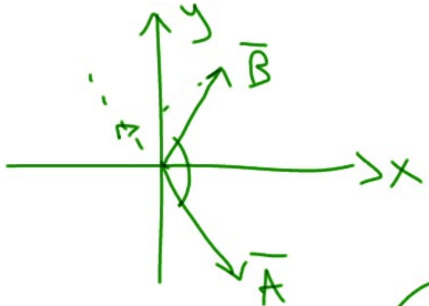


$$\bar{B}_{\parallel} = (\bar{B}\cdot\hat{A})\hat{A} = \frac{(\bar{B}\cdot\bar{A})\bar{A}}{|\bar{A}|^2} =$$

$$= \frac{2\cdot 3 + 3\cdot(-4)}{3^2+4^2}(3\hat{x} - 4\hat{y}) = +\frac{6}{5}\cdot\frac{-3\hat{x}+4\hat{y}}{5}$$



$$\cos(\widehat{\bar{A},\bar{B}}) = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|} < 0$$ άρα ∢ αμβλεία.



$\bar{B}_{\perp} = \bar{B} - \bar{B}_{\parallel} = \dots$ $\left((\hat{A}\times\bar{B})\times\hat{A} \right)$

3. Αν $\bar{v} = -x\hat{x} + t\hat{y}$ και $\bar{r}\big|_{t=0} = 0$ ποιο το $\bar{r}(t)$;

Λύση:

$$\dot{x} = -x \Leftrightarrow \frac{dx}{dt} = -x \Leftrightarrow \int \frac{dx}{x} = -\int dt \Leftrightarrow \ln|x| = -t + \ln|C|$$

$$\cancel{\int_0^x \frac{dx}{x} = -\int_0^t dt \Leftrightarrow \Big[\ln|x|\Big]_0^x = -t}$$

$$|x| = e^{-t}\underbrace{e^{\ln|C|}}_{|C|} \Leftrightarrow x = C e^{-t}$$

$C = 0$ από αρχικές

$$\dot{y} = t$$

$$\int_0^y dy = \int_0^t t\,dt$$

$$y = t^2/2$$

$$\boxed{\bar{r} = \frac{t^2}{2}\hat{y}}$$

Αν αρχικά $\bar{r}\big|_{t=0} = -\hat{x}$ ποιο το $\bar{r}(t)$;

Αρχικά $x = -1$, $y = 0$ άρα $C = -1$ και $\boxed{\bar{r}(t) = -e^{-t}\hat{x} + \frac{t^2}{2}\hat{y}}$

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$$\bar{r} = \left(\frac{8}{3}t^3 - 12t^2\right)\hat{x} + \left(4t^2 - \lambda t\right)\hat{y}$$

(α) $\bar{\upsilon} = \dot{\bar{r}} = \left(8t^2 - 24t\right)\hat{x} + \left(8t - \lambda\right)\hat{y}$

$$|\bar{\upsilon}| = 0 \Leftrightarrow \begin{cases} 8t^2 - 24t = 0 \\ 8t - \lambda = 0 \end{cases} \Leftrightarrow \begin{cases} t = 0 \\ \lambda = 0 \end{cases} \text{ ή } \begin{cases} t = 3 \\ \lambda = 24 \end{cases}$$

(β) $\bar{\alpha} = \dot{\bar{\upsilon}} = \left(16t - 24\right)\hat{x} + 8\hat{y}$

$\bar{\alpha}\cdot\hat{x} = 0 \Leftrightarrow t = 3/2$ Τότε $\bar{\alpha} \,//\, \hat{y}$

Αλλιώς : $\bar{\alpha} \,//\, \hat{y} \Leftrightarrow \bar{\alpha} \times \hat{y} = 0$

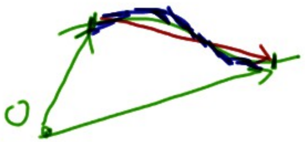
(γ) $\bar{\alpha}\big|_{t=1} \cdot \bar{\upsilon}\big|_{t=1} = 0 \Leftrightarrow \dots \Leftrightarrow \lambda = 24$.

(δ) $\bar{\alpha}\big|_{t=1} \times \bar{\upsilon}\big|_{t=1} = 0 \Leftrightarrow \dots \Leftrightarrow \lambda = -8$

(ή $\frac{\alpha_x}{\alpha_y} = \frac{\upsilon_x}{\upsilon_y}$)

(ε) $\Delta s = \int_0^t |\bar{\upsilon}|\, dt = \int_0^t \sqrt{\left(8t^2 - 24t\right)^2 + \left(8t - 24\right)^2}\, dt$

$\int \bar{\upsilon}\, dt = \Delta\bar{r}$



0

$ds = |d\bar{r}|$

ή $\int \sqrt{(dx)^2 + (dy)^2}$