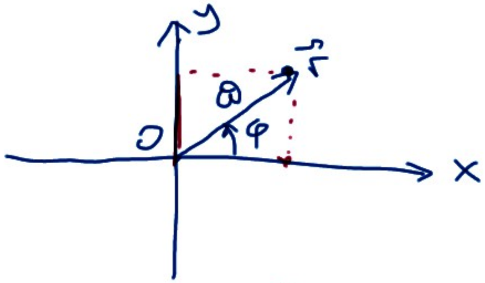
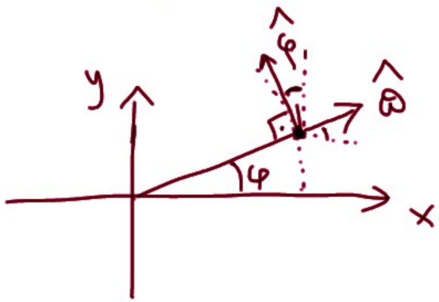
Πολικές συντεταγμένες (ϖ, φ)



$0 \le \varpi < \infty$

$0 \le \varphi < 2\pi$ (όχι πάντα, μπορώ να έχω $-\infty < \varphi < \infty$)

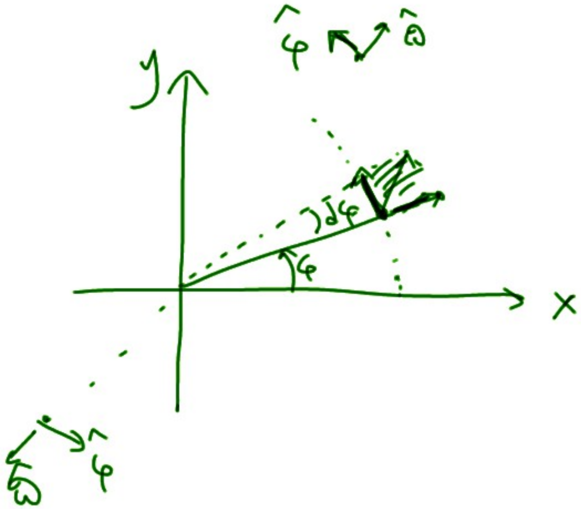
$$\left.\begin{aligned} x &= \varpi \cos\varphi \\ y &= \varpi \sin\varphi \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} &\varpi = \sqrt{x^2+y^2} \\ &\varphi = \text{κοινή λύση των } \begin{cases} \sin\varphi = \dfrac{y}{\sqrt{x^2+y^2}} \\ \cos\varphi = \dfrac{x}{\sqrt{x^2+y^2}} \end{cases} \end{aligned}\right.$$



$$\left.\begin{aligned} \hat{\varpi} &= \hat{x}\cos\varphi + \hat{y}\sin\varphi \\ \hat{\varphi} &= -\hat{x}\sin\varphi + \hat{y}\cos\varphi \end{aligned}\right\} \Leftrightarrow \begin{aligned} \hat{x} &= (\hat{x}\cdot\hat{\varpi})\hat{\varpi} + (\hat{x}\cdot\hat{\varphi})\hat{\varphi} = \cos\varphi\,\hat{\varpi} - \sin\varphi\,\hat{\varphi} \\ \hat{y} &= \quad \cdots \quad = \sin\varphi\,\hat{\varpi} + \cos\varphi\,\hat{\varphi} \end{aligned}$$

$$\bar{r} = \varpi\cos\varphi\,\hat{x} + \varpi\sin\varphi\,\hat{y}, \qquad \hat{\varpi} \uparrow\uparrow \frac{\partial\vec{r}}{\partial\varpi} = \cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}$$

$$\hat{\varphi} \uparrow\uparrow \frac{\partial\vec{r}}{\partial\varphi} = -\varpi\sin\varphi\,\hat{x} + \varpi\cos\varphi\,\hat{y}, \quad \hat{\varphi} = \frac{\frac{\partial\bar{r}}{\partial\varphi}}{\left|\frac{\partial\bar{r}}{\partial\varphi}\right|}$$



$$\boxed{\vec r = \varpi\,\hat\varpi}$$

$$d\vec r = d\varpi\,\hat\varpi + \varpi\,d\varphi\,\hat\varphi$$

$$ds = \sqrt{(d\varpi)^2 + \varpi^2(d\varphi)^2}$$

$$d\bar S = \varpi\,d\varpi\,d\varphi\,\hat z$$

$$\bar v = \frac{d\bar r}{dt} = \frac{d\varpi\,\hat\varpi + \varpi\,d\varphi\,\hat\varphi}{dt} \Leftrightarrow \boxed{\bar v = \dot\varpi\,\hat\varpi + \varpi\dot\varphi\,\hat\varphi}$$

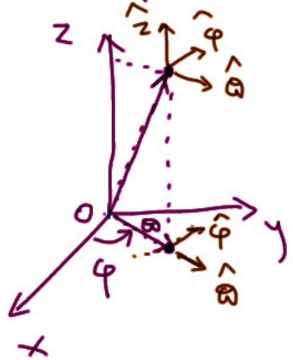
ή $\bar v = \dot{\bar r} = \frac{d}{dt}(\varpi\,\hat\varpi) = \dot\varpi\,\hat\varpi + \varpi\,\underbrace{\dot{\hat\varpi}} = \dot\varpi\,\hat\varpi + \varpi\dot\varphi\,\hat\varphi$

$$\frac{d}{dt}(\cos\varphi\,\hat x + \sin\varphi\,\hat y) = -\sin\varphi\,\dot\varphi\,\hat x + \cos\varphi\,\dot\varphi\,\hat y = \dot\varphi\,\hat\varphi$$

$$\bar\alpha = \frac{d\bar v}{dt} = \frac{d}{dt}\left(\dot\varpi\,\hat\varpi + \varpi\dot\varphi\,\hat\varphi\right) = \ddot\varpi\,\hat\varpi + \dot\varpi\,\underbrace{\dot{\hat\varpi}}_{\dot\varphi\hat\varphi} + \dot\varpi\dot\varphi\,\hat\varphi + \varpi\ddot\varphi\,\hat\varphi + \varpi\dot\varphi\,\underbrace{\dot{\hat\varphi}}_{-\dot\varphi\hat\varpi} \Leftrightarrow$$

$$\Leftrightarrow \boxed{\bar\alpha = \left(\ddot\varpi - \varpi\dot\varphi^2\right)\hat\varpi + \left(2\dot\varpi\dot\varphi + \varpi\ddot\varphi\right)\hat\varphi}$$ ή $\bar\alpha = \left(\ddot\varpi - \varpi\dot\varphi^2\right)\hat\varpi + \frac{1}{\varpi}\frac{d}{dt}\left(\varpi^2\dot\varphi\right)\hat\varphi$

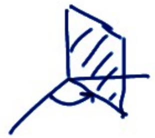
Κυλινδρικές (ϖ, φ, z) $\quad 0 \le \varpi < \infty, \quad 0 \le \varphi < 2\pi, \quad -\infty < z < \infty$



$$\boxed{\vec{r} = \varpi\,\hat{\varpi} + z\,\hat{z}}$$

$$\hat{\varpi} \times \hat{\varphi} = \hat{z}, \quad \hat{z} \times \hat{\varpi} = \hat{\varphi}$$

$$\boxed{d\bar{r} = d\varpi\,\hat{\varpi} + \varpi\, d\varphi\,\hat{\varphi} + dz\,\hat{z}}$$

$ds = |d\bar{r}|, \quad d\bar{S} =$  $\varpi\, d\varphi\, dz\,\hat{\varpi}$ ή  $d\varpi\, dz\,\hat{\varphi}$ ή  $\varpi\, d\varpi\, d\varphi\,\hat{z}$

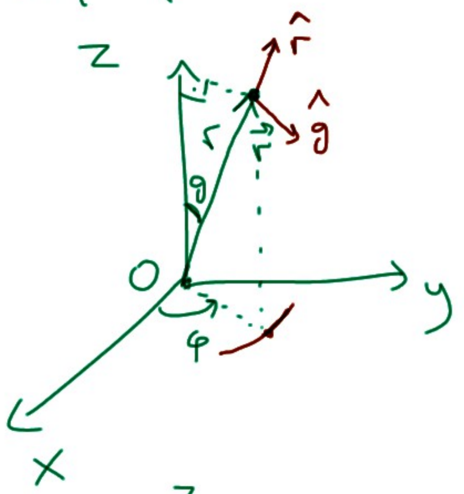
$$dV = \varpi\, d\varpi\, d\varphi\, dz$$

$$\boxed{\bar{v} = \dot{\varpi}\,\hat{\varpi} + \varpi\dot{\varphi}\,\hat{\varphi} + \dot{z}\,\hat{z}}$$

$$\vec{L} = \bar{r} \times m\bar{v} = \begin{vmatrix} \hat{\varpi} & \hat{\varphi} & \hat{z} \\ \varpi & 0 & z \\ m\dot{\varpi} & m\varpi\dot{\varphi} & m\dot{z} \end{vmatrix}, \quad \boxed{L_z = m\varpi^2\dot{\varphi}}$$

$$\boxed{\bar{\alpha} = \left(\ddot{\varpi} - \varpi\dot{\varphi}^2\right)\hat{\varpi} + \underbrace{\left(2\dot{\varpi}\dot{\varphi} + \varpi\ddot{\varphi}\right)}_{\frac{1}{\varpi}\frac{d}{dt}\left(\varpi^2\dot{\varphi}\right)}\hat{\varphi} + \ddot{z}\,\hat{z}}$$

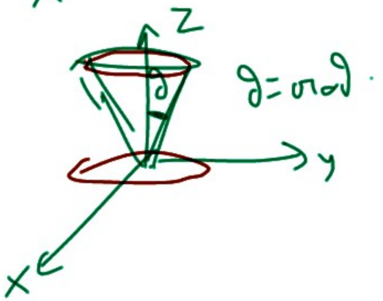
Σφαιρικές συντεταγμένες (r, θ, φ):

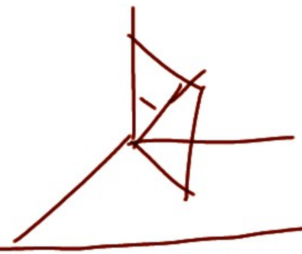


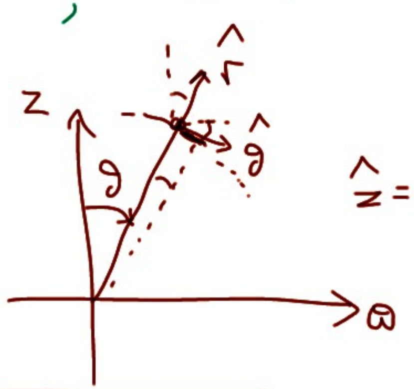
$$\begin{cases} x = \varpi \cos\varphi = r \sin\theta \cos\varphi \\ y = \varpi \sin\varphi = r \sin\theta \sin\varphi \\ z = r \cos\theta \end{cases}$$

(όπου $\varpi = r \sin\theta$)

$0 \le r < \infty$, $\quad 0 \le \theta \le \pi$, $\quad 0 \le \varphi < 2\pi$







$\hat{z} = \hat{r} \cos\theta - \hat{\theta} \sin\theta$

$$\boxed{d\bar{r} = dr\, \hat{r} + r\, d\theta\, \hat{\theta} + \underbrace{\varpi}_{r \sin\theta} d\varphi\, \hat{\varphi}}$$

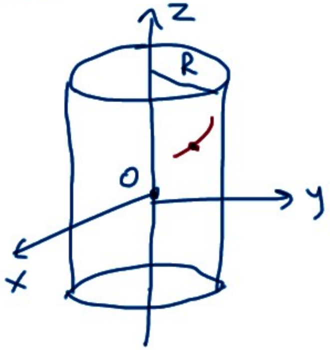
$$\boxed{\bar{v} = \dot{r}\, \hat{r} + r \dot{\theta}\, \hat{\theta} + r \sin\theta\, \dot{\varphi}\, \hat{\varphi}}$$

$\bar{a}$ Δείτε άσκηση 4 στο εργασία #1, 2014-2015

Σώμα κινείται πάνω σε κύλινδρο ακτίνας R, με $|\bar{v}| = \text{σταθ}$ και $v_z = \text{σταθ}$ (παράλληλα στον άξονα του κυλίνδρου).

Ποιο το $\bar{r}(t)$;

Λύση:



$$\boxed{\varpi = R}$$

$$\bar{r} = R\,\hat{\varpi} + z\,\hat{z}$$

$$\bar{v} = \cancel{\dot{\varpi}\,\hat{\varpi}} + \varpi\dot{\varphi}\,\hat{\varphi} + \dot{z}\,\hat{z}$$

$$v_z = \text{σταθ} \Leftrightarrow \dot{z} = \text{σταθ} = v_z \Leftrightarrow \frac{dz}{dt} = v_z \Leftrightarrow \int dz = \int v_z\, dt$$

$$\Leftrightarrow \boxed{z = v_z t + z_0}$$ ← σταθερά ολοκλήρωσης

$$|\bar{v}| = \text{σταθ} \Leftrightarrow \sqrt{R^2\dot{\varphi}^2 + v_z^2} = \text{σταθ} \Leftrightarrow |\dot{\varphi}| = \text{σταθ} \Leftrightarrow \dot{\varphi} = \text{σταθ} = \omega$$

$$\boxed{\varphi = \omega t + \varphi_0}$$ ← σταθερά ολοκλ.

Ελικοειδής κίνηση

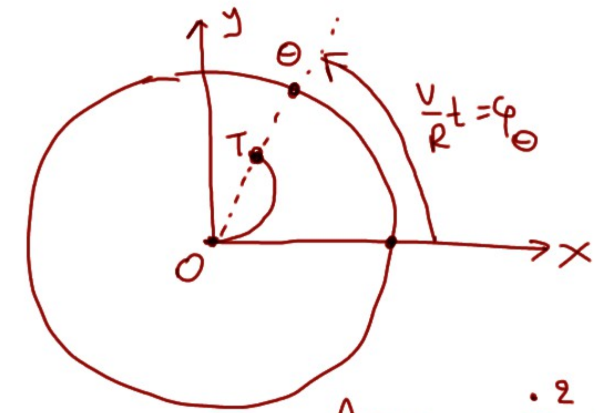
$$b = \text{βήμα έλικας} = \Delta z\Big|_{\Delta\varphi = 2\pi} = v_z \cdot \frac{2\pi}{\omega}$$

Σε καρτεσιανές:

$$\left.\begin{aligned} &\sqrt{x^2 + y^2} = R \\ &\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} = \text{σταθ} \\ &\dot{z} = \text{σταθ} \end{aligned}\right\} \quad \begin{aligned} x &= R\cos(\omega t + \varphi_0) \\ y &= R\sin(\omega t + \varphi_0) \\ z &= z_0 + v_z t \end{aligned}$$

2η από 2η εργασία 2011-2012

$(r=\varpi)$



$$r_\Theta = R, \quad \varphi_\Theta = \frac{v}{R}t$$

$$\varphi_T = \varphi = \frac{v}{R}t$$

$$r_T = r(t) = \, ?$$

$\bar{v}_T = \dot{r}\,\hat{r} + r\dot{\varphi}\,\hat{\varphi}$ με $|\bar{v}_T|^2 = U^2$ και $\varphi = \frac{v}{R}t$

Άρα $\dot{r}^2 + r^2\left(\frac{v}{R}\right)^2 = U^2 \Leftrightarrow \dot{r} = \pm\sqrt{U^2 - r^2\left(\frac{v}{R}\right)^2}$

$$\frac{dr}{dt} = \sqrt{\dots} \Leftrightarrow \int_0^r \frac{dr}{\sqrt{U^2 - r^2\left(\frac{v}{R}\right)^2}} = \int_0^t dt \qquad (1)$$

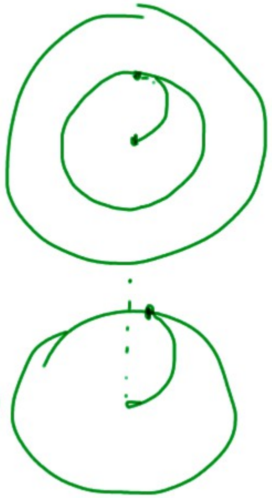
$$\int \frac{d\xi}{\sqrt{1-\xi^2}} \quad \frac{\xi = \sin w}{d\xi = \cos w\, dw} \quad \int \frac{\cos w\, dw}{|\cos w|} = w + \text{σταθ}$$

Θέτω $r\frac{v}{RU} = \xi \Leftrightarrow r = \frac{RU}{v}\xi$ οπότε (1) $\rightsquigarrow$ $\int_0^\xi \frac{\frac{R}{v}\,d\xi}{\sqrt{1-\xi^2}} = t \Leftrightarrow$

$$\frac{R}{v}\left[\arcsin\xi\right]_0^\xi = t \Leftrightarrow \arcsin\xi = \frac{vt}{R} \Leftrightarrow \xi = \sin\frac{vt}{R} \Leftrightarrow \boxed{r = \frac{RU}{v}\sin\frac{vt}{R}}$$

V<v

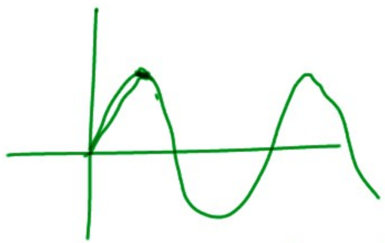
V>v



$$r = \frac{RV}{v}\sin\left(\frac{vt}{R}\right) = \frac{RV}{v}\sin\varphi$$

$$r = r_0 \Leftrightarrow \frac{RV}{v}\sin(\ ) = R \Leftrightarrow t = \frac{R}{v}\arcsin\frac{v}{V}$$

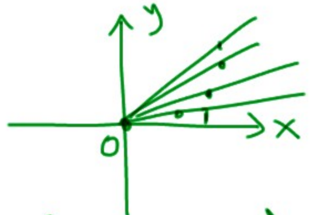
(υπάρχει μόνο αν $v < V$)

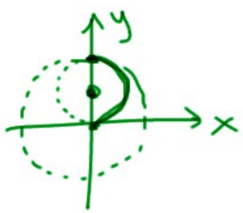


Αν $V < v$ τότε $r = \frac{RV}{v}\sin\varphi$ για $0 \le \varphi \le \frac{\pi}{2}$ και μετά

$$r = \frac{RV}{v} \qquad \left(\dot{r}^2 + r^2\left(\frac{v}{R}\right)^2 = V^2\right)$$

Τροχιά; $r = \overbrace{\frac{RV}{V}}^{\to 2\alpha}\sin\varphi$





(είναι τμήμα κύκλου γιατί $\left.\begin{array}{l} x = 2\alpha\sin\varphi\cos\varphi \\ y = 2\alpha\sin^2\varphi \end{array}\right\} \rightsquigarrow \ldots \quad x^2 + (y-\alpha)^2 = \alpha^2$

Σώμα κινείται στο επίπεδο και έχει

$$r = r_0 + ct \quad , \quad \varphi = \omega_0 \frac{r_0 t}{r_0 + ct} \quad , \quad r_0, c, \omega_0 \text{ θετικές σταθερές.}$$

Βρείτε την ταχύτητα και την επιτάχυνση.

Διατηρείται η στροφορμή;

Περιγράψτε την κίνηση για $\dfrac{\omega_0 r_0}{c} = \dfrac{\pi}{2}, \pi, 2\pi, 4\pi$.

<u>Λύση:</u> $\bar{r} = (r_0 + ct)\hat{r}$

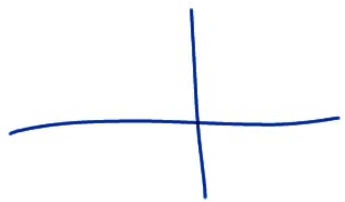
$$\bar{v} = \dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi} = c\hat{r} + \frac{\omega_0 r_0^2}{r_0 + ct}\hat{\varphi} \quad , \quad \bar{\alpha} = (\ddot{r} - r\dot{\varphi}^2)\hat{r} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi})\hat{\varphi}$$

$$\bar{\alpha} = -\frac{\omega_0^2 r_0^4}{(r_0 + ct)^3}\hat{r}$$

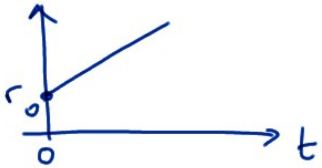
$$\dot{L} = \bar{r} \times m\bar{\alpha} = 0 \quad \text{διότι } \bar{\alpha} \,/\!/\, \hat{r}$$

Αλλιώς: $\bar{L} = \bar{r} \times m\bar{v} = \begin{vmatrix} \hat{r} & \hat{\varphi} & \hat{z} \\ r & 0 & 0 \\ mc & \dfrac{m\omega_0 r_0^2}{r} & 0 \end{vmatrix} = m\omega_0 r_0^2 \hat{z} = \text{σταθ}$

$$\left.\begin{aligned} x &= r\cos\varphi = (r_0 + ct)\cos\left(\omega_0 \frac{r_0 t}{r_0 + ct}\right) \\ y &= r\sin\varphi = (r_0 + ct)\sin\left(\omega_0 \frac{r_0 t}{r_0 + ct}\right) \end{aligned}\right\}$$

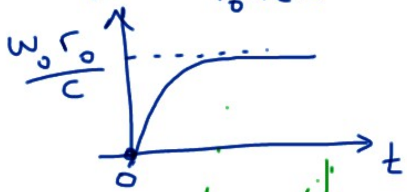


$r = r_0 + ct$



$\varphi = \omega_0 \frac{r_0 t}{r_0 + ct} = \frac{\omega_0 r_0}{c} \frac{ct}{r_0 + ct}$

$\frac{\xi}{1+\xi} = 1 - \frac{1}{1+\xi}$



- $\frac{\omega_0 r_0}{c} = \frac{\pi}{2}$

$\lim_{t \to \infty} x(t) = \lim_{t \to \infty} (r_0 + ct) \cos\left(\frac{\pi}{2} \frac{ct}{r_0 + ct}\right) =$

$= \infty \cdot 0 = \frac{\pi}{2} r_0$

- $\frac{\omega_0 r_0}{c} = 4\pi$