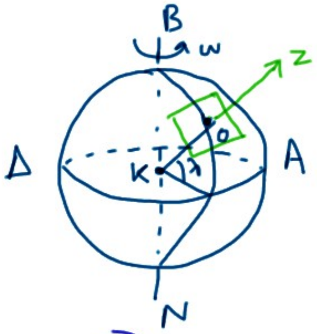
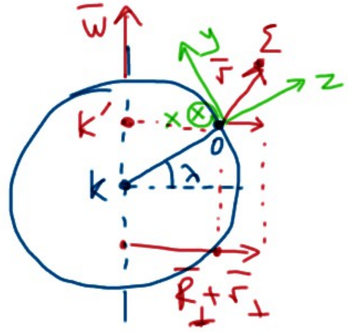
Κίνηση σώματος κοντά στην επιφάνεια της Γης





$$m\bar{\alpha}_\sigma = \bar{B} + \bar{F}_{\text{άλλες}} - m\bar{\alpha}_0 - m\bar\omega\times(\bar\omega\times\bar r) - 2m\bar\omega\times\bar v_\sigma - \cancel{m\dot{\bar\omega}\times\bar r}$$

βάρος $\bar B = -\dfrac{GMm}{|\overline{K\Sigma}|^3}\overline{K\Sigma} = -\dfrac{GMm}{|\overline{KO}+\bar r|^3}(\overline{KO}+\bar r) \approx -\dfrac{GMm}{R^2}\hat z$

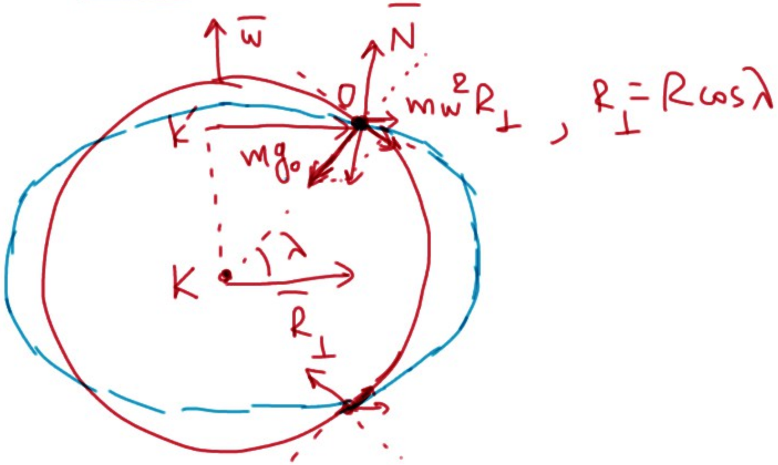
$\bar B = m\bar g_0$ με $\bar g_0 \approx -\dfrac{GM}{R^2}\hat z$

$-m\bar\alpha_0 = -m\left(-\omega^2\,\overrightarrow{K'O}\right) = m\omega^2\underbrace{\bar R_\perp}_{\overrightarrow{K'O}}$ (το O κινείται κυκλικά με ω, $R_\perp = K'O = R\cos\lambda$)

$-m\bar\omega\times(\bar\omega\times\bar r) = m\omega^2\bar r_\perp$ αμελητέο σε σχέση με την $-m\bar\alpha_0 = m\omega^2\bar R_\perp$

$\bar B - m\bar\alpha_0 - m\bar\omega\times(\bar\omega\times\bar r) \approx m(\bar g_0 + \omega^2\bar R_\perp) = m\vec g_{eff}$ Άρα $\boxed{m\bar\alpha_\sigma = m\bar g_{eff} - 2m\bar\omega\times\bar v_\sigma + \bar F_{\text{άλλες}}}$

Σχήμα της Γης:



Στο σύστημα με κέντρο το K που περιστρέφεται,

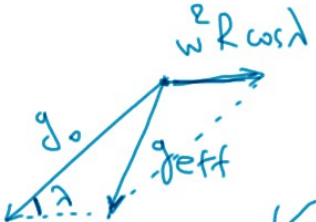
$$\cancel{m\bar{\alpha}_\sigma} = m\bar{g}_0 - \cancel{m\bar{\alpha}_0} + m\omega^2\bar{R}_\perp - \cancel{2m\bar{\omega}\times\bar{v}_\sigma}$$

$$\cancel{- m\dot{\bar{\omega}}\times\bar{r}} + \bar{N}$$

$$-\bar{N} = m\left(\bar{g}_0 + \omega^2\bar{R}_\perp\right) = m\,\bar{g}_{eff}$$

$$\frac{m\omega^2 R_\perp}{m g_0} = \frac{\left(\frac{2\pi}{1\,day}\right)^2 \cdot 6400\,km\,\cos\lambda}{9{,}81\,m/s^2} = 3\cdot 10^{-3}\cos\lambda$$

η επιφάνεια σε κάθε τόπο είναι κάθετη στο $\vec{g}_{eff}$.



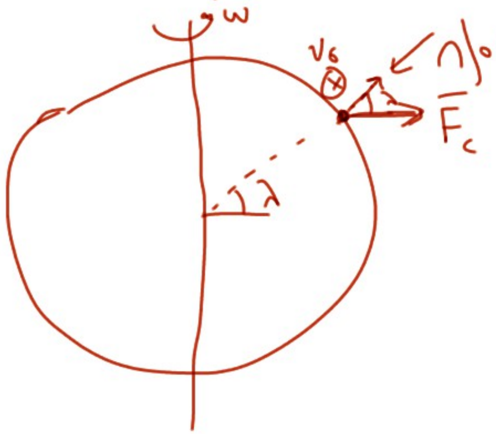
$$g_{eff} = \sqrt{g_0^2 + (\omega^2 R\cos\lambda)^2 - 2 g_0 \omega^2 R\cos^2\lambda}$$

(όμοια από $g_{eff} = \sqrt{(\bar{g}_0 + \omega^2\bar{R}_\perp)^2}$)

$$\boxed{m\bar{\alpha}_\sigma = m\bar{g} - 2m\bar{\omega}\times\bar{v}_\sigma + \bar{F}_{\alpha\lambda\lambda\epsilon\varsigma}}$$

eff

Eötvös effect:



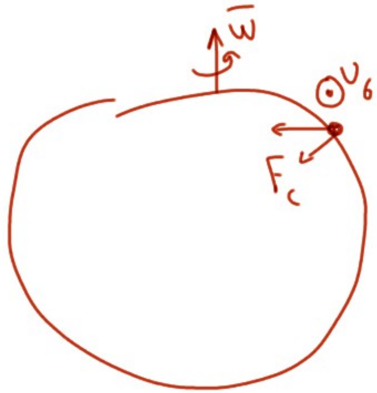
πλοίο κινείται προς ανατολάς

$$\bar F_c = -2m\bar\omega\times\bar v_\sigma$$

$$\cancel{m\bar\alpha_\sigma}^{\,0} = m\bar g + \bar F_c + \bar N$$

$$-\bar N = m\bar g + \bar F_c$$

άρα "μικρότερο" βάρος



Αν το πλοίο κινείται προς δυσμάς

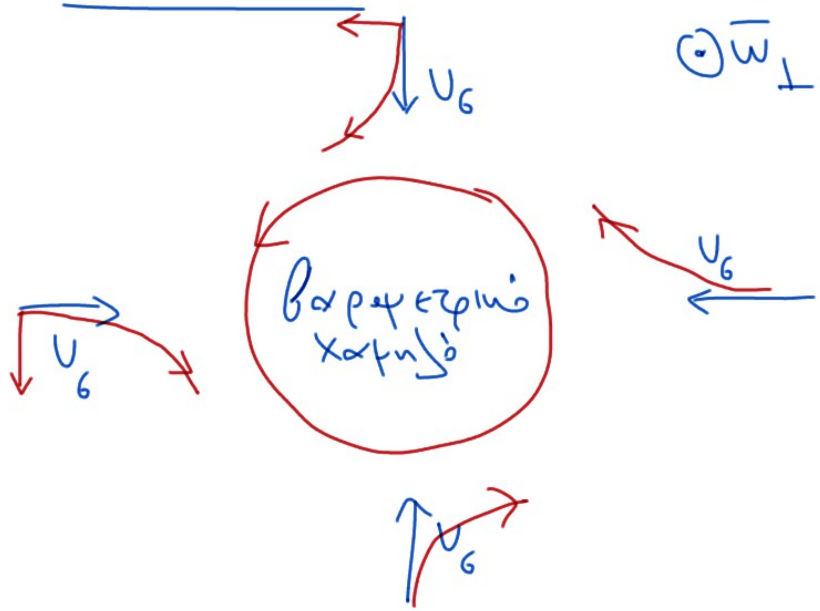
"μεγαλύτερο" βάρος.

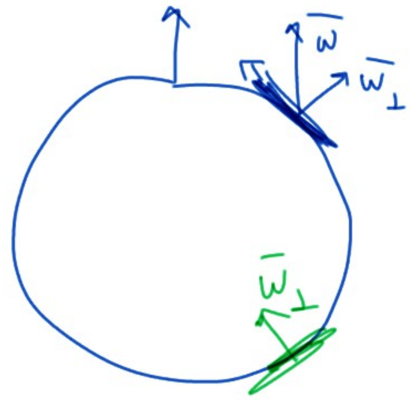
$$\frac{F_{c\perp}}{mg} = \frac{2\cancel{m}\omega v_\sigma\cos\lambda}{\cancel{m}g} = 4\cdot10^{-4}\left(\frac{v_\sigma}{100\,\frac{km}{h}}\right)\cos\lambda$$

Κυκλώνες:

Στο βόρειο ημισφαίριο

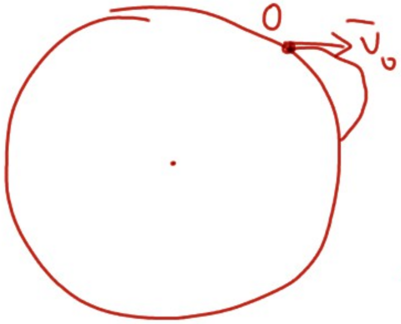
$\odot \bar{\omega}_\perp$





$-2m\bar{\omega}_\perp \times \bar{\upsilon}_\sigma$

Πλάγια βολή σε ένα τόπο O της περιστρεφόμενης Γης:



$$m\bar{\alpha}_\sigma = m\bar{g}_{eff} - 2m\bar{\omega}\times\bar{v}_\sigma + \cancelto{0}{\bar{F}_{\text{άλλες}}}$$

ή απλούστερα $\quad \bar{\alpha} = \bar{g} - 2\,\bar{\omega}\times\bar{v} \quad (*)$

Διαταρακτικά: $\quad \bar{r} = \bar{r}^{(0)} + \underbrace{\bar{r}^{(1)}}_{\text{ανάλογο του } \omega} \quad , \quad \bar{v} = \underbrace{\bar{v}^{(0)}}_{\dot{\bar{r}}^{(0)}} + \underbrace{\bar{v}^{(1)}}_{\dot{\bar{r}}^{(1)}}$

$$\bar{\alpha} = \underbrace{\bar{\alpha}^{(0)}}_{\dot{\bar{v}}^{(0)}} + \underbrace{\bar{\alpha}^{(1)}}_{\dot{\bar{v}}^{(1)}}$$

Σε μηδενική τάξη: $(*) \rightarrow \bar{\alpha}^{(0)} = \bar{g} \Leftrightarrow \bar{v}^{(0)} = \bar{v}_0 + \bar{g}t \Leftrightarrow \bar{r}^{(0)} = \bar{r}_0 + \bar{v}_0 t + \frac{\bar{g}t^2}{2}$

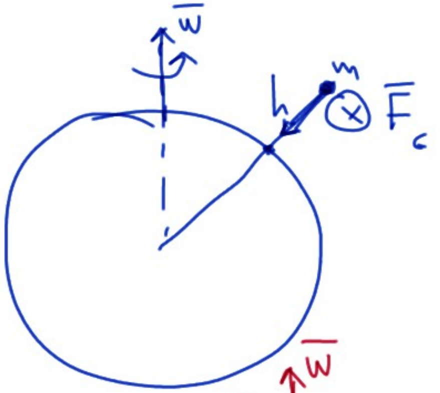
Σε 1η τάξη: $(*) \rightarrow \bar{\alpha}^{(0)} + \bar{\alpha}^{(1)} = \bar{g} - 2\bar{\omega}\times\left[\bar{v}^{(0)} + \cancel{\bar{v}^{(1)}}\right] \Leftrightarrow$

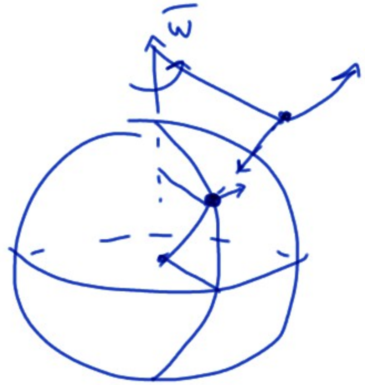
$\Leftrightarrow \dot{\bar{v}}^{(1)} = -2\bar{\omega}\times(\bar{v}_0 + \bar{g}t) \Leftrightarrow \bar{v}^{(1)} = -2\bar{\omega}\times\left(\bar{v}_0 t + \frac{\bar{g}t^2}{2}\right) + \cancelto{0}{\vec{c}}$ ώστε $\dot{\bar{v}}^{(1)}\Big|_{t=0} = 0$

$\Leftrightarrow \bar{r}^{(1)} = -2\bar{\omega}\times\left(\bar{v}_0\frac{t^2}{2} + \frac{\bar{g}t^3}{6}\right) + \cancelto{0}{\vec{D}}$ ώστε $\bar{r}^{(1)}\Big|_{t=0} = 0$

Δηλ. $\boxed{\vec{r} = \bar{r}_0 + \bar{v}_0 t + \frac{\bar{g}t^2}{2} - \bar{\omega}\times\bar{v}_0 t^2 - \underbrace{\bar{\omega}\times\bar{g}\,\frac{t^3}{3}}_{\text{προς ανατολή}}}$

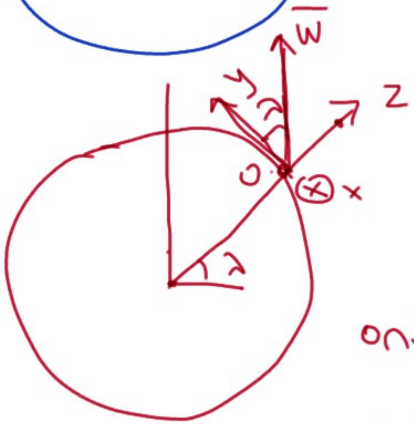
Παράδειγμα: Σώμα αρχικά ακίνητο σε ύψος h.





$$\bar{\alpha} = \bar{g} - 2\bar{\omega}\times\bar{v}$$

Απόκλιση προς ανατολή



$$\bar{\omega} = \omega\cos\lambda\,\hat{y} + \omega\sin\lambda\,\hat{z}$$

$$\bar{r}_0 = h\hat{z}\ ,\quad \bar{v}_0 = 0\ ,\quad \bar{g} = -g\hat{z}$$

οπότε $\bar{r} = \bar{r}_0 + \cancel{\bar{v}_0 t} + \dfrac{\bar{g}t^2}{2} - \cancel{\bar{\omega}\times\bar{v}_0\, t^2} - \bar{\omega}\times\bar{g}\,\dfrac{t^3}{3}$

$$\Leftrightarrow \begin{cases} x = \omega\cos\lambda\,\dfrac{gt^3}{3} \\ y = 0 \\ z = h - \dfrac{gt^2}{2} \end{cases}$$

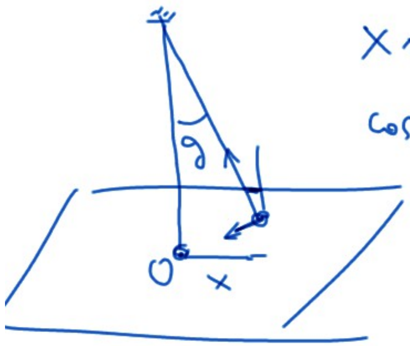
Όταν το σώμα πέσει $z = 0 \Leftrightarrow t = \sqrt{\dfrac{2h}{g}}$

και $x = \omega\cos\lambda\,\dfrac{g}{3}\left(\dfrac{2h}{g}\right)^{3/2}$

Αν π.χ. $h = 125\,\text{m}$, $t = 5\,\text{s}$, $x = 2.92\cos\lambda\ \text{cm}$

Για Αθήνα $\lambda = 38^\circ$ $\quad x = 2.3\ \text{cm}$

Εκκρεμές του Φουκώ:



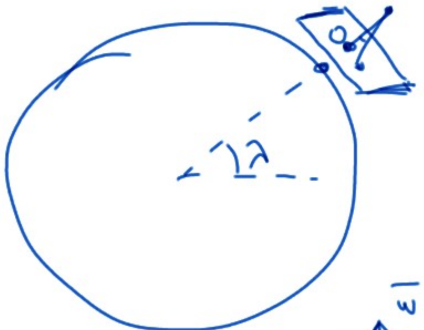
$x \approx R\theta$

$\cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1$

Δύναμη επαναφοράς $-mg\underbrace{\sin\theta}_{\approx\theta} \approx -\frac{mg}{R}x$

Ισοδύναμο με οριζόντιο ελατήριο με $\bar F_{ελ} = -\frac{mg}{R}\vec r$

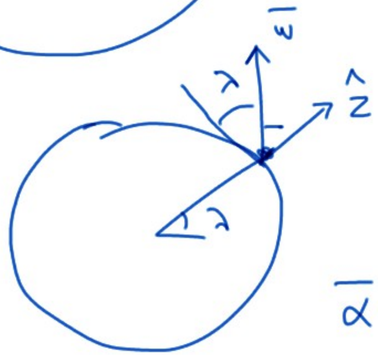
απομάκρυνση από Θ.Ι.



Δυνάμεις $m\bar g$, $\bar T$, $-2m\bar\omega\times\bar v$, $-\underbrace{\frac{mg}{R}}_{k}\bar r$

Προβολές τους στο xy επίπεδο:

$m\bar\alpha = -k\bar r - 2m\,\bar\omega_\perp\times\bar v$ με $k = \frac{mg}{R}$



$\bar\omega_\perp = \omega\sin\lambda\,\hat z$

Σε πολικές $\bar r = \varpi\hat\varpi$, $\bar v = \dot\varpi\hat\varpi + \varpi\dot\varphi\hat\varphi$

$\bar\alpha = \left(\ddot\varpi - \varpi\dot\varphi^2\right)\hat\varpi + \frac{1}{\varpi}\frac{d}{dt}\left(\varpi^2\dot\varphi\right)\hat\varphi$, $\bar\omega_\perp\times\bar v = \begin{vmatrix} \hat\varpi & \hat\varphi & \hat z \\ 0 & 0 & \omega\sin\lambda \\ \dot\varpi & \varpi\dot\varphi & 0 \end{vmatrix} =$

$= -\varpi\dot\varphi\,\omega\sin\lambda\,\hat\varpi + \dot\varpi\,\omega\sin\lambda\,\hat\varphi$

$\hat{\varpi}$: $\quad \ddot{\varpi} - \varpi\dot{\varphi}^2 = -\frac{g}{R}\varpi + 2\omega\sin\lambda\,\varpi\dot{\varphi}$ (1)

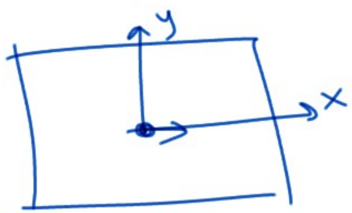
$\hat{\varphi}$: $\quad \frac{1}{\varpi}\frac{d}{dt}\left(\varpi^2\dot{\varphi}\right) = -2\omega\sin\lambda\,\dot{\varpi}$ (2)

(2) → $\frac{d}{dt}\left(\varpi^2\dot{\varphi}\right) = -2\omega\sin\lambda\,\underbrace{\varpi\dot{\varpi}}_{\frac{d}{dt}\left(\varpi^2/2\right)}$ $\Leftrightarrow$ $\boxed{\varpi^2\left(\dot{\varphi} + \omega\sin\lambda\right) = \ell = \text{σταθ}}$ (2)'

(1) $\xrightarrow{\text{αντικαθ. } \dot{\varphi}}$ $\ddot{\varpi} + \left(\frac{g}{R} + \overbrace{\omega^2\sin^2\lambda}^{\text{αμελητέο}}\right)\varpi = \frac{\ell^2}{\varpi^3}$ (1)'

(ισοδύναμη με ολοκλήρωμα ενέργειας $\frac{m\dot{\varpi}^2}{2} + \frac{m\varpi^2\dot{\varphi}^2}{2} + \frac{1}{2}k\varpi^2 = \text{σταθ}$)

Έστω αρχικά $\varpi = 0$, $\dot{\varpi} = v_0$ με $\bar{v} = v_0\hat{x}$ (δηλ. $\varphi = 0$)



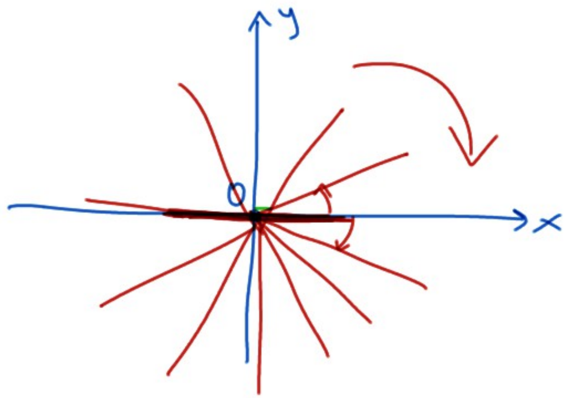
Τότε (2)' → $\ell = 0$ δηλ. $\dot{\varphi} = -\omega\sin\lambda$ $\Leftrightarrow$ $\boxed{\varphi = -\omega t\sin\lambda}$

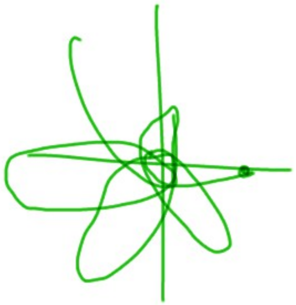
και (1)' → $\varpi = C_1\cos\left(\sqrt{\frac{g}{R}}t\right) + C_2\sin\left(\sqrt{\frac{g}{R}}t\right)$ με $C_1 = 0$, $C_2 = v_0\sqrt{\frac{R}{g}}$

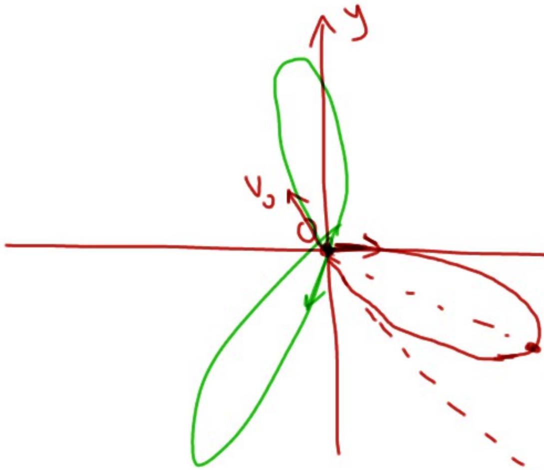
δηλ. $\varpi = v_0\sqrt{\frac{R}{g}}\sin\left(\sqrt{\frac{g}{R}}t\right)$

$$\theta = v_0 \sqrt{\frac{R}{g}} \sin\left(\sqrt{\frac{g}{R}}\, t\right)$$

$$\varphi = -\omega t \sin\lambda$$

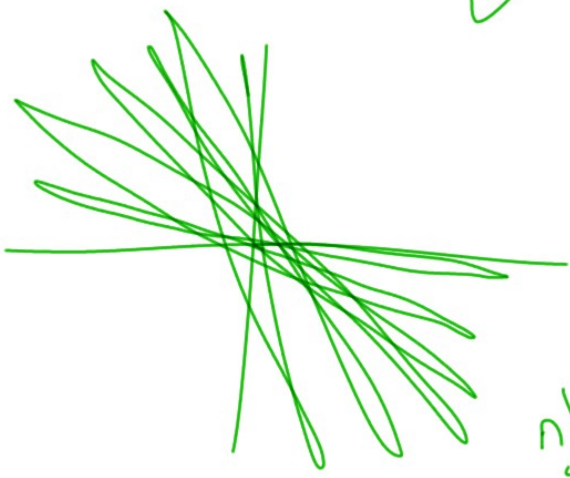






$$\sqrt{\frac{g}{R}}\, t = \frac{\pi}{2}, \quad t = \frac{T}{4}, \quad T = 2\pi\sqrt{R/g}$$

$$\varphi = -\omega \frac{T}{2} \sin\lambda$$



Μια πλήρης περιστροφή του επιπέδου ταλάντωσης γίνεται σε χρόνο $\dfrac{2\pi}{\omega \sin\lambda} \overset{\lambda = 49^\circ}{=\!=\!=} 32h \dfrac{0.75}{\sin\lambda}$

πλήθος λοβών $= \dfrac{2\pi}{\omega \frac{T}{2} \sin\lambda}$