για να είναι $|\vec{a}|^2 = σταθ = a^2$.

$$a_1 = \frac{d^2x}{dt^2} = \underline{όχι}\ σταθερό$$

$$\gamma_1^3 a_1 = a = σταθ \qquad a^\mu = \begin{pmatrix} \gamma_1^4 a_1 u_1 \\ \gamma_1^4 a_1 \end{pmatrix}$$

για να είναι σταθ.
μέτρου θα πρέπει

αν $u_1 = 0$ $a_1 = a$, αν $u_1 \neq 0$ $a_1 \neq a$

Οποιαδήποτε 4-ταχύτ.

$$v^\mu = \begin{pmatrix} \operatorname{ch}\varphi(\tau) \\ \operatorname{sh}\varphi(\tau)\hat{n}(\tau) \end{pmatrix}$$

$|v|^2 = -1$

$\varphi(\tau)$ = άγνωστο

$\tanh\varphi(\tau) = v(\tau)$

ιδιοχρ. σωματ.

$$a^\mu = \begin{pmatrix} \dot\varphi \operatorname{sh}\varphi(\tau) \\ \dot\varphi \operatorname{ch}\varphi(\tau) + \operatorname{sh}\varphi(\tau)\ \dot{\hat{n}} \end{pmatrix}$$

$\bullet = \frac{d}{d\tau}$

$|a|^2 = a_0^2 =$ σταθ $\quad \hat{n} =$ σταθ $\Rightarrow \varphi(\tau) = \overset{a_0}{\overset{\shortparallel}{\alpha}}\tau + \beta \underset{v^\mu}{\Rightarrow} x^\mu(\tau)$

$$a^\mu = \begin{pmatrix} \gamma^4 \vec{v}\cdot\underline{\underline{\vec{a}}} \\ \gamma^4 (\vec{v}\cdot\vec{a})\vec{v} + \gamma^2 \vec{a} \end{pmatrix} \qquad \vec{v} = \frac{d\vec{x}}{dt}$$

$$u^\mu = \begin{pmatrix} \gamma \\ \gamma\vec{v} \end{pmatrix} \quad \frac{du^\mu}{d\tau} = \ldots\ldots \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

[illegible] $\vec{v} \parallel \vec{a} \parallel \hat{x}$ $|a|^2 = a_0^2 =$ קבוע

$$a^\mu = \begin{pmatrix} \gamma^4 v a \\ \gamma^4 a \hat{x} \end{pmatrix} \qquad |a|^2 = (\gamma^4 a)^2(-v^2+1)$$

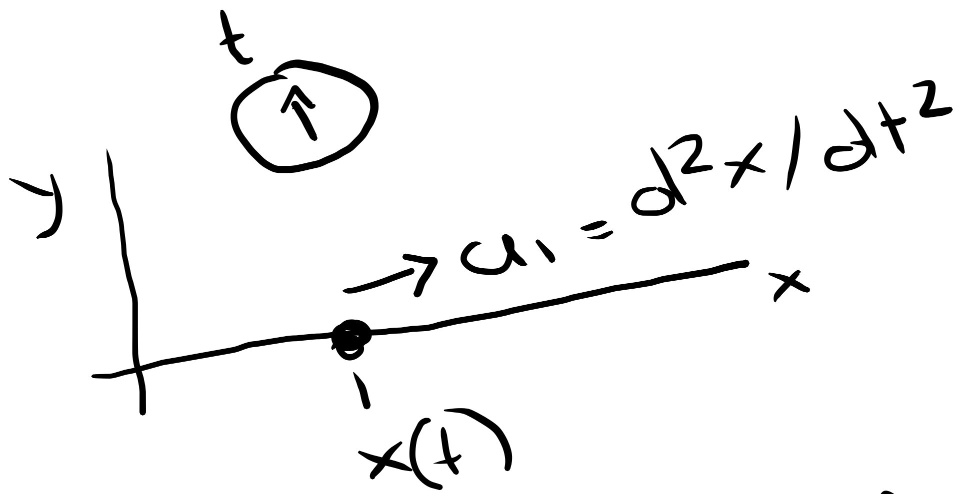
$$|a|^2 = a_0^2 = \gamma^6 \underline{\underline{a^2}}$$

ox1
קבוע

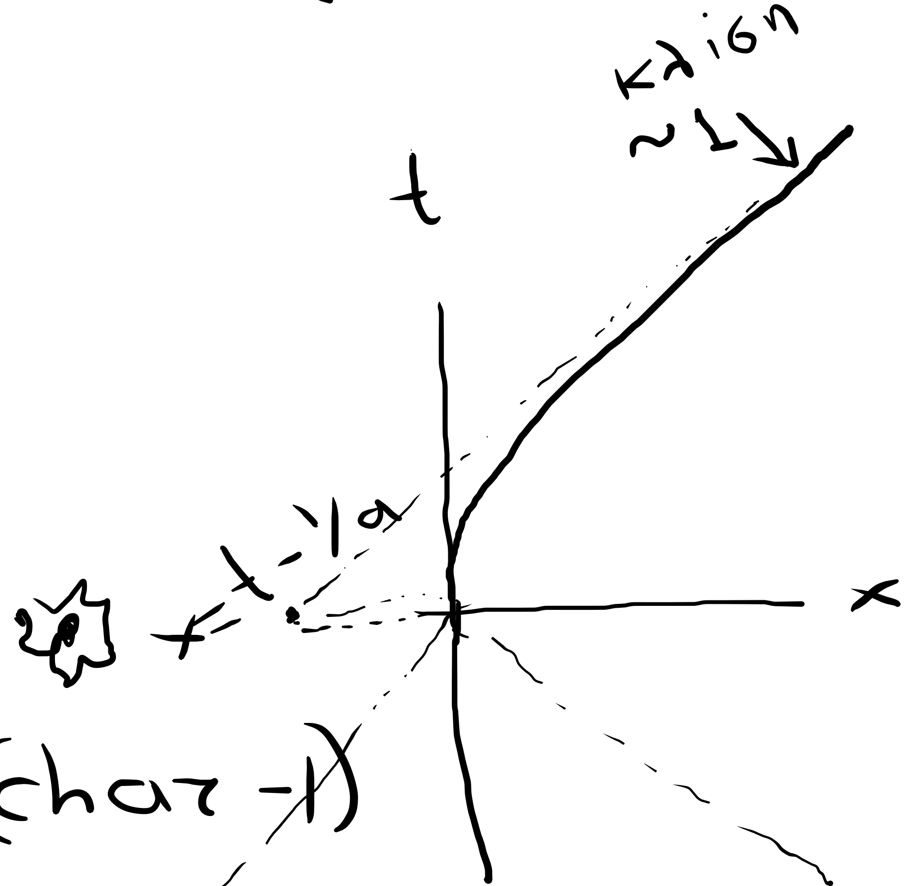
קבוע $= a_0 = \gamma^3 a$

$$x = \sqrt{t^2 + 1/a_0^2} - 1/a_0 \qquad \frac{d^2x}{dt^2} \neq$$ קבוע

$$\frac{dx}{dt} = v \rightarrow \gamma$$



$$\gamma_1^3 a_1 = a$$



$$a_1 = \frac{a}{\gamma_1^3} \xrightarrow[\substack{\gamma_1 \uparrow \infty \\ v_1 \uparrow 1}]{} 0$$

$$\frac{dx}{dt} \sim 1 \quad (\text{για } t \text{ μεγάλο})$$

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) \to 0$$

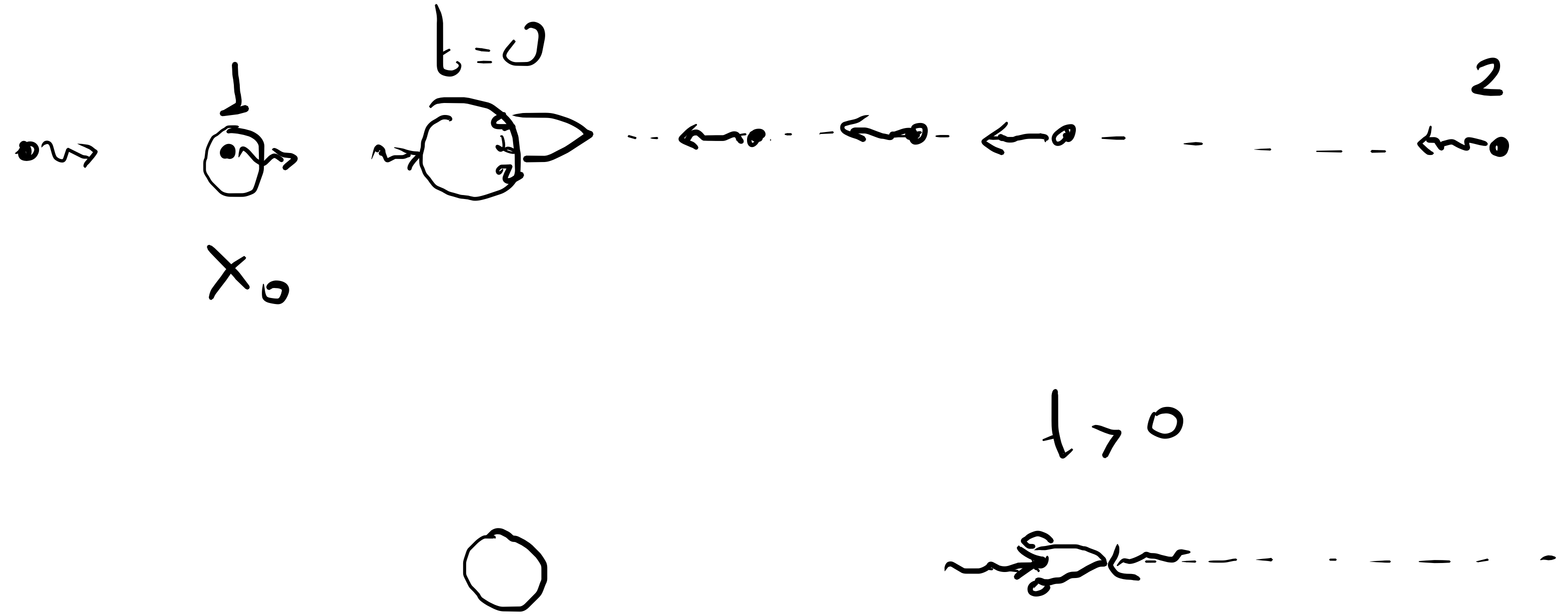
$$x = \frac{1}{a}(\operatorname{ch} a\tau - 1)$$

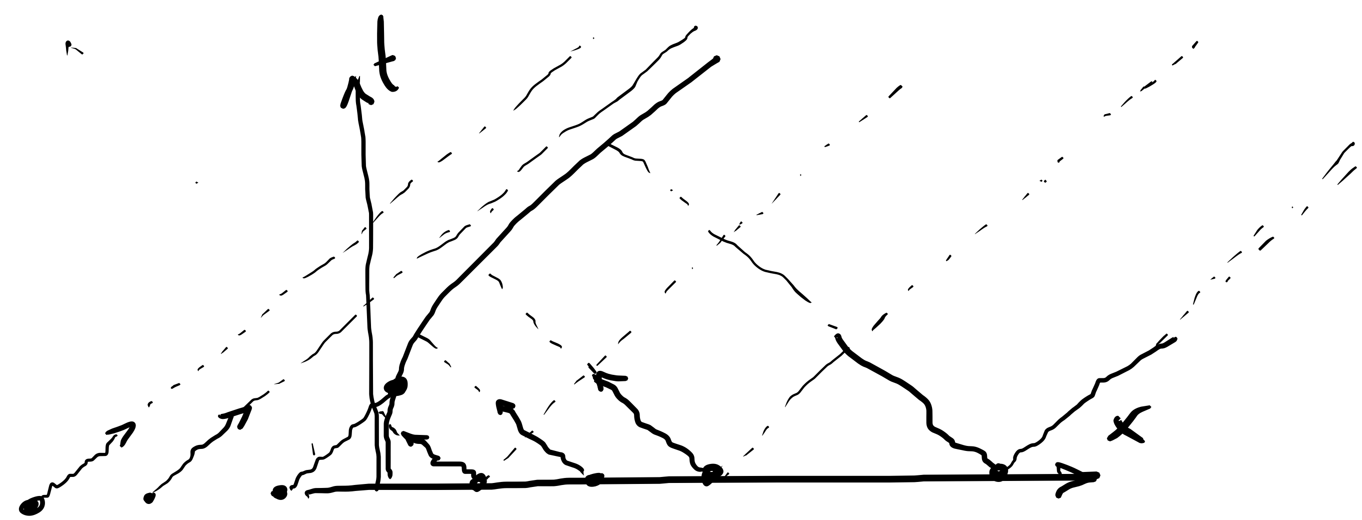
$$t = \frac{1}{a} \operatorname{sh} a\tau$$

$$\operatorname{ch} = \sqrt{\operatorname{sh}^2 + 1}$$

$$x = \frac{1}{a}\left(\left((ta)^2 + 1\right)^{1/2} - 1\right) = \sqrt{t^2 + 1/a^2} - 1/a \underset{t \ll 1/a}{\simeq} \frac{1}{2} a t^2$$

$$\underset{t \gg 1/a}{\simeq} t - 1/a$$





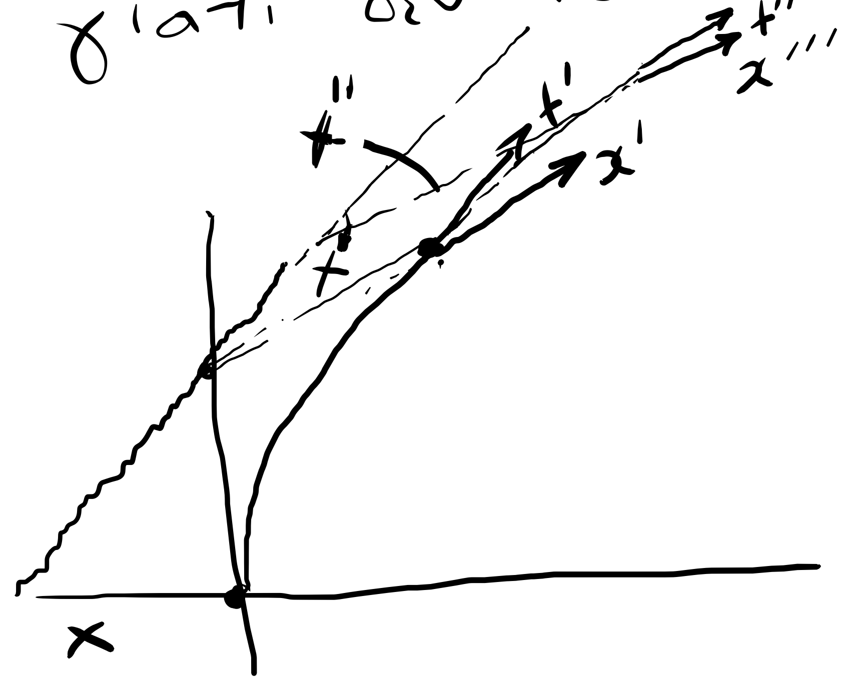
$x_0 < -1/a$

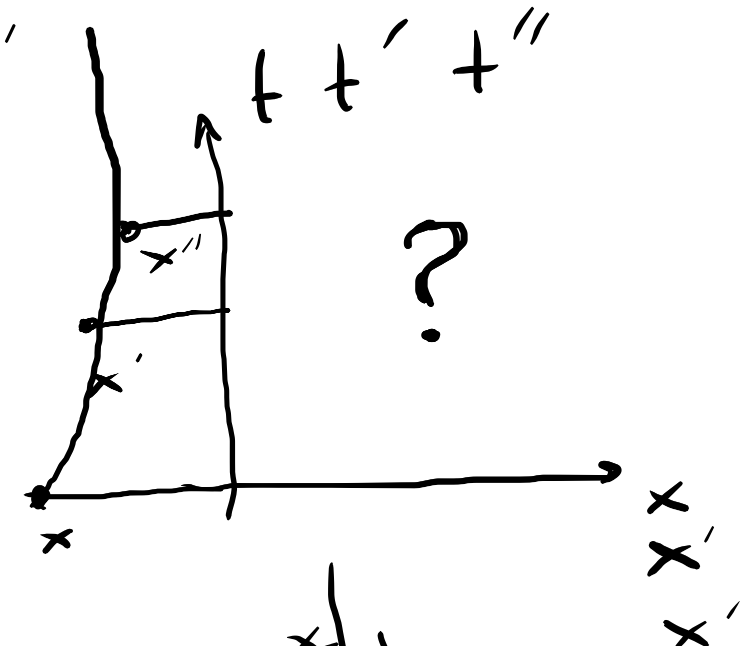
Το φως με αρχή το $(0, x_0)$ ΔΕΝ θα φτάσει ποτέ τον αστρον.

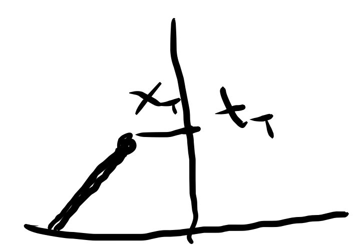
$$v_{ασ}'' - 1 = \frac{v_{αστ} - 1}{1 - v_{ακτ}} \overset{!}{=} -1$$

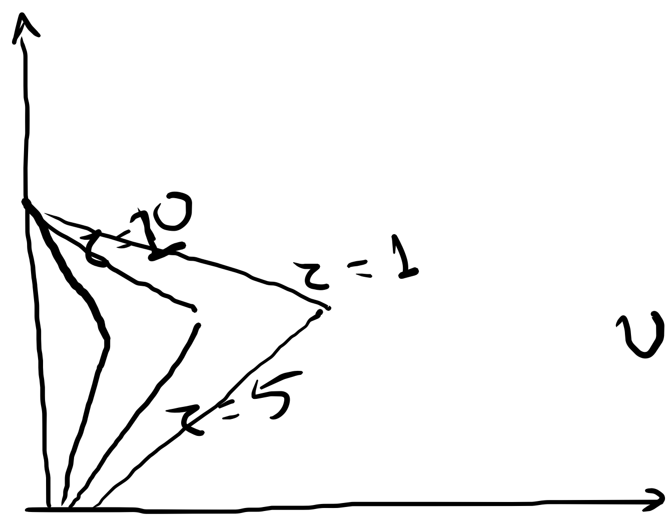
αφού τον πλησιάζει με σταθερή ταχυτ.

γιατί δεν τον φτάνει ποτέ ?

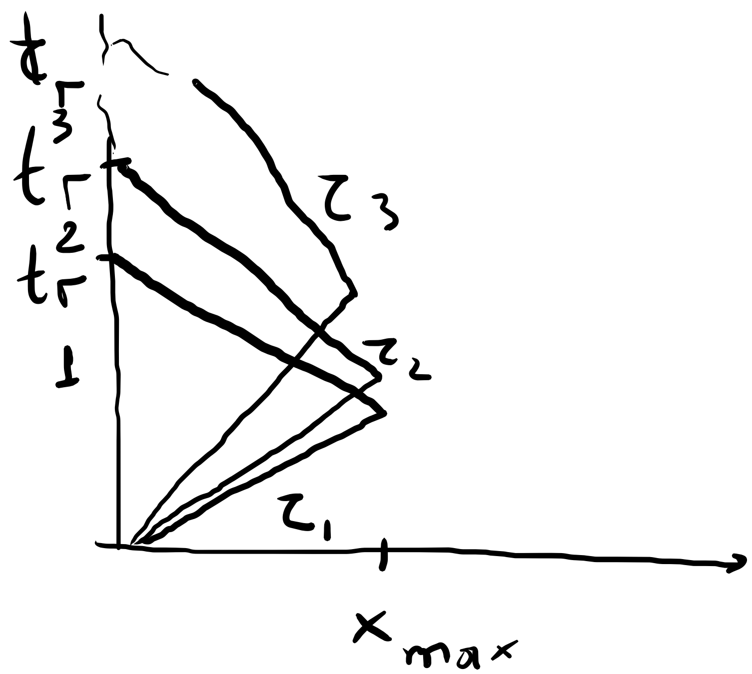








$v \to 1 \quad \tau \to 0$



$t_r > \tau$

$\downarrow v\uparrow$

$t_r >> \tau \to 0$

$\downarrow$

$2x_{max}$

Μηχ 2

$$x \to x' = x + \varepsilon\, \eta(x,t)$$
$$t \to t' = t + \varepsilon\, \tau(x,t)$$

οποιοσδήποτε μετασχ.

$$L(x,v,t) \Longrightarrow \underbrace{\int L' dt}_{S'} = \underbrace{\int L\, dt}_{S} + \varepsilon \int \frac{d}{dt}\big(p\eta + \tau(L - pv)\big) + O(\varepsilon^2)$$

$$L' = L(\underbrace{x'}_{x+\varepsilon\eta}, \underbrace{v'}_{\frac{d(x+\varepsilon\eta)}{d(t+\varepsilon\tau)} = \dots}, t') = L(x,v,t) + \varepsilon \dots$$

$L' \neq L \quad S' \neq S$

αν

αν $S' - S = O(\varepsilon^2)$

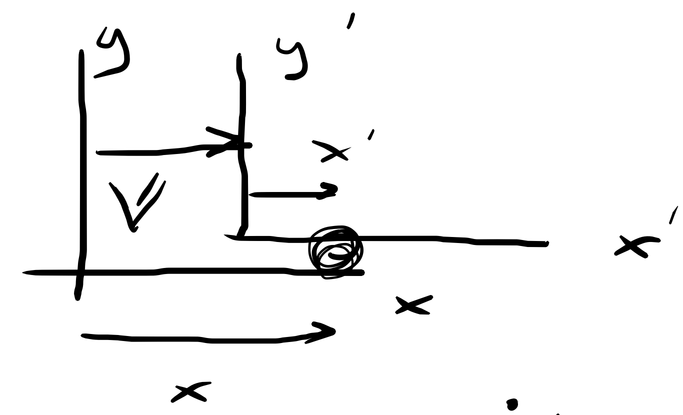
$$L = \frac{v^2}{2} - \frac{x^2}{2} \qquad L' = \frac{v'^2}{2} - \frac{x'^2}{2}$$

$$\Rightarrow \varepsilon \int \quad = 0$$

$p\eta + \tau(\quad) = \text{στα}\vartheta$

$$L = \frac{1}{2} m \dot{x}^2$$

σε άλλο σύστημα: $x' = x - \int_{t=0}^{t} V(t)\,dt'$ $V(t)$ γνωστή συνάρτ.



$$L(x,v,t) \underset{\text{αριθμ.}}{=} L'(x',v',t') \qquad \dot{x}' = \dot{x} - V(t)$$

$$L' = \frac{1}{2} m \left(\dot{x}' + V(t)\right)^2$$

$$x \longrightarrow x'$$

$L = L'$

αριθμ

αλλαγή συντετ.

$\Downarrow$

ίδιες $\in L$

$L \xrightarrow[t \to t']{x \to x'} L' \neq L$

$\upsilon \to \upsilon'$

αν τύχει ο μετασχ. $L' - L \simeq O(\varepsilon^2)$

$\Downarrow$

συμμετρία

$\rightsquigarrow$ διατήρηση

$$L = \frac{1}{2} m \dot{x}^2$$

$$x \rightarrow x' = x + \varepsilon t$$

$$t \rightarrow t' = t$$

ιδια
συναρτ.

$$L' = \frac{1}{2} m \left(\frac{dx'}{dt'}\right)^2$$

διαφορ
νουμερ

$$= \frac{1}{2} m (\dot{x} + \varepsilon)^2$$

$$= \underbrace{\frac{1}{2} m \dot{x}^2}_{L} + \dots$$

ιδιο
νουμερ

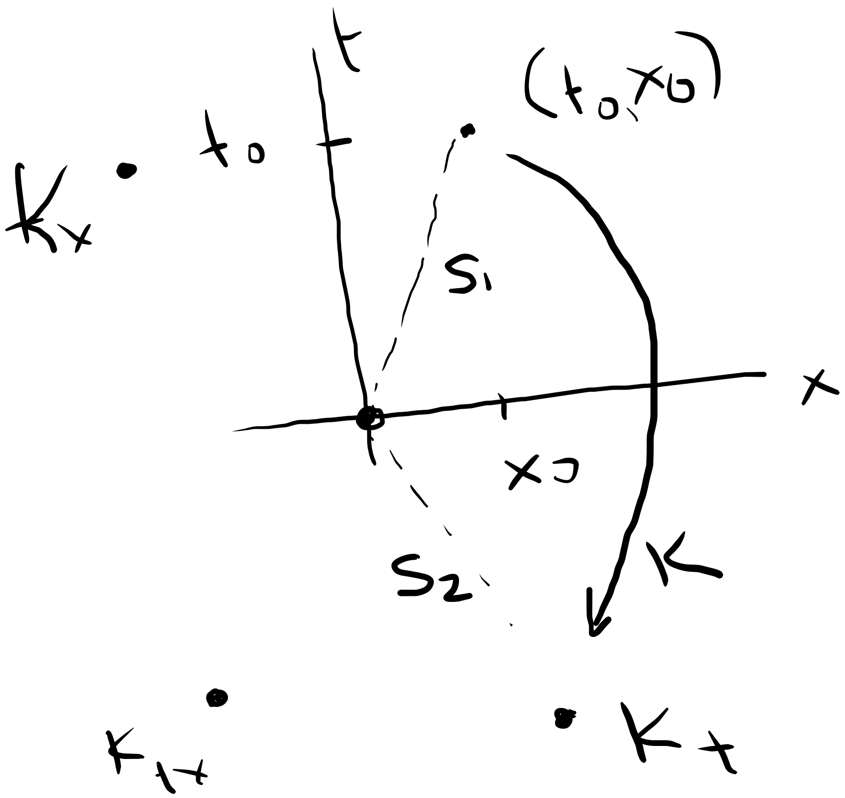
$$L' = L$$

διαφορ.
συναρτηση

$$\underbrace{\frac{1}{2} m \dot{x}^2}_{L(x,v,t)} = \frac{1}{2} m \left[\frac{d}{dt}(x' - \varepsilon t)\right]^2$$

$$= \underbrace{\frac{1}{2} m \dot{x}'^2 - m \dot{x}' \varepsilon + \frac{m \varepsilon^2}{2}}_{L(x', v', t'=t)}$$

ΕΘΣ



$$K_t = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

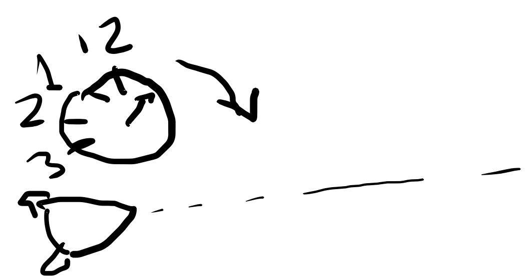
$$K x_0^\mu = \begin{pmatrix} -t_0 \\ x_0 \end{pmatrix}$$

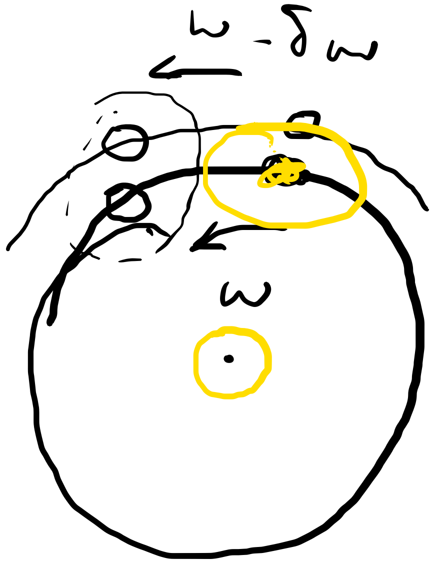
$$K_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$K_{xt} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$s_1^2 = -(t_0)^2 + x_0^2$$

$$s_2^2 = -(-t_0)^2 + x_0^2$$





$$\omega^2 r = \frac{GM}{r^2}$$

$$\omega^2 \sim 1/r^3$$

$$r + \delta r \ , \quad \omega - \delta\omega$$

- για τη Γη ο αστρον **ΔΕΝ** είναι αδραν

$\vec{v}_{αστρ.}$ αλλάζει

- για τον αστροναύτη και το "μικρό" περιβάλλον του <u>είναι</u> αδραν. ό,τι οτιδήποτε πετάξει θα κινείται ομαλά

$2L/c = \Delta t$

$\omega \Delta t = \Delta\varphi$

$\Delta\varphi = \omega \frac{2L}{c}$

$c = \ldots$

$\delta\varphi$

$c_{\to} \neq c_{\leftarrow}$

180°

$c_{\to} \to c_{\leftarrow}$

$c_{\leftarrow} \to c_{\to}$

$$x^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |x|^2 = -1$$

$$\tilde{x}^\mu = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix} \qquad |\tilde{x}|^2 = 5$$

$$x^\mu(\ ) = \begin{pmatrix} \tau \\ 2\tau \\ 0 \\ 0 \end{pmatrix} \qquad |x|^2 = +3\tau^2$$

$\gamma = 2/\sqrt{3}$

$\uparrow$

$v = \frac{2-1}{3-1} = 1/2$

$$v^\mu = \begin{pmatrix} \gamma \\ \gamma v \hat{x} \end{pmatrix}$$

$\gamma = v^0 = \frac{dt}{d\tau}$

$\downarrow$

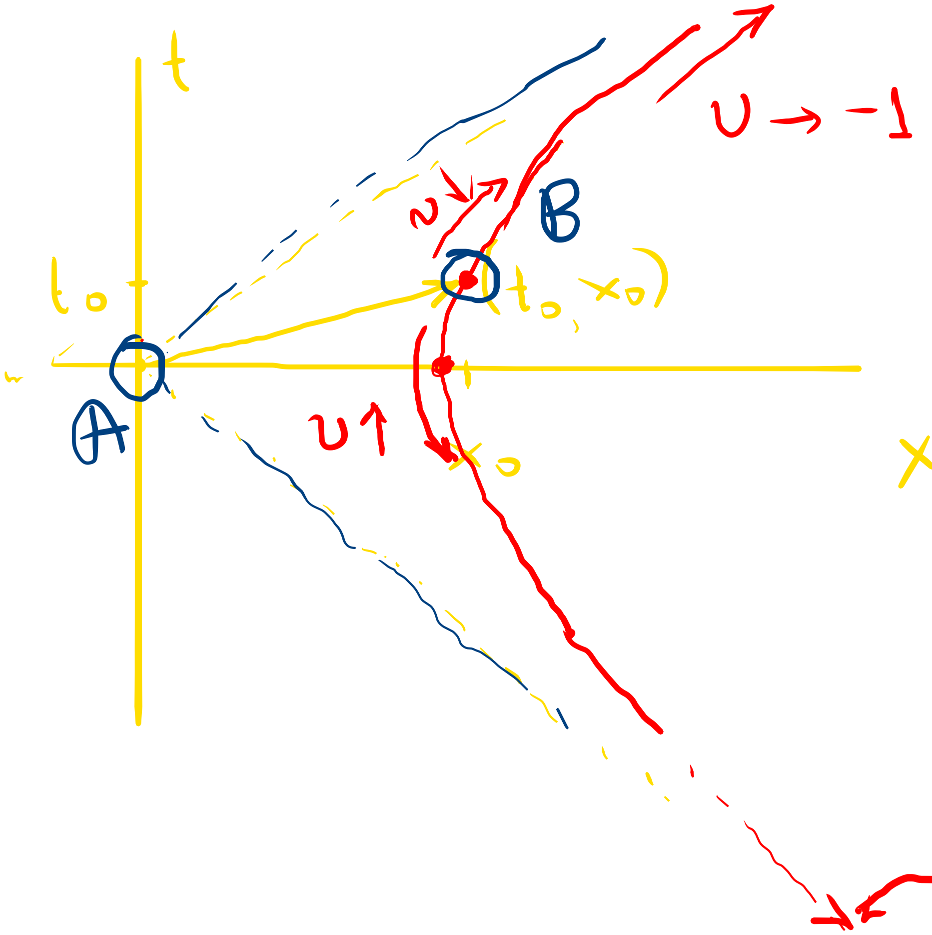
$dt/d\tau = 2/\sqrt{3}$

$x=1$	$x=3/2$	$x=2$
$\cdot$	$\cdot$	$\cdot$
$t=1$	$t=2$	$t=3$

Sim exp. $\rightarrow \tau = 0$ $\quad \tau = ?$

$t = 2 \qquad \Delta t = 1 = 2-1$

$\Delta \tau = \frac{1}{2/\sqrt{3}}$



$$x' = \gamma(x_0 - v t_0)$$
$$t' = \gamma(t_0 - v x_0)$$

$$|x_0| > |t_0|$$
$$x_0 > t_0$$

$$\underline{x'^2 - t'^2} = x_0^2 - t_0^2$$

~~[illegible]~~ $\underline{v_0 = t_0/x_0} < 1$ $\exists$ [illegible]
[illegible]

$$t'(v_0) = 0$$

$v \to 1 \quad \gamma \to +\infty$

$$x' \to \gamma(x_0 - t_0) \to +\infty$$
$$t' \to \gamma(t_0 - x_0) \to -\infty$$

$v \to -1 \quad \gamma \to +\infty$

$$x' = \gamma(x_0 + t_0) \to +\infty$$
$$t' = \gamma(t_0 + x_0) \to +\infty$$

Σ

$t_A = 0 \quad x_A = 0$ (πυροβολισμός του X)

$t_B = 2 \quad x_B = 4$ (θάνατος του X)

το A είναι αίτιο του B ?

γιατί αν πάω σε ένα άλλο συστ. αναφοράς Σ_1 που τρέχει ως προς το Σ με $v_1 = 1/2$

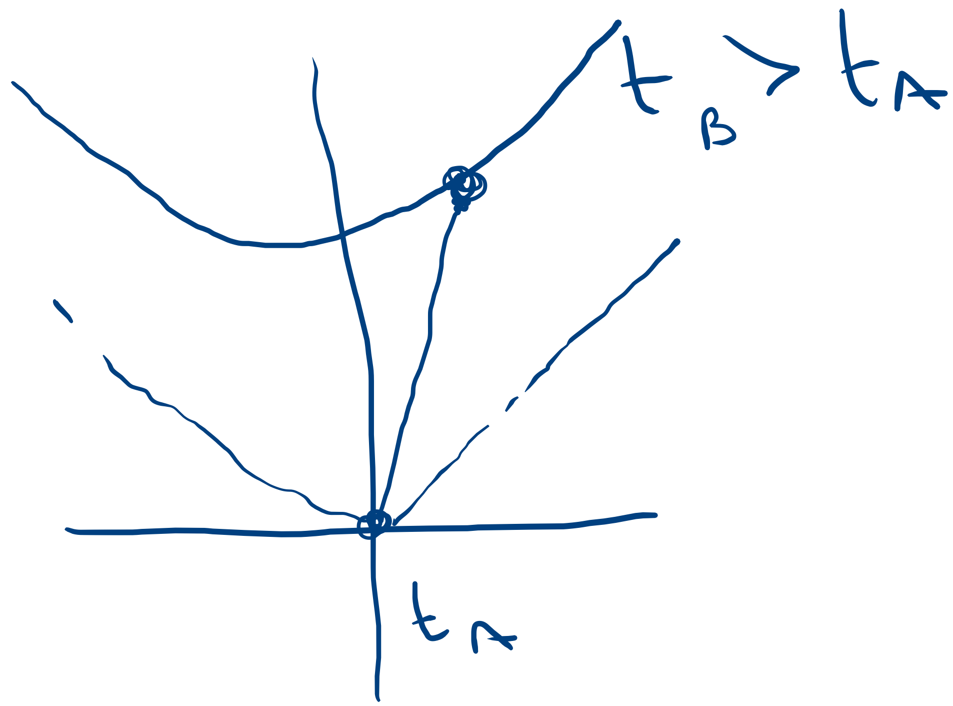
και μετά πάω σε άλλο Σ_2 που τρέχει με $v_2 = 2/3$

Σ_1: $t_{A_1} \quad x_{A_1}$, $t_{B_1} \quad x_{B_1}$

Σ_2: $t_{A_2} \quad x_{A_2}$, $t_{B_2} \quad x_{B_2}$

$t_{A_1} = t_{B_1}$! αποκλείεται ο πυροβ $\Rightarrow$ θάνατο

$t_{B_2} < t_{A_2}$ πρώτη πέθανε μετά πυροβολ.


$t_B > t_A$
t_A