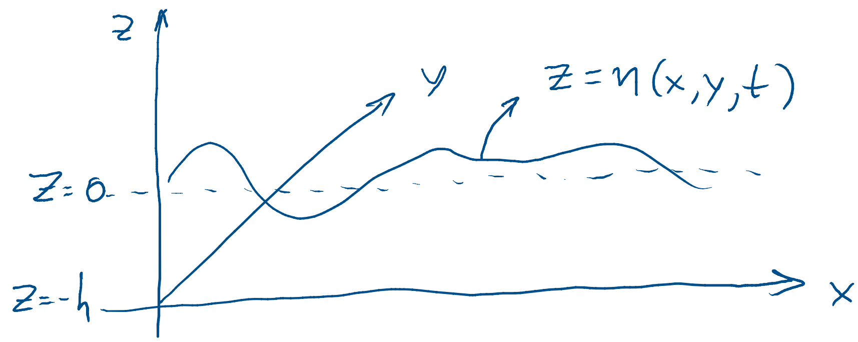
Τρίτη, 16 Μαρτίου 2021 6:33 μμ



$\vec{v} = \nabla\phi$

$\therefore$ $\Delta\phi = 0$ [εξ. Laplace στο νερό], $-h < z < \eta(x,y,t)$

$\therefore$ $\phi_z = 0$, $z = -h$ [κατακόρυφη συνιστώσα της ταχύτητας $= 0$ στον πυθμένα]

$\therefore$ $\phi_t + \frac{1}{2}|\nabla\phi|^2 + g\eta = 0$, $z = \eta(x,y,t)$ [εξ. Bernoulli]

$\therefore$ $\phi_z = \eta_t + \vec{v}\cdot\nabla\eta$ [kinematic condition], $z = \eta(x,y,t)$

Linear regime: $|\eta| \ll 1$, $|\nabla\phi| \ll 1$

$\therefore$ $\Delta\phi = 0$, $-h < z < \eta(x,y,t)$ (1)

$\therefore$ $\phi_z = 0$, $z = -h$ (2)

$$\left.\begin{array}{l} \therefore\ \phi_t + g\eta = 0 , \quad z = 0 \Rightarrow \phi_{tt} + g\eta_t = 0 \\ \therefore\ \phi_z = \eta_t , \quad z = 0 \end{array}\right\} \Rightarrow \phi_{tt} + g\phi_z = 0 , \quad z = 0 \quad (3)$$

Λύση: $\phi = f(z)\, e^{i(k_x x + k_y y - \omega t)}$ (επ. κύμα)

(1): $f'' - k^2 f = 0$, $k^2 \equiv k_x^2 + k_y^2$ (✱)

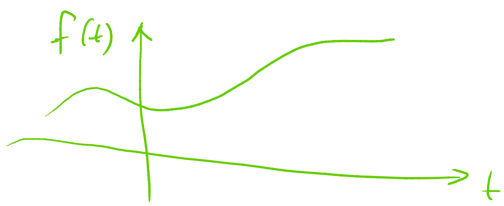
(2): $f' = 0$, $z = -h$ (ΣΣ Neumann)

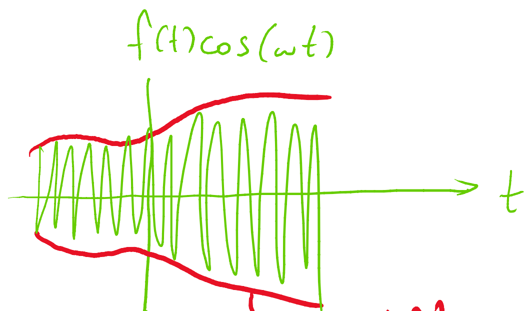
(3): $-\omega^2 f + g f' = 0$, $z = 0$ (ΣΣ Robin) (*)

(**): $f(z) = A\cosh[k(z+h)]$ και $f'(-h) = 0$

(*) $$-\omega^2 \cancel{A}\cosh[k(z+h)]\Big|_{z=0} + gk\cancel{A}\sinh[k(z+h)]\Big|_{z=0} = 0$$

$$\Rightarrow \omega^2 = gk\tanh(kh) \Rightarrow \underline{\omega = \sqrt{gk\tanh(kh)}}$$





περιβάλλουσα (envelope)