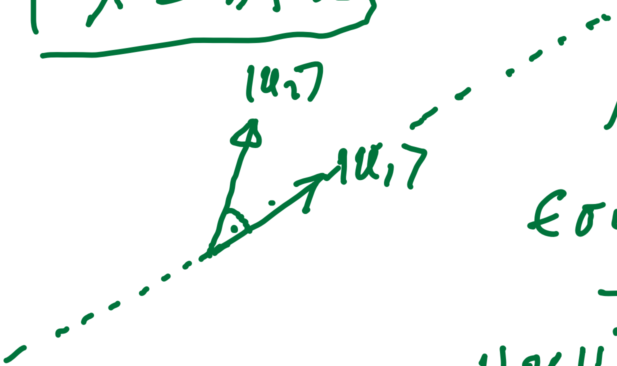
Παρασκευή 27 Μαΐου

$$\boxed{\dot{x} = A x}$$



$A|u_i\rangle = \lambda_i |u_i\rangle$

εσωτερικό γινόμενο

$\longrightarrow$ μέτρο.

$\|u\|^2 = \langle u|u\rangle$

[illegible] $\langle u,v\rangle$

$$\cos\theta = \frac{\langle u|v\rangle}{\sqrt{\langle u|u\rangle\langle v|v\rangle}}$$

Hilbert

μέτρο χωρίς εσωτερικό γινόμενο

$\|u\|_\infty = \max |u_i|$ $\qquad \|\ \|_\infty$ δηλ. ορίζω τη $\|\ \|_\infty$ νόρμα

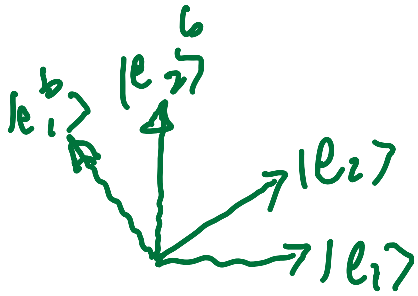
Banach

$\|u\|_1 = \sum_i |u_i|$ $\quad$ (δηλ. ορίζω τη $\|\cdot\|_1$ norm 1

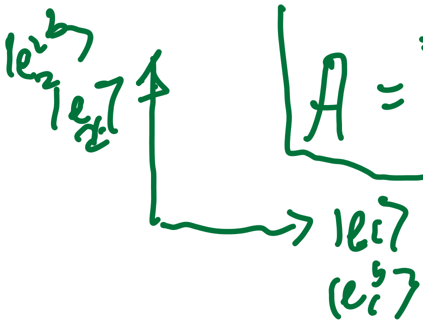
$$\left[\ \|u\|_p = \left[\sum |u_i|^p\right]^{1/p}\ \right]$$

$$A = U \Lambda U^{-1} \qquad U^{-1} U = I$$

$$= \sum \lambda_i$$



A



$$\boxed{A = \sum \lambda_i |e_i\rangle\langle e_i^b|}$$

$\rightarrow \|x\|_2 = \sqrt{\langle x|x\rangle}$

$$\|A\|_2 = \max_{\|x\|=1} \|Ax\|_2$$

$$x(t) = e^{At} x(0)$$

$$\|e^{At}\|$$

πολική ανάλυση
ή ιδιάζουσα ανάλυση
(singular value decomposition)
Schmidt

$$J = \left(A^\dagger A\right)^{1/2}$$ ερμιτιανός θετικός

ιδιάζουσες τιμές είναι
ιδιοτιμές του πίνακα $\sigma_i \geq 0$
και οι ιδιοκαταστάσεις κάθετες

$$z = \sqrt{z^* z}\,\underbrace{\left(\frac{z}{\sqrt{z^* z}}\right)}_{e^{i\theta}}$$

ιδιοκαταστάσεις
του J

$$|\psi_1\rangle = A|v_1\rangle$$
$$|\psi_2\rangle = A|v_2\rangle$$
$$\langle\psi_1|\psi_2\rangle = 0 \quad !$$
$$\langle v_1|A^\dagger A|v_2\rangle = \lambda_2\langle v_1|v_2\rangle = 0$$
$$\langle\psi_1|\psi_1\rangle = \sigma_1^2$$

Τα $|\psi_i\rangle, |v_i\rangle$ είναι
διαφορετικά

v_1, v_2 ιδιοχαρακτηριστικά

τι J

εδώ A εμφανίζονται τα

v_1, v_2 είναι e_1, e_2 και

u_1, u_2 είναι e_1, e_2

$\|A\|_2 = \sigma_1$ $\qquad \sigma_1 > \sigma_2 > \dots \sigma_n$

$$A = \sum_{i=1}^{n} \sigma_i \, |u_i\rangle\langle v_i|$$

$$= U \Sigma V^{\dagger}$$

ιδιάζουσα ανάλυση

$\|e^{At}\|_2$

A $\quad n \times n$ $\qquad A_{m\times m}$ $\qquad 100 \times 100$

$$\boxed{A_{m\times m}} = \sum_{i=1}^{m} \sigma_i |u_i\rangle\langle v_i| \qquad \|A - A_{m\times m}\| = \sigma_{m+1}$$

10^2

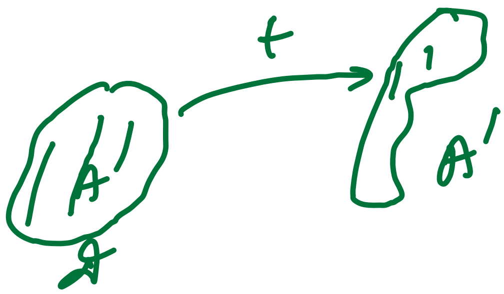
$1002 \times 100 \qquad 10^4$

$\sigma_i \, |u_i\rangle\langle v_i|$

$1 + 100 + 100 = 201$

$\nabla \cdot \vec{V} \qquad \dot{\vec{x}} = \vec{V}(x)$

$$A = \int d^N x$$



$$\frac{dA}{dt} = \int_{\partial} \nabla \cdot \vec{v} \, d^N x$$

$$\frac{1}{\delta A} \frac{d\,\delta A}{dt} = \nabla \cdot \vec{V}$$

σε 2-διαστάσεις!

οριακς

$$\frac{dA}{dt} = 0 \qquad \int_{\partial_{\text{οριχ}}} \nabla \cdot \vec{v} \, d^2 x = 0$$

$\dot{x} = Ax \qquad E = x^{+}x$

$\langle x | x \rangle$

$x_0 \qquad \frac{dE}{dt}$ μέγιστο ή ελάχιστο

$\dot{x} = Ax$

$\dot{x}^{+} = x^{+}A^{+}$

$E = x^{+}x$

$$\frac{dE}{dt} = \dot{x}^{+}x + x^{+}\dot{x}$$

$$= x^{+}A^{+}x + x^{+}Ax$$

$$= x^{+}(\underbrace{A^{+}+A}_{\text{είναι ερμιτιανό}})x$$

ιδιοτιμή $|\phi_i\rangle$

$\mu_1 \quad \in \mathbb{R}$

$\mu_2 \quad |\phi_1\rangle$

$\mu_3 \quad |\phi_2\rangle$

$\vdots \quad \dots$

$\mu_n \quad \phi_n$

$\left.\frac{dE}{dt}\right|_{max}$ είναι η ιδιοτιμή

$|\phi_1\rangle \quad \mu_1$

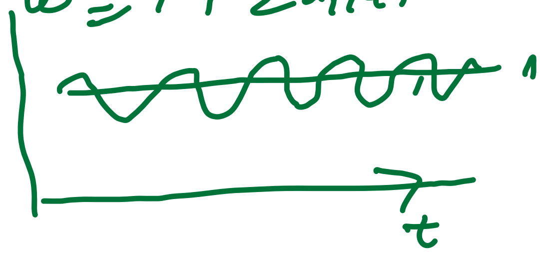
$\left.\frac{dE}{dt}\right|_{min}$

$|\phi_n\rangle \quad \mu_n$

$$e^{A^\dagger t} e^{At} = (I + A^\dagger t + \dots)(I + At + \dots)$$

$$= I + (A + A^\dagger)t + O(t^2)$$

$\omega^2 = 1 + \varepsilon \omega_1(t)$



$$\ddot{x} + \omega^2(t)x = 0$$

$$E = \dot{x}^2 + x^2$$

$$\dot{x} = y$$
$$\dot{y} = -\omega^2(t)x$$

$$A = \begin{bmatrix} 0 & 1 \\ -\omega^2(t) & 0 \end{bmatrix}$$

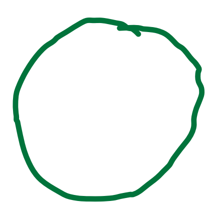
$$\text{trace}(A) = 0$$

$$\nabla \cdot \vec{v} = 0$$

$$E = x^2 + y^2$$

$$\not\equiv \quad \frac{dE}{dt}$$

$$A + A^\dagger$$



$$A + A^\dagger = \begin{bmatrix} 0 & 1-\omega^2 \\ 1-\omega^2 & 0 \end{bmatrix}$$

$$\lambda = \pm|1-\omega^2|$$

$$-|1-w^2| \le \frac{dE}{dt} \le |1-w^2|$$

$$\boxed{e^{-\int_0^t |1-w^2(s)|\,ds} E(0) \le \underline{\underline{E(t)}} \le e^{\int_0^t |1-w^2(s)|\,ds} E(0)}$$

[illegible]

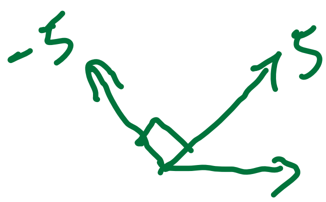
$$A = \begin{bmatrix} -0 & 10 \\ 0 & -0 \end{bmatrix}$$

$$\frac{A+A^t}{2} = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$$

$$(\lambda + 1)(\lambda + \quad) - 25 = 0$$

$$\lambda^2 \qquad -25 = 0$$

$$\lambda = \pm 5 \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



$\dot{x} = v(x) \qquad x \in \mathbb{R}^n$

x_e σημείο ισορροπίας.

$v(x_e) = 0$

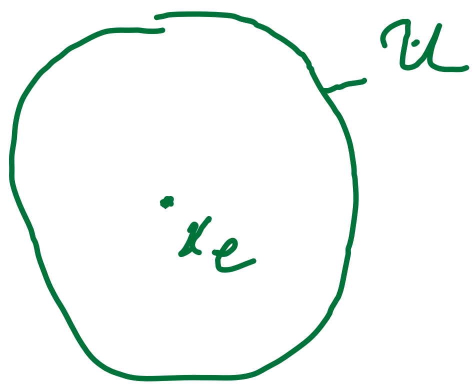
Επ. να προσδιορισθεί η έννοια της x_e κατά Lyapunov

δηλ. εάν

$\forall \varepsilon > 0, \exists \delta$ τέτοιο ώστε

αν $|x(0) - x_e| < \delta$ τότε

$|x(t, x(0)) - x_e| < \varepsilon$



Lyapunov (1905)

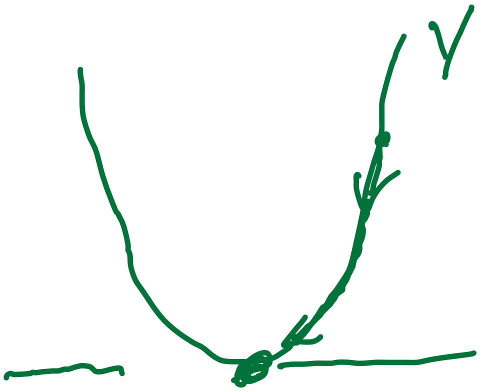
$V(x) > 0 \ \forall x \in \mathcal{U}$

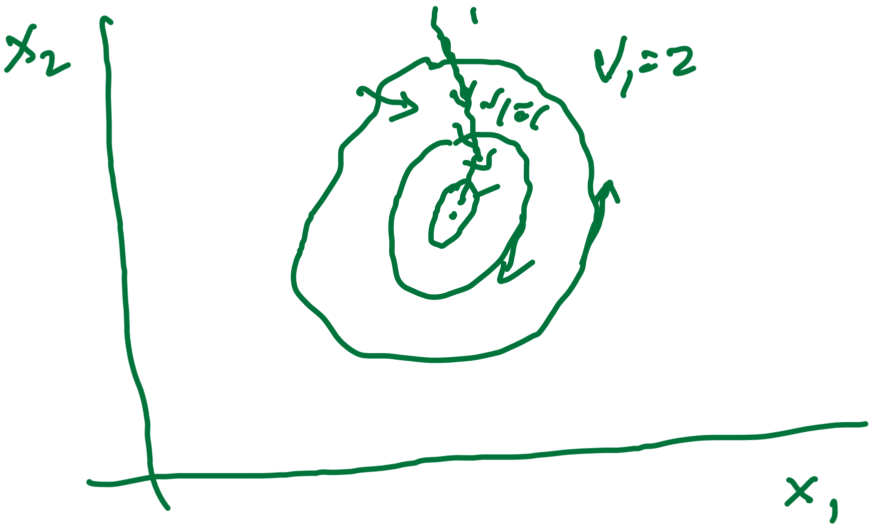
αλλά $V(x_e) = 0$

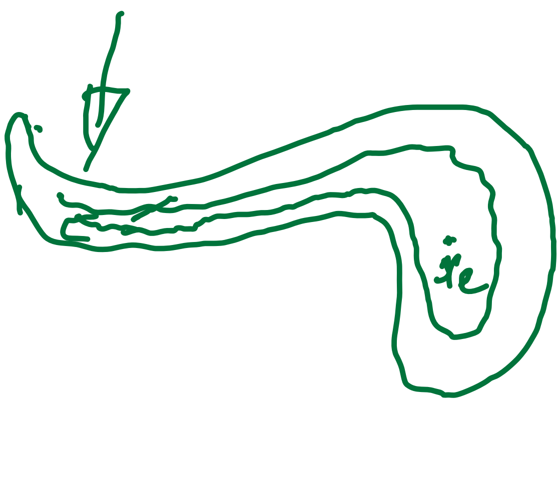
να είναι

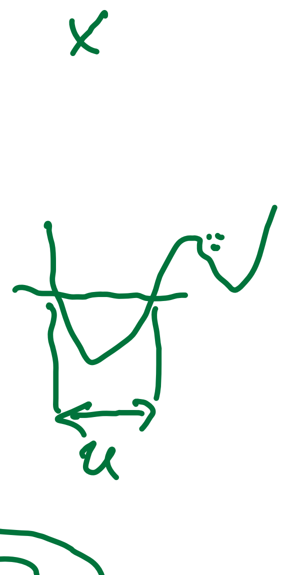
και επίσης να είναι $\dot{x} = v(x)$

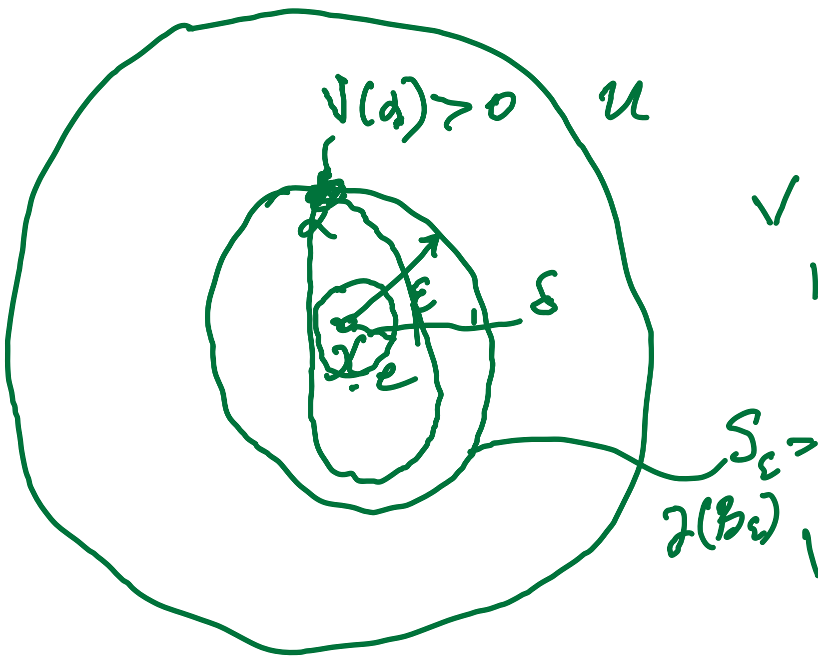
$\dfrac{dV}{dt} \leq 0$ τότε το x_e είναι Lyapunov ευσταθές


V


x_2
$V_1=2$
$V_1=1$
x_1




x
u



δ + δοξεύν τι

ε

V είναι συνεχής και διαφορίσιμη

V ελάχιστο

$S_\varepsilon \Rightarrow \partial(B_\varepsilon)$

$V(\alpha) = \min(V_{x \in S_\varepsilon})$

$U' = \{x : V(x) \leq V(\alpha)\}$

$U' \subset B_\varepsilon \qquad x_e \in U' \qquad V(x_e) = 0 \leq \alpha$

$B_\delta \subset U'$

$x_e \in B_\delta \qquad \forall \; x(0) \in B_\delta$

$V(x(0)) < \alpha$

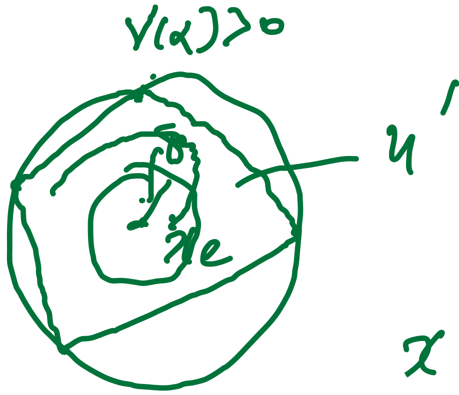
$\frac{dV}{dt} \leq 0 \qquad V(x(t)) < \alpha$

$\forall \; x(t) \in U' \subset B_\varepsilon$

$\forall t \quad |x(t, x_0) - x_e| < \varepsilon$

$0 - \epsilon - \delta -$

$\frac{dV}{dt} < 0 \qquad x(t) \to x_e$



$V(x_e) = 0$

$x(0) \in u'$

$|x(t) - x_e| < \varepsilon$

$\frac{dV}{dt} \leq 0$

$V(x_0) \leq \alpha \Rightarrow V(t) \leq \alpha$

$\frac{dV}{dt} < 0 \qquad x(t) \to x_e$

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$$

$$\dot{x} = y$$

$$\dot{y} = -\gamma y - \omega^2 x$$

$$(x_e, y_e) = (0,0)$$

$$V = \frac{y^2 + \omega^2 x^2}{2}$$

$$V(x) > 0 \quad \forall \; x \neq (x_e, y_e)$$

$$V(0,0) = 0$$

$$\frac{dV}{dt} \leq 0$$

$$\frac{dV}{dt} = y\dot{y} + \omega^2 x\dot{x}$$

$$= -\gamma y^2 - \omega^2 xy + \omega^2 xy$$

$$= -\gamma y^2 \leq 0$$

άρα η V είναι συνάρτηση Lyapunov

$\forall \; x, y$

$$\frac{dV}{dt} < 0 \qquad x(t) \to (0,0)$$

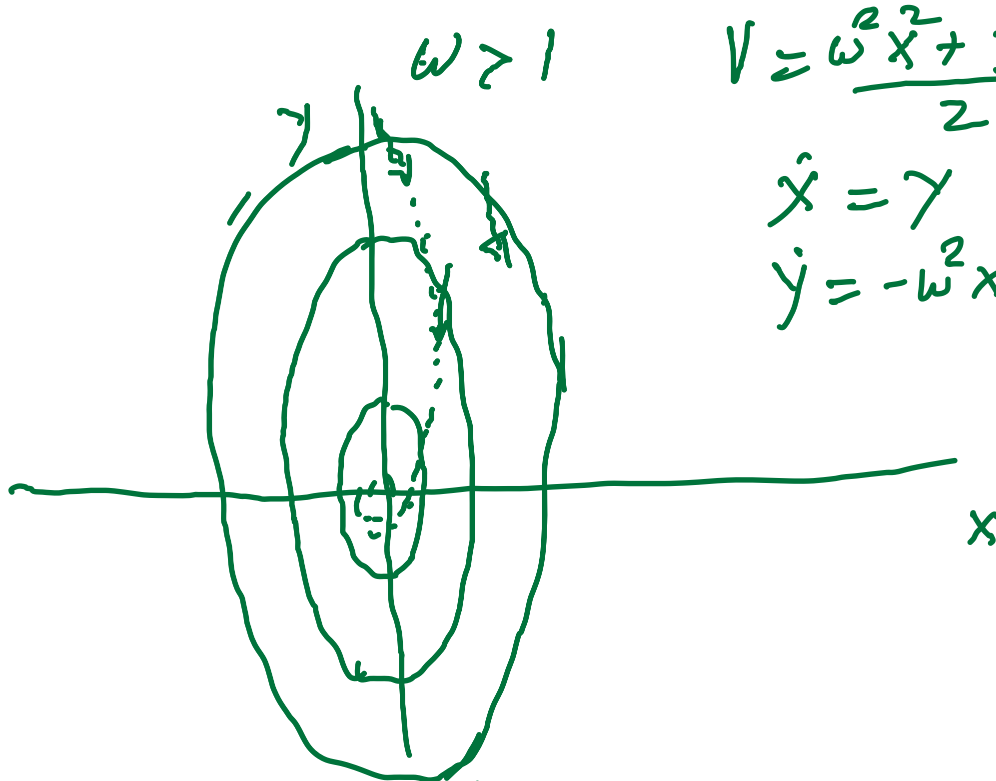
$\forall \; (x_0, y_0)$

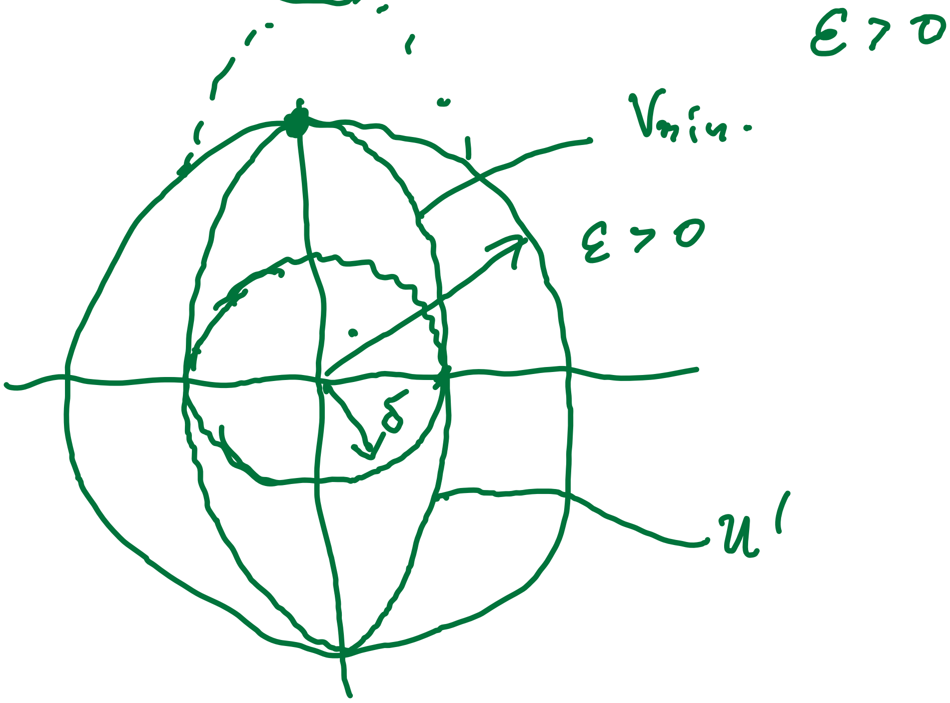
ορίζεται έτσι το πεδίο έλξης του σημείου ισορροπίας.

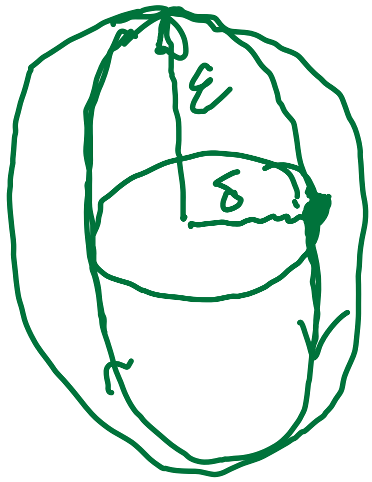
$$V = \frac{\omega^2 x^2 + y^2}{2}$$

$$\dot{x} = y$$

$$\dot{y} = -\omega^2 x - \gamma y$$







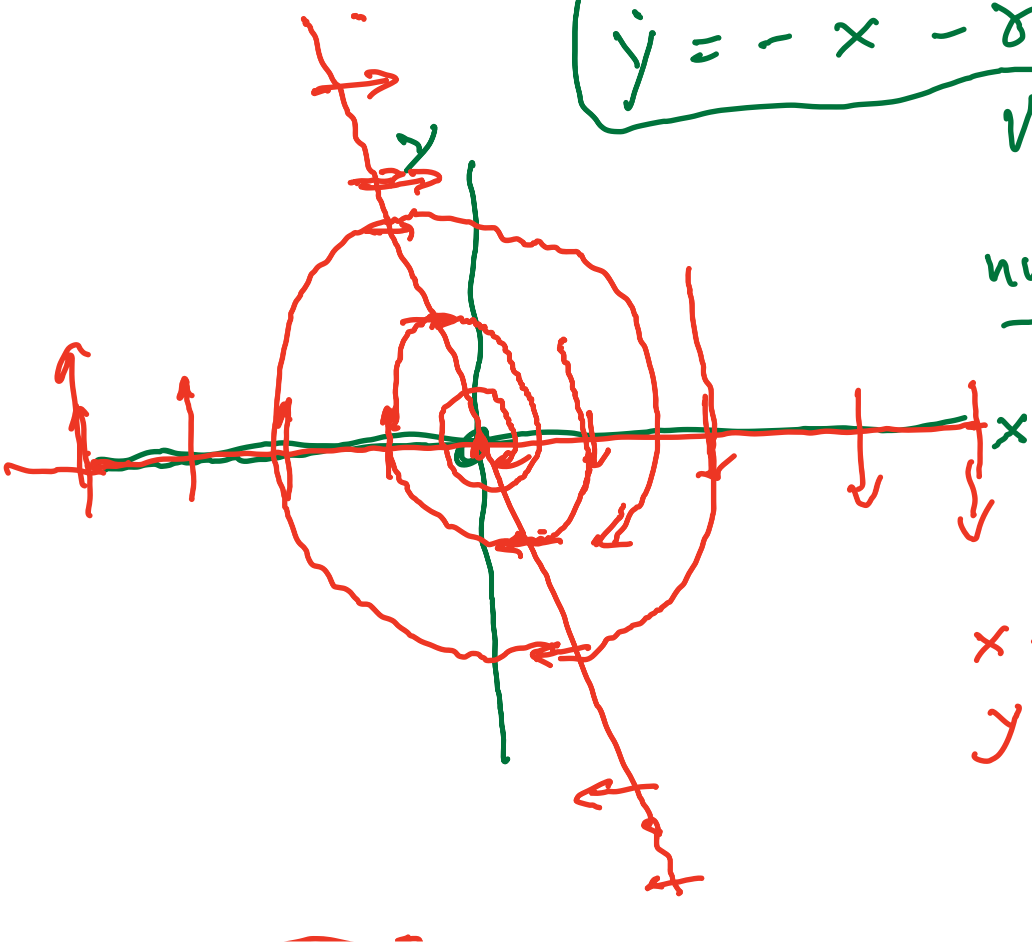
$\omega = 1$

$\ddot{x} + \gamma \dot{x}$ $\quad \underline{\gamma > 0}$

$$\boxed{\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x - \gamma y \end{aligned}}$$

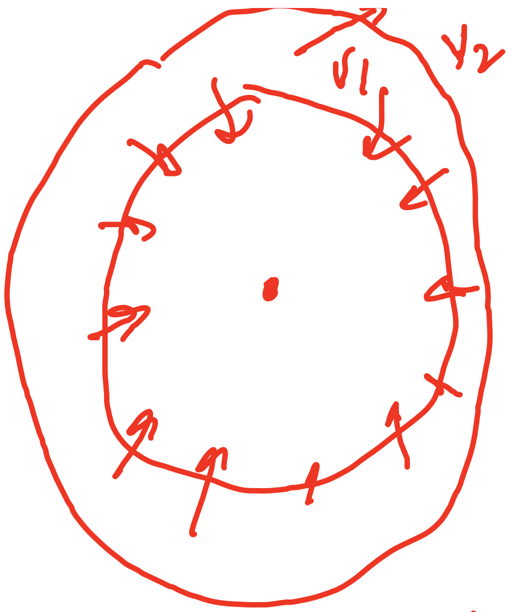
$V = \dfrac{x^2 + y^2}{2}$

<u>nullclines</u>



$x + \gamma y = 0$

$y = -\frac{1}{\gamma} x$



$$\frac{dV}{dt} < 0$$

$\sigma > 0$

$r > 0$

$\beta > 0$

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = rx - y - xz.$$
$$\dot{z} = -\beta z + xy$$

$$x^2 + y^2 + z^2$$

$$x = 0,\ y = 0,\ z = 0$$

$$V = \frac{1}{2}\left(\frac{1}{\sigma}x^2 + y^2 + z^2\right) > 0 \qquad V(0,0,0) = 0$$

$$\frac{dV}{dt} = \frac{1}{\sigma} x\dot{x} + y\dot{y} + z\dot{z}$$
$$= x(y - x) + y(rx - y - xz) + z(-\beta z + xy)$$
$$= -x^2 - y^2 + xy(1 + r) - \beta z^2$$
$$= -\left(x + \frac{r+1}{2}y\right)^2 + y^2\frac{(r+1)^2}{4} - y^2 - \beta z^2$$
$$= -\left(x + \frac{r+1}{2}y\right)^2 - y^2\left(1 - \left(\frac{r+1}{2}\right)^2\right) - \beta z^2$$

θ; είναι η V Lyapunov

Εάν $1-\left(\frac{r+1}{2}\right)>0$

$2-r-1>0$

Εάν $r<1$

Εάν $r<1$ $V=\frac{1}{\sigma}x^2+y^2+z^2$

εν Lyapunov ⇒

$x_e=(0,0,0)$ είναι

καθολικώς ελκυστικό

$\forall (x,y,z) \to x_e$