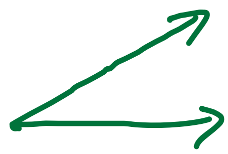
Τρίτη 24 Μαΐου

$\dot{x} = A x$ $\qquad \mathrm{Re}(\mathrm{eis}(A)) < 0$

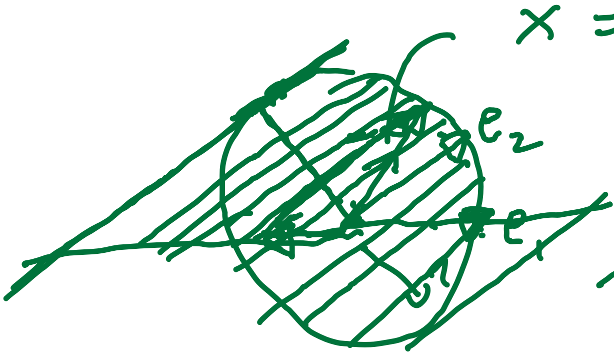
$\underline{x = 0}$

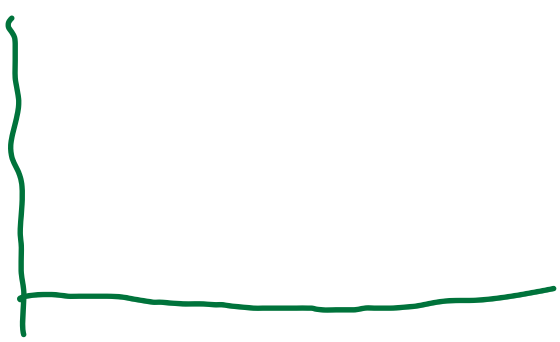
$\|x\|$

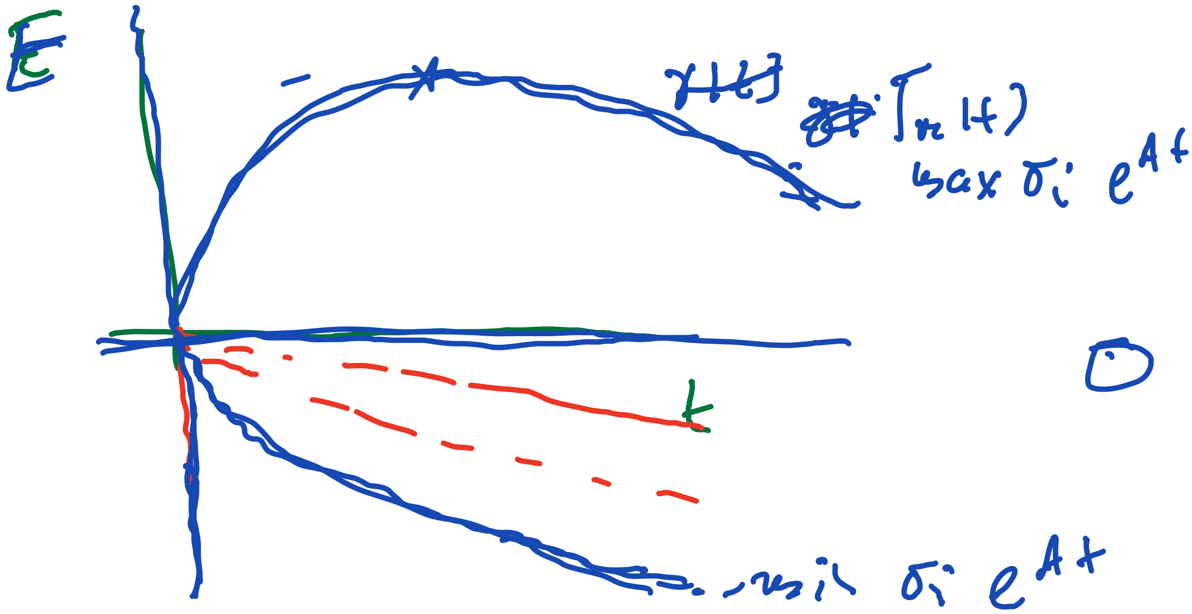
$x = 0$



$x = \alpha e_1 + \beta e_2$

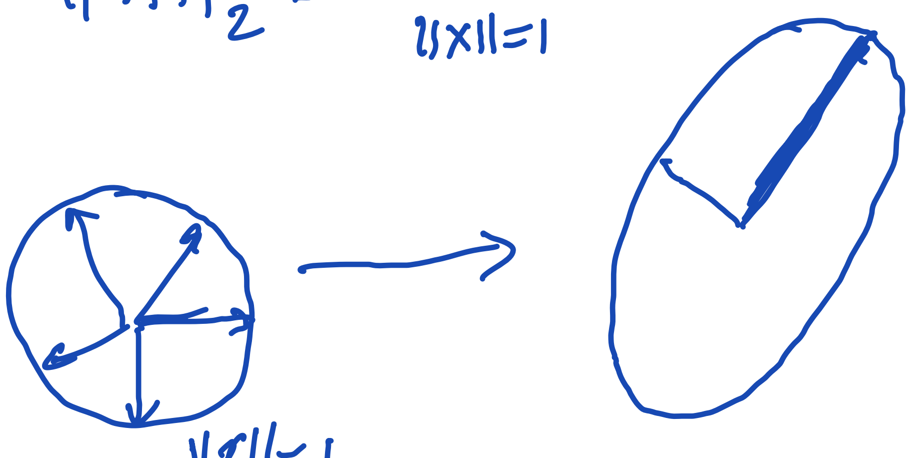






$$\|A\|_2 = \max_{\|x\|=1} \|Ax\|$$

$\|A\|_2$



$\|x\|=1$

$\boxed{\text{Πολική ανάλυση πινάκων}}$ singular value decomposition

$$x_0 \to e^{At} x_0$$

$$F_m(t) = \|e^{At}\|_2^2 = \max_{\|x\|_2 = 1} \|e^{At} x_0\|_2^2$$

$$z = |z|\, e^{i\theta} = \left(\frac{z}{|z|}\right) |z|$$

$$\boxed{A = J\,U} \qquad UJ \qquad UU^+ = I$$

θετικά ερμιτιανός πίνακας.

$$A \qquad J = \sqrt{A^+ A} \qquad K = \sqrt{A A^+}$$

$A^+ A$ ερμιτιανός πίνακα

θετικά είναι ($x^+ A^+ A x = \|Ax\|^2 \geq 0$ για $x \neq 0$

$A^{+}A = V \lambda V^{+}$ $\qquad \lambda$ θετικά

$\sqrt{A^{+}A} = V\sqrt{\lambda}\, V^{+}$

οι ιδιοκαταστάσεις τις J

είναι ορθογώνιες.

$A v_1 = \psi_1$ $\qquad \lambda_1 > \lambda_2$

$A v_2 = \psi_2$

$(\psi_1, \psi_2) = \psi_1^{+} \psi_2 = v_1^{+} (A^{+} A) v_2$

$= \lambda_2 (v_1^{+} v_2) = 0$

$(\psi_1, \psi_2) = \lambda_1$ $\qquad |\psi_1|^2 = \lambda_1$

$(\psi_i, \psi_j) = \delta_{ij} \lambda_i$ $\qquad |\psi_2|^2 = \lambda_2$

$\|A\| = \sqrt{\lambda_1}$

$V = \left[\frac{\psi_1}{\sqrt{\lambda_1}}, \frac{\psi_2}{\sqrt{\lambda_2}} \right]$

$$Ax = U \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix} V^\dagger x$$

$$\boxed{A = U \Sigma V^\dagger}$$ πολικί SVD

$$J = \sqrt{A^\dagger A}$$

$$A^\dagger A = V \Sigma \underbrace{U^\dagger U}_{I} \Sigma V^\dagger = V \Sigma^2 V^\dagger$$

$$\sqrt{A^\dagger A} = V \Sigma V^\dagger$$

$$A = U \Sigma V^\dagger$$

$$= U V^\dagger \underbrace{V \Sigma V^\dagger} = (U V^\dagger) \sqrt{A^\dagger A}$$

$$U V^\dagger = \mathcal{U}$$

$$\mathcal{U}^\dagger \mathcal{U} = V \underbrace{U^\dagger U}_{I} V^\dagger = I$$

$$A = UJ$$

$$K = \sqrt{AA^{\dagger}}$$

$$A = U\Sigma V^{\dagger}$$

ο δείκτης σε κανονικές μετρικές
ενώ πίνακα.

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \quad \begin{bmatrix} \alpha' & \beta' \\ \gamma' & \delta' \end{bmatrix}$$

$A \quad n \times 4$

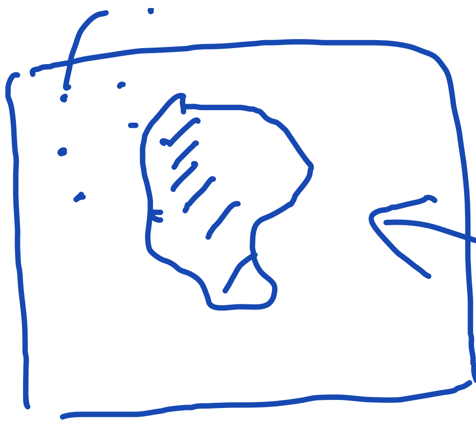
$A_k \quad n \times k$

$$A = U(1{:}k)\begin{pmatrix} \Sigma_1 & & & 0 \\ & \ddots & & \\ 0 & & \Sigma_k & \\ & & & 0 \\ & & & & \ddots \end{pmatrix}(V(1{:}k))^{\dagger}$$

Σ_{k+1}

$$\| A - A_k \| = \Sigma_{k+1}$$

[illegible]



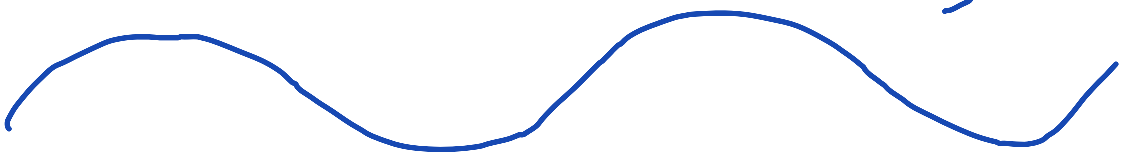
Σ_1

$A = U \Sigma V^T$

V_1

V_2

V_3



$\cancel{A} e^{At} \cancel{B}$

$e^{At} = U \Sigma V^T$

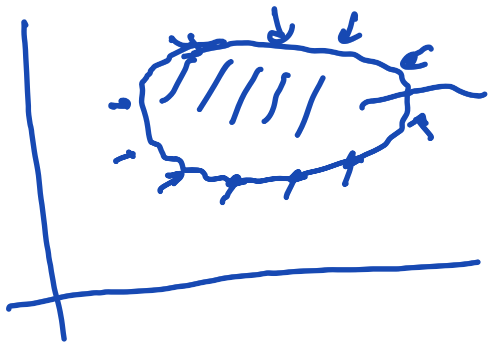
$\max(\sigma_i) = \sigma_1(t)$

$\min(\sigma_i) = \sigma_k(t)$

$$\sigma_i = \left(\frac{x^T A^T A x}{x^T x} \right)$$

$\frac{dE}{dt}$

$\| e^{At} \|$



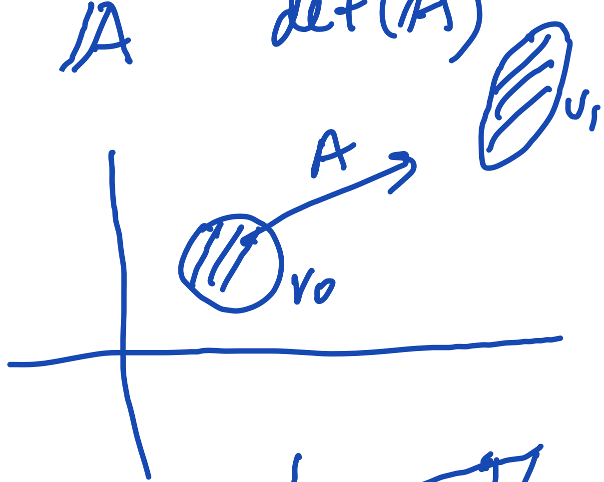
$\dot{x} = V(x)$

$\frac{dV}{dt}$

$V = \int_{\Omega} dx_0^{(N)}$

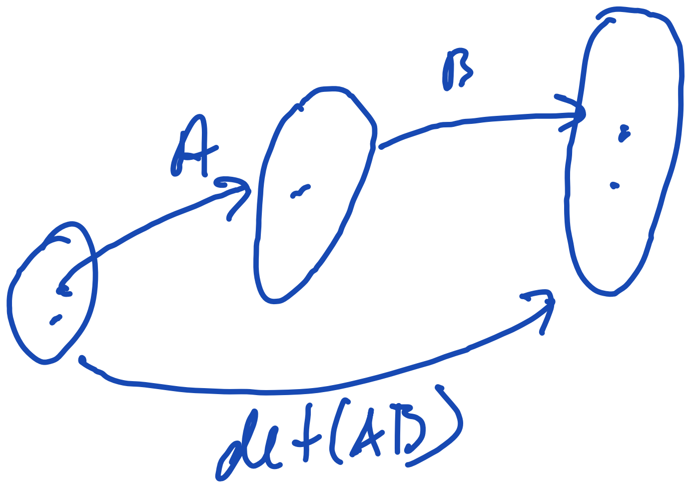
$\cancel{d}\ \frac{V_1}{V_0} = \det(A)$

$A \qquad \det(A)$



$\det A$

$$\underline{\det(AB)} = \underline{\det(A)\cdot\det(B)}$$



$$\underbrace{\det(e^{A})} = e^{\mathrm{trace}(A)}$$

$$e^{A} = \lim_{n\to\infty}\left(I + \frac{A}{n}\right)^{n}$$

$$\det(e^{A}) = \lim_{n\to\infty}\left(\underbrace{\det\left(I+\frac{A}{n}\right)}\right)^{n}$$

$$\det\left(I + \frac{A}{n}\right) = 1 + \frac{1}{n}\mathrm{trace}(A) + O\left(\frac{1}{n^2}\right)$$

$$\begin{vmatrix} 1+\frac{\alpha_{11}}{n} & \frac{\alpha_{12}}{n} \\ \frac{\alpha_{21}}{n} & 1+\frac{\alpha_{22}}{n} \end{vmatrix} = \left(1+\frac{\alpha_{11}}{n}\right)\left(1+\frac{\alpha_{22}}{n}\right) - \frac{\alpha_{12}\alpha_{21}}{n^2}$$

$$= 1 + \frac{1}{n}(\alpha_{11}+\alpha_{22}) + O\left(\frac{1}{n^2}\right)$$

$$\lim_{n\to\infty}\left[1+\frac{1}{n}\operatorname{trace}(A) + O\left(\frac{1}{n^2}\right)\right]^n = e^{\operatorname{trace}(A)}$$

$$\frac{1}{n} = \varepsilon$$

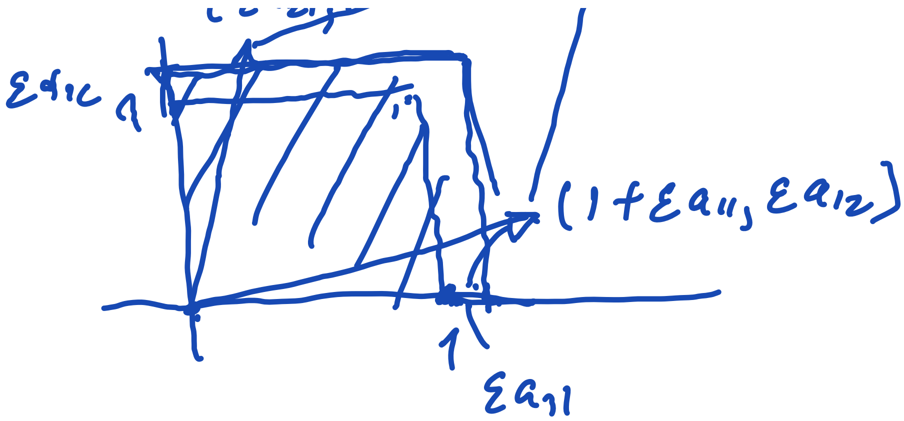
$$\det(I + \varepsilon A) =$$

$$= \varepsilon_{i_1 i_2 \dots i_n}\left(\delta_{1i_1} + \underline{\varepsilon A_{1i_1}}\right)\cdots\left(\delta_{ni_n} + \varepsilon A_{ni_n}\right)$$

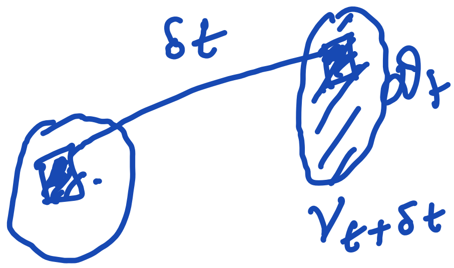
$$= 1 + \varepsilon\left(A_{11} + A_{22} + \dots + A_{nn}\right) + O(\varepsilon^2)$$

$$\boxed{\det(\mathbb{I} + \underbrace{\varepsilon A}) = 1 + \varepsilon \operatorname{trace}(A)} + O(\varepsilon^2)$$

$(\varepsilon\alpha_{21}) \cancel{+ \varepsilon\alpha_{22}}$



$$(1+\varepsilon a_{11})(1+\varepsilon a_{22})$$
$$= 1 + \varepsilon(a_{11}+a_{22})$$



$$V_{t+\delta t} = \int_{D_{t+\delta t}} d^n x(t+\delta t)$$

$$V_t = \int_{D_t} d^n x(t)$$

$$= \int_{D_t} d^N x(t) \left\| \frac{\partial x_i(t+\delta t)}{\partial x_j(t)} \right\|$$

$$\dot{x} = v(x) \qquad \int dy = \int dx \left| \frac{dy}{dx} \right|$$

$$x_i(t+\delta t) = \underline{x_i(t)} + \delta t\, v_i(x) + O(\delta t^2)$$

$$\frac{\partial x_i(t+\delta t)}{\partial x_j(t)} = \delta_{ij} + \delta t \frac{\partial v_i}{\partial x_j} + O(\delta t^2)$$

$$\det\left(\frac{\partial x_i(t+\delta t)}{\partial x_j(t)}\right) = \det\left(I + \delta t A + O(\delta t^2)\right)$$

$$A_{ij} = \frac{\partial v_i}{\partial x_j} \qquad \mathrm{trace}(A) = \frac{\partial v_i}{\partial x_i} = \nabla\cdot\vec{v}$$

$$= 1 + \delta t(\nabla\cdot\vec{v}) + O(\delta t^2)$$

$$V(t+\delta t) = \int_{\Omega_t} \left(1 + \delta t(\nabla\cdot\vec{v}) + O(\delta t^2)\right) d^N x(t)$$

$$= V(t) + \delta t \int_{\Omega_t} (\nabla\cdot\vec{v})\, d^N x + O(\delta t^2)$$

$$\frac{V(t+\delta t) - V(t)}{\delta t} = \int_{\Omega_t} (\nabla\cdot\vec{v})\, d^N x + O(\delta t)$$

$$\lim_{\delta t\to 0} \quad \boxed{\frac{dV}{dt} = \int_{\Omega} (\nabla\cdot\vec{v})\, d^N x}$$

είναι
και δείκτης
ευστάθειας;

$$\underbrace{\dot{x} = A^{(t)} x}_{\vec{v} = Ax}$$

$$\nabla\cdot\vec{v} = \frac{\partial}{\partial x_i}(A_{ij} x_j) = A_{ij}\delta_{ij} = A_{ii} = \mathrm{trace}(A)$$

$$\frac{dV}{dt} = \int_{\Omega_0} \mathrm{trace}(A)\, d^N x = \mathrm{trace}(A)\, V$$

$$V(t) = e^{\int_0^t \mathrm{trace}(A(s))\,ds}\, V_0$$

$$= e^{t\,\mathrm{trace}(A)}\, V_0$$

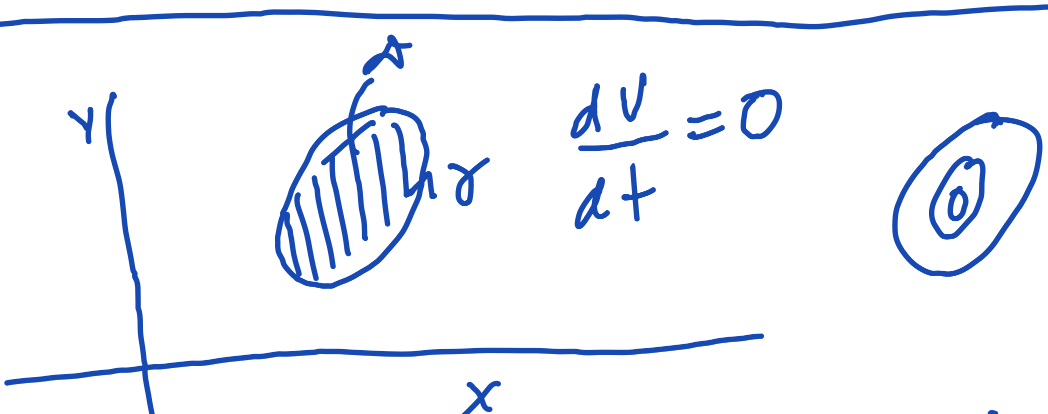
Εάν A είναι ανεξάρτητο του χρόνου

$\dot{x} = A(t)x$

$$X(t) = e^{At} = \lim \left(\prod e^{A(\bar{t}_i)\delta t} \right)$$

$$e^{A(t_i)\delta t} \qquad \underbrace{\qquad\qquad}_{\Phi}$$

$$\boxed{x_0 \qquad \det \Phi = e^{\int_0^t \mathrm{trace}(A(s))ds}}$$



σε μία περιοχή που η ροή
είναι $\nabla \cdot v$ > ή < 0
δεν μπορεί να υπάρχει
οριακός κύκλος

οριακί $\boxed{\int_V \nabla \cdot \vec{v} = 0}$ αν υπάρχει οριακή κίνηση

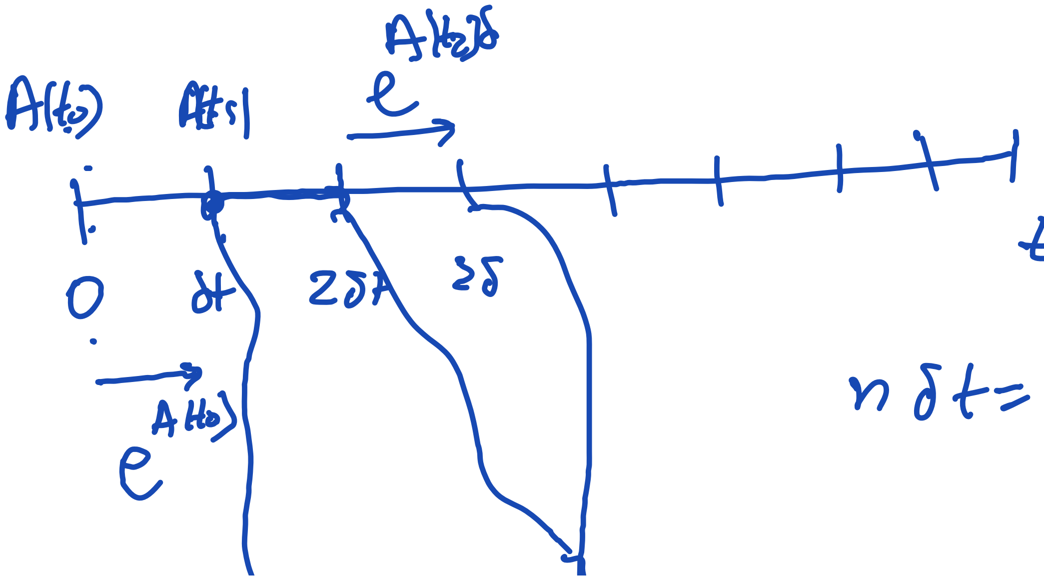
$$\boxed{e^{A_1} e^{A_2} \neq e^{A_1 + A_2}}$$

είναι $e^{A_1} e^{A_2} = e^{A_1 + A_2}$

όταν $[A_1, A_2] = 0$

$$\dot{x} = A(t)\, x$$

$$x(t) = \underbrace{\Phi(t,0)}\, x_0$$



$n \delta t = t$

$$e^{A(t_0)\delta t} x_0$$

$$e^{A(t_1)\delta t} e^{A(t_0)\delta t} x_0$$

$$\underbrace{e^{A(t_{n-1})\delta t} \cdots\cdots e^{A(t_2)\delta t} e^{A(t_1)\delta t} \cdot e^{A(t_0)\delta t} x_0}_{\Phi_n}$$

$$\boxed{\Phi = \lim_{n\to\infty} \left(e^{A(t_{n-1})\delta t} \cdots\cdots e^{A(t_0)\delta t} \right)}$$

$n\delta t = t$

$$\neq e^{\int A(s)\,ds}$$

δt_i 1 9x*in dcm'

$$\bar{A} = \frac{\int_0^t A(s)\,ds}{t}$$

$$\det \Phi = e^{\int_0^t trace(A)}$$

$$= e^{Tr(A(t_{n-1}))\delta} \cdot e^{Trc(A(t_{n-2}))\delta t} \cdots\cdots e^{Trs(A(t_0))\delta t}$$

$$\simeq e^{\left\{ \mathrm{Trace}(A(t_n)) + \dots \mathrm{Trace}(A(t_0)) \right] \delta t}$$

$$\simeq e^{\int_0^t \mathrm{trace}(A(s))\, ds}$$